

Thermal Conductivity Accumulation in Amorphous Materials

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Abstract

By definition, phonons are non-localized propagating vibrations. For amorphous solids, with the exception of very long-wavelength (low-frequency) modes, the vibrational modes are non-propagating. Recently, experimental measurements of the thermal conductivity of amorphous materials demonstrate that propagating (phonon-like) modes with large mean free paths contribute significantly to thermal transport in amorphous materials [Regner et al., Nat. Commun. 4, 1640 (2013) and Sultan et al., Phys. Rev. B 87, 214305 (2013)]. Using broadband frequency domain thermal-reflectance, Regner et al. measured how the thermal conductivity of a-SiO₂ and a-Si thin films change with the penetration depth associated with the heating laser pulse. Regner et al. argue that their measurements probe the mean free paths of propagating vibrational modes to measure the thermal conductivity accumulation function. Using lattice dynamics calculations and molecular dynamics simulations on realistic models of a-SiO₂ and a-Si, we predict and characterize the contributions from propagating and non-propagating vibrations to thermal conductivity. The vibrational mean free paths and the thermal conductivity accumulation functions are predicted for a-SiO₂ and a-Si and compared with experimental results of Regner et al. and varying thin film measurements. For a-SiO₂, the propagating modes are found to contribute negligibly for both bulk and thin films. We demonstrate the scaling of the low-frequency propagating mode diffusivities using (to our knowledge) the largest model of bulk a-Si. For a-Si, the propagating modes contribute significantly to thermal transport. We consider two scalings of the propagating mode lifetimes (and from them, the mean free paths) to compare with the results of Regner et al. and thin film measurements, and find qualitative agreement for both. Further experiments are suggested based on our predictions and the comparisons.

I. INTRODUCTION

The vibrational modes in disordered solids (i.e., alloys, amorphous materials) can be classified as propagons (propagating and de-localized, i.e., phonon-like), diffusons (non-propagating and de-localized), and locons (non-propagating and localized).^{1,2} Diffusons are de-localized but non-propagating, and contribute to thermal conductivity by harmonic coupling with other modes due to the disorder. Locons are localized and non-propagating, and do not contribute significantly to thermal transport in three dimensional systems.³

Experimental measurements, estimates based on experiments, and modeling have demonstrated that propagating modes contribute significantly to the thermal conductivity of amorphous silicon (a-Si)⁴⁻¹⁰ and amorphous silicon nitride,¹¹ but not to that of amorphous silica (a-SiO₂).^{10,12-14} A recent modeling study of a-SiO₂ and a-Si found the contribution from propagating modes to be inconclusive due to limited system sizes which were accessible.¹⁵ (ALAN: I can't lump this in with the previous sentence, but I also don't know how to put this any better) Notably, using broadband frequency domain thermal-reflectance, Regner et al. measured how the thermal conductivity of a-SiO₂ and a-Si thin films change with the penetration depth associated with the heating laser pulse.¹⁰ Following the suggestion of Koh and Cahill, they interpret the measured value at a given penetration depth to be representative of the phonons with mean free path (MFP) less than that value, allowing for the construction of the thermal conductivity accumulation function.¹⁶⁻¹⁸ For a-SiO₂, the thermal conductivity of a 1000 nm thick film did not vary for penetration depths between 57 and 960 nm, suggesting that any propagating modes that contribute to thermal conductivity have MFPs below 57 nm. For a-Si, they find that the thermal conductivities of films with thicknesses of 500 and 2000 nm vary by 40% between penetration depths of 44 and 968 nm, suggesting that propagating modes with MFPs in this range contribute significantly to thermal conductivity.¹⁰

To understand the results of Regner et al. requires knowledge of the MFPs of the propagating modes and the contribution to thermal conductivities from non-propagating modes. Experimentally, inelastic neutron scattering can be used to measure phonon lifetimes (and from these, MFPs) in materials, but this technique is suited to single crystal samples.¹⁹ Traditionally, empirical expressions and simple models have been the only means to estimate MFPs in amorphous materials,²⁰⁻²⁴ while the Allen Feldman (AF) theory can be used to

model the non-propagating modes.^{1,4}

Predicting the vibrational MFPs in an amorphous solid requires the group velocity and lifetime of the low-frequency propagating modes.^{4-8,12,14,21,22} It is typically assumed that the group velocity of the low-frequency modes is equal to the sound speed.^{4-6,12,20-24} The MFPs are then determined by the vibrational lifetimes.

To evaluate the expressions and models for the low-frequency mode lifetimes requires knowledge of how the lifetimes scale with frequency.^{4-8,12,14,21,22,25,26} The scaling for a-SiO₂ has only recently been understood by experimental measurements, with evidence of ω^{-2} , ω^{-4} , and a second ω^{-2} regime as the mode frequency, ω , increases from π to $2\pi \times 10^{12}$ rads/s.²⁷⁻³⁰ For a-Si, the scaling is less understood,^{4-9,25,26} and temperature-dependent^{7,25,31} and film thickness-varying measurements^{5,7,8,25,32-36} suggest both ω^{-2} and ω^{-4} scalings.^{4,6}

The objective of this work is to investigate the propagating and non-propagating contributions to the thermal conductivity of a-SiO₂ and a-Si by predicting the MFPs and thermal conductivity accumulation functions for realistic models. The results will be compared with the measurements by Regner et al.¹⁰ and experimental measurements for varying film thicknesses at a temperature of 300 K.^{5,7,8,12-14,21-23,37,38} Two of the most widely-studied amorphous materials, a-SiO₂ and a-Si, are examined in parallel to understand the experimental results.

The paper is organized as follows. The theoretical formulation and modeling framework is discussed in Section II. The sample preparation for the a-SiO₂ and a-Si bulk models and the computational details are discussed in Section III. In Sections IV A, IV B, and IV C, harmonic lattice dynamics calculations are performed on large models to predict the vibrational density of states, the plane-wave character of the vibrational modes, and the group velocity of the low-frequency propagating modes (i.e., the sound speed).

The vibrational mode lifetimes are predicted using the molecular dynamics-based normal mode decomposition method (NMD) in Section IV D. Using the sound speeds and lifetimes, the vibrational mode diffusivities are calculated and compared with predictions from the AF theory in Section IV E. Predicting the diffusivities (i.e., the product of the square of the group velocity and the lifetime) of the propagating modes allows for a direct comparison with those of the non-propagating modes, where group velocity and lifetime are not well-defined.¹ Using this comparison, a cutoff frequency between propagating and non-propagating modes is specified.

The properties of the propagating and non-propagating modes are then used to predict the total thermal conductivity from the bottom-up in Section V A. Using the Green-Kubo method and molecular dynamics simulations of very large models of bulk a-SiO₂ and a-Si (including, to our knowledge, the largest molecular dynamics simulation for a model of a-Si^{9,39}), we also predict the total thermal conductivity from the top-down. The spectrum of vibrational MFPs and the thermal conductivity accumulation functions are predicted for the first time in Section V B and the results are compared with the measurements of Regner et al.¹⁰ and thermal conductivity experiments on thin films.^{5,7,8,12–14,21–23,37,38}

II. THEORETICAL FORMULATION OF VIBRATIONAL THERMAL CONDUCTIVITY

We calculate the total vibrational thermal conductivity, k_{vib} , of an amorphous solid from

$$k_{vib} = k_{pr} + k_{AF}, \quad (1)$$

where k_{pr} is the contribution from propagating modes^{40–42} and k_{AF} is the contribution from non-propagating modes predicted by the AF theory.⁴ Mode-level properties obtained from molecular dynamics (MD) simulations and lattice dynamics calculations will be used as inputs. Equation (1) has been used in previous studies of amorphous materials,^{4–8,12,14,21,22} leading to predictions that while k_{pr} is a negligible fraction of k_{vib} for a-SiO₂ ($< 10\%$),^{12,14,21,22} it is non-negligible for a-Si ($20 - 80\%$).^{4–9}

The propagating contribution is modeled as^{4,6}

$$k_{pr} = \frac{1}{V} \int_0^{\omega_{cut}} DOS(\omega) C(\omega) D_{pr}(\omega) d\omega, \quad (2)$$

where V is the system volume, ω is the mode frequency, ω_{cut} is the maximum frequency of propagating modes, $DOS(\omega)$ is the vibrational density of states (DOS), $C(\omega)$ is the mode specific heat, and $D_{pr}(\omega)$ is the mode diffusivity. When using mode properties obtained from calculations on finite-sized systems, it is common to write Eq. (2) as a summation over the available modes.^{4,6} We choose the integral form because the required use of finite-sized simulation cells limit the lowest frequency modes that can be accessed. An extrapolation must be made to the zero frequency limit that is more easily handled with the integral.^{4–8,12,14} Equation (2) is obtained by using the single-mode relaxation time approximation to solve the

