1. Flowchart

[10 points] Write a function Print_values with arguments a, b, and c to reflect the following flowchart. Here the purple parallelogram operator on a list [x, y, z] is to compute and print x+y-10z. Try your output with some random a, b, and c values. Report your output when a = 10, b = 5, c = 1.

1.1 Codes for problem1

My basic idea can be seen behind "#" in the codes.

```
# Ask users to input 3 numbers to assign a, b, c
a=float(input("please input the 1st number as the value of a:"))
b=float(input("please input the 2nd number as the value of b:"))
c=float(input("please input the 3rd number as the value of c:"))
# Compare number size for a, b, c
if(a>b):
    if(b>c):
        # Based on the number size of a, b, c,  assign x, y, z
        x=a
        y=b
        z=c
        # Output the value of (x+y-10*z)
        print("x+y-10*z = ", x+y-10*z)
    elif(a>c):
        x=a
        y=c
        z=b
        print("x+y-10*z = ", x+y-10*z)
    else:
        x=c
        y=a
        z=b
        print("x+y-10*z = ", x+y-10*z)
elif(b>c):
    print(None)
else:
    x=c
    y=b
    z=a
    print("x+y-10*z = ", x+y-10*z)
```

1.2 Output for problem1

As the output displays, when a = 10, b = 5, c = 1, x+y-10z= 5.

```
please input the 1st number as the value of a:10
please input the 2nd number as the value of b:5
please input the 3rd number as the value of c:1
x+y-10*z =  5.0
```

2. Continuous celing function

[10 points] Given a list with N positive integers. For every element x of the list, find the value of continuous ceiling function defined as F(x) = F(ceil(x/3)) + 2x, where F(1) = 1.

2.1 Codes for problem2

```
from math import *
import numpy as np
# Ask users to input the range and the item number to create a random list
M=int(input("Please input the range of the positive intergers (>0):"))
N=int(input("Please input the number of positive intergers:"))
list=np.random.randint(1,M+1,N)
print("The random list is shown below:\n",list)
# Define a function F(x) to get the value of F(x) = F(ceil(x/3)) + 2x, where F(1) = 1
y=0
def F(x):
    if(x==1):
        return 1
    else:
        y=F(ceil(x/3))+2*x
        return y
# Use a "for" loop and the function F(x) to get the value of F(list[i])
for i in range(N):
    print ("x=", list[i],",F(x)=", F(list[i]))
```

First, randomly generate a positive integer list based on its range and element number, which are decided by users. Then, define a function to achieve F(x) = F(ceil(x/3)) + 2x, where F(1) = 1, and at last, put each element of the list into the function. More details are shown behind "#" in the codes.

2.2 Output for problem2

When the range and the number of the positive integer list were assigned 50 and 9, a list with 9 elements was outputted and the F(x) value of each elements were calculated.

```
Please input the range of positive intergers (>0):50
Please input the number of positive intergers:9
The random list is shown below:
 [40 49 27  5 40  4 42 15 24]
x= 40 ,F(x)= 123
x= 49 ,F(x)= 149
x= 27 ,F(x)= 79
x= 5 ,F(x)= 15
x= 40 ,F(x)= 123
x= 4 ,F(x)= 13
x= 42 ,F(x)= 127
x= 15 ,F(x)= 45
x= 24 ,F(x)= 71
```

3. Dice rolling

[15 points] Given 10 dice each with 6 faces, numbered from 1 to 6. Write a function Find_number_of_ways to find the number of ways to get sum x, defined as the sum of values on each face when all the dice are thrown.

[5 points] Count the number of ways for any x from 10 to 60, assign the number of ways to a list called Number_of_ways, so which x yields the maximum of Number_of_ways?

