CHAPTER 4:

DISCRETE-TIME FOURIER TRANSFORM (DTFT)

Lecture 6: Discrete-Time Fourier Transform (DTFT)

Lecture 7: Digital signal spectra

Lecture 8: Frequency response and filter

Lecture 6

Discrete-Time Fourier Transform (DTFT)

- **Outline:**

1. DTFT basics

2. Inversion of DTFT

3. Properties of DTFT

DTFT formula

$$\boxed{X(\Omega) = \mathrm{DTFT}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}}$$

- Continuous function in frequency and periodic with period of 2π
- Gives the complex frequency spectrum of DT signal
- Not all DTFT is converge

Convergence of the DTFT

We always have:

$$\left|\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}\right| \le \sum_{n=-\infty}^{\infty} \left|x[n]e^{-j\Omega n}\right|$$

$$\le \sum_{n=-\infty}^{\infty} \left|x[n]\right|\left|e^{-j\Omega n}\right|$$

$$= \sum_{n=-\infty}^{\infty} \left|x[n]\right|$$

→ DTFT exists when:

$$\boxed{\sum_{n=-\infty}^{\infty} \left|x[n]\right| < \infty}$$

From ZT to DTFT

Recall ZT of **x[n]**:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Evaluating X(z) on the unit circle (if the unit circle is in the ROC of X(z)):

$$X(z)\Big|_{z=e^{j\Omega}} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = X(\Omega)$$

From ZT to DTFT

DTFT is the Z-transform of x[n] evaluated on the unit circle

$$\boxed{X(\Omega) = X(z)\Big|_{z=e^{j\Omega}}}$$

If the ROC of the ZT contains the unit circle, we can get the DTFT from the ZT by substitution **$z = e^{j\Omega}$**

Example of calculating DTFT

Find DTFT of x(n) where $x[n] = a^n u[n]$

If |a|≥1, DTFT does not exist

If |a| < 1:

$$X(\Omega) = \sum_{n=0}^{\infty} a^n e^{-j\Omega n} = \sum_{n=0}^{\infty} (ae^{-j\Omega})^n = \frac{1}{1 - ae^{-j\Omega}} = \frac{e^{j\Omega}}{e^{j\Omega} - a}$$

$$X(z) = \frac{z}{z-a}, ROC: |z| > |a| \Rightarrow X(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega} - a}, |a| < 1$$

Lecture 6

DTFT

- **Outline:**

  1. DTFT basics

  2. Inversion of DTFT

  3. Properties of DTFT

Inversion of DTFT

$$X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]\,e^{-j\Omega n}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega l}d\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}\right]e^{j\Omega l}d\Omega$$

$$= \sum_{n=-\infty}^{\infty} x[n]\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\Omega(l-n)}d\Omega\right]$$

$$\boxed{x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\,e^{j\Omega n}d\Omega}$$

Examples of calculating the inverse DTFT

1. Find x[n] from its DTFT X(Ω): $X(\Omega) = \begin{cases} 1, & |\Omega| \le \Omega_c \\ 0, & \Omega_c < |\Omega| < \pi \end{cases}$

$$x[n] = \frac{1}{2\pi} \int_{-\Omega_c}^{\Omega_c} 1.e^{j\Omega n} d\Omega$$

Examples of calculating the inverse DTFT

2. Find x(n) from its DTFT X(Ω): $X(\Omega) = \cos^2 \Omega$

Examples of calculating of the inverse of DTFT

3. Find x(n) from its DTFT X(Ω): $$X(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega} - 2}$$

Lecture 6
DTFT

- **Outline:**

1. DTFT basics

2. Inversion of DTFT

3. Properties of DTFT

Linearity

$$\boxed{ax[n] + by[n] \overset{\text{DTFT}}{\longleftrightarrow} aX(\Omega) + bY(\Omega)}$$

The DTFT of a linear combination of two or more signals is equal to the same linear combination of the DTFT of the individual signals.

Time shifting

$$\boxed{x[n-n_0] \overset{\text{DTFT}}{\leftrightarrow} e^{-j\Omega n_0} X(\Omega)}$$

Proof: infer from the shifting property of ZT

$$x[n-n_0] \overset{\text{ZT}}{\leftrightarrow} z^{-n_0} X(z)$$

→ A shift in time causes **a linear phase shift** in frequency – **no change in DTFT magnitude**

Frequency shifting and modulation

$$e^{j\Omega_0 n}x[n] \overset{\text{DTFT}}{\longleftrightarrow} X(\Omega-\Omega_0)$$

$$\cos(\Omega_0 n)x[n] \overset{\text{DTFT}}{\longleftrightarrow} \frac{1}{2}X(\Omega-\Omega_0)+\frac{1}{2}X(\Omega+\Omega_0)$$

$$e^{j\Omega_0 n}x[n] \overset{\text{DTFT}}{\longleftrightarrow} \sum_{n=-\infty}^{\infty}(e^{j\Omega_0 n}x[n])e^{-j\Omega n} = \sum_{n=-\infty}^{\infty}x[n]e^{-j(\Omega-\Omega_0)n} = X(\Omega-\Omega_0)$$

→ Modulation causes **a shift** in frequency

Convolution in time domain

$$\boxed{x_1[n] * x_2[n] \quad \overset{\text{DTFT}}{\longleftrightarrow} \quad X_1(\Omega).X_2(\Omega)}$$

Convolution in time ←→ Multiplication in frequency

Convolution in frequency domain

$$x_1[n].x_2[n] \overset{\text{DTFT}}{\leftrightarrow} \frac{1}{2\pi}\int_{2\pi} X_1(\lambda)X_2(\Omega-\lambda)d\lambda = \frac{1}{2\pi}X_1(\Omega)*X_2(\Omega)$$

$$x_1[n].x_2[n] \overset{\text{DTFT}}{\leftrightarrow} \sum_{n=-\infty}^{\infty}(x_1[n].x_2[n])e^{-j\Omega n}$$

Multiplication in time

$$= \sum_{n=-\infty}^{\infty}\left(\frac{1}{2\pi}\int_{2\pi} X_1(\lambda)e^{j\lambda n}d\lambda\right)x_2[n]e^{-j\Omega n}$$

$$= \frac{1}{2\pi}\int_{2\pi} X_1(\lambda)\left(\sum_{n=-\infty}^{\infty}x_2[n]e^{-j(\Omega-\lambda)n}\right)d\lambda$$

$$= \frac{1}{2\pi}\int_{2\pi} X_1(\lambda)X_2(\Omega-\lambda)d\lambda$$

Convolution in frequency

HW

Prob.1 Compute the DTFT of the following signals

a) $x[n] = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

b) $y[n] = \begin{cases} 2 - (\frac{1}{2})n & |n| \leq 4 \\ 0 & |n| > 4 \end{cases}$

