Supplementary Materials

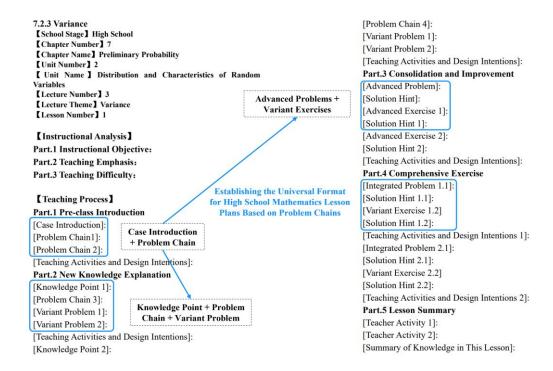


Fig.1 Instructional Design Structure

Table.1 Example Content of Knowledge Points for Each Unit in Chapter 13 "Statistics" of High School Mathematics Textbook

Chapters	Units	Course Themes	Knowledge Point Content
Chapter 13 "Statistics"	Population and Sample	Population and Sample	Population, Individual, Population Size, Sample, Sample Size or Sample Capacity, Sample Statistics, Population Numerical Characteristics in Statistical Problems
	Data Acquisition	Data Acquisition	Observational Data, Experimental Data, Census, Sampling, Sample Survey
	Sampling Methods	Simple Random Sampling	Simple Random Sampling, Lottery and Number Tag Methods, Random Number Table Method, Random Number Method
		Stratified Random Sampling	Stratified Random Sampling or Stratified Sampling, Methods of Stratified Sampling
	Statistical Charts	Frequency Distribution Table and Frequency Histogram	Steps to Plot a Frequency Distribution Table, Range, Class Interval and Number of Classes, Frequency Histogram, Title, Annotations, Frequency Polygon or Frequency Line Graph
		Stem-and-Leaf Plots and Scatter Plots	Stem-and-Leaf Plots, Scatter Plots
	Statistical Estimation	Estimating Population Distribution	Population Distribution Density Curve
		Estimating Population Numerical Characteristics	Numerical Characteristics of a Data Set, Central Tendency and Dispersion of a Data Set, Summation Symbol, Standard Deviation of Sample Data
		Estimating Percentiles	Percentiles, kth Percentile
	Statistical Activities	Statistical Activities	Main Processes of Statistical Activities, Concept of Statistical Estimation

Table.2 Common Types of Elements in High School Mathematics Instructional Design

Element	Туре
Lesson Introduction	Question-driven Introduction, Problem-driven Introduction, History of
	Mathematics Introduction, Math Story Introduction, Scenario Introduction, Media
	Introduction, Chart Introduction, Physical Object Introduction, Preview
	Introduction, Comparison Introduction, Music or Art Introduction
Teaching	Heuristic Teaching, Cooperative Learning, Situational Learning, Project-based
C	Learning, Tiered Instruction, Blended Learning, Inquiry-based Teaching,
Methods	Gamified Learning, Story-based Teaching, Direct Instruction
	Group Discussion, Guided Activities, Interactive Q&A, Online Learning,
Toolhing	Brainstorming, Role-playing, Experimental Activities, Gaming Activities,
Teaching Activities	Demonstration and Display, Field Trips, Classroom Debate, Creative Activities,
	Self-reflection, Note-taking Activities, Writing Stories or Diaries, Making Charts
	or Models, Reading Aloud, Knowledge Contests, Collaborative Learning
Class	Student Speech Summary, Group Summary Discussion, Mind Mapping, Quick
Summary	Quiz