3.1 Codes for problem3

In total, the idea for solving this problem is to first calculate the probability of each sum X, and then multiply the total number of ways. Please see behind "#" to find out each step in the codes.

```
# 3.1 Find_number_of_ways
# Set the total number of dices(D)
D=10
# Define a function to calculate the probability of sum X for D dices
def Prob(D,X):
    prob=0
    # For each dice, the probability to toss "0/1/2/3/4/5/6" is (1/6).
    # When sum X is equal to the number of dices(D), the probability is (1/6)^D
    if X==D:
        return pow(1/6,D)
    # When sum X is smaller than D or larger than 6D, the probability is 0
    if X<D or X>6*D:
        return 0
    else:
    # The probability of sum X for D dices is equal to the probability of
    # sum (X-a) for (D-1) dices, where "a" is the potint of the last dice, belonging to 1-6.
    # Inspired from https://blog.csdn.net/yue_luo_/article/details/95517498
        for i in range(6):
            prob += Prob(D-1, X-i-1)
        return prob/6
# Define a function to calculate the number of ways to reach sum X
def Find_number_of_ways(X):
    # The total number of ways is (6^10). The number of ways for each sum(X) is probability*(6^10).
    print("The number of ways is",round(Prob(D,X)*(6**10)))
# Ask users to input a sum X and find its number of ways
x=int(input("Please input a sum X:"))
Find_number_of_ways(x)
```

Count number of ways for sum 10-60.

```
# 3.2 List: Number_of_ways
# Use a "for" loop to calculate 51 times to get Number_of_ways for sum X, 10-60 respectively.
# Consume around 2.5 min
Number_of_ways=[]
for i in range(10,61):
    ways_number=round(Prob(D,i)*(6**10))
    Number_of_ways.append(ways_number)
print("Number of ways are listed below\n", Number_of_ways)
```

3.2 Output for problem3

When sum X = 13, the number of ways is 220, and number of ways to achieve sum 10-60 were listed. As shown in the list, we can find a digital symmetrical rule, so the sum 35, the middle number between 10 and 60, yields the maximum of Number_of_ways.

```
Please input a sum X:13
The number of ways is 220
Number of ways are listed below
 [1, 10, 55, 220, 715, 2002, 4995, 11340, 23760, 46420, 85228, 147940, 243925, 38347
0, 576565, 831204, 1151370, 1535040, 1972630, 2446300, 2930455, 3393610, 3801535, 412
1260, 4325310, 4395456, 4325310, 4121260, 3801535, 3393610, 2930455, 2446300, 197263
0, 1535040, 1151370, 831204, 576565, 383470, 243925, 147940, 85228, 46420, 23760, 113
40, 4995, 2002, 715, 220, 55, 10, 1]
```

4. Dynamic programming

[5 points] Write a function Random_integer to fill an array of N elements by randomly selecting integers from 0 to 10.

[15 points] Write a function Sum_averages to compute the sum of the average of all subsets of the array. For example, given an array of [1, 2, 3], you Sum_averages function should compute the sum of: average of [1], average of [2], average of [3], average of [1, 2], average of [1, 3], average of [2, 3], and average of [1, 2, 3].

[5 points] Call Sum_averages with N increasing from 1 to 100, assign the output to a list called Total_sum_averages. Plot Total_sum_averages, describe what do you see.

4.1 Codes for problem4

Behind "#" were explanations for each step.

```python
import numpy as np
import math
# 4.1 Random_integer function
# Ask users to decide the size of an array and create an array
N=int(input("Please input the number of elements to create an array:"))
Random_interger=np.random.randint(0,11,N)
print(Random_interger)
```

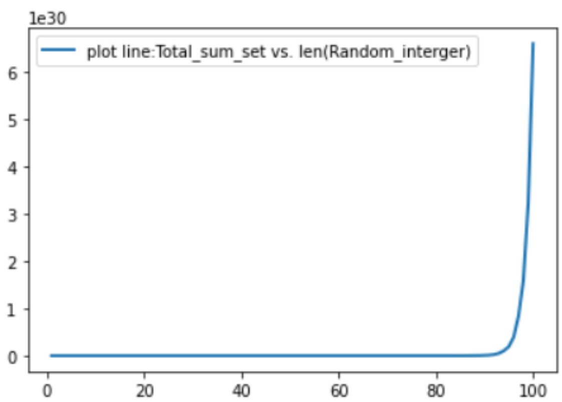
```python
# 4.2 Sum_averages function
# Define a function to calculate the sum of subset average(SA)
def Sum_averages(array):
    # For an array with N elements, sum of all subset average with n elements (n<=N) is
    # equal to (array.sum() * C(N-1,n-1)/n).Here use a "for" loop to achieve the sum of C(N-1,n-1)/n
    # for all subsets, whose element number increasing from 1 to N
    # Discussed with Wenting Yuan and inspired from https://www.geeksforgeeks.org/sum-average-subsets/
    SA=0
    for i in range(0,len(array)):
        SA+=(math.factorial(len(array)-1)/math.factorial(len(array)-1-i)/math.factorial(i))/(i+1)
    SA=SA*array.sum()
    return SA
print("The Sum_averages for each subset is:\n", Sum_averages(Random_interger))
```

```
# 4.3 Total_sum_set function
import matplotlib.pyplot as plt
Total_sum_set=[]
# Use a "for" loop to calculate Sum_averages for random list with N increasing from 1 to 100
# Each result of Sum_averages(np.random.randint(0,11,j+1)) appended in the Total_sum_set
for j in range(100):
    sum=Sum_averages(np.random.randint(0,11,j+1))
    Total_sum_set.append(sum)
print("Sum_averages with element number increasing from 1 to 100 are shown below:\n",Total_sum_set)
# Plot Total_sum_set
x=np.arange(1,101,1)
y=Total_sum_set
plt.plot(x, y, ls="-", lw=2, label="plot line:Total_sum_set vs. len(Random_interger)")
plt.legend()
plt.show()
```

4.2 Output for problem4

The figure of the Total_sum_set shows, a sudden sharp increase appear after N=85, because Sum_averages of those arrays with more elements has higher orders of magnitude.

```
Please input the number of elements to create an array:5
[5 5 9 5 6]
The Sum_averages for each subset is:
 186.0
Sum_averages with element number increasing from 1 to 100 are shown below:
 [7.0, 12.0, 28.0, 63.75, 155.0, 283.5, 489.85714285714283, 1211.25, 2668.555555555556, 5626.5, 8374.09090909
091, 17744.999999999996, 49776.07692307692, 57340.5, 185679.66666666666, 245756.25, 693905.2941176471, 85924
6.5, 2621435.0, 5347732.5, 10885212.333333332, 21924765.68181818, 46319699.52173913, 91575631.875, 144955141.
92, 283922112.6923077, 830161496.6296295, 1370938216.6071432, 2776918505.172414, 4617089838.900001, 130927228
80.096773, 23890755578.4375, 41387866665.72727, 97521022127.02942, 185542587181.80002, 376048247688.74994, 68
3480201044.973, 1540763004706.8157, 2551430828552.4873, 6542094185261.251, 9332440157704.244, 20419501658692.
5, 43366784202508.93, 84362528531171.9, 164975611349852.03, 304421306333803.0, 838436100840246.0, 14953358137
75354.5, 2665395697831513.0, 4661225614328459.0, 1.0243481505391714e+16, 2.3384074988269876e+16, 4.6225626363
95377e+16, 8.373359307185142e+16, 1.8603960642519578e+17, 3.1010500291322554e+17, 6.8517922751854 3e+17, 1.391
4569883186086e+18, 2.960467931320971e+18, 6.302637558517427e+18, 1.0659798829479703e+19, 2.179393553869717e+1
9, 4.6116860184273895e+19, 8.81984951024238e+19, 1.8333225648640567e+20, 3.9576650921776854e+20, 6.9161524049
49014e+20, 1.4496970636750558e+21, 3.39633966249864e+21, 5.801764536096992e+21, 1.087474535139 7e+22, 2.289035
975724314e+22, 4.864684376873936e+22, 1.0618943010128495e+23, 1.863760638572553e+23, 4.0065551423083514e+23,
8.183799785378254e+23, 1.4065386939747132e+24, 2.961103115132035 5e+24, 5.984182807092415e+24, 1.2417608418757
67e+25, 2.317599254188046e+25, 4.812398684345463e+25, 1.0293139835575988e+26, 1.961588812249996e+26, 3.598662
9048993615e+26, 7.932776688524129e+26, 1.7162350544638217e+27, 2.82361604466216e+27, 6.712385990791839e+27,
1.2352192919462915e+28, 2.4382036425925093e+28, 5.026302783163007e+28, 1.0388160670088383e+29, 1.909815706922
7936e+29, 3.895384656951331e+29, 8.347544545317335e+29, 1.584563250285287e+30, 3.220344706640401 3e+30, 6.5917
83121186792e+30]
```



5. Path counting

[5 points] Create a matrix with N rows and M columns, fill the right-bottom corner and top-left corner cells with 1, and randomly fill the rest of matrix with integer 0 or 1.

