Phase Retrieval Problem

AAIS in PKU   Weijie Chen 1901111420

May 22, 2020

1 Problem Settings

Considering the phase retrieval problem

$$y_r = | < \boldsymbol{a}_r, x > |^2, r = 1, 2, ..., m \tag{1}$$

where $x \in \mathbb{C}^n$ is the decision variable, $\boldsymbol{a}_r \in \mathbb{C}^n$ are known sampling vectors, and $y_r \in \mathbb{R}$ are the observed measurements. We want to recover the true signal x using the finite mearsuerments $\boldsymbol{a}_r$.

2 Algorithm

In this problem, we are going to solve for x in the following problem

$$\text{minimize} \quad f(x) = \frac{1}{2m} \sum_{r=1}^{m} \left(y_r - \left| \boldsymbol{a}_r^T x \right|^2 \right)^2, x \in \mathbb{C}^n \tag{2}$$

And the **Wirtinger Flow** algorithm will use to solve it.

The first step is to carefully initialize x_0 as the initial guess via a spectral method , see Algorithm 1.

Algorithm 1: Wirtinger Flow : Initialization

Input : Observations $\{y_r\}_{r=1}^m \in \mathbb{R}$

Output: Initial guess x_0

1 Set $\lambda^2 = n \frac{\sum_r y_r}{\sum_r \|\boldsymbol{a}_r\|^2}$

2 Set x_0, normalized to λ, to be the eigenvector corresponding to the largest eigenvalue of

$$\boldsymbol{Y} = \frac{1}{m} \sum_{r=1}^{m} y_r \boldsymbol{a}_r \boldsymbol{a}_r^T$$

Then from the initial guess x_0, for $t = 0, 1, 2...$, we update x as the following:

$$x_{t+1} = x_t - \frac{\mu_{t+1}}{\|x_0\|^2} \left(\frac{1}{m} \sum_{r=1}^{m} \left(\left| \boldsymbol{a}_r^T x \right|^2 - y_r \right) \left(\boldsymbol{a}_r \boldsymbol{a}_r^T \right) x \right) := x_t - \frac{\mu_{t+1}}{\|x_0\|^2} \nabla f\left(x_t\right) \tag{3}$$

where μ_{t+1} can be interpreted as a step size. This gradient-descent-like algorithm can iteratively refine our guess x and finnaly we get our recovered signal.

3 Numerical Result

To implement this algorithm, we choose the input data as phaseless measure- ments about a Gaussian complex valued 1D signal. During experiments, we found that the algorithm would work well when m is larger than approximately $4n$, and would crash if m is relatively small.

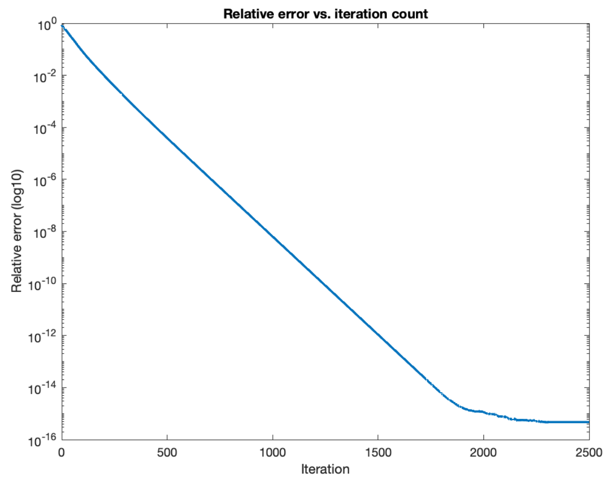

Figure 1: Wirtinger Flow phase retrieval ($n = 20, m = 100$)

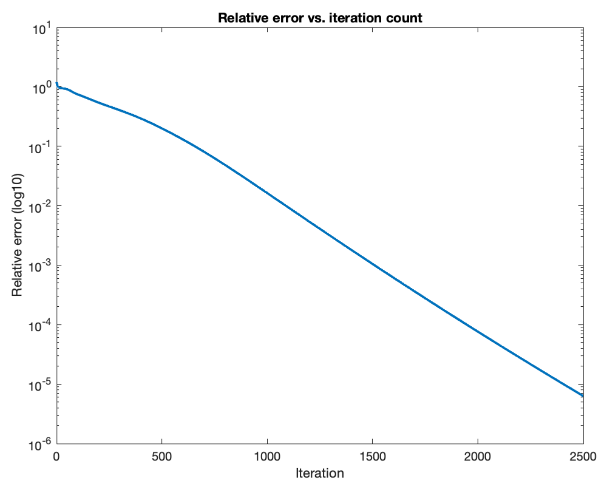

Figure 2: Wirtinger Flow phase retrieval ($n = 60, m = 200$)

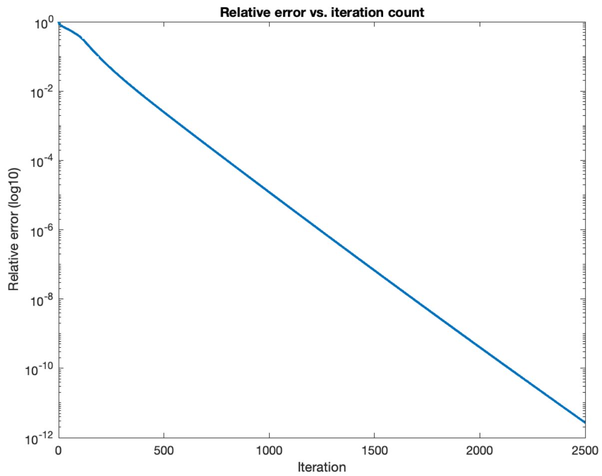

Figure 3: Wirtinger Flow phase retrieval ($n = 100, m = 400$)

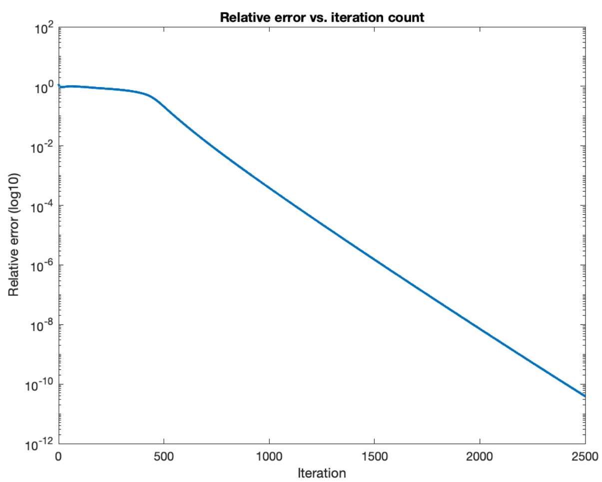

Figure 4: Wirtinger Flow phase retrieval ($n = 200, m = 800$)

4 README

The environment of numerical experiment as following:

- MacBook Pro(15-inch, 2019)
- 2.6 GHz 6-Core Intel Core i7
- 16 GB 2400 MHz DDR4
- macOS Catalina version 10.15.1
- Matlab 2019a

Reference

[1] E. J. Candes, X. Li and M. Soltanolkotabi. Phase retrieval via Wirtinger flow: theory and algorithms. IEEE Transactions on Information Theory 61(4), 1985–2007.