Boltzmann transport equation for a phonon gas.⁴² In the derivation of Eq. (2), the system is assumed to be isotropic (valid for an amorphous material) and have a single polarization, making the mode properties only a function of frequency. The choice of a single polarization (i.e., an averaging of the transverse and longitudinal branches) does not significantly change the results predicted in this work or that of others.^{4–8,14} We will evaluate Eq. (2) under the Debye approximation, which assumes isotropic and linear dispersion such that the the DOS is

$$DOS(\omega) = \frac{3\pi\omega^2}{2v_s^3}, \quad (3)$$

where v_s is an appropriate sound speed.⁴⁰

The specific heat in the classical, harmonic limit is k_B , where k_B is the Boltzmann constant.⁴³ Taking this classical limit allows for a direct comparison between the lattice dynamics-based predictions and those from the classical MD simulations. The harmonic approximation has been found to be valid for systems ranging from Lennard-Jones (LJ) argon,⁴⁴ to crystalline Stillinger-Weber silicon and carbon nanotubes⁴⁵ at temperatures below half the melting temperature. The full quantum expression for the specific heat is⁴²

$$C(\omega) = k_B \left[\frac{\hbar\omega/2k_BT}{\sinh(\hbar\omega/2k_BT)} \right]^2, \quad (4)$$

where \hbar is the Planck constant.⁴⁰ The quantum specific heat will be used for the high-frequency non-propagating modes to compare the k_{AF} predictions to experimental measurements in Sections V A and V B.

The diffusivity of the propagating modes is

$$D_{pr}(\omega) = \frac{1}{3}v_s^2\tau(\omega), \quad (5)$$

where $\tau(\omega)$ is the frequency-dependent mode lifetime.⁴² An equivalent physical picture in terms of a scattering length is

$$D_{pr}(\omega) = \frac{1}{3}v_s\Lambda(\omega), \quad (6)$$

where $\Lambda(\omega)$ is the phonon mean free path (MFP), defined as

$$\Lambda(\omega) = v_s\tau(\omega). \quad (7)$$

The lifetimes will be modeled using

$$\tau(\omega) = B\omega^{-n}. \quad (8)$$

By using a constant sound speed, the lifetime and diffusivity scalings will be the same. For amorphous materials, the scaling exponent n has been found experimentally and numerically to be between two and four.^{4,6–9,27–30,46–64} A value of two corresponds to Umklapp scattering,⁶⁵ while a value of four corresponds to Rayleigh scattering from point defects.⁶⁶ Combined with Eq. (3), choosing $n \leq 2$ ensures that the thermal conductivity evaluated from Eq. (2) is finite. Choosing $n > 2$ causes the thermal conductivity to diverge, which can be fixed using additional anharmonic^{4,6} or boundary scattering terms.^{5,7,8}

The AF diffuson contribution to thermal conductivity is^{4,6}

$$k_{AF} = \frac{1}{V} \sum_{i, \omega_i > \omega_{cut}} C(\omega_i) D_{AF}(\omega_i), \quad (9)$$

where ω_i is the frequency of the i th diffuson mode, $C(\omega_i)$ is the diffuson specific heat, and $D_{AF}(\omega_i)$ is the diffuson diffusivity. Equation (9) is written as a sum because there are enough high-frequency diffusons in the finite-size systems studied here to ensure a converged value.^{4,6} The AF diffusivities are calculated from¹

$$D_{AF}(\omega_i) = \frac{\pi V^2}{\hbar^2 \omega_i^2} \sum_{j \neq i} |S_{ij}|^2 \delta(\omega_i - \omega_j), \quad (10)$$

where δ is the Dirac delta function.⁶⁷ The heat current operator S_{ij} , which measures the thermal coupling between vibrational modes i and j based on their frequencies and spatial overlap of eigenvectors, can be calculated from harmonic lattice dynamics theory.^{1,4,6} For Eq. (10), S_{ij} is directionally averaged because amorphous materials are isotropic.

III. CALCULATION DETAILS

A. Sample Preparation

The three smallest a-SiO₂ samples are the same as those used in Ref. 68 and contain 288, 576, and 972 atoms at a density of 2350 kg/m³. The atomic interactions are modeled using the modified Beest-Kramer-van Santen (BKS) potential^{69,70} from Ref. 68, except that the 24-6 LJ potential⁷¹ is changed to a 12-6, which has a negligible effect on the predictions. The LJ potentials use a cutoff of 8.5 Å and the Buckingham potentials use a cutoff of 10 Å. The electrostatic interactions are handled using the Wolf direct summation method with a damping parameter of 0.223 Å⁻¹ and a cutoff of 12 Å.⁷² Larger systems of 2880, 4608,

and 34,562 atoms were created by tiling the smaller samples, melting at a temperature of 10,000 K, and quenching instantaneously to a temperature of 300 K at constant volume. The melt-quench procedure and subsequent MD simulations were performed using the MD package LAMMPS and a time step of 0.905 fs.⁷³ The resulting a-SiO₂ structure is built from a network of rigidly-bonded SiO₄ tetrahedral sub-units that are weakly bonded via shared oxygen atoms, as shown in Fig. 1 (a). The 34,562 atom sample has a supercell side length of 8.052 nm.

For a-Si, we use samples with 216, 1000, 4096, and 100,000 atoms generated from the modified Wooten-Winer-Weaire (WWW) algorithm from Ref. 39. The resulting a-Si structure is a predominantly tetrahedrally-bonded network³⁹ and is shown in Fig. 1 (b). A larger sample was created from the 100,000 atom sample by tiling it twice in all directions to create an 800,000 atom sample with a side length of 24.81 nm. All a-Si structures have a density of 2330 kg/m³, equivalent to the perfect crystal with a lattice constant of 5.43 Å. The SW potential is used to model the atomic interactions.⁷⁴ The MD simulations are performed using LAMMPS with a time step of 0.5 fs.

Amorphous materials may have many different atomic configurations with nearly equivalent potential energies, leading to potential metastability during MD simulations.^{6,9,75–77} This meta-stability can cause errors when predicting vibrational lifetimes using normal mode decomposition (NMD, see Section IV D). To remove metastability, all a-SiO₂ and a-Si samples were annealed at a temperature of 1100 K for 10 ns.^{6,9} The removal of meta-stability is demonstrated by a decrease and plateau of the sample’s potential energy during the annealing.

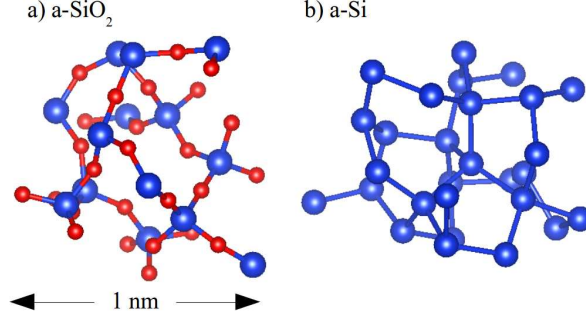


FIG. 1: (a) Small sample of a-SiO₂ created from a melt-quench technique showing the Si-O tetrahedral bond network. Bond lengths range between 1.6 and 1.8 Å. (b) Small sample of a-Si created by the modified WWW algorithm. Bond lengths range between 2.3 and 2.7 Å. Visualizations using the VESTA package.⁷⁸

B. Simulation Details

Before data collection, all MD simulations are first equilibrated in an NVT (constant number of atoms, volume, and temperature) ensemble for 10^6 time steps at a temperature of 300 K. Data are then collected from simulations in the NVE (constant number of atoms, volume, and total energy) ensemble for 2^{21} time steps, where the atomic trajectories are sampled every 2^8 time steps. Ten MD simulations with different initial conditions are run and the predictions are ensemble-averaged.

The Green-Kubo (GK) method is used to predict a top-down thermal conductivity k_{GK} [i.e., without using Eq. (1)]⁴³ using the first-avalanche method to specify the converged value of the integral of the heat current autocorrelation function (Section V A).⁷⁹ For system sizes of 4,608 (a-SiO₂, supercell side length of 4.026 nm) and 4,096 (a-Si, supercell side length of 4.344 nm) atoms, the trajectories from the MD simulations are also used to predict the vibrational mode lifetimes using the NMD method (Section IV D).

