# 代数 2 期中考试

Yueke

2022/4/14

几点声明:

1、此试卷为考生回忆版，不保证问题顺序的准确性与问题陈述的严谨性。

2、考试时间为 2022 年 4 月 14 日 9:50 至 11:50。

---

1.State and prove Sylow's 3rd theorem.

2.Classify group with order 455.

3.For group $G$, $C$ denote the minimal subgroup of $G$ which contains all the element of the form $xyx^{-1}y^{-1}$. Find the subgroup $C$ when $G = S_n$,$n \geq 4$.

4.G is a finite non-abelian group, $N = \{(x, y) \in G \times G | xy = yx\}$, $c(G)$ denote the number of conjugacy classes of G.

(a)Prove $|N| = c(G) \cdot |G|$.

(b)Prove $[G : Z(G)] \geq 4$.

(c)Prove $c(G) \leq \frac{5}{8}|G|$.

5. $v_1, ..., v_k \in \mathbb{R}^n$, orthonormal respect to dot product in $\mathbb{R}^n$, $M = [v_1, ..., v_k]$. Find $det(I_k - 2M^T \cdot M)$ and $det(I_n - 2M \cdot M^T)$.

6.$A$ is a real positive definite symmetric matrix. Prove that for any integer $m \neq 0$,$\exists$ matrix $B$ s.t. $B^m = A$.

7.$S = \mathbb{Z}/n\mathbb{Z}$,$V = \mathcal{L}(S, \mathbb{C})$, a Hermitian form defined on $V$ is given by:

$$< f, g >= \frac{1}{n} \sum_{x=0}^{n-1} f(\bar{x}) g(x)$$

(a)Prove that $\{e_x\}_{0 \le x \le n-1}$ is an orthonormal basis of $V$ where $e_x(y) = e^{2\pi i \frac{xy}{n}}$.

(b)$W = \{f \in V | f(x) = f(-x)\}$. Find a orthonormal basis for $W$ and $W^{\perp}$.

(c)$f(x) = x$, find $\sum_{x=0}^{n-1} |\hat{f}(x)|^2$ where $\hat{f}(x) =< e_x, f >$.