附录I: 无约束优化检验题目

下面给出无约束优化问题的检验题目，每一题给出的内容包括：

- n,m. 若n或m未给定，则为正整数变量. 变量的维数或规模可以自选.
- $r_i(x)$, $i = 1, ...m$,
- x_0, x^*,$f(x^*)$.

从而优化问题为

$$\min \sum_{i=1}^{m} r_i^2(x).$$

上机习题

1. Rosenbrock 函数

$$\begin{aligned} &n = 2, m = 2, \\ &r_1(x) = 10(x_2 - x_1^2),\ r_2(x) = 1 - x_1, \\ &x_0 = (-1.2, 1), \quad x^* = (1, 1), \quad f(x^*) = 0. \end{aligned}$$

2. Powell badly scaled 函数

$$\begin{aligned} &n = 2, m = 2, \\ &r_1(x) = 10^4 x_1 x_2 - 1,\ r_2(x) = e^{-x_1} + e^{-x_2} - 1.0001, \\ &x_0 = (0, 1), \quad x^* = (1.098...10^{-5}, 9.106...), \quad f(x^*) = 0. \end{aligned}$$

3. Wood 函数

$n = 4, m = 6,$

$r_1(x) = 10(x_2 - x_1^2),\ r_2(x) = 1 - x_1,\ r_3(x) = (90)^{1/2}(x_4 - x_3^2),$

$r_4(x) = 1 - x_3,\ r_5(x) = (10)^{1/2}(x_2 + x_4 - 2),\ r_6(x) = (10)^{-1/2}(x_2 - x_4),$

$x_0 = (-3, -1, -3, -1)^T,\ x^* = (1, 1, 1, 1),\ f(x^*) = 0.$

4. Watson 函数

$2 \le n \le 31,\ m = 31,$

$$r_i(x) = \sum_{j=2}^{n}(j-1)x_j t_i^{j-2} - (\sum_{j=1}^{n} x_j t_i^{j-1})^2 - 1,$$

其中$t_i = i/29, 1 \le i \le 29,$

$f_{30}(x) = x_1, \quad f_{31} = x_2 - x_1^2 - 1,$

$x_0 = (0, ...0),$

若$n = 6, f^* = 2.28767...10^{-3},$

若$n = 9, f^* = 1.39976...10^{-6},$

若$n = 12, f^* = 4.72238...10^{-10}.$

5. Trigonometric 函数

$$
\begin{aligned}
&m = n,\\
&r_i(x) = n - \sum_{j=1}^{n} cos\ x_j + i(1 - cos\ x_i) - sin\ x_i,\\
&x_0 = (1/n, ..., 1/n),\ f^* = 0.
\end{aligned}
$$

6. Discrete boundary value 函数

$$
\begin{aligned}
&m = n,\\
&r_i(x) = 2x_i - x_{i-1} - x_{i+1} + h^2(x_i + t_i + 1)^3/2,\\
&\quad 其中 h = 1/(n+1),\ t_i = ih,\ x_o = x_{n+1} = 0.\\
&x_0 = (\xi_j), 其中 \xi_j = t_j(t_j - 1),\ f^* = 0.
\end{aligned}
$$

7. Beale 函数

$$
\begin{aligned}
&n = 2, m = 3,\\
&r_i(x) = y_i - x_1(1 - x_2^i),\ 其中\ y_1 = 1.5,\ y_2 = 2.25,\ y_3 = 2.625\\
&x_0 = (1, 1), \quad x^* = (3, 0.5), \quad f(x^*) = 0.
\end{aligned}
$$

8. Biggs EXP6 函数

$$n = 6, m \geq n,$$
$$r_i(x) = x_3 e^{-t_i x_1} - x_4 e^{-t_i x_2} + x_6 e^{-t_i x_5} - y_i,$$
其中$t_i = 0.1i,\ y_i = e^{-t_i} - 5e^{-10t_i} + 3e^{-4t_i}$
$$x_0 = (1, 2, 1, 1, 1, 1).\text{若 } m = 13, \quad f(x^*) = 5.65565...10^{-3},$$
$$x^* = (1, 10, 1, 5, 4, 3), f(x^*) = 0.$$

9. Extended Rosenbrock 函数

$$n\text{为偶数}, m = n,$$
$$r_{2i-1}(x) = 10(x_{2i} - x_{2i-1}^2), \quad r_{2i}(x) = 1 - x_{2i-1},$$
$$x_0 = (\xi_j),\ \text{其中 } \xi_{2j-1} = -1.2, \xi_{2j} = 1$$
$$x^* = (1, ..., 1), \quad f(x^*) = 0.$$

10. Extended Powell singular 函数

$$n\text{为4的整数倍}, m = n,$$
$$r_{4i-3}(x) = x_{4i-3} + 10x_{4i-2},$$
$$r_{4i-2}(x) = 5^{1/2}(x_{4i-1} - x_{4i}),$$
$$r_{4i-1}(x) = (x_{4i-2} - 2x_{4i-1})^2,$$
$$r_{4i}(x) = 10^{1/2}(x_{4i-3} - x_{4i})^2.$$
$$x_0 = (\xi_j),\ \text{其中 } \xi_{4j-3} = 3, \xi_{4j-2} = -1, \xi_{4j-1} = 0, \xi_{4j} = 1,$$
$$x^* = (0, ..., 0), \quad f(x^*) = 0.$$