Table.3 Instructional Objectives Prompt Command and Generated Result Example

Table.3 Instructional Objectives Prompt Command and Generated Result Example		
Teaching Steps	Prompt Design & Question-Answer Examples	
	Prompt: Lesson Name + Knowledge Points + Quantity Limits + Core Disciplinary Literacy + Examples of Instructional Objectives	
Instructional Objective	Question: Assuming you're a high school mathematics teacher, design the "instructional objectives" for the lesson on "Stratified Random Sampling" in the "Statistics" unit. Please summarize the "instructional objectives" in 3 points. The knowledge content should include: stratified random sampling or stratified sampling, methods of stratified sampling, etc. Please refrain from going beyond the given knowledge content. Here are examples of "instructional objectives": (1) Utilize relationships between the sine, cosine, tangent, and cotangent of the same angle for evaluations, deepen the understanding of formulas, and enhance mathematical computational proficiency. (2) Through simplification and proof of trigonometric identities, appreciate the intrinsic connection between the sine, cosine, tangent, and cotangent of the same angle. Grasp the mathematical concept of reduction, and develop logical reasoning skills. (3) Experience the derivation process of the cosine formula for the difference between two angles, understand the significance of the cosine formula for the angle difference, and appreciate the combined idea of mathematics and geometry. (4) From the cosine formula for the difference between two angles, derive the cosine formula for the sum of two angles, and comprehend the inherent connection between the formulas. (5) Flexibly apply the cosine formulas for angle differences and sums to conduct simple identity transformations. Solve related evaluation and simplification problems, enhancing mathematical computational proficiency. Available core competencies to choose from: Mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, mathematical operations, data analysis. Answer: (1) Students should understand and be able to clearly explain what "Stratified Random Sampling" is, and elaborate on its applications in data collection and research. Emphasize how to utilize stratified sampling more effectively to obtain samples in specific scenarios, enhancing data analysis	

mathematical modeling proficiency.

(3) Students should be able to apply the knowledge of stratified sampling to analyze and solve some practical problems. For instance, in a survey study, how to design an effective sample using stratified sampling. They should not only be capable of application but also understand and explain the results, fostering intuitive imagination and mathematical computational skills.

Table.4 Teaching Difficulties and Emphasis Prompt Command and Generated Result Example

Teaching Steps	Prompt Design & Question-Answer Examples
	Prompt: Instructional Objectives + Lesson Name
	Question:
	Based on the previously generated "Instructional Objective", analyze and design the
	Teaching Emphasis and Difficulty for the "Stratified Random Sampling" lesson in the
	"Statistics" unit. Summarize into one key point for Teaching Emphasis and one for
	Teaching Difficulty, and express it concisely.
Teaching	Answer:
Emphasis	(1) Teaching Emphasis: Understand and master the principles and applications of
and	"Stratified Random Sampling", clarify its importance in data collection and research, and
Difficulty	proficiently use logical reasoning and mathematical modeling skills to design a
	reasonable and effective stratified sampling scheme.
	(2) Teaching Difficulty: Effectively applying "Stratified Random Sampling" when
	analyzing and solving real-world problems, including issues related to sample design and
	result interpretation. This requires a deep understanding and mastery of stratified random
	sampling knowledge, while also strengthening intuitive imagination and mathematical
	computation skills.

Table.5 Pre-class Introduction Phase Prompt Command and Generated Result Example

Teaching Steps Prompt Design & Qu	Prompt Design & Question-Answer Examples	
Prompt: Lesson Name + Instructional Or Pre-Class Introduction + Examples of Case Format Question: You are currently designing the "Pre Subsequently, there will be segment "Consolidation and Improvement," "Compt that you need to design. Please integrate the previously generated design the "Pre-class Introduction" teast Sampling" lesson in the "Statistics" unit. It with the segments that will be designed lated lesson includes: stratified random samplistratified sampling. The "Pre-class Introduction inquiry introduction, problem introduction, object introduction, preview introduction, object introduction, preview introduction. Please provide a detailed and specific teand a problem chain should contain specific Please do not exceed the content knowled list the mathematical formulas and plug in Below is an example of the "Pre-class Integrated to?"	bjectives + Knowledge Points + Methods of se Introductions and Problem Chains + Output declared in the second plan. The second plan in the second	

angle using the Cartesian coordinate system?

[Problem Chain 2]: Does the sine, cosine, tangent, and cotangent of angle α have meaning for any angle?

[Teaching Activities and Design Intentions]: By creating relevant scenarios to arouse students' interest, new concepts are introduced in specific contexts, enabling students to better understand and accept definitions and formulas.

Answer:

[Case Introduction]: Suppose we are a market research company commissioned by a large supermarket to conduct a study on consumer purchasing habits. Our task is to sample the supermarket's customers, but the supermarket's customers are numerous and diverse. We could simply randomly select some people, but this may result in underrepresentation of certain groups, such as young people or the elderly. How can we design a more effective sampling scheme?

[Problem Chain 1]: For such a complex population, what issues might arise if we only use simple random sampling? Is it possible that some groups may be under-sampled? [Problem Chain 2]: How can we avoid this issue in simple random sampling? Can we classify the population based on certain characteristics and then draw a proportional sample from each category?

[Problem Chain 3]: Does this grouped sampling method have a specific name? What are its main steps and methods?