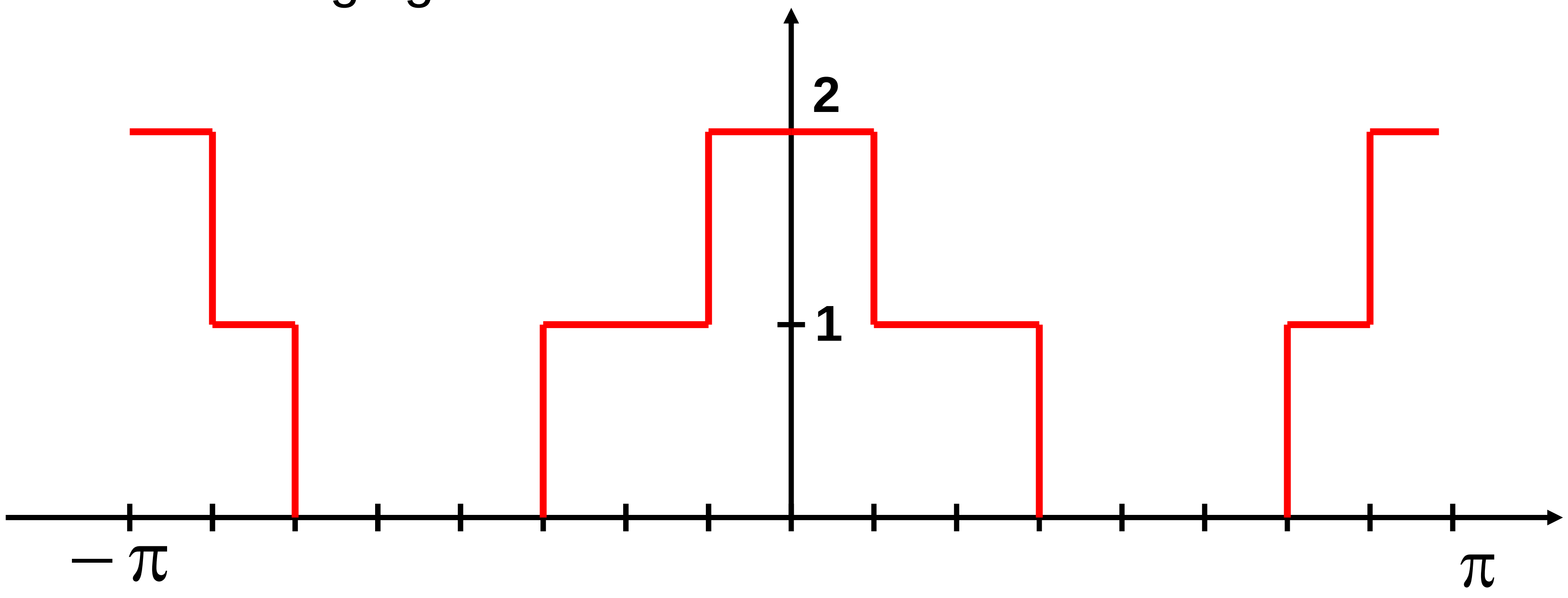
c) $v[n] = 2^n u[-n]$

d) $h[n] = a^n \sin(\Omega_0 n) u[n]$

e) $w[n] = u[n] - u[n-6]$

HW

Prob.2 Determine the signal having the DTFT shown in following figure



HW

Prob.3 A signal x(n) has the following DTFT:

$$X(\Omega) = \frac{1}{1 - ae^{-j\Omega}}$$

Determine the DTFT of the following signals:

$$(a)\ x[n] * x[n-1]$$

$$(b)\ x[n]\cos(0.3\pi n)$$

$$(c)\ e^{j\pi n/2} x[n+2]$$

Lecture 7

Digital signal spectra

- **Outline:**

1. Frequency spectrum

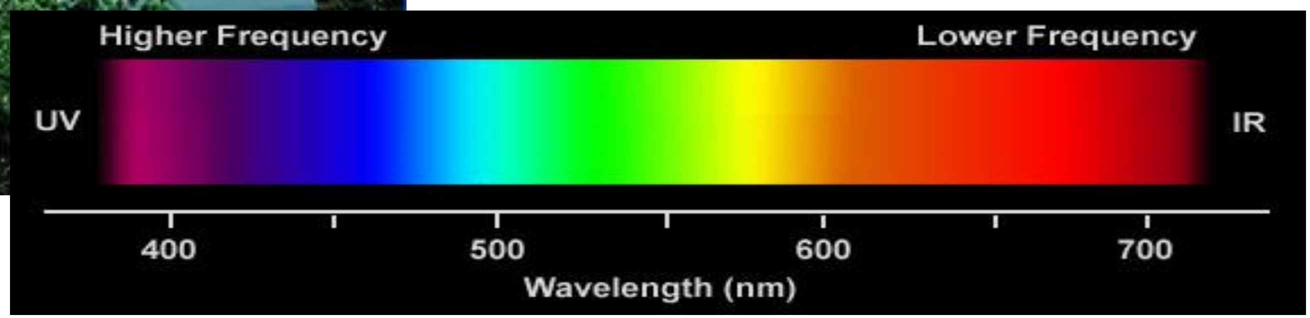
2. Nonperiodic signals

3. Periodic signals

Frequency spectrum

- Detailed description of the frequency components the signal contains.
- Two parts: magnitude spectrum & phase spectrum.
- Tools for calculating the spectrum: DTFT for nonperiodic signals and DFS for periodic signals.

Frequency spectrum



Lecture 7

Digital signal spectra

- **Outline:**

  1. Frequency spectrum

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DTFT to calculate spectrum of a nonperiodic signal

$$\boxed{X(\Omega) = \left|X(\Omega)\right| e^{j\theta(\Omega)}}$$

Amplitude spectrum **Phase spectrum**

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} \quad ; \quad X(-\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{j\Omega n}$$

$$\Rightarrow \quad X(\Omega) = X^{*}(-\Omega)$$

$$\Rightarrow \quad |X(\Omega)| = |X(-\Omega)| \quad \text{and} \quad \angle X(\Omega) = -\angle X(-\Omega)$$

Example of finding amplitude spectrum and phase spectrum

Find and plot amplitude spectrum and phase spectrum:

