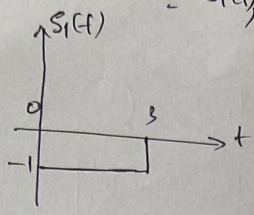
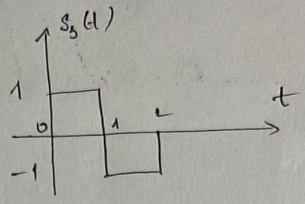
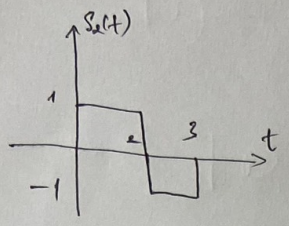
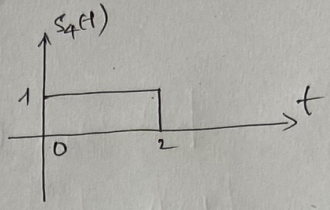
Phạm Quốc Bảo
2080701

Cho tín hiệu $S_1(t)$, $S_2(t)$, $S_3(t)$, $S_4(t)$



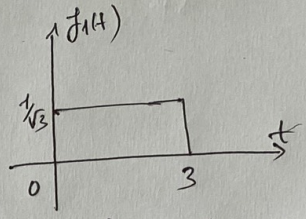






a) $E_1 = \int_{-\infty}^{+\infty} |S_1(t)|^2 dt = \int_{-\infty}^{+\infty} |(-1)|^2 dt = \int_0^3 1.dt = t\Big|_0^3 = 3$

$f_1(t) = \dfrac{S_1(t)}{\sqrt{E_1}} = \dfrac{S_1(t)}{\sqrt{3}} \Rightarrow$



$C_{12} = \int_{-\infty}^{+\infty} f_1(t).S_2(t)dt = f_1(t).\left[\int_0^2 1.dt + \int_2^3 (-1)dt\right] = f_1(t)$

$f_2'(t) = S_2(t) - C_{12}.f_1(t) = S_2(t) - f_1^2(t)$

$E_2 = \int_{-\infty}^{+\infty} |f_2(t)|^2 dt = \int_{-\infty}^{+\infty} |S_2(t) - f_1^2(t)|^2 dt$

$= \int_{-\infty}^{+\infty} [S_2^2(t) - 2.S_2(t).f_1^2(t) + f_1^4(t)]dt$

$= \int_{-\infty}^{+\infty} S_2^2(t).dt - \int_{-\infty}^{+\infty} 2S_2(t).f_1^2(t)dt + \int_{-\infty}^{+\infty} f_1^4(t)dt$

$= 3 - 2.f_1^2(t) + 3.f_1^4(t) = \dfrac{8}{3}$

$f_2(t) = \dfrac{S_2(t)}{\sqrt{E_2}} = \dfrac{\sqrt{6}}{4}.S_2(t)$

$C_{13} = \int_{-\infty}^{+\infty} f_1(t).S_3(t)dt = f_1(t).\left[\int_0^1 S_3(t).dt + \int_1^2 (-1)S_3(t)dt\right] = f_1(t).0 = 0$

$C_{23} = \int_{-\infty}^{+\infty} f_2(t).S_3(t)dt = 0$

$$f_3'(t) = S_3(t) - C_{13}\cdot f_1(t) - C_{23}\cdot f_2(t) = S_3(t)$$

$$E_3 = \int_{-\infty}^{+\infty} |f_3'(t)|^2 dt = \int_{-\infty}^{+\infty} |S_3(t)|^2 dt = 2$$

$$f_3(t) = \frac{S_3(t)}{\sqrt{E_3}} = \frac{S_3(t)}{\sqrt{2}}$$

$$C_{14} = \int_{-\infty}^{+\infty} f_1(t).\, S_4(t)\, dt = f_1(t).\int_{-\infty}^{+\infty} S_4(t).dt = 2 f_1(t)\,dt$$

$$C_{24} = \int_{-\infty}^{+\infty} f_2(t).S_4(t)\,dt = f_2(t).\int_{-\infty}^{+\infty} S_4(t)\,dt = 2 f_2(t)\,dt$$

$$C_{34} = \int_{-\infty}^{+\infty} f_3(t).\, S_4(t).dt = f_3(t)\cdot\int_{-\infty}^{+\infty} S_4(t)\,dt = 2.f_3(t)\,dt$$

$$f_4'(t) = S_4(t) - C_{14}\cdot f_1(t) - C_{24}\cdot f_2(t) - C_{34}\cdot f_3(t)$$

$$= S_4(t) - 2.f_1^2(t) - 2.f_2^2(t) - 2.f_3^2(t)$$

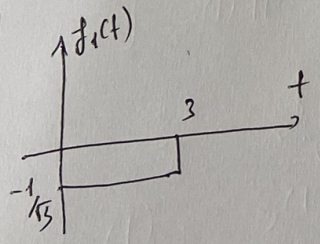
$$E_4 = \int_{-\infty}^{+\infty} |f_4'(t)|^2 dt = \int_{-\infty}^{+\infty} \left[S_4(t) - 2f_1^2(t) - 2f_2^2(t) - 2f_3^2(t)\right]^2 dt$$