For an amorphous supercell, the only allowed wave vector is the Gamma point (i.e., $\kappa = 0$), where κ is the wavevector and there are $3N_a$ polarization branches labeled by ν , where N_a is the number of atoms. Calculation of the vibrational modes at the Gamma point requires the eigenvalue solution of a dynamical matrix of size $(3N_a)^2$ that scales as $[(3N_a)^2]^3$, limiting the system sizes that can be considered to 4,608 (a-SiO₂) and 4,096 (a-Si) atoms. The eigenvalue solution is also required to predict the vibrational DOS (Section IV A) and

structure factors (Section IV B), and to perform the NMD calculations (Section IV D) and the AF calculations (Section IV E). The frequencies and eigenvectors were computed using harmonic lattice dynamics calculations with GULP.⁸⁰ The calculation of the AF thermal diffusivities [Eq. (10)] is performed using GULP and a Lorentzian broadening of $14\delta\omega_{avg}$ for a-SiO₂ and $5\delta\omega_{avg}$ for a-Si, where $\delta\omega_{avg}$ is the average mode frequency spacing [$\delta\omega_{avg} = 1.8 \times 10^{10}$ (a-SiO₂) and 1.0×10^{10} (a-Si) rads/s].^{4,6} Varying the broadening by 10% around these values does not change k_{AF} within its uncertainty (see Section V A).

IV. VIBRATIONAL MODE PROPERTIES

A. Density of States

The vibrational DOS is computed from

$$DOS(\omega) = \sum_i \delta(\omega_i - \omega), \quad (11)$$

where a unit step function of width $100\delta\omega_{avg}$ is used to broaden $\delta(\omega_i - \omega)$. The results for a-SiO₂ and a-Si are plotted in Fig. 2. The DOS for a-Si is similar to that of crystalline silicon,^{2,81} with peaks at mid- and high-frequencies. The DOS for a-SiO₂ is constant over most of the frequency-range, with a gap that separates the high-frequency Si-O interactions.⁶⁸ There is a clear ω^{-2} scaling for both a-Si and a-SiO₂ at the lowest frequencies. The onset of this scaling occurs at a higher frequency for a-Si ($\sim 1.5 \times 10^{13}$ rads/s) than a-SiO₂ ($\sim 4.5 \times 10^{12}$ rads/s). This low-frequency scaling is predicted by the Debye model [Eq. (3)] and suggests that these modes may be propagating (i.e., phonon-like).

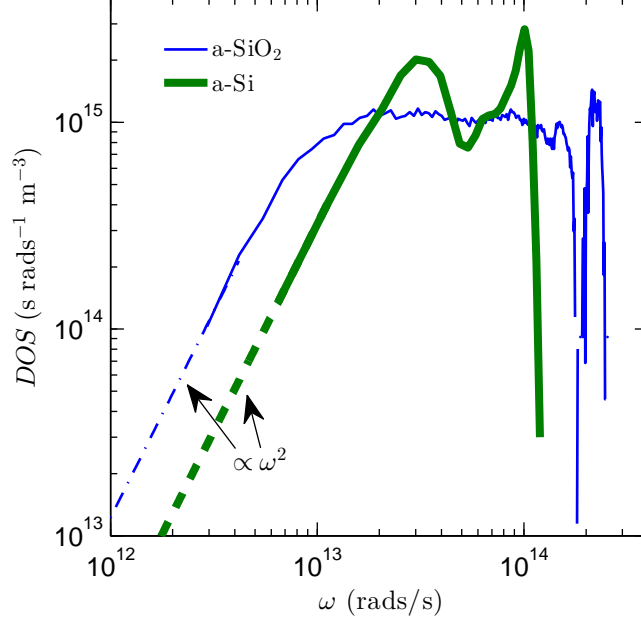


FIG. 2: Vibrational DOS of a-SiO₂ and a-Si plotted on a log-log scale. Both models show an ω^{-2} scaling at low frequency. The DOS for a-Si has two peaks similar to the DOS of the crystalline phase.⁸² The DOS for a-SiO₂ is flat over most of the spectrum, with a high frequency gap that separates the Si-O interactions.⁶⁸

B. Structure Factor

Calculating the structure factors of the supercell Gamma modes is a method to test for their propagating (i.e., plane-wave) character at a particular wavevector and polarization. This approach has been previously used to predict effective dispersion curves of disordered and amorphous materials experimentally^{14,28,30,47,51,52,83–86} and numerically.^{2,4,6,48–50,53,55,58,59,63,87–92} The structure factor at a wavevector $\boldsymbol{\kappa}$ is defined as²

$$S^{L,T}(\boldsymbol{\kappa}) = \sum_{\nu} E^{L,T}(\boldsymbol{\kappa}_{\nu}) \delta(\omega - \omega(\boldsymbol{\kappa}_{\nu}^{\mathbf{0}})), \quad (12)$$

where the summation is over the Gamma modes, E^T refers to the transverse polarization and is defined as

$$E^L(\boldsymbol{\kappa}) = \left| \sum_b \hat{\boldsymbol{\kappa}} \cdot e(\boldsymbol{\kappa}_{\nu}^{\mathbf{0}} \begin{smallmatrix} b \\ a \end{smallmatrix}) \exp[i\boldsymbol{\kappa} \cdot \mathbf{r}_0^{(l=0)} \begin{smallmatrix} l=0 \\ b \end{smallmatrix}] \right|^2 \quad (13)$$

and E^L refers to the longitudinal polarization and is defined as

$$E^T(\boldsymbol{\kappa}) = \left| \sum_b \hat{\boldsymbol{\kappa}} \times e(\boldsymbol{\kappa}=\mathbf{0} \atop \nu \atop \alpha) \exp[i\boldsymbol{\kappa} \cdot \mathbf{r}_0^{(l=0)}] \right|^2. \quad (14)$$

In Eqs. (13) and (14), the b summations are over the atoms in the disordered supercell, $\mathbf{r}_0^{(l=0)}$ refers to the equilibrium atomic position of atom b , l labels the unit cells ($l = 0$ for the supercell), α labels the Cartesian coordinates, and $\hat{\boldsymbol{\kappa}}$ is a unit vector. The vibrational mode shape is contained in the $3N_a$ components of its eigenvector, $e(\boldsymbol{\kappa}=\mathbf{0} \atop \nu \atop \alpha)$.⁴¹

The transverse and longitudinal structure factors are plotted in Figs. 3(a) and 3(b) for a-SiO₂ and a-Si for wavevectors along the [100] direction of the supercells. Because amorphous structures are isotropic, the structure factors are direction-independent. Mode frequencies $[\omega_0(\boldsymbol{\kappa})]$ and linewidths $[\Gamma(\boldsymbol{\kappa})]$ can be predicted by fitting each structure factor peak $S^{L,T}(\boldsymbol{\kappa})$ to a Lorentzian function of the form

$$S^{L,T}(\boldsymbol{\kappa}) = \frac{C_0(\boldsymbol{\kappa})}{[\omega_0(\boldsymbol{\kappa}) - \omega]^2 + \Gamma^2(\boldsymbol{\kappa})}, \quad (15)$$

where $C_0(\boldsymbol{\kappa})$ is a constant related to the DOS.⁹⁰ A dispersion relation is identified by plotting the $\omega_0(\boldsymbol{\kappa})$ values in the middle panels of Figs. 3(a) and 3(b), where the error bars indicate the linewidths. For a-Si, Lorentzian fits to the structure factor peaks have coefficients of determination⁹³ greater than 0.8 for $|\boldsymbol{\kappa}|/\kappa_{max} \leq 0.75$ and less than 0.7 for $|\boldsymbol{\kappa}|/\kappa_{max} > 0.75$, where $\kappa_{max} = 2\pi/a$ and a is the lattice constant of crystalline silicon (5.43 Å).⁷⁴ For a-SiO₂, the coefficients of determination are greater than 0.8 for $|\boldsymbol{\kappa}|/\kappa_{max} \leq 0.2$ and less than 0.7 for larger wavevectors, where the structure factors peaks are less than an order of magnitude larger than the background. To evaluate κ_{max} for a-SiO₂, we use a lattice constant of 4.8 Å, which corresponds to the a direction of quartz.⁹⁴

For a-Si, the extracted dispersion is nearly linear at small wavevectors with a slight decrease in slope at the largest values.^{4,6} For a-SiO₂, the dispersion is concave-down for the smallest wavevectors considered, transitioning to a strong concave-up dispersion at intermediate wavevectors. For the intermediate wavevectors, the longitudinal dispersion for a-SiO₂ is well-described by the so-called “dispersion law for diffusons,” where $\omega \propto \kappa^2$.⁹⁰ This large concave-up dispersion has been observed in experimental measurements and numerical models of amorphous materials^{14,48,53,55,84} including a-SiO₂.^{14,48,53,84} We note that at frequencies lower than $\sim 1 \times 10^{12}$ rads/s, experimental measurements of a-SiO₂ recover a

linear dispersion.^{14,28,30,52,84} This frequency range is not accessible with the models studied in this work.

