Convex Optimization Project Midterm Report

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We are going to solve the second problem about optimal transport. Because many hard problems in varied fields (e.g. biological feature identification, medical imaging, and some particular neural networks) can be reformulated into optimal transport problems, to find an efficient and robust way to solve this problem with large-scale data is pivotal.

1 Description to the Problem

For the purpose of finding a solution to this problem, we first formulate the original problem into the linear programming form as follows:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\pi_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{n} \pi_{ij} = \mu_i \ \forall i = 1, \cdots, m \\
& \sum_{i=1}^{m} \pi_{ij} = \nu_i \ \forall j = 1, \cdots, n \\
& \pi_{ij} \geq 0
\end{aligned}
\tag{1}
$$

where c_{ij} denote the cost between i and j, and π_{ij} are variables to solve.

2 Solving LP by existing methods

We have solved the previous linear programming problem using interior method and simplex method by calling Gurobi directly, and compared the efficiency between two methods.

In our numerical experiments, we used a simple undirected graph i.e. $N \times N$ square lattice. Each point can be denoted by the i-th point in $\mathbb{Z}$ or $\left(i_x, i_y\right)$ in $\mathbb{Z} \times \mathbb{Z}$.Thus, the size of corresponding solution matrix π is $N^2 \times N^2$. We used Euclidean distance $d_{ij} = \sqrt{(i_x - j_x)^2 + (i_y - j_y)^2}$ to represent the distance between two points $i = \left(i_x, i_y\right)$ and $j = \left(j_x, j_y\right)$, and define the cost function as $c_{ij} = d_{ij} + \epsilon\delta_{ij}$, where ϵ is similar to the storage cost for each warehouse. To guarantee the existence of solutions, we generated a uniformly distributed random matrix π_0, whose elements are in the interval $(0, 1)$, then deduced μ and ν by definition. We then applied interior point methods and simplex methods in Gurobi to solve the discrete optimal transport problem described in (1). Three sizes (16×16, 32×32 and 64×64) are tested. Results of running time are included in Table 1.

Table 1 Running Time (s) on Each Method

Graph Size	Solution Size	Simplex Method	Interior Point Method
16×16	256×256	0.17	0.41
32×32	1024×1024	3.68	10.55
64×64	4096×4096	1635.87	2269.76

3 Further Research

In our research in this field in the future, we are going to do more numerical experiments on different experimental subjects at the first time, such as the test objects given in *DOTmark*. The implementation for some other methods to solve this problem is also under our consideration, and in the meantime we will analyze the properties of them. What's more, we are going to read *DOTmark —A Benchmark for Discrete Optimal Transport* and *Multiscale Strategies for Computing Optimal Transport*, to compare the methods in these papers with existing ones and find out their advantages and disadvantages. After that, we plan to read *Computational Optimal Transport* or *Vector and Matrix Optimal Mass Transport: Theory, Algorithm, and Applications*, hoping to find some intriguing problems from them to solve if time permits.

In another aspect, our research will consider the solutions to some particular problems where the standard solvers cannot run fast, because to figure out the non-standard solutions is another quite interesting subject in the field of optimization.