[Teaching Activities and Design Intentions]: By setting up a real-world scenario, students' interest in stratified random sampling is sparked. It allows students to ponder how to address practical challenges, naturally leading to the introduction of the concept of stratified random sampling. This approach enables students to better grasp the principles and applications of stratified random sampling, and they can effectively apply it to solve real-world problems in subsequent learning.

Table.6 New Knowledge Explanation Phase Prompt Command and Generated Result Example

Teaching Steps	Prompt Design & Question-Answer Examples	
_	Prompt: Lesson Name + Pre-class Introduction + Knowledge Points + Teaching Activities + Teaching Methods + Examples of Problem Chains and Variant Problems + Output Format Question: Now, please design the content for "New Knowledge Explanation" for the "Stratified Random Sampling" lesson in the "Statistics" unit, incorporating the context from the previously generated pre-class introduction. The new knowledge to be taught in this lesson includes: stratified random sampling or simply stratified sampling, and methods of stratified sampling. The design for "New Knowledge Explanation" can refer to the following learning activities: group discussions, guided activities, interactive Q&A, online learning, brainstorming, role-playing, experimental activities, game-based activities, demonstrations and presentations, field observations, classroom debates, creative activities, self-reflection, shorthand activities, writing stories or diaries, creating charts or models, reading aloud, knowledge contests, cooperative learning, etc. Teaching methods to refer to include: heuristic teaching, cooperative learning, situational learning, project-based learning, tiered instruction, blended learning, inquiry-based teaching, gamified learning, and	
	storytelling teaching methods, etc. Please design detailed and specific activities for each knowledge point in the form of a detailed teaching plan. Ensure coherence with the pre-class introduction and the design of the problem chain. Generate variant questions based on the content of the pre-class introduction scenario, and present them in the form of problem chains and variant questions. After each question, provide possible teaching activities and design intentions. Please refrain from designing beyond the content scope. If calculations are needed, list the mathematical formulas and plug in the numbers using the relevant tools. Here is the content format: Knowledge Point 1: [Problem Chain 3]: Given that the terminal side of angle α passes through point P(1,2),	

find the sine, cosine, tangent, and cotangent values of angle α .

[Variant Question 1]: If the coordinates of point P in Problem Chain 3 are changed to P(a,-2a) (a>0), will the sine, cosine, tangent, and cotangent values of angle α change? Why?

[Variant Question 2]: Given that the terminal side of angle α passes through point P(a,-2a) (a<0), find the sine, cosine, tangent, and cotangent values of angle α .

[Teaching Activities and Design Intentions]:

Knowledge Point 2:

[Problem Chain 4]:

[Variant Question 1]:

[Variant Question 2]:

[Teaching Activities and Design Intentions]:

Knowledge Point 3:

[Problem Chain 5]:

[Variant Question 1]:

[Variant Question 2]:

[Teaching Activities and Design Intentions]:

Answer:

Knowledge Point 1: What is stratified random sampling?

[Problem Chain 4]: Based on the pre-class introduction, we have already categorized the supermarket's customer groups. Can such an operation solve the potential problems we discussed in simple random sampling?

[Variant Question 1]: Suppose we categorize based on two dimensions: gender and age, and randomly select a certain number of people from each category for the survey. Can this method better represent the overall customer group? What is this sampling method called?

[Teaching Activities and Design Intentions]: Interactive Q&A activity, encouraging students to think and answer questions, guiding them to discover the concept of stratified random sampling on their own. The goal of the instructional design is to let students understand through this activity that stratified random sampling is a more effective sampling method that can better represent the entire research group.

Knowledge Point 2: How to carry out stratified random sampling?

[Problem Chain 5]: We already understand the basic concept of stratified random sampling. So, how do we proceed with stratified random sampling? If we categorize the supermarket's customers by gender and age, how many people should we select in each category?

[Variant Question 1]: Suppose we categorize male customers by age groups: 18-25 years, 26-35 years, 36-45 years, and over 46 years. Female customers are also categorized in the same age brackets. How many people should we select in each category? How do we decide?

[Teaching Activities and Design Intentions]: Group discussion activity, encouraging students to understand and grasp the specific methods of stratified random sampling through discussion and exploration. Through this activity, students understand the factors and steps to be considered in stratified random sampling, enhancing their logical reasoning and mathematical modeling abilities.