The atomic structures of a-SiO₂ and a-Si play an important role in determining the differences in the low-frequency mode properties. The weakly-bonded network of tetrahedra in a-SiO₂^{68–71} results in a Debye scaling of the DOS which occurs at a lower frequency than a-Si (Fig. 2), which is a network of strongly-bonded tetrahedra.^{2,39,74,95} The lower-frequency onset of the Debye-scaling of the DOS for a-SiO₂ is also related to the strong non-linear dispersion seen in Fig. 3(a). The behavior of the DOS and structure factors demonstrate a clear difference in the properties of the low-frequency modes for our models of a-SiO₂ and a-Si, which is investigated in the following sections.

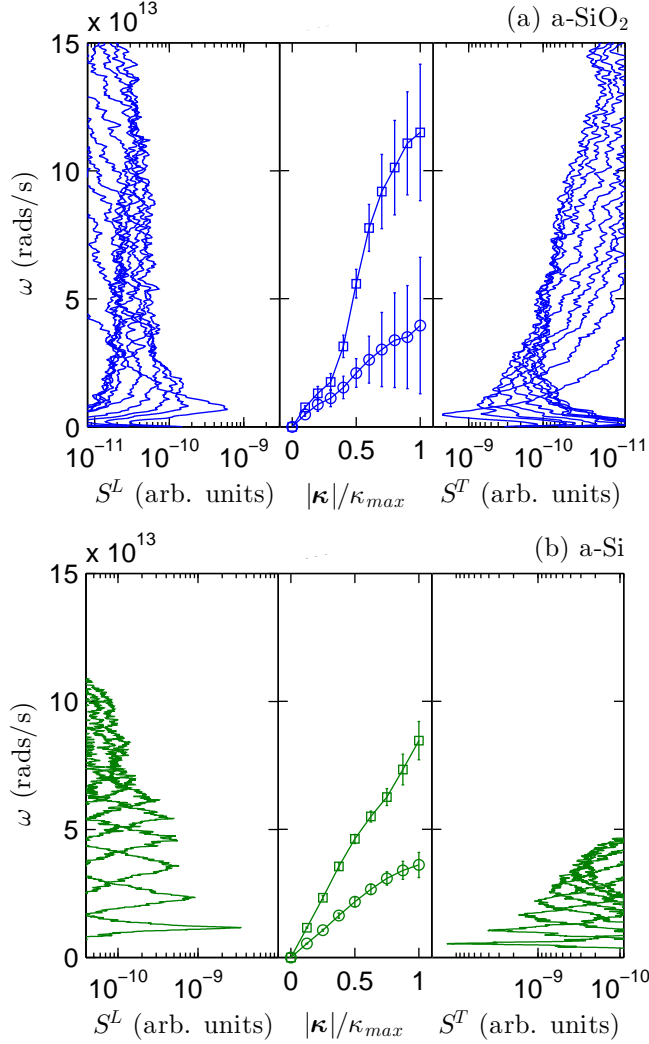


FIG. 3: Longitudinal (left panel) and transverse (right panel) structure factors [Eq. (12)] for (a) a-SiO₂ and (b) a-Si. The wavevectors are normalized by $\kappa_{max} = 2\pi/a$, where a is 4.8 (a-SiO₂) and 5.43 (a-Si) Å, based on the lattice constants of the crystalline phases.^{74,94}

C. Sound Speed

For a disordered solid, except for the transverse and longitudinal sound speeds, there is not an accepted method to predict the group velocity of individual vibrational modes. While the structure factor gives the frequency spectrum needed to construct a propagating state with pure wavevector κ , the individual mode spectra $E^T(\kappa)$ and $E^L(\kappa)$ predict the plane-wave character of each mode.^{2,95} It is not generally possible to assign a unique

wavevector to individual modes, even at low frequency,^{2,95} which makes predicting their group velocities challenging. While attempts have been made to predict individual mode group velocities,^{9,81,96–99} there is no theoretical basis for the proposed methods.

We now use the DOS and structure factors predicted in Sections IV A and IV B to predict the group velocities of the low-frequency modes for a-SiO₂ and a-Si (i.e., the sound speeds). By fitting the DOS from Fig. 2 to Eq. (3), a sound speed is obtained and is reported in Table I. Because the DOS is a mixture of transverse and longitudinal modes, only a single sound speed can be predicted.

Both longitudinal and transverse sound speeds can be predicted from the structure factor peaks by forward differencing the dispersion relation as

$$v_s = \frac{\omega_0(\kappa_{min})}{\kappa_{min}}, \quad (16)$$

where κ_{min} is 0.1 for a-SiO₂ and 0.125 κ_{max} for a-Si. The results are provided in Table I.

The transverse and longitudinal sound speeds of a material can also be predicted from the material's bulk (G) and shear (K) moduli from

$$v_{s,T} = \left[\frac{G}{\rho} \right]^{1/2} \quad (17)$$

and

$$v_{s,L} = \left[\frac{4G + 3K}{3\rho} \right]^{1/2}. \quad (18)$$

Using the bulk and shear moduli defined in terms of the elastic constants according to the Voigt convention,⁸⁰ the corresponding sound speeds are reported in Table I.

The longitudinal and transverse sound speeds for a-SiO₂ predicted using the bulk and shear moduli are 10-20% lower than predictions made using a linear fit to the peaks of the current correlation function for a model with 8,016 atoms using the BKS potential [3,568 m/s (transverse) and 5,937 m/s (longitudinal)]. The smaller values predicted by the structure factors and DOS result from the concave-down dispersion seen at low wavevector, particularly for a-SiO₂ (i.e., we are not able to reach the linear portion of the dispersion curve).⁵³ Experimental measurements of the sound speeds using Brillouin light and inelastic x-ray scattering range between 3,800 to 4,000 m/s (transverse) and 6,000 to 6,400 m/s (longitudinal).^{47,52,84,100,101} Differences between our predictions and experimental measurements may be related to limitations of the BKS potential.

The effect of the concave-down dispersion is less pronounced for a-Si than for a-SiO₂, where the sound speeds predicted by all three methods are within five percent of each other. Our sound speed predictions for a-Si using all three methods are within 10% of predictions made using the elastic moduli^{102,103} and structure factor⁵⁵ from models created by the original WWW algorithm.¹⁰⁴ The 4096 atom model created by the modified WWW algorithm³⁹ predicted a longitudinal sound speed of 7,670 m/s from the structure factor,⁵⁸ within 5% of our prediction. In an attempt to explain the anomalously high longitudinal sound speed (8,300 m/s) and thermal conductivity measurements in Ref. 7, three 1,000 atom models relaxed using a tight-binding electron structure method predicted an average of 4,740 (transverse) and 7,830 (longitudinal) m/s.⁷ By annealing our structures (Section III A), the sound speeds predicted by the elastic moduli are increased, but not by the amount reported in Ref. 7. Experimental transverse sound speeds measurements using Rayleigh wave scattering are 3,420 and 4,290 m/s for sputtered and ion-bombarded a-Si thin films,¹⁰⁵ which is within 15% of the predictions from our models. It is clear that the experimentally-measured sound speeds for a-Si show a wide range.

The sound speed $v_{s,DOS}$ will be used for both a-SiO₂ and a-Si for the rest of this work, allowing for the use of a single polarization for the propagating contribution [Eq. (2)]. By comparing the sound speeds in Table I, it is clear that the low-frequency DOS of our models for a-Si and a-SiO₂ are dominated by transverse modes, which is expected due to their degeneracy and lower frequencies compared to the longitudinal modes. The transverse sound speed predicted for our model of a-SiO₂ is 85% of that predicted by the other methods (Table I) and that measured by experiment.^{47,52,84,100,101} While using a smaller transverse sound speed leads to an underprediction of the mode diffusivities [Eq. (5)], it leads to an overprediction of the DOS [Eq. (3)]. Holding all other input parameters in Eq. (1) constant, a smaller sound speed leads to a larger k_{pr} because the DOS scales as $1/v_s^3$. We can thus regard our k_{pr} prediction as an upper bound.