约束优化检验问题

11.

$$\begin{aligned} \min \quad & f(x) = -1, \\ s.t. \quad & x_1^2 + x_2^2 - 25 = 0, \\ & x_1 x_2 - 9 = 0. \end{aligned}$$

$x_0 = (2,1)^T$为非可行点,

$x^* = (a, 9/a),\ (-a, -9/a),\ (b, 9/b),\ (-b, -9/b),$

其中$a = \sqrt{\dfrac{25+\sqrt{301}}{2}},\ b = \sqrt{\dfrac{25-\sqrt{301}}{2}},$

$f(x^*) = -1.$

12.

$$\begin{aligned} \min \quad & f(x) = ln(1 + x_1^2) - x_2, \\ s.t. \quad & (1 + x_1^2)^2 + x_2^2 - 4 = 0. \end{aligned}$$

$x_0 = (2,2)$为非可行点,

$x^* = (0, \sqrt{3}),\ f(x^*) = -\sqrt{3}.$

13.

$$\begin{aligned}
&\min \quad f(x) = .5x_1^2 + x_2^2 - x_1x_2 - 7x_1 - 7x_2,\\
&s.t. \quad 25 - 4x_1^2 - x_2^2 \geq 0.\\
&x_0 = (0,0)\text{为可行点}, \quad x^* = (2,3),\ f(x^*) = -30.
\end{aligned}$$

14.

$$\begin{aligned}
\min \quad & f(x) = -x_1x_2x_3,\\
s.t. \quad & -x_1^2 - 2x_2^2 - 4x_3^2 + 48 \geq 0.
\end{aligned}$$

$$x_0 = (1,1,1)\text{为可行点}$$

$$x^* = (a,b,c,), (a,-b,-c), (-a,b,-c), (-a,-b,c$$

其中$a = 4, b = 2\sqrt{2}, c = 2,$

$$f(x^*) = -16\sqrt{2}.$$

15.

$$\begin{aligned}
\min \quad & (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^4,\\
s.t \quad & x_1(1 + x_2^2) + x_3^4 - 4 - 3\sqrt{2} = 0,\\
& -10 \leq x_i \leq 10, \quad i = 1,2,3.
\end{aligned}$$

$$\begin{aligned}
&x_0 = (2,2,2)^T\text{为非可行点},\\
&x^* = (1.104859024, 1.196674194, 1.535262257)^T,\\
&f(x^*) = 0.03256820025.
\end{aligned}$$

16.

$$\begin{aligned} \min \quad & x_1x_4(x_1+x_2+x_3)+x_3, \\ s.t \quad & x_1x_2x_3x_4-25\geq 0, \\ & x_1^2+x_2^2+x_3^2+x_4^2-40=0, \\ & 1\leq x_i\leq 5, \quad i=1,...,4. \end{aligned}$$

$x_0=(1,5,5,1)^T$为可行点,

$f(x_0)=16,$

$x^*=(1,4,7429994,3.8211503,1.3794082)^T,$

$f(x^*)=17.014.173.$

17.

$$
\begin{aligned}
\min \quad & 5.04x_1 + 0.035x_2 + 10x_3 + 3.36x_5 - 0.063x_4x_7, \\
s.t \quad & c_1(x) = 35.82 - 0.222x_{10} - bx_9 \geq 0, \\
& c_2(x) = -133 + 3x_7 - ax_{10} \geq 0, \\
& c_3(x) = -c_1(x) + x_9(1/b - b) \geq 0, \\
& c_4(x) = -c_2(x) + (1/a - a)x_{10} \geq 0, \\
& c_5(x) = 1.12x_1 + 0.13167x_1x_8 - 0.00667x_1x_8^2 - ax_4 \geq 0, \\
& c_6(x) = 57.425 + 1.098x_8 - 0.038x_8^2 + 0.325x_6 - ax_7 \geq 0, \\
& c_7(x) = -c_5(x) + (1/a - a)x_4 \geq 0, \\
& c_8(x) = -c_6(x) + (1/a - a)x_7 \geq 0, \\
& c_9(x) = 1.22x_4 - x_1 - x_5 = 0, \\
& c_{10}(x) = 98000x_3/(x_4x_9 + 1000x_3) - x_6 = 0, \\
& c_{11}(x) = (x_2 + x_5)/x_1 - x_8 = 0,
\end{aligned}
$$

其中$a = 0.99, b = 0.9$.

$x_0 = (1745, 12000, 110, 3048, 1974, 89.2, 92.8, 8, 3.6, 145)^T$为非可行点,

$f(x_0) = -872.3872$.

$$
\begin{aligned}
x^* = & (1698.096, 15818.73, 54.10228, 3031.226, 2000, 90.11537, 95, \\
& 10.49336, 1.561636, 153.53535)^T, \\
& f(x_0) = -1768.80696.
\end{aligned}
$$