[25 points] Consider a cell marked with 0 as a blockage or dead-end, and a cell marked with 1 is good to go. Write a function Count_path to count total number of paths to reach the right-bottom corner cell from the top-left corner cell.

Notice: for a given cell, you are **only allowed** to move either rightward or downward.

[5 points] Let N = 10, M = 8, run Count_path for 1000 times, each time the matrix (except the right-bottom corner and top-left corner cells, which remain being 1) is re-filled with integer 0 or 1 randomly, report the mean of total number of paths from the 1000 runs.

5.1 Codes for problem5

Detailed statement behind "#" helps to understand each step.

```
# 5.1 Create a matrix
import numpy as np
# Ask users to decide the size of a matrix and create a matrix
N = int(input("Please input the number of rows:"))
M = int(input("Please input the number of columns:"))
arr=np.random.randint(2,size=(N,M))
arr[0,0]=1
arr[N-1,M-1]=1
print(arr)
```

```
# 5.2 Count_path function
# Define a function Count_path(matrix,a,b) to count the pathway for a specific matrix,
# where "a" is row number, "b" is column number.
# Regard each element as a point, the number of pathways to a point is equal to
# the pathway number of its upper point plus that of its left point, because
# only moving either rightward or downward is allowed.
# Inspired from https://www.geeksforgeeks.org/count-number-of-ways-to-reach-destination-in-a-maze/?ref=rp
def Count_path(matrix,a,b):
    # Substract 1 from each element for counting purposes, and the matrix is
    # refilled by -1 or 0, which were 0 or 1 before, respectively.
    matrix=np.add(matrix,-1)
    # Initialize the leftmost column
    for i in range(a):
        # If meet a blockage, break the loop
        if(matrix[i,0] != 0):
            break
        # If meet an access, each element plus 1, which means
        # the number of pathways from entrance[0,0] to the element[i,0] is 1
        else:
            matrix[i,0] +=1
    # Initialize the uppermost row, whose solution is similar to the leftmost column
    for j in range(b):
        if(matrix[0,j] != 0):
            break
        else:
            matrix[0,j] +=1
```

Joint to the above codes:

```
    # Since the pathway number of a point is equal to the pathway number of
    # its upper point plus that of its left point,
    # two "for" loops were used to count the pathways for each point, up to the last element[a-1,b-1]
    for i in range(1,a,1):
        for j in range(1,b,1):
            if(matrix[i,j] != 0):
                continue
            if(matrix[i-1,j] >0):
                matrix[i,j] += matrix[i-1,j]
            if(matrix[i,j-1] >0):
                matrix[i,j] += matrix[i,j-1]
    Count_path = matrix[a-1,b-1]
    # Return the pathway number of the last element[a-1,b-1]
    if (Count_path >= 0):
        return Count_path
    # If the value of last element[a-1,b-1] is smaller than zero, return 0
    else:
        return 0
# Print the pathway number of the matrix in 5.1
print(Count_path(arr,N,M))
```

```
# 5.3 Count_path function
# Calculate the mean pathway number for 1000 runs by refilling new random matrix
sum_path=0
for k in range(1000):
    sum_path += Count_path(np.random.randint(2,size=(10,8)),10,8)
print("The mean of Count_path for matrixes with size(10,8) from the 1000 runs is\n", sum_path/1000)
```

5.2 Output for problem5

The mean of total number of paths from the 1000 runs is close to "0".

```
Please input the number of rows:6
Please input the number of columns:7
[[1 1 1 1 0 0 1]
 [0 0 0 1 1 0 1]
 [1 0 0 1 0 1 0]
 [1 0 1 0 1 0 1]
 [0 1 0 0 0 0 1]
 [1 0 0 0 1 0 1]]
0
The mean of Count_path for matrixes with size(10,8) from the 1000 runs is
 0.05
```