TABLE I: Longitudinal and transverse sound speed estimated from the elastic moduli [Eqs. (18) and (17)], structure factors [Eq. (16)], and DOS [Eq. (3)]. The pre-annealed group velocities predicted by the elastic constants are $v_{s,T} = 2,541$ m/s, $v_{s,L} = 4,761$ m/s for a-SiO₂ and $v_{s,T} = 3,670$ m/s, $v_{s,L} = 7,840$ m/s for a-Si (see Section IV B).

method	moduli (m/s)	S^T, S^L (m/s)	DOS (m/s)
a-SiO ₂			
Transverse	3,161	2,732	2,528
Longitudinal	5,100	4,779	
a-Si			
Transverse	3,886	3,699	3,615
Longitudinal	8,271	8,047	

D. Lifetimes

We now predict the lifetimes of all vibrational modes in our models of a-SiO₂ and a-Si using the MD simulation-based NMD method,^{9,44,45,99,106–108} which explicitly includes the disorder in the supercell.^{9,91,97,98,109} In NMD, the atomic trajectories from an MD simulation are first mapped onto the vibrational mode coordinate time derivatives,⁴¹

$$\dot{q}(\boldsymbol{\kappa}=\mathbf{0}; t) = \sum_{\alpha, b, l}^{3, n, N} \sqrt{\frac{m_b}{N}} \dot{u}_{\alpha}(b; t) e^{*}(\boldsymbol{\kappa}=\mathbf{0} \mid_{\alpha}^b) \exp[i(\mathbf{0} \cdot \mathbf{r}_0(b; t))]. \quad (19)$$

Here, m_b is the mass of the b_{th} atom in the supercell, \dot{u}_{α} is the α -component of the atomic velocity, and t is time. Because the supercells of a-SiO₂ and a-Si are disordered, the NMD method can only be performed at the Gamma point ($\boldsymbol{\kappa} = \mathbf{0}$). The spectral energy of each vibrational mode, $\Phi(\nu, \omega)$, is calculated from

$$\Phi(\nu, \omega) = \lim_{\tau_0 \rightarrow \infty} \frac{1}{2\tau_0} \left| \frac{1}{\sqrt{2\pi}} \int_0^{\tau_0} \dot{q}(\boldsymbol{\kappa}=\mathbf{0}; t) \exp(-i\omega t) dt \right|^2. \quad (20)$$

We choose the frequency-domain representation of the normal mode energy because we find it to be less sensitive to metastability of the amorphous structure than the time-domain representation.

The vibrational mode frequency and lifetime are predicted by fitting each mode's spectral

energy to a Lorentzian function,

$$\Phi(\nu, \omega) = \frac{C_0(\nu)}{[\omega_0(\nu) - \omega]^2 + \Gamma^2(\nu)}, \quad (21)$$

where the constant $C_0(\nu)$ is related to the average energy of each mode. This expression is valid when the linewidth $\Gamma(\nu) \ll \omega_0(\nu)$.⁴⁵ The mode lifetime is^{106,108}

$$\tau(\nu) = \frac{1}{2\Gamma(\nu)}. \quad (22)$$

The NMD-predicted lifetimes are plotted in Figs. 4(a) and 4(b) for a-SiO₂ and a-Si. Also plotted are the timescales extracted from the structure factor linewidths, $1/[2\Gamma(\kappa)]$ (Section IV B). For a-SiO₂, the NMD lifetimes are larger than the Ioffe-Regel (IR) limit $\tau = 2\pi/\omega$,⁴⁸ and are bounded by this limit at low frequencies. There is no clear evidence for an ω^{-2} scaling, which would correspond to propagating modes. At mid-frequencies, the NMD lifetimes are approximately constant and there is a peak near 2×10^{14} rads/s, which corresponds to the peak in the DOS (see Fig. 2). The lifetimes predicted from the structure factor fall below the NMD-predicted lifetimes and the IR limit. These low values result because the structure factor for a-SiO₂ is evaluated for wavevectors where the resulting wavepackets are formed by non-propagating modes.^{2,4,6}

For a-Si, the NMD lifetimes show a clear ω^{-2} scaling at low frequency. The lifetimes plateau at higher frequencies, over a wider range of frequencies than for a-SiO₂, with two peaks corresponding to the peaks in the DOS (see Fig. 2). A similar plateau of lifetimes at high frequencies has been reported for disordered lattices^{91,109,110} and other models of a-Si.⁹ The transition from the low-frequency scaling to the plateau region occurs near 10^{13} rads/s, which corresponds to where the DOS first peaks in Fig. 2. Similar behavior has been observed for models of disordered lattices.⁹¹ The lifetimes predicted by the structure factors are in good agreement with those predicted by NMD at low frequencies. Similar agreement has been reported in other models of amorphous materials.^{6,111–113} The agreement between the NMD-predicted lifetimes and the structure factor timescales for a-Si at low frequencies indicates that these modes are plane-wave-like and that the wavepackets formed by these modes are propagating.^{2,4,6}

The NMD-predicted lifetimes for a-Si range from 0.5 to 10 ps and are similar in magnitude to those predicted for previous WWW-generated models of a-Si.^{112–115} We note that one previous study of a-Si modeled using the Tersoff potential predicted vibrational lifetimes

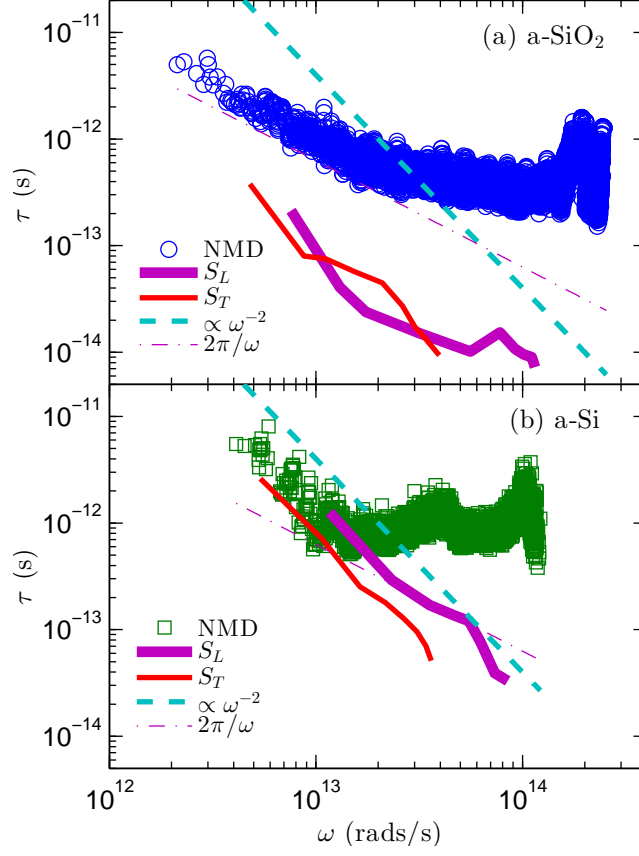


FIG. 4: Vibrational mode lifetimes predicted by NMD [Eq. (22)] and the structure factors [Eq. (15)] for (a) a-SiO₂ and (b) a-Si. The NMD-predicted lifetimes are larger than the IR limit. The lifetimes predicted from the structure factors fall below the IR limit at high-frequency for a-Si and for all frequencies for a-SiO₂. For a-Si, a clear ω^{-2} scaling is observed at low frequencies, while the lifetimes plateau at higher frequencies, over a wider range of frequencies than for a-SiO₂, with two peaks corresponding to the peaks in the DOS (Fig. 2). The transition from the low-frequency scaling to the plateau region for a-Si occurs near 10^{13} rads/s, which corresponds to where the DOS first peaks in Fig. 2.

on the order of 100 ps, an order of magnitude larger than the values reported here and in previous studies.^{112–115} It is unclear what the source of this discrepancy is, although in Ref. 9 the NMD analysis was performed in the time domain, where metastability can be more strongly pronounced. Using the Tersoff potential on the WWW a-Si models in this work, we predict similar lifetimes to those from the SW potential.

