Solution of Mathematical Analysis

Baozhuo Su

September 2025

Chapter 1. Set

1. Prove that a set contains n elements has 2^n subsets.

Solution: Let set $A = \{a_1, a_2, a_3...a_n\}$. The amount of the subset that contains **k** elements: $C_n^k, 0 \leq k \leq n$, then the total amount of the subset is given by:

$$\sum_{k=0}^{n} C_n^k = \sum_{k=0}^{n} \binom{n}{k}$$

and known that $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} * x^k y^{n-k}$, let $x = y = 1$,we will have:

$$(1+1)^n = \sum_{k=0}^{n} \binom{n}{k} 1^k 1^{n-k} = \sum_{k=0}^{n} \binom{n}{k} = 2^n$$

2. Set A and B are two countable set, prove that $A \cup B$ is also countable.

Solution: Let set $A = \{a_1, a_2, a_3, ...a_n\}$ has n elements, set $B = \{b_1, b_2, b_3...b_n\}$ also has n elements, then $A \cup B = \{a_1, a_2, a_3, ..a_n, b_1, b_2, b_3, ...b_n\}$ which contains $2n$ elements, it is also countable.

3. Use set symbols to represent the following set of numbers.

(1) Real numbers that satisfy $\frac{x-3}{x+2} \leq 0$

Solution: The inequality can be transformed to $(x-3)(x+2) \leq 0$, then the solution is $\{x \mid -2 \leq x \leq 3, x \in \mathbb{R}\}$

(2) Points in the first quadrant in $\mathbb{R}^2$.

Solution: $\{(x, y) \mid 0 < x \text{ and } 0 < y\}$

(3) Rational numbers that greater than 0 and less than 1.

Solution: $\{x \mid 0 < x < 1 \text{ and } x \in \mathbb{Q}\}$

(4) Real number solution of function $sin(x)cot(x) = 0$
Solution: $\sin(x)\cot(x) = \cos(x) = 0$, $\{x \mid x = k\pi + \frac{\pi}{2}, k \in \mathbb{Z}\}$

4. Prove that $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$
Solution 1: The equation can be easily proved by Figure 1.

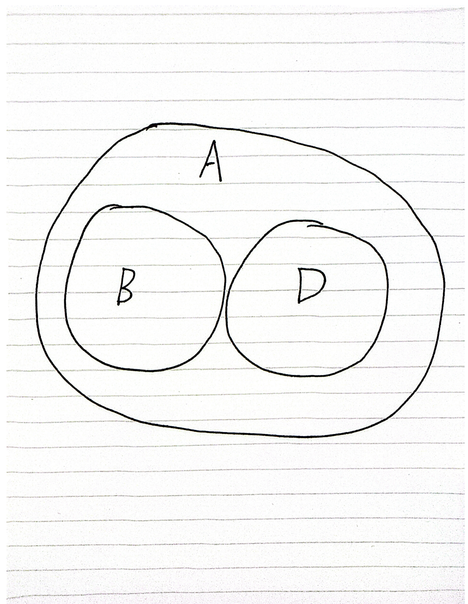

Figure 1: Ch1.4

Solution 2: If we want to prove 2 sets are equivalent, we need to prove that 2 sets are mutually inclusive.
(1) Let $x \in A \cap (B \cup D)$, then x in A, and in B or in D, then x is either in $A \cap B$ or in $A \cap D$, either $x \in A \cap B$ or $x \in A \cap D$, then

$$A \cap (B \cup D) \subset (A \cap B) \cup (A \cap D)$$

(2) If $x \in (A \cap B) \cup (A \cap D)$, then x must in A and x is either in B or in D, then $x \in A \cap (B \cup D)$, which means that

$$(A \cap B) \cup (A \cap D) \subset A \cup (B \cap D)$$

Then

$$A \cap (B \cup D) = (A \cap B) \cap (A \cup D)$$

5. Prove that $(A \cup B)^c = A^c \cap B^c$

Solution : Let $x \in (A \cup B)^c$, $x \notin A$ and $x \notin B$, then $x \in A^c \cap B^c$, then

$$(A \cup B)^c \subset A^c \cap B^c$$

Let $x \in A^c \cap B^c$, $x \notin A \cup B$, then $x \in (A \cup B)^c$, then: $A^c \cap B^c \subset (A \cup B)^c$, finally, $(A \cup B)^c = A^c \cap B^c$