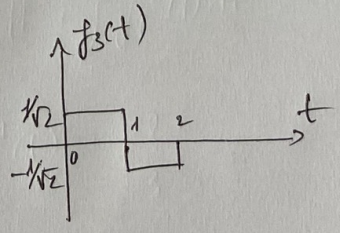
~~$\Rightarrow E_4 = 2 - 3 f_1^4(t) - 3 f_2^4(t) -$~~

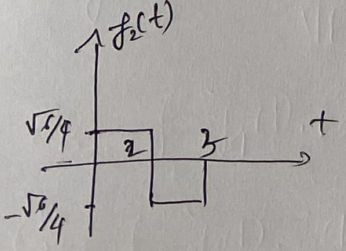
$$\Rightarrow E_4 = \frac{289}{32}$$

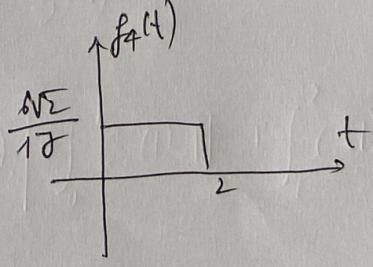
$$f_4(t) = \frac{S_4(t)}{\sqrt{E_4}} = \frac{S_4(t).6\sqrt{2}}{17}$$

b)









c)

$$\int_{-\infty}^{+\infty} |f_1(t)|^2 dt = 1 \qquad \int_{-\infty}^{+\infty} |f_2(t)|^2 dt = \frac{9}{8} \qquad \int_{-\infty}^{+\infty} |f_3(t)|^2 dt = 1$$

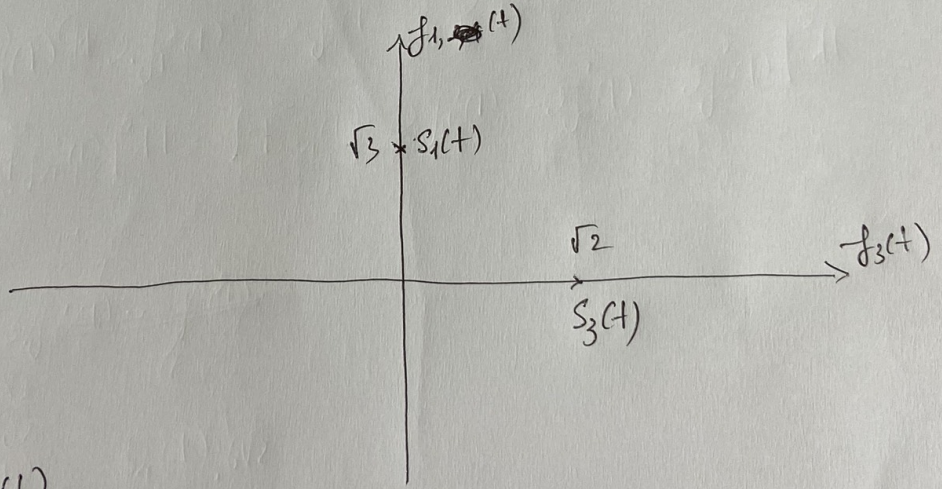
$$\int_{-\infty}^{+\infty} |f_4(t)|^2 dt = \frac{144}{289}$$

$$\int_{-\infty}^{+\infty} f_1(t) \cdot f_2(t)\, dt = -0{,}353 \qquad \int_{-\infty}^{+\infty} f_2(t) \cdot f_3(t)\, dt = 0$$

$$\int_{-\infty}^{+\infty} f_1(t) \cdot f_3(t)\, dt = 0 \qquad \int_{-\infty}^{+\infty} f_2(t) \cdot f_4(t)\, dt = 0{,}611$$

$$\int_{-\infty}^{+\infty} f_1(t) \cdot f_4(t)\, dt = -0{,}576 \qquad \int_{-\infty}^{+\infty} f_3(t) \cdot f_4(t)\, dt = 0$$

d)



$$s_1 = \sqrt{3} f_1(t)$$

$$s_3 = \sqrt{2} f_3(t)$$

e) $d_{13} = \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2} = \sqrt{5} = 2{,}236$