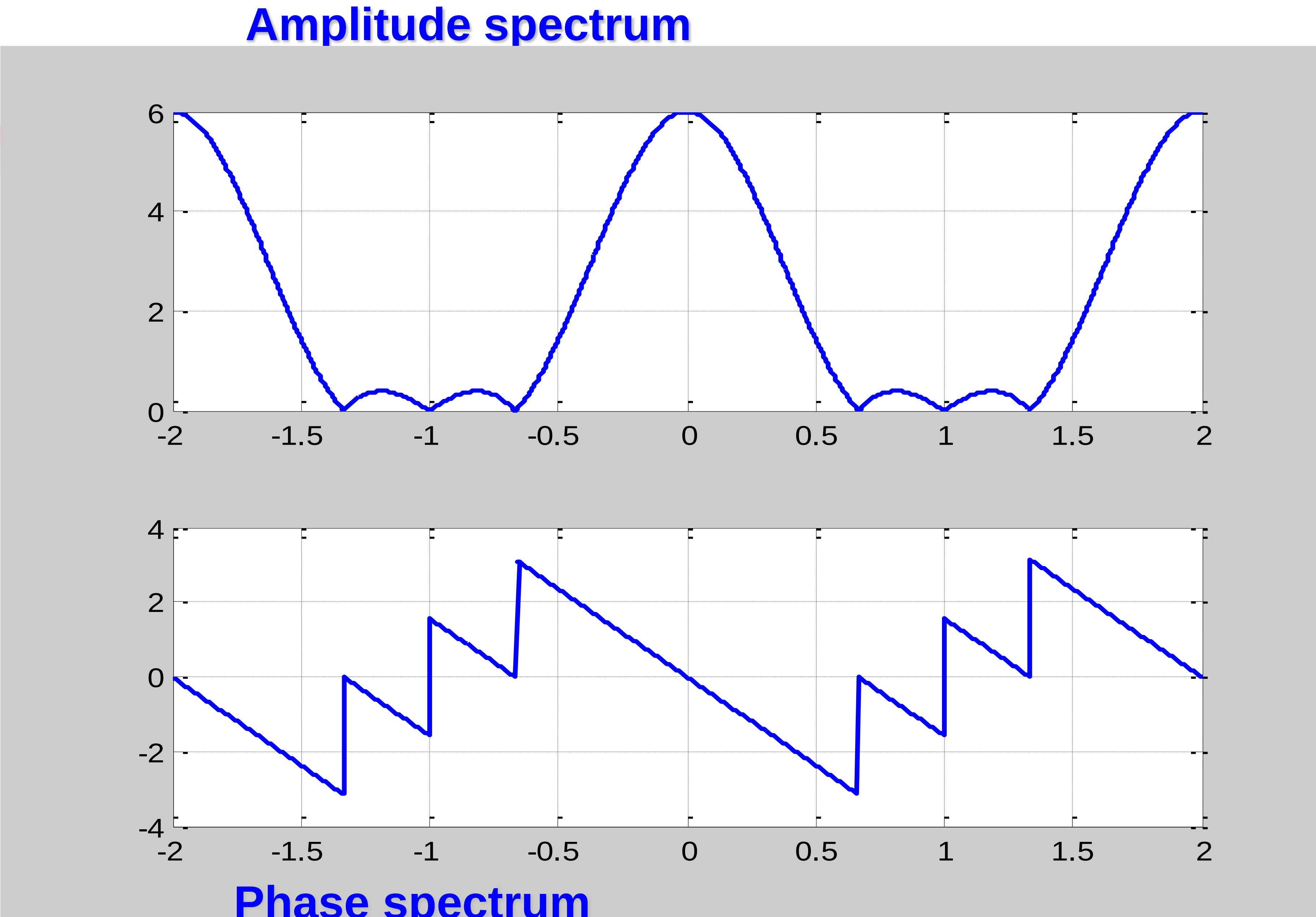
x[n] = u[n] - u[n - 4]

$$X(\Omega) = \sum_{n=0}^{3} e^{-j\Omega n} = \frac{1 - e^{-j4\Omega}}{1 - e^{-j\Omega}}$$

$$= \frac{\sin(2\Omega)}{\sin(\Omega/2)} e^{-j3\Omega/2}$$

Using Matlab to plot amplitude spectrum and phase spectrum

```
w = -2*pi:pi/255:2*pi; % freq. -2π → 2π, resolution of π/255

X =4*sinc(2*w/pi)./sinc(w/(2*pi)).*exp(-j*1.5*w);

subplot(2,1,1);

plot(w/pi,abs(X)); % plot amplitude spectrum

subplot(2,1,2);

plot(w/pi,phase(X)); % plot phase spectrum
```


Amplitude spectrum
6
4
2
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
4
2
0
-2
-4
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Phase spectrum

Lecture 7

Digital signal spectra

- **Outline:**

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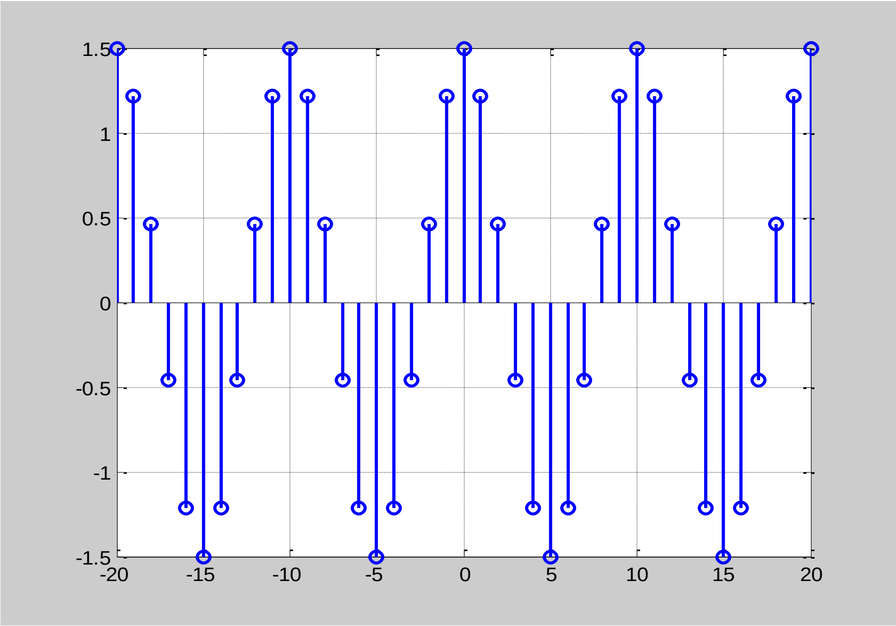
Periodic sinusoidal signals

- Digital sinusoidal signal is periodic only if its frequency **F** is rational number

$$x(n) = A\cos(\Omega n + \theta), \quad -\infty < n < +\infty$$
$$= A\cos(2\pi F n + \theta), \quad -\infty < n < +\infty$$



Fourier expansion

- **CT periodic signals with period T:**

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}; \quad a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt} \qquad \omega_0 = \frac{2\pi}{T}$$

- **DT periodic signals with period N:**

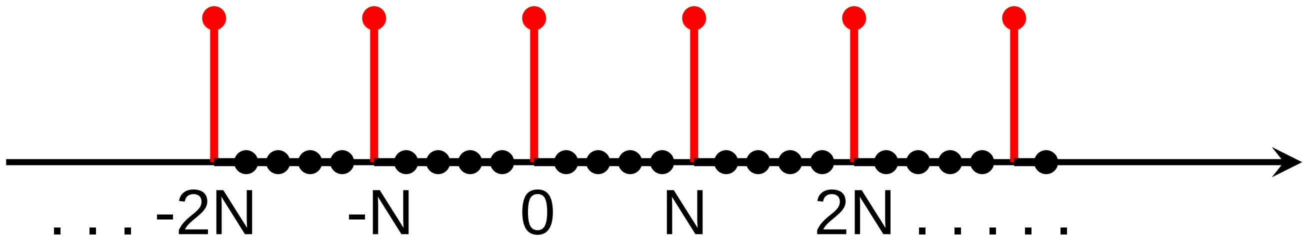
$$\boxed{x[n] = \sum_{k\in<N>} a_k e^{jk\Omega_0 n}; \quad a_k = \frac{1}{N}\sum_{n\in<N>} x[n] e^{-jk\Omega_0 n}} \qquad \Omega_0 = \frac{2\pi}{N}$$