Knowledge Point 3: Advantages and disadvantages of stratified random sampling [Problem Chain 6]: Now that we know how to perform stratified random sampling, is this sampling method necessarily better than simple random sampling? If so, what are its advantages? If not, what are its disadvantages?

[Variant Question 1]: Suppose in our supermarket survey, we find that most shoppers are middle-aged females. In this case, which method is better, stratified random sampling or simple random sampling? Why?

[Teaching Activities and Design Intentions]: Classroom debate activity, allowing students to discuss and debate the pros and cons of stratified random sampling, enhancing their logical reasoning and data analysis abilities. Through this activity, students deeply understand the strengths and weaknesses of stratified random sampling and can better select and apply the appropriate sampling method to solve practical problems.

Prompt Design & Question-Answer Examples
Prompt: New Knowledge Explanation + Lesson Name + Knowledge Points + Teaching Activities + Teaching Methods + Output Format (Advanced Problems and Advanced Exercises)
Question: Now, combining the content of the "Introduction to New Knowledge" you generated earlier with the context, please analyze and design the "Consolidation and Enhancement" section for the "Stratified Random Sampling" lesson in the "Statistics" unit. Be sure to design the problem chain that connects before and after the "Introduction to New Knowledge" section. The new knowledge taught in this lesson includes: stratified random sampling or stratified sampling, methods of stratified sampling, etc. The design of the "Consolidation and Enhancement" section can refer to the following learning activities: group discussions, experimental activities, demonstrations and presentations, interactive Q&A, field investigations, classroom debates, creative activities, self-reflection, online learning, note-taking activities, brainstorming, writing stories or diaries, creating charts or models, knowledge competitions, collaborative learning, complex problem-solving learning, project-based learning, tiered grouping learning, blended learning, inquiry-based teaching, and transfer analogy learning methods, etc. Please design the activities in detail, in the form of a detailed lesson plan. Ensure the coherence with the problem chain of the "Introduction to New Knowledge" section. Present it in the form of advanced questions and answer hints. At the end, include possible teaching activities and design intentions. Please do not go beyond the scope of the knowledge content in the design. If calculations are needed, please list the mathematical formulas and plug the numbers into the formulas for calculation. Here is the content format: [Advanced Question]: [Answer Hint 1]: [Advanced Exercise 1]: [Answer Hint 1]:
Teaching Activities and Design Intentions]: Answer: [Advanced Question]: Suppose a supermarket has four main customer groups: students, office workers, retirees, and freelancers. Their consumption habits and needs in the supermarket might differ. Now, if the supermarket wants to understand the consumption needs and habits of each customer group, how should stratified sampling be carried out? And how should the number of samples to be drawn from each group be determined? [Answer Hint]: First, stratify based on the supermarket's customer groups (students, office workers, retirees, and freelancers). Then, randomly draw a certain number of samples from each stratum. When determining the number of samples to be drawn from each stratum, consider the proportion of that stratum in the overall population, and factors like the importance of each group can also be considered. [Advanced Exercise 1]: To gain a deeper understanding of customer shopping habits, the supermarket decided to add a question to the sample survey: What type of products do you usually purchase in the supermarket? Please design a simple survey form and explain your design rationale. [Answer Hint 1]: When designing the survey form, it should include questions related to the customer's personal information (such as gender, age, occupation, etc.) as well as questions about shopping habits (like the most commonly purchased product types, shopping frequency, etc.). The design rationale is to understand the customer's shopping habits in as much detail as possible, providing the supermarket with a more effective basis for decision-making.

you use the results of stratified random sampling to formulate more effective sales strategies? Please explain your rationale in as much detail as possible.

[Answer Hint 2]: The results of stratified random sampling can reveal the shopping needs and habits of various customer groups. This can assist the supermarket in formulating more accurate sales strategies targeting different groups. For instance, if survey results show that the student group prefers to buy snacks, while retirees prefer health foods, the supermarket could make corresponding adjustments in product arrangement, promotional activities, etc.

[Teaching Activities and Design Intentions]: In this section, through designing advanced questions and exercises, students are encouraged to apply the knowledge they've learned to real-world problems, enhancing their problem-solving and logical reasoning abilities. At the same time, these questions and exercises also offer students an opportunity to test and consolidate their knowledge.