E. Diffusivities

Using the sound speeds predicted from the DOS (Table I), the NMD-predicted lifetimes for a-SiO₂ and a-Si are used to predict the mode diffusivities with Eq. (5). The results are plotted in Figs. 5(a) and 5(b). We note that the sound speed is most appropriate for the lowest-frequency modes, where the DOS scales as ω^2 (Fig. 2). The AF theory is also used to predict the mode diffusivities and the results are also plotted in Figs. 5(a) and 5(b).

For a-SiO₂, the mode diffusivities predicted by NMD and AF agree well over the majority of the frequency range. The AF diffusivities at the highest frequencies show a sharp decrease, which is an indication that these modes are localized.⁴ The low- and mid-frequency diffusivities are above the high-scatter limit,

$$D_{HS} = \frac{1}{3}v_s a, \quad (23)$$

which assumes that all vibrational modes travel with the sound speed and scatter over a distance of the lattice constant.²³ In evaluating Eq. (23), we use the lattice constant of the crystalline phases (see Section IV B). The low-frequency NMD diffusivities do not show a definitive scaling. Based on the results in Ref. 14, we choose a propagating/non-propagating cutoff frequency of 4.55×10^{12} rads/s. The constant B in Eq. (8) for $n = 2$ is then fit to the AF-predicted diffusivities for frequencies below the cutoff by dividing the diffusivities by $v_{s,DOS}$. The fit value is $B = 2.95 \times 10^{13} \text{ s}^{-2}$. This choice is discussed in Section V A.

For a-Si, the mode diffusivities predicted by NMD at low frequencies show a clear ω^{-2} scaling. The NMD-predicted diffusivities are larger and show less scatter than those predicted by the AF theory, which is due to the finite-size system and the broadening that is required to evaluate Eq. (10).⁴ By using a larger broadening ($100\delta\omega_{avg}$), the scatter in the AF-predicted diffusivities at low frequency can be smoothed, but at the cost of decreasing the diffusivities at intermediate and high frequencies, which affects the predicted diffuson contribution to thermal conductivity (see Section V A). It is possible that a frequency-dependent broadening may be necessary for a-Si and the AF theory, but determining this dependence is not necessary for interpreting our results. For a-Si the NMD- and AF-predicted diffusivities diverge near a frequency of 10^{13} rads/s. The NMD-predicted diffusivities are relatively constant above this frequency, indicating that the sound speed is no longer an applicable scaling. The AF diffusivities are larger than the high-scatter limit [Eq. (23)], except for the highest frequencies, which are localized.⁴

For a-Si, we choose ω_{cut} and B so that Eq. (5) is equal to the average AF-predicted diffusivity at the cutoff frequency. The resulting values are $\omega_{cut} = 1.16 \times 10^{13}$ rads/s and $B = 2.76 \times 10^{14} \text{ s}^{-2}$. This choice allows Eq. (5) to pass reasonably well through both the AF- and NMD-predicted diffusivities.

While experiments show that there is a cross-over region for the low-frequency lifetime scaling from ω^{-2} to ω^{-4} ,²⁷ and back to ω^{-2} for a-SiO₂,²⁷⁻³⁰ our present model is not large enough to investigate the mode-by-mode properties in this cross-over region. Because experiments are limited for a-Si thin films,²⁶ we consider a ω^{-4} scaling for Eq. (8) that is discussed in Section V A. Because this scaling is not clear from the data in Fig. 5 (b), we use a cutoff frequency of 1.52×10^{13} rads/s based on Refs. 4 and 5 and choose $B = 2.07 \times 10^{40} \text{ s}^{-4}$ so that Eq. (5) is equal to the average AF-predicted diffusivity at the cutoff frequency.

Both a-SiO₂ and a-Si have a region at higher frequencies where the AF-predicted mode diffusivities are relatively constant. This behavior has been reported for model disordered systems such as disordered lattices^{90,91,110} and jammed systems.^{60,62} While diffusons are non-propagating modes whose MFPs are not well-defined,⁴ a diffuson MFP can be defined as

$$\Lambda_{AF}(\omega_i) = [3D_{AF}(\omega_i)\tau(\omega_i)]^{1/2}, \quad (24)$$

where $\tau(\omega_i)$ is the NMD-predicted lifetime for that mode. Using this definition, $\Lambda_{AF}(\omega_i)$ for both a-SiO₂ and a-Si is found to vary between the crystal lattice constant (~ 0.5 nm) and the supercell size (~ 5 nm) for modes with frequency above the cutoff. Similar MFPs have been estimated for diffusons in a-Si in previous studies.^{4,6} For modes with frequency below the cutoff, the NMD-predicted MFPs from Eq. (7) range up to 16 nm (a-SiO₂) and 43 (a-Si) nm. This result is in contrast to the MFPs estimated in Ref. 9 for a-Si, which ranged up to 500 nm. We believe that the origin of the large MFPs in Ref. 9 is a combination of the predicted lifetimes (see Section IV D) and the method used to estimate the mode group velocities.

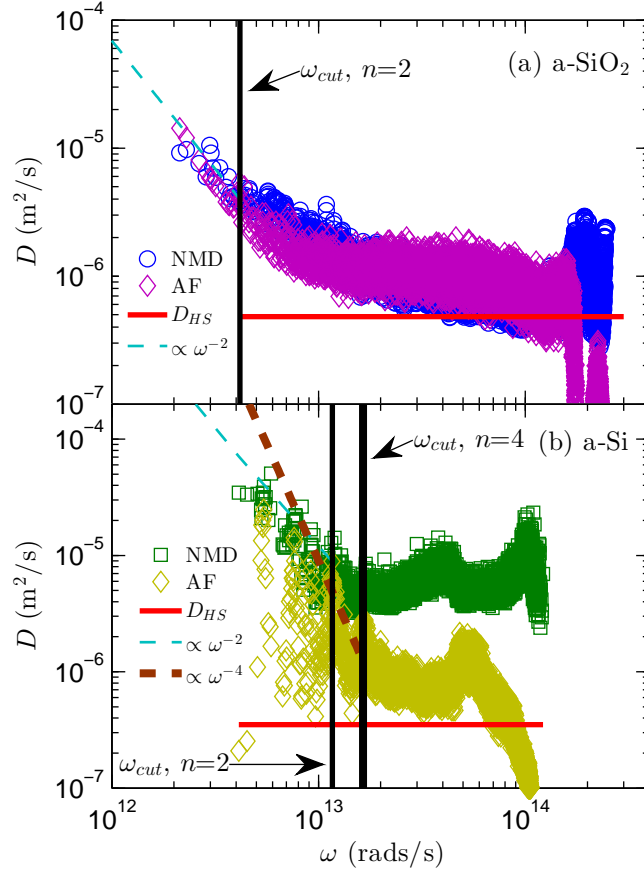


FIG. 5: Vibrational mode diffusivities predicted from NMD [using Eqs. (5) and (22) with the DOS sound speed from Table I] and the AF theory [Eq. (10)]. Also shown are extrapolations based on an ω^{-2} scaling with Eqs. (5) and (8) for a-SiO₂ and a-Si, and an additional ω^{-4} scaling for a-Si. For both systems, the diffusivities are larger than the high-scatter limit [Eq. (23)] except at high frequencies, where the modes are localized.

V. THERMAL CONDUCTIVITY

A. Bulk

To predict the bulk thermal conductivity for our models of a-SiO₂ and a-Si, we use both Eq. (1) and the GK method. The GK method is computationally inexpensive compared to the NMD and AF methods so that larger system sizes can be accessed. The GK-predicted thermal conductivities for a-SiO₂ and a-Si are plotted in Fig. 6 versus the inverse of the system size. For a-SiO₂, there is no system-size dependence. The bulk thermal conductivity

is estimated to be 2.1 ± 0.2 W/m-K by averaging over all the samples. This prediction is in agreement with the GK predictions in Ref. 68 within the uncertainties, but larger than the MD-based direct-method predictions in Ref. 116. Shenogin et al. predicted the total thermal conductivity of a-SiO₂ using non-equilibrium MD simulations of the same small structures used in this work, finding 2.0 W/m-K for their largest system which was based on a 972 atom model tiled in one direction six times.¹⁵ Our GK-predicted value is larger than experimental measurements, which range between 1.3 and 1.5 W/m-K,^{10,13,23,38} because of the classical nature of the MD simulation and the suitability of the BKS interatomic potential for modeling thermal transport in a-SiO₂.^{68,116} Quantum statistical effects are considered later in this section.

For a-Si, there is a clear system-size dependence. Because the low-frequency DOS has the form of Eq. (3) and the diffusivities scale as ω^{-2} , the thermal conductivity will scale as the inverse of the system size. The bulk value can be found by extrapolating to an infinite system size.^{45,117,118} The extrapolation is performed using the three largest system sizes,¹¹⁹ leading to a bulk value of 2.0 ± 0.2 W/m-K, where the uncertainty is estimated from the ensemble averaging for each system size. While it is difficult to create bulk a-Si experimentally,¹⁰⁵ our extrapolated bulk value is in reasonable agreement with experimental values for a wide range of thin film thicknesses (see Fig. 7 in Section V B).

To predict thermal conductivity from Eq. (1), we use the parameters B and ω_{cut} specified in Section IV E assuming an ω^{-2} scaling below ω_{cut} and the AF-predicted diffusivities. For a-SiO₂, the propagating, non-propagating, and total thermal conductivities are 0.10 ± 0.05 , 1.9 ± 0.1 , and 2.0 ± 0.1 W/m-K (see Table II). The uncertainties are estimated by varying ω_{cut} and the AF broadening by 10%. The total value agrees with the GK value within the uncertainties. For the propagating contribution using an expression similar to Eq. (2), Baldi et al. estimated¹⁴ 0.1 W/m-K and Love and Anderson¹² estimated 0.03 W/m-K.

By using the ω^{-2} diffusivity scaling for a-Si, the propagating, non-propagating, and total thermal conductivities are 0.6 ± 0.2 , 1.2 ± 0.2 , and 1.8 ± 0.2 W/m-K. This value for total thermal conductivity is in agreement with the GK-predicted bulk value within the uncertainties. Earlier studies using similar models of a-Si find that k_{pr} is less than half of k_{vib} ,^{4,6} in agreement with our results. A recent study of a-Si modeled using the Tersoff potential found $k_{pr} \approx k_{AF}$.⁹ Estimates based on experimental measurements have found k_{pr} to be as low as^{5,6} 20% and as high as 80% of k_{vib} .^{7,8}

If an ω^{-4} lifetime scaling is assumed for a-Si, the thermal conductivity diverges at low frequency. We bound the thermal conductivity by assuming the sample to be a thin film of thickness t_f and modify the lifetimes using the Matthiessen rule,¹²⁰

$$\frac{1}{\tau_{eff}} = \frac{1}{\tau_{bulk}} + \frac{2v_s}{t_f}. \quad (25)$$

Using the largest film thickness from the experimental literature ($80 \mu\text{m}$)⁷ gives a propagating contribution to thermal conductivity of $3.0 \pm 0.4 \text{ W/m-K}$. Using the ω^{-2} scaling and this film thickness gives a propagating contribution of 0.6 W/m-K (i.e., there is no change from the bulk value). While predictions for k_{pr} for a-Si vary based on the assumed scaling of the low-frequency vibrational lifetimes⁴⁻⁹ all evidence supports that k_{pr} is a significant fraction of the total thermal conductivity.⁴⁻¹⁰

In Section II, we approximated the specific heat of the propagating and non-propagating modes by the classical, harmonic-limit value of k_B . At a temperature of 300 K, the quantum heat capacity [Eq. (4)] at the largest cutoff frequency for either a-SiO₂ or a-Si is $0.98k_B$, justifying the use of the classical specific heat in the propagating term in Eq. (2). For the AF contribution, however, the effect of the quantum specific heat is important. At the highest frequencies in a-SiO₂ and a-Si, the specific heat is $0.073k_B$ and $0.47k_B$. Using Eq. (4) in Eq. (9) gives AF thermal conductivities of 1.4 ± 0.1 and $1.0 \pm 0.1 \text{ W/m-K}$ for a-SiO₂ and a-Si (Table II). This correction brings the estimate of k_{vib} for a-SiO₂ into good agreement with experimental measurements.^{10,13,23,38} For a-Si, the modified k_{AF} is 20% lower than the classical-limit value.

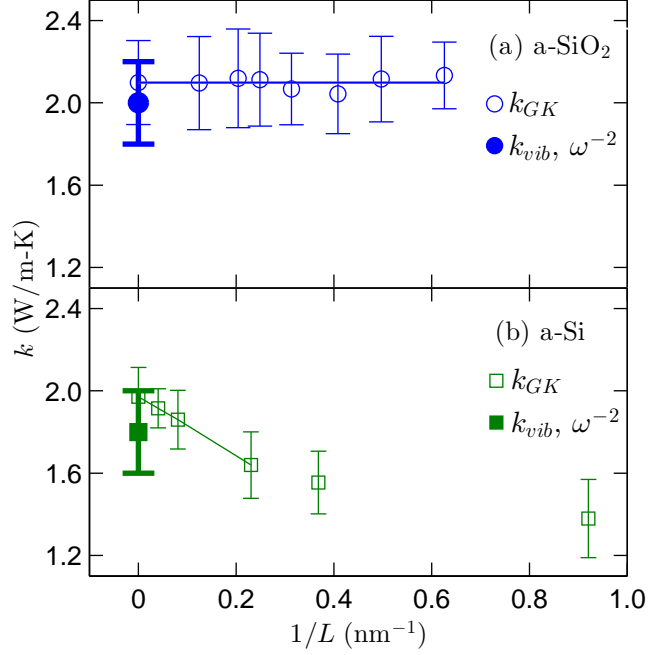


FIG. 6: Thermal conductivities of a-SiO₂ and a-Si predicted using the GK method and Eq. (1). For a-SiO₂, the GK-predicted thermal conductivity is size-independent, indicating that there is not an important contribution from propagating modes. For a-Si, there is a clear size dependence, indicating the importance of propagating modes.

TABLE II: Thermal conductivities for bulk a-SiO₂ and a-Si predicted by the GK method (k_{GK}) and Eqs. (1) (k_{vib}), (2) (k_{pr}), and (9) (k_{AF}). For the non-propagating contribution, classical and quantum specific heats are considered.

Thermal Conductivity (W/m-K)	a-SiO ₂	a-Si
(1) k_{GK}	2.1 ± 0.2	2.0 ± 0.2
(2) k_{vib} (classical)	2.0 ± 0.1	1.8 ± 0.2
(3) k_{pr}	0.10 ± 0.05	0.6 ± 0.2
(4) k_{AF} (classical)	1.9 ± 0.1	1.2 ± 0.1
(5) k_{AF} (quantum)	1.4 ± 0.1	1.0 ± 0.1
(6) k_{vib} (quantum)	1.5 ± 0.1	1.6 ± 0.2

B. Accumulation Function

In their broadband frequency domain thermal-reflectance measurements, Regner et al.,¹⁰ following the suggestion of Koh and Cahill,¹²¹ interpret the measured thermal conductivity at a given penetration depth to be representative of the thermal conductivity accumulation function at a MFP equal to the penetration depth.^{16,18} The vertical coordinate of any point on the accumulation function represents the thermal conductivity that comes from phonons with MFPs less than the horizontal coordinate at that point. Their results are plotted in Fig. 7(a) for a 1000 nm thick film of a-SiO₂ and in Fig. 7(b) for 500 nm and 2000 nm thick films of a-Si. Also plotted in Figs. 7(a) and 7(b) are experimental measurements of thin film thermal conductivities. For a-Si, the experimental measurements are grouped broadly by sample preparation technique: (A) chemical vapor deposition^{7,8,35} and (B) sputtered.^{5,33,34} When using the Matthiessen rule to predict thermal conductivity accumulation functions for varying film thicknesses, there are two approximations when plotting the film thickness and Λ^* on the same horizontal scale. First, for varying film thickness, Eq. (25) predicts varying lifetimes, though we note that the thermal conductivity accumulation for varying t_f follows closely the accumulation for an infinite film thickness. Second, the assumption ignores the ability of modes to propagate in a direction which is not perpendicular to the thin film direction, which can lead to MFPs larger than the film thickness.^{120,122,123}

Based on the results in Section IV E, we build thermal conductivity accumulation functions for a-SiO₂ and a-Si from

$$k(\Lambda^*) = k_{AF} + \frac{1}{V} \int_{\Lambda_{cut}}^{\Lambda^*} k(\Lambda) d\Lambda, \quad (26)$$

where Λ_{cut} is the MFP at the cut-off frequency, Λ^* is the maximum MFP considered in the thermal conductivity accumulation, $k(\Lambda)$ is the thermal conductivity as a function of MFP, and the propagating mode MFPs are calculated using lifetimes from Eq. (25). The non-propagating contribution k_{AF} is evaluated using the quantum specific heat (see Section V A). The results are plotted for a-SiO₂ in Fig. 7(a) using an infinite film thickness and for a-Si in Fig. 7(b) using a film thickness of 80 μm .

The predicted thermal conductivity accumulation function for a-SiO₂ saturates at a MFP of 10 nm, which is on the order of the finite size of our model. This result is in accord with the penetration depth-independent thermal conductivity measurements using broadband FDTR¹⁰ and experimental measurements that show minimal film thickness dependance.^{13,38}

The predicted accumulated thermal conductivity is in good quantitative agreement with both Regner et al. and film thickness dependent measurements.

For a-Si, the low-MFP plateau of thermal conductivity in the measurements of Regner et al. is consistent with our predicted k_{AF} . The propagating contribution to the accumulation is predicted using both ω^{-2} and ω^{-4} lifetime scalings. For the ω^{-2} scaling, the thermal conductivity accumulation saturates at $1\ \mu\text{m}$, in good agreement with where the measurements of Regner et al. saturate. As discussed in Section V, the ω^{-2} scaling best describes the propagating contribution for our model of bulk a-Si. It is worth noting again that the results from thin film experiments have been interpreted using both ω^{-2} and ω^{-4} scalings.^{4-8,25} The measurements of Regner et al. show much sharper accumulations than either the ω^{-2} or ω^{-4} scalings, particularly for a film of thickness $2\ \mu\text{m}$. Predictions for both the ω^{-2} and ω^{-4} scalings pass reasonably through the experimental measurements. We note that thermal conductivities as high as $6\ \text{W/m-K}$ [not plotted in Fig. 7(b)] have been measured for a-Si thin films deposited using hot-wire chemical vapor deposition (HWCVD).⁸ Based on time domain thermal reflectance measurements, the frequency dependence of thermal conductivity of these HWCVD a-Si thin films was found to be similar to SiGe alloys, suggesting an ω^{-4} scaling of the lifetimes of the propagating modes.

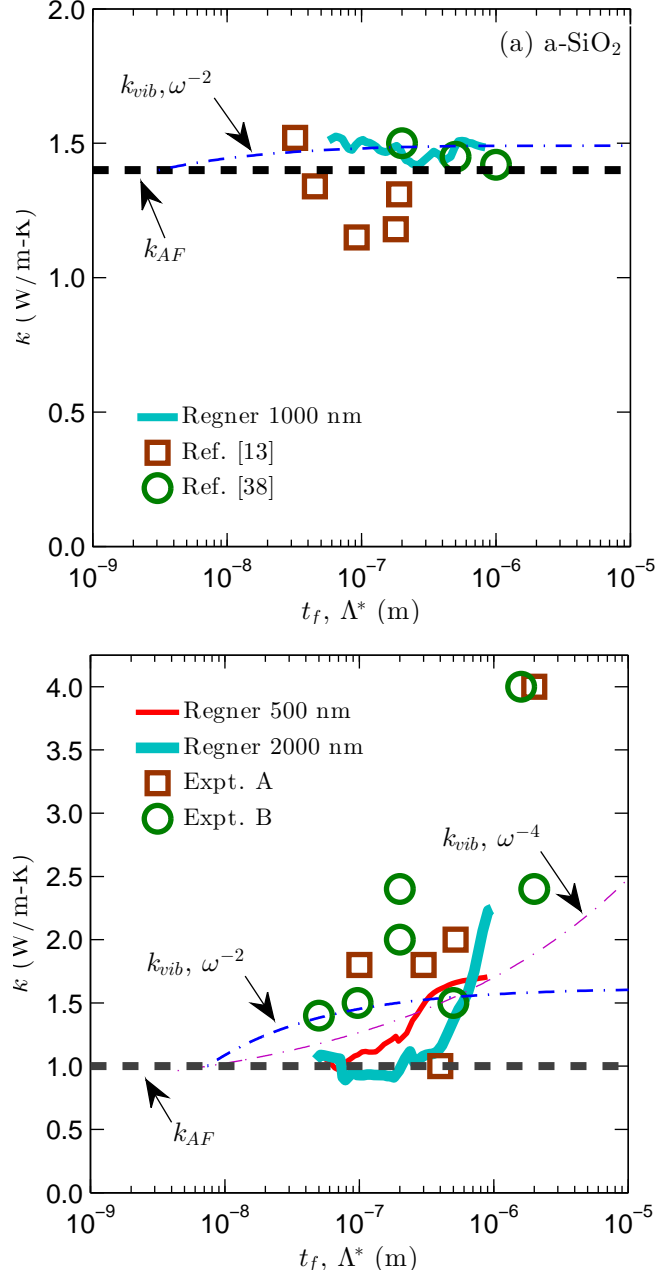


FIG. 7: (a) Predicted thermal conductivity accumulation function [Eq. (26)] for a-SiO_2 compared with experimental broadband frequency domain reflectance measurements by Regner et al.¹⁰ and thin film measurements from Refs. 13 and 38. The predicted thermal conductivity accumulation demonstrates that the propagating contribution is negligible in our model, which is in accord with the experimental measurements. (b) Predicted thermal conductivity accumulation function for a-Si compared with experimental measurements by Regner et al. and thin film measurements fabricated by sputtering (Expt. A)^{5,33,34} and chemical vapor deposition (Expt. B)^{7,8,32,35} for a wide-range of film thicknesses. The predicted thermal conductivity accumulation demonstrates that the propagating contribution is significant for a-Si .

VI. SUMMARY

We investigated the contributions of propagating (k_{pr}) and non-propagating (k_{AF}) modes to the total vibrational thermal conductivity (k_{vib}) of a-SiO₂ and a-Si using the NMD method (Section IV D), AF theory (Section IV E), and the GK method (Section V A). The atomic structures of a-SiO₂ and a-Si play an important role in determining the mode-by-mode properties that describe the propagating and non-propagating contributions. The onset of the propagating regime occurs at a lower frequency for a-SiO₂, which is evident from the scaling of the low-frequency DOS (Fig. 2) and the effective dispersion extracted from the structure factors (Fig. 3). This is due, in part, to the weak bonding that exists between the SiO₄ tetrahedra in a-SiO₂,^{68–71} while a-Si is formed by a network of strongly-bonded tetrahedra.^{2,39,74,95} The structural differences are also apparent in the low-frequency scalings of the mode lifetimes (Fig. 4) which show a clear ω^{-2} dependence for a-Si, but not for a-SiO₂. The combined differences of all the mode-by-mode properties results in a significant difference in the propagating (k_{pr}) and non-propagating (k_{AF}) contributions to thermal conductivity for a-SiO₂ and a-Si (Table II).

For our model of a-SiO₂, the contribution from propagating modes was shown to be negligible (5-7%). Our predictions align with experimental measurements of the film thickness-independence of thermal conductivity^{13,38} and penetration depth-independence in the measurements by Regner et al.¹⁰ While the finite size of our model makes it difficult to identify a clear scaling of the low-frequency lifetime scaling, experiments show that both ω^{-2} and ω^{-4} scalings exist in a-SiO₂.^{27,28,30} In all cases, the propagating contribution is negligible.^{12–14,38}

For our model of bulk a-Si, the thermal conductivity has a significant (33-38%) contribution from propagating modes that are best described by a lifetime scaling of ω^{-2} . Our predicted thermal conductivity accumulation function saturates at a MFP equal to the penetration depth where measurements by Regner et al. saturate for a 500 nm thick film. However, for all film thicknesses studied by Regner et al. the thermal conductivities accumulate with penetration depth much faster than our thermal conductivities accumulate with MFP, including that based on an ω^{-4} scaling.

Our bulk and thin-film thermal conductivity predictions agree with a subset of the widely-varying experimental measurements. The large range of thermal conductivity measurements on a-Si thin films suggest that a comprehensive experimental study using the recently de-

veloped frequency-varying techniques^{10,17,121,124} on varying film thicknesses and preparation techniques is necessary. It may be particularly helpful to perform the experiments at temperatures less than 10 K, where the propagating contribution dominates for both a-SiO₂ and a-Si and the low-frequency lifetime scaling is still under debate.^{4-8,12,14,22,23,37}

Acknowledgments

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