Note: finite sums over an interval length of one periodic **N**

$$e^{jk\Omega_0 n} = e^{jk\frac{2\pi}{N}n} = e^{j(k+N)\frac{2\pi}{N}n} = e^{j(k+N)\Omega_0 n}$$

Example of Fourier Series expansion

Given a DT periodic signals with period N: $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-kN]$.



$$p[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}$$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} p[n] e^{-jk2\pi n/N} = \frac{1}{N}$$

Spectrum of periodic signals

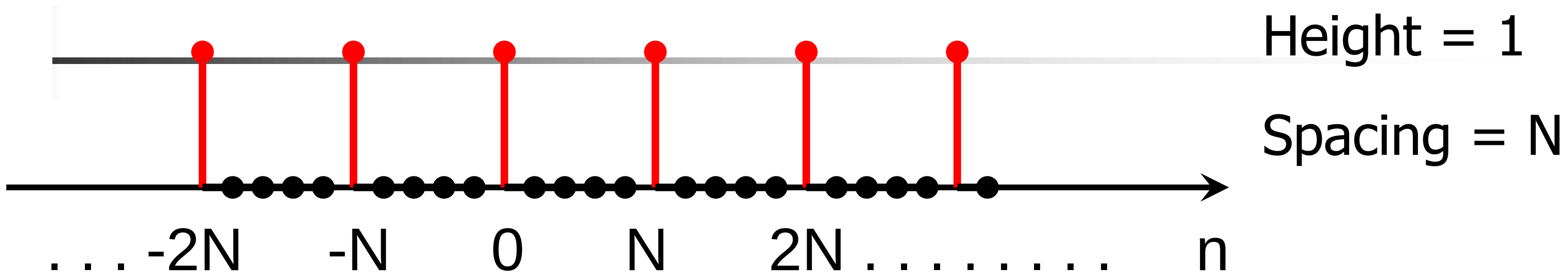
- CT periodic signals with period T:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \stackrel{F}{\longleftrightarrow} X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$$

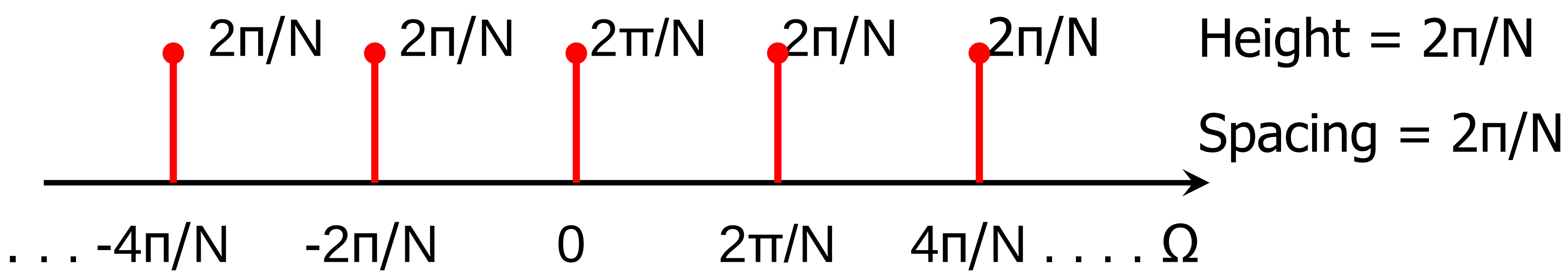
- DT periodic signals with period N:

$$\boxed{x[n] \stackrel{F}{\longleftrightarrow} X(\Omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\Omega - k\Omega_0)}$$

Example of calculating DTFT of periodic signals (DFS)

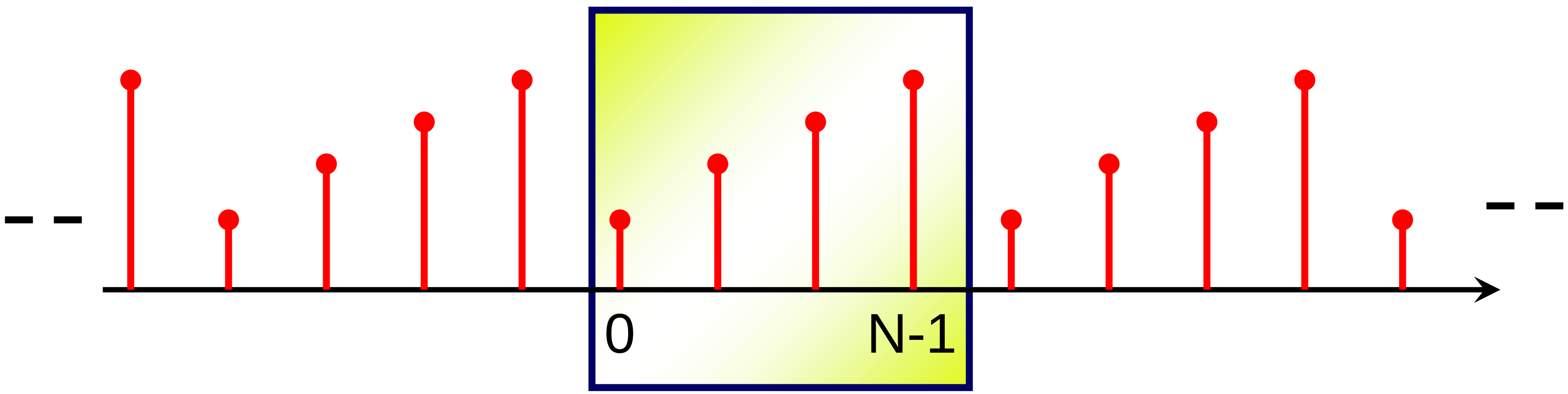


$P(\Omega)$



Another approach to get DTFT of periodic signals (DFS)

x(n) is periodic signal; $x_0(n)$ is a part of x(n) that is repeated



$$x_0[n] = \begin{cases} x[n], & 0 \le n \le N-1 \\ 0, & \text{otherwise.} \end{cases}$$

Another approach to get DTFT of periodic signals (DFS)

$$x_0[n] = \begin{cases} x[n], & 0 \le n \le N-1 \\ 0, & \text{otherwise.} \end{cases}$$

$$x[n] = \sum_{k=-\infty}^{\infty} x_0[n-kN] = \sum_{k=-\infty}^{\infty} x_0[n] * \delta[n-kN] = x_0[n] * \sum_{k=-\infty}^{\infty} \delta[n-kN]$$

p(n) in previous example

$$x[n] = x_0[n] * p[n] \xleftrightarrow{F} X_0(\Omega)P(\Omega) = X(\Omega)$$

Another approach to get DTFT of periodic signals (DFS)

$$x[n] = x_0[n] * p[n] \overset{F}{\longleftrightarrow} X_0(\Omega)P(\Omega) = X(\Omega)$$

$$\boxed{\begin{aligned} X(\Omega) &= X_0(\Omega)\left(\frac{2\pi}{N}\sum_k \delta(\Omega - k\frac{2\pi}{N})\right) \\ &= \frac{2\pi}{N}\sum_k X_0(k\frac{2\pi}{N})\delta(\Omega - k\frac{2\pi}{N}) \end{aligned}}$$

N samples—you are sampling the DTFT at
N equal intervals around the unit circle

→ It has **N** distinct values at k = 0, 1, …, N-1

Procedure to calculate spectrum (DFS) of periodic signals

Step 1:

Start with $\mathbf{x_0(n)}$ – one period of $\mathbf{x(n)}$**,** with zero everywhere else

Step 2:

Find the DTFT $\mathbf{X_0(\Omega)}$ of the signal $x_0[n]$ above

Step 3:

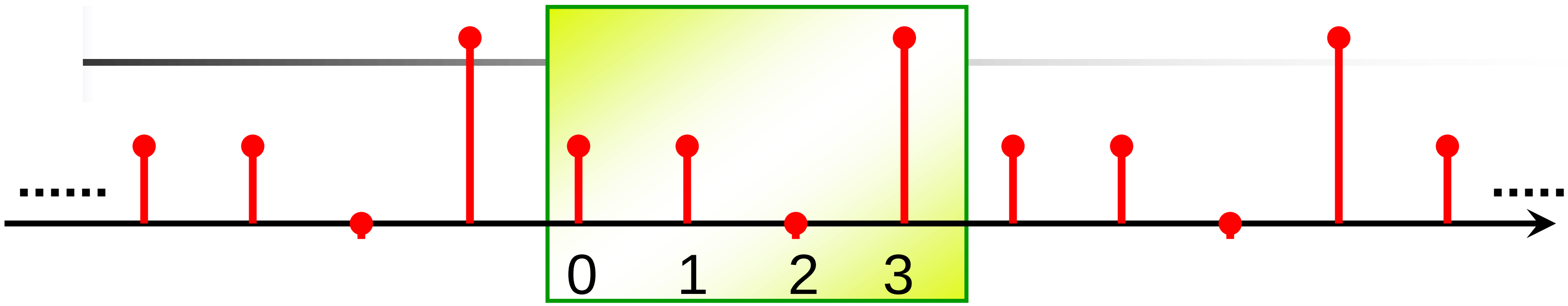
Find $X_0(\Omega)$ at **N** equally spacing frequency points $\mathbf{X_0(k2\pi/N)}$

Step 4:

Obtain the DTFT(DFS):

$$\boxed{X(\Omega) = \frac{2\pi}{N}\sum_{k} X_0\left(k\frac{2\pi}{N}\right)\delta\left(\Omega - k\frac{2\pi}{N}\right)}$$

Example of calculating DFS of periodic signals



$$X_0(\Omega) = \sum_{n=0}^{3} x_0(n) e^{-j\Omega n} = 1 + e^{-j\Omega} + 2e^{-j3\Omega}$$

$$X_o\left(\frac{2\pi k}{4}\right) = 1 + e^{-j\frac{2\pi k}{4}} + 2e^{-3j\frac{2\pi k}{4}} \qquad k = 0,1,2,3$$

k = 0 → $X_0(0) = 4$; k = 1 → $X_0(1) = 1+j$

k = 2 → $X_0(2) = -2$; k = 3 → $X_0(3) = 1-j$

Example of calculating DFS of periodic signals

$$X(\Omega) = \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} X_o\left(\frac{2\pi k}{N}\right) \delta\left(\Omega - \frac{2\pi k}{N}\right)$$

$$= \frac{2\pi}{4} \sum_{k=-\infty}^{\infty} X_o\left(\frac{2\pi k}{4}\right) \delta\left(\Omega - \frac{2\pi k}{4}\right)$$

For one period $0 \leq \Omega < 2\pi$

$$\frac{\pi}{2}\left\{4\delta(\Omega) + (1+j)\delta\left(\Omega - \frac{\pi}{2}\right) - 2\delta(\Omega - \pi) + (1-j)\delta\left(\Omega - \frac{3\pi}{2}\right)\right\}$$

HW

Prob.4 Determine and sketch the magnitude and phase response of the following signals

$$a)\quad x[n]=\frac{1}{2}\left(\delta[n]-\delta[n-2]\right)$$

$$b)\quad x[n]=\frac{1}{2}\left(\delta[n]-2\delta[n-1]+\delta[n-2]\right)$$

$$c)\quad x[n]=\frac{1}{8}\left(\delta[n]+3\delta[n-1]+3\delta[n-2]+\delta[n-3]\right)$$

HW

Prob.5

Given a periodic signal y(n) with N = 3 with associated

$$y_0[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$$

Find $Y_0(\Omega)$ and $Y(\Omega)$. Do the test with the inverse DTFT

Lecture 8

Frequency response and filter

- **Outline:**

1. Frequency response

2. Response to complex exponential signals

3. Filters

Frequency response

- For impulse response, **h(n),** its DTFT is often called **frequency response H(Ω)**
- **H(Ω)** completely characterizes a LTI system in the frequency domain
- **H(Ω)** allows us to determine the steady-state response of the system to any arbitrary weighted linear combination of sinusoids or complex exponential

Example of frequency response

- A LTI causal system is described by the following equation:

$$y[n]+0.1y[n-1]+0.85y[n-2]=x[n]-0.3x[n-1]$$

- First, checking the stability of the system (by using Matlab):

```
b = [1 -0.3];
a = [1 0.1 0.85];
zplane(b,a) % plot zeros and poles to check if all poles are inside the unit circle
```

- Second, take DTFT for two sides:

$$H(\Omega)=\frac{1-0.3e^{-j\Omega}}{1+0.1e^{-j\Omega}+0.85e^{-j2\Omega}}$$

Example of amplitude & phase responses

$$H(\Omega) = \frac{1}{1 - 0.4e^{-j\Omega}}$$

$$|H(\Omega)| = \frac{1}{|1 - 0.4e^{-j\Omega}|} = \frac{1}{|1 - 0.4\cos\Omega + j0.4\sin\Omega|}$$

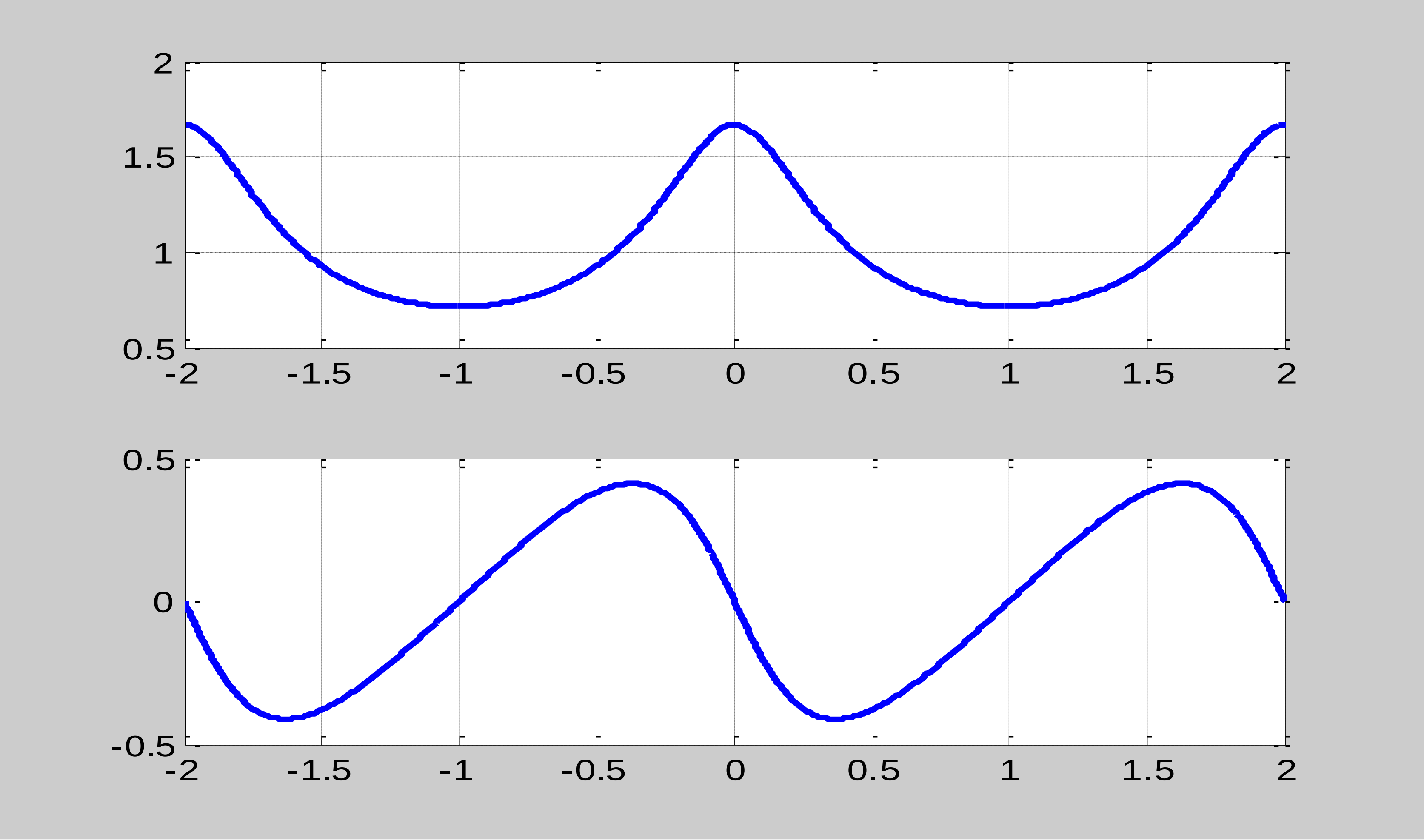
$$= \frac{1}{\sqrt{(1 - 0.4\cos\Omega)^2 + (0.4\sin\Omega)^2}}$$

$$= \frac{1}{\sqrt{1.16 - 0.8\cos\Omega}}$$

$$\angle H(\Omega) = 0 - \angle(1 - 0.4e^{-j\Omega}) = -\mathrm{arctg}\left(\frac{0.4\sin\Omega}{1 - 0.4\cos\Omega}\right)$$

Example of amplitude & phase responses

$$H(\Omega) = \frac{1}{1 - 0.4e^{-j\Omega}}$$



Lecture 8

Frequency response and filter

- **Outline:**

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Response to complex exponential signals

$$x[n] = Ae^{j\Omega_0 n}, \quad -\infty < n < \infty$$

$$\Rightarrow \quad y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\left(Ae^{j\Omega_0 (n-k)}\right)$$

$$= A\left[\sum_{k=-\infty}^{\infty} h[k]\left(e^{-j\Omega_0 k}\right)\right]e^{j\Omega_0 n}$$

$$= (Ae^{j\Omega_0 n})H(\Omega_0) = x[n]H(\Omega_0)$$

Example of determining response to complex exponential signals

Determine the output signal of system $h[n]=(1/2)^n u[n]$ to the input signal below

$$x[n]=Ae^{j\frac{\pi}{2}n}, \quad -\infty<n<\infty$$

$$H\left(\frac{\pi}{2}\right)=\frac{1}{1+j\frac{1}{2}}=\frac{2}{\sqrt{5}}e^{-j26.6^0}$$

$$y[n]=$$

Response to sinusoidal signals

$$x[n] = A\cos(\Omega_0 n) = \frac{A}{2}e^{j\Omega_0 n} + \frac{A}{2}e^{-j\Omega_0 n}, \quad -\infty < n < \infty$$

$$y[n] = \frac{A}{2}e^{j\Omega_0 n}H(\Omega_0) + \frac{A}{2}e^{-j\Omega_0 n}H(-\Omega_0)$$

$$= \frac{A}{2}e^{j\Omega_0 n}\,|H(\Omega_0)|\,e^{j\angle H(\Omega_0)} + \frac{A}{2}e^{-j\Omega_0 n}\,|H(\Omega_0)|\,e^{-j\angle H(\Omega_0)}$$

$$= A\,|H(\Omega_0)|\cos\left(\Omega_0 n + \angle H(\Omega_0)\right)$$

Example of determining response to sinusoidal signals

$$h[n] = (1/2)^n u[n]$$

$$x[n] = 10 - 5\sin\frac{\pi}{2}n + 20\cos\pi n, \quad -\infty < n < \infty$$

$$H(0) = \frac{1}{1-0.5} = 2;\ H\left(\frac{\pi}{2}\right) = \frac{2}{\sqrt{5}}e^{-j26.6^0};\ H(\pi) = \frac{1}{1+0.5} = \frac{2}{3}$$

$$y[n] = 10\,|H(0)| - 5\left|H\left(\frac{\pi}{2}\right)\right|\sin\left(\frac{\pi}{2}n + \angle H(\tfrac{\pi}{2})\right)$$

$$+ 20\,|H(\pi)|\cos(\pi n)$$

$$= 20 - 2\sqrt{5}\sin\left(\frac{\pi}{2}n - 26.6^0\right) + \frac{40}{3}\cos(\pi n)$$

Lecture 8

Frequency response and filter

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3. Filters

What is digital filters?

- Systems that perform mathematical operations on a DT signal to reduce or enhance certain aspects of that signal.
- Provide a convenient means to change the nature of a signal.
- Change the frequency characteristics of a signal in a specific way, letting some frequencies in the signal pass while blocking other

B
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E

A
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E
R

Characterizations of digital filter

1. Impulse response **h(n)**: FIR and IIR filters

2. Transfer function **H(z)**

3. Frequency response **H(Ω)**: amplitude response and phase response

4. Difference equation:

$$y[n]+a_1 y[n-1]+...+a_N y[n-N]=b_0 x[n]+b_1 x[n-1]+...+b_M x[n-M]$$

Typical applications of digital filters

- Noise Suppression: radio signal, biomedical signal, analog media signal...
- Enhancement of Selected Frequency Range: treble/bass control, equalizers in audio, image edge enhancement…
- Bandwidth Limiting: aliasing prevention, interference avoidance…
- Removal of Specific Frequencies: DC removal, 60 Hz signal removal, notch filter…
- Special Operations: differentiation, integration, Hilbert transform…

Basic digital filter types

- Low pass filter (LPF): lets low frequencies through while blocking high frequencies;
- High pass filter (HPF): lets high frequencies while blocking low frequencies;
- Band pass filter (BPF): allows a band of frequencies to pass;
- Band stop filter (BSF): allows all frequencies outside a band to pass

Ideal digital filters

$$y[n]=\begin{cases} Cx[n-n_0], & \omega_1<\omega<\omega_2 \\ 0, & \omega \neq \end{cases}$$

$$\Rightarrow \quad Y(e^{j\omega})=CX(e^{j\omega})e^{-j\omega n_0}=X(e^{j\omega})H(e^{j\omega}), \quad \omega_1<\omega<\omega_2$$

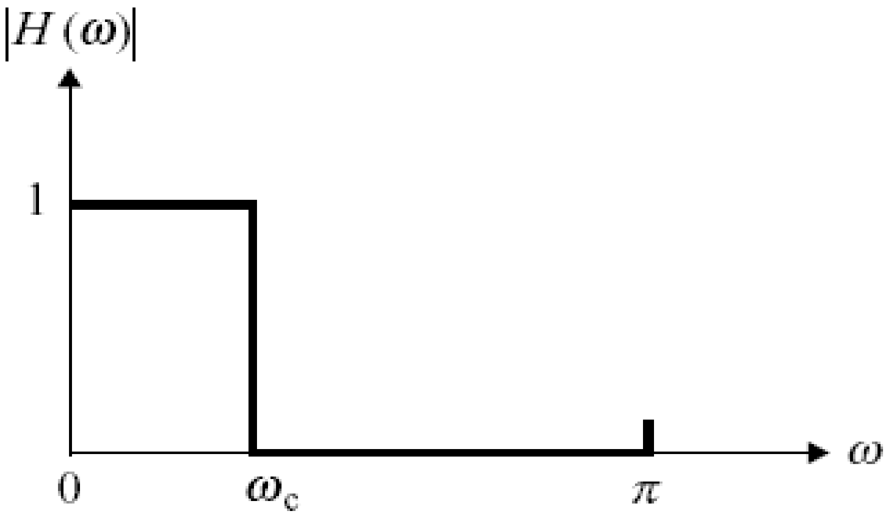
$$\Rightarrow \quad H(e^{j\omega})=\begin{cases} Ce^{-j\omega n_0}, \; \omega_1<\omega<\omega_2 \\ 0, \quad \omega \neq \end{cases}$$

$$\Rightarrow \quad \boxed{\begin{aligned} &|H(e^{j\omega})|=C, \quad \omega_1<\omega<\omega_2 \\ &\theta(\omega)=-\omega n_0, \quad \omega_1<\omega<\omega_2 \end{aligned}}$$

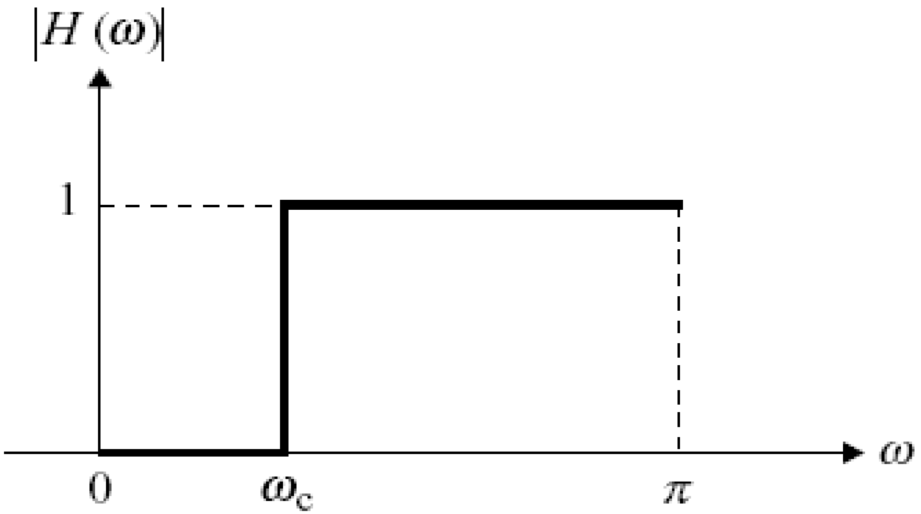
Linear phase response

Constant amplitude response

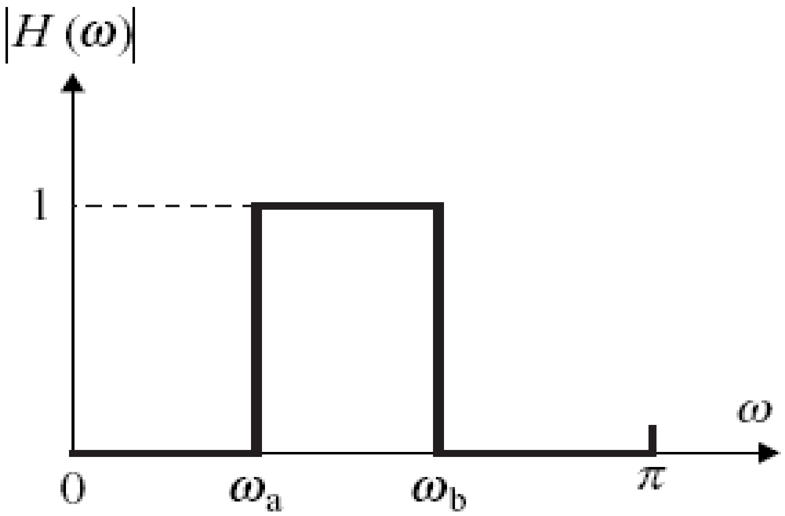
Ideal digital filters



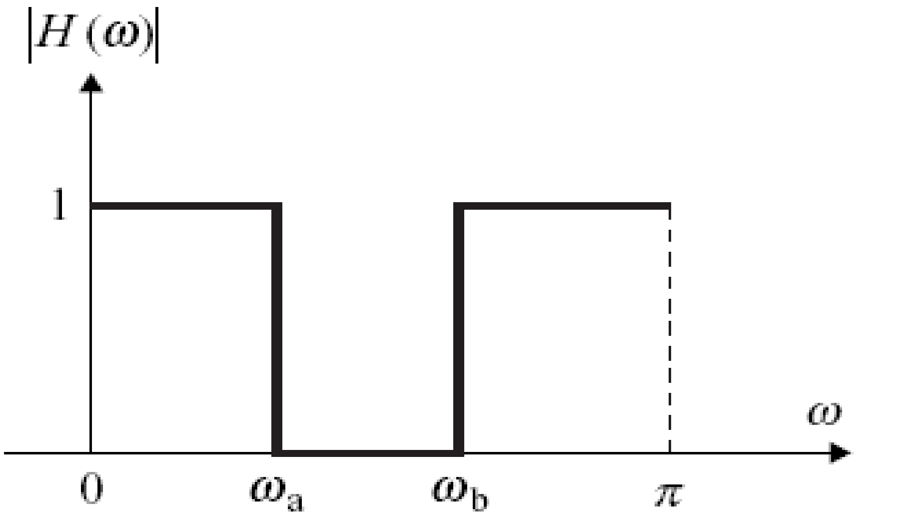
(a) Lowpass filter



(b) Highpass filter



(c) Bandpass filter



(d) Bandstop filter

Block diagram representation

- Being built based on input-output relation or transform function by connecting the basic components in the specific way:

$$y[n]+a_1y[n-1]+...+a_Ny[n-N]=b_0x[n]+b_1x[n-1]+...+b_Mx[n-M]$$

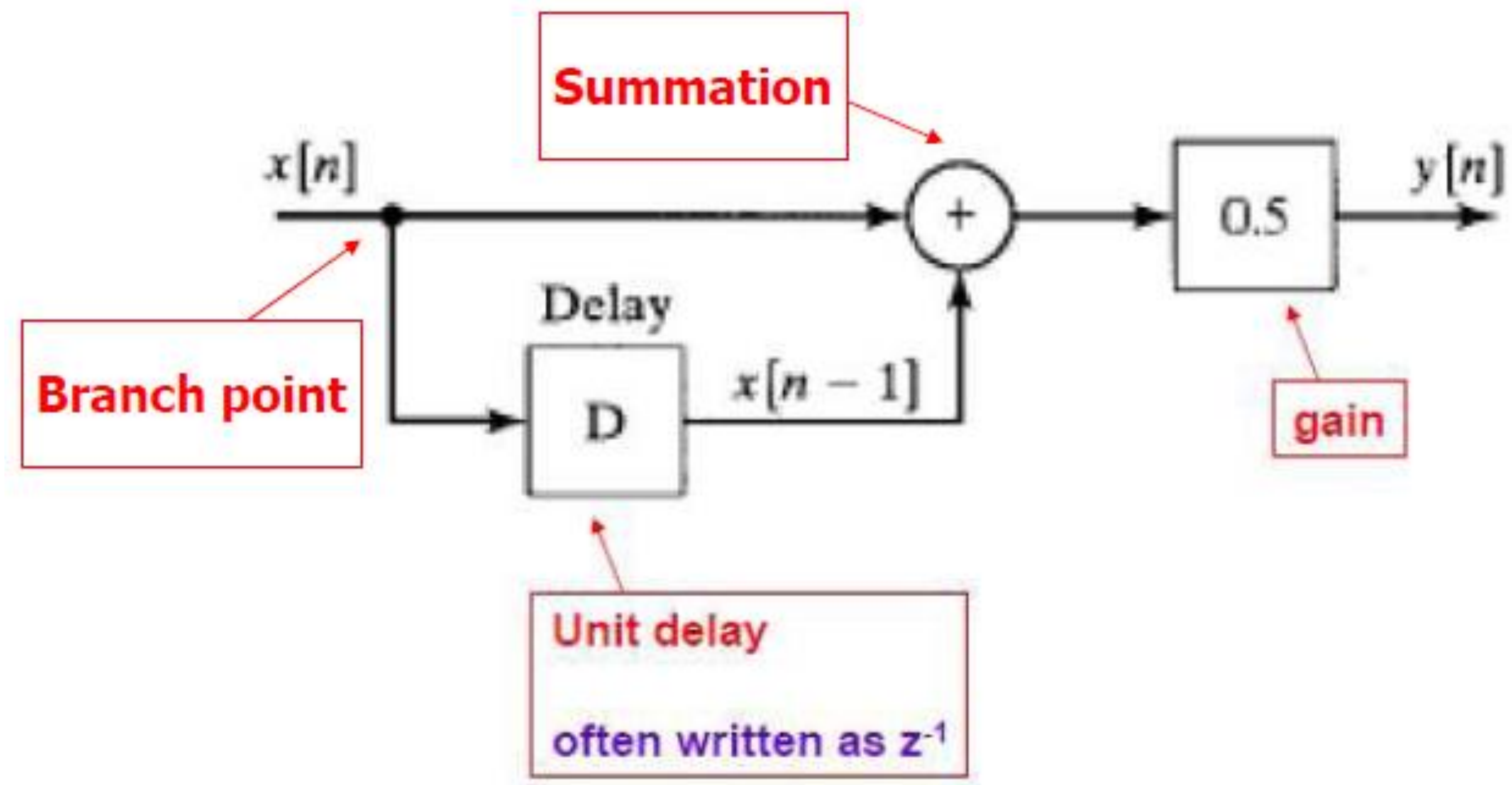
$$\leftrightarrow \quad \sum_{k=0}^{N} a_k y[n-k] = \sum_{r=0}^{M} b_r x[n-r]$$

$$\leftrightarrow \quad H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{r=0}^{M} b_r z^{-r}}{\sum_{k=0}^{N} a_k z^{-k}}$$

- Some advantages: easy to determine the relation between input and output, easy to manipulate the diagram to the other equivalent one, easy to determine the hardware…

Components for block diagram

EX: an averaging system y[n] = 0.5(x[n] + x[n-1])



HW

Prob.6 An FIR filter is described by the difference equation

$$y[n] = x[n] - x[n-10]$$

a) Compute and sketch its magnitude and phase response

b) Determine its response to the inputs

$$(1) \quad x[n] = \cos\frac{\pi}{10}n + 3\sin\left(\frac{\pi}{3}n + \frac{\pi}{10}\right) \quad -\infty < n < \infty$$

$$(2) \quad x[n] = 10 + 5\cos\left(\frac{2\pi}{5}n + \frac{\pi}{2}\right) \quad -\infty < n < \infty$$

HW

Prob.7 (a) Determine the coefficients of a linear-phase FIR filter

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

such that:

- It rejects completely a frequency component at $\Omega_0 = 2\pi/3$

- Its frequency response is normalized so that H(0) = 1

(b) Compute and sketch the magnitude and phase response of the filter to check if it satisfies the requirements

- w = -pi:pi/255:pi;
- b0 =1/3 ; b1 =1/3 ; b2 =1/3   ;
- H = b0 + b1*exp(-j*w) + b2*exp(-j*2*w);
- Habs = abs(H);
- Hphase = phase(H);
- subplot(2,1,1);
- plot(w/pi,Habs,'linewidth',2);grid;
- subplot(2,1,2);
- plot(w/pi,Hphase,'linewidth',2);grid;