Table.8 Comprehensive Exercise Phase Prompt Command and Generated Result Example

Teaching	Property Design & Occasion Angurer Francisco	
Steps	Prompt Design & Question-Answer Examples	
	Prompt: Consolidation and Improvement + Lesson Name + Knowledge Points + Teaching Methods + Output Format (Comprehensive Examples and Variant Exercises)	
	Question:	
	Now, please combine the content and context of the previously generated "Consolidation and Improvement" section, analyze and design two classic examples for the "Statistics" unit's lesson on "Stratified Random Sampling" and their corresponding instructional activities. Pay attention to the linkages before and after the "Consolidation and Improvement" section and the design of the question chain.	
	The new knowledge taught in this lesson includes: stratified random sampling or stratified sampling, methods of stratified sampling, etc. For the design of example explanation activities, you can refer to teaching methods such as cooperative learning, complex problem-solving learning, project-based learning, tiered grouping learning, blended learning, inquiry-based teaching, and transfer analogy	
	learning. Please design in the form of a detailed lesson plan, detailing the integrated examples and variant exercises. After each example explanation activity, provide possible teaching activities and design intentions.	
Comprehensive	Please do not design beyond the scope of the content. If calculations are needed, please list the mathematical formulas and input the values into the formulas using	
Exercise	plugins for calculation.	
	Here is the content format:	
	[Integrated Example 1.1]:	
	[Answer Hint 1.1]:	
	[Variant Exercise 1.2]:	
	[Answer Hint 1.2]:	
	[Teaching Activity and Design Intent 1]:	
	[Integrated Example 2.1]: [Answer Hint 2.1]:	
	[Variant Exercise 2.2]:	
	[Answer Hint 2.2]:	
	[Teaching Activity and Design Intent 2]:	
	Answer:	
	[Integrated Question 1.1]: Suppose a large city has 10 districts, each with a different	
	population. Now the city government wishes to survey the quality of life of its	
	residents and has decided to use the stratified random sampling method. Assuming	
	the populations of the districts are A1, A2,, A10 respectively, if the city	
	government decides to draw a total of 1,000 samples, how should it determine the	
	number of samples drawn from each district?	
	[Answer Hint 1.1]: When performing stratified random sampling, the number of samples drawn from each stratum is usually proportional to its share of the overall	

population. Therefore, the number of samples drawn from each district should be the proportion of that district's population to the total population multiplied by the total number of samples. The specific formula is: Number of samples drawn from each district = (District's population / Total population) x Total number of samples. [Variant Exercise 1.2]: If the city government decides to draw an equal number of samples from each district, how many samples should each district draw?

[Answer Hint 1.2]: If the city government decides to draw an equal number of samples from each district, then the number of samples each district should draw is the total number of samples divided by the number of districts. The specific formula is: Number of samples drawn from each district = Total number of samples / Number of districts.

[Teaching Activity and Design Intent 1]: Through specific examples and variant exercises, students are introduced to the operation method of stratified random sampling, understanding how to determine the number of samples drawn from each stratum in different situations. This also provides a practical application scenario to help students better understand and master the principles and methods of stratified random sampling.

[Integrated Question 2.1]: Suppose you are conducting a study on the physical health of middle school students nationwide and plan to randomly draw 1,000 middle school students as samples across the country. Considering differences in economy, culture, geography, etc., across the country, you decide to use the stratified random sampling method. How should you stratify? And determine the number of samples drawn from each stratum?

[Answer Hint 2.1]: Firstly, you can stratify based on geographic location (e.g., coastal eastern region, central region, western region), economic development level (e.g., first-tier cities, second-tier cities, third-tier cities, rural areas), or other important factors. Then, the number of samples drawn from each stratum can be determined based on its proportion in the total population. The specific formula is the same as in Question 1.1.

[Variant Exercise 2.2]: If you decide to draw an equal number of samples from each province, how many samples should you draw from each province?

[Answer Hint 2.2]: If you decide to draw an equal number of samples from each province, then the number of samples each province should draw is the total number of samples divided by the number of provinces. The specific formula is the same as in Ouestion 1.2.

[Teaching Activity and Design Intent 2]: Through this question and variant exercise, students understand how to stratify when performing stratified random sampling and how to determine the number of samples drawn from each stratum based on the stratification. This also presents a practical research scenario, helping students recognize the application of stratified random sampling in real research.

Table.9 Lesson Summary Phase Prompt Command and Generated Result Example

Teaching Steps	Prompt Design & Question-Answer Examples
	Prompt: Pre-class Introduction + New Knowledge Explanation + Consolidation and Improvement + