In [145]:
```python
import numpy as np
import matplotlib.pyplot as plt
import numpy as np
%matplotlib inline
from scipy import integrate
from numpy import exp
import pandas as pd
```

In [146]:
```python
# Read the csv file global.1751_2008 to obtain the gama value(Carbon emissions from fossil-fuels)
Carbon_emissions_fossil_fuels=pd.read_csv("global.1751_2008.csv")
# As required, select data rows from 1987 to 2004
df=Carbon_emissions_fossil_fuels.tail(22).head(18)
df.reset_index(drop=True, inplace=True)# Reset the dataframe index
# Transfer the Carbon emissions into the ppm units
df["gama_ppm"]=df["Total carbon emissions from fossil-fuels (million metric tons of C)"
                 ].astype(int)*(10**12)/(10**15)/2.13
df
```

Out[146]:

	Year	Total carbon emissions from fossil-fuels (million metric tons of C)	carbon emissions from gas fuel consumption	carbon emissions from liquid fuel consumption	carbon emissions from solid fuel consumption	carbon emissions from cement production	carbon emissions from gas flaring	Per capita carbon emissions (metric tons of carbon; after 1949 only)	gama_pp
0	1987	5755	894	2309.0	2364.0	143.0	44.0	1.15	2.7018
1	1988	5968	937	2414.0	2414.0	152.0	50.0	1.17	2.8018
2	1989	6088	973	2462.0	2457.0	156.0	41.0	1.17	2.8582
3	1990	6151	1020	2515.0	2419.0	157.0	40.0	1.16	2.8877
4	1991	6239	1062	2624.0	2348.0	161.0	44.0	1.16	2.9291
5	1992	6178	1094	2511.0	2372.0	167.0	35.0	1.13	2.9004
6	1993	6172	1119	2541.0	2301.0	176.0	36.0	1.11	2.8976
7	1994	6284	1133	2566.0	2361.0	186.0	38.0	1.12	2.9502
8	1995	6422	1154	2588.0	2446.0	197.0	38.0	1.12	3.0150
9	1996	6550	1208	2627.0	2473.0	203.0	39.0	1.13	3.0751
10	1997	6663	1210	2703.0	2500.0	209.0	41.0	1.13	3.1281
11	1998	6638	1243	2755.0	2395.0	209.0	37.0	1.11	3.1164
12	1999	6584	1270	2703.0	2356.0	217.0	37.0	1.09	3.0910
13	2000	6750	1288	2818.0	2370.0	226.0	48.0	1.10	3.1690
14	2001	6916	1311	2827.0	2494.0	237.0	46.0	1.12	3.2469
15	2002	6981	1346	2810.0	2525.0	252.0	48.0	1.11	3.2774
16	2003	7397	1391	2935.0	2747.0	276.0	47.0	1.16	3.4727
17	2004	7782	1431	3027.0	2971.0	298.0	55.0	1.21	3.6535

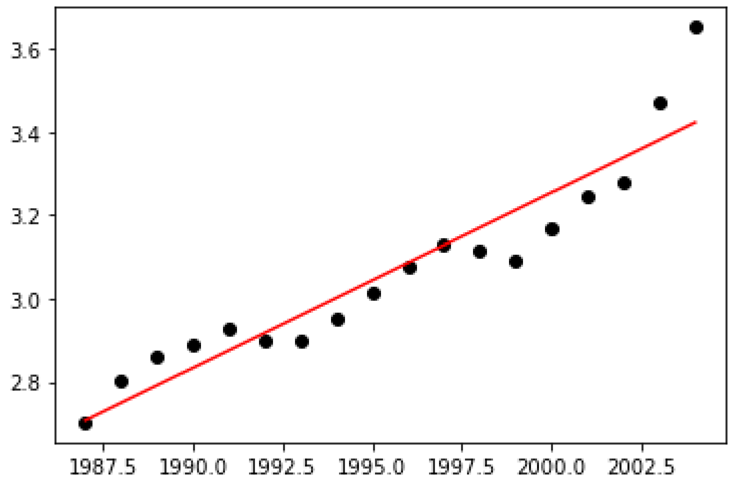
In [147]:

```
# Import stats
from scipy import stats
# Apply Linear Regression to gama for continuous data
time=np.array([i for i in range(1987,2005)])
gama=np.array(df["gama_ppm"])
plt.plot(time, gama, 'ko')
res = stats.linregress(time, gama)# Linear Regression
print(res)
plt.plot(time, res.slope*time+res.intercept, 'r-')
# Linregress Result shown below
```

```
LinregressResult(slope=0.042129488316206165, intercept=-81.00424004870877, rvalue=0.93982556144
74387, pvalue=7.150076443522602e-09, stderr=0.003828831926443388, intercept_stderr=7.6404599321
23952)
```

Out[147]:

```
[<matplotlib.lines.Line2D at 0x2506cd06fa0>]
```



In [148]:

```
# Read the csv file co2_annmean_mlo to obtain CO2 annual observation values
observations=pd.read_csv("co2_annmean_mlo.csv")
# Set two arrays to contain CO2 observation values and their corresponding time,
# used for comparisons with modeling results
time=np.array([i for i in range(1987,2005)])
CO2_observations=np.array(observations["mean(ppm)"][(observations["year"].astype(int)< 2005)
                                                     &(observations["year"].astype(int)> 1986)])
print(time)
print(CO2_observations)
```

```
[1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000
 2001 2002 2003 2004]
[349.31 351.69 353.2  354.45 355.7  356.54 357.21 358.96 360.97 362.74
 363.88 366.84 368.54 369.71 371.32 373.45 375.98 377.7 ]
```

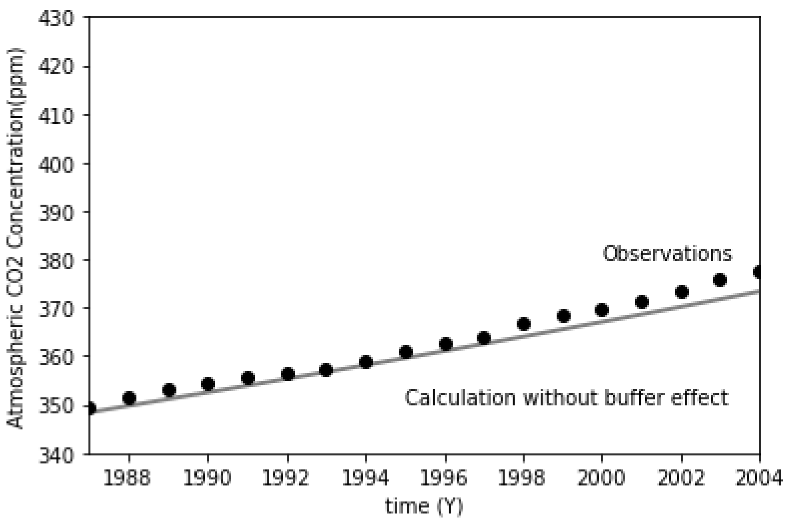
In [149]:

```
# 1.1 Solving ODEs: Following equation 1-2(without the buffer effect)
# Define the function "model(t,N)"
def model(t,N):
    N1,N2=N
    # N1 and N2 denote the carbon concentration of atmosphere and ocean respectively; t is time
    dN1dt = -(105/740) * N1 + (102/900) * N2 + res.slope*t+res.intercept # Gama value
    dN2dt = (105/740) * N1 - (102/900) * N2
    dNdt =[dN1dt,dN2dt]
    return dNdt

# Initial condition
N1986 = [347,423]
t = np.arange(1986,2005)

# Solve ODE
Sol = integrate.odeint(model,N1986,t,tfirst= True)

# Plot results
plt.plot(t, Sol[:,0],color="gray",linewidth =2.0)
# Compare with observation value
plt.plot(time,CO2_observations, "ko")
plt.xlabel("time (Y)")
plt.ylabel("Atmospheric CO2 Concentration(ppm)")
plt.ylim(340,430)
plt.xlim(1987,2004)
# Add annotation
plt.annotate("Calculation without buffer effect", xy=(2000,367.04), xytext=(1995,350),fontsize=10)
plt.annotate("Observations", xy=(2000,369.71),xytext=(2000,380),fontsize=10)
plt.show()
```



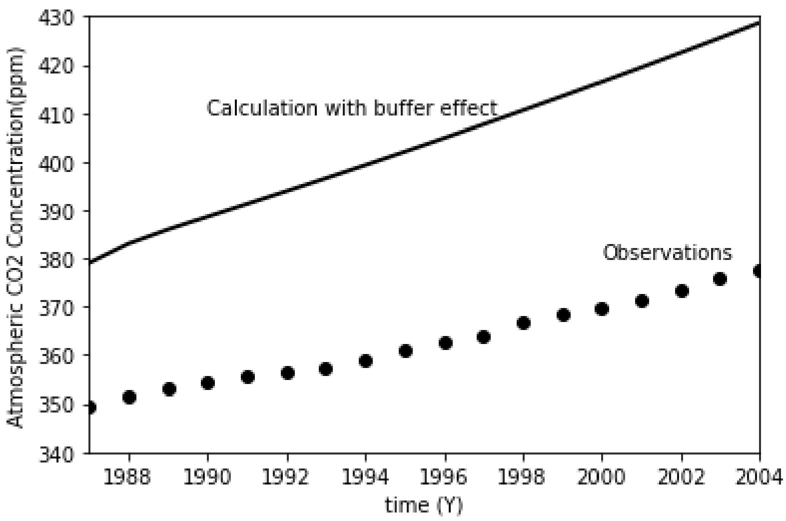
In [150]:

```python
# 1.2 Solving ODEs: Following equation 1-2(with the buffer effect)
# Define the function "model2(t,N)"
def model2(t,N):
    N1,N2=N
    # Take buffer effect (ksi) into consideration
    ksi=3.69+1.86*10**(-2)*N1-1.80*10**(-6)*N1**2
    dN1dt = -(105/740) * N1 + (102/900) *(385+ksi*(N2-385)) + res.slope*t+res.intercept # Gama
    dN2dt = (105/740) * N1 - (102/900) *(385+ksi*(N2-385))
    dNdt =[dN1dt,dN2dt]
    return dNdt

# Initial condition
N1986 = [347,423]
t = np.arange(1986,2005)

# Solve ODE
Sol2 = integrate.odeint(model2,N1987,t,tfirst= True)

# Plot results
plt.plot(t, Sol2[:,0],color="black",linewidth =2.0)
# Compare with observation value
plt.plot(time,CO2_observations, "ko")
plt.xlabel("time (Y)")
plt.ylabel("Atmospheric CO2 Concentration(ppm)")
plt.ylim(340,430)
plt.xlim(1987,2004)
# Add annotation
plt.annotate("Calculation with buffer effect", xy=(1992,390),xytext=(1990,410),fontsize=10)
plt.annotate("Observations", xy=(2000,369.71),xytext=(2000,380),fontsize=10)
plt.show()
```



In [151]:
```
# 1.3 Reproduce the results of 1.1 and 1.2
plt.plot(t, Sol[:,0],color="gray",linewidth =2.0)
plt.plot(t, Sol2[:,0],color="black",linewidth =2.0)
plt.plot(time,CO2_observations, "ko")
plt.xlabel("time (Y)")
plt.ylabel("Atmospheric CO2 Concentration(ppm)")
plt.ylim(340,430)
plt.xlim(1987,2004)
plt.annotate("Calculation with buffer effect", xy=(1992,390),xytext=(1990,410),fontsize=10)
plt.annotate("Calculation without buffer effect", xy=(2000,367.04), xytext=(1995,350),fontsize=10)
plt.annotate("Observations", xy=(2000,369.71),xytext=(2000,380),fontsize=10)
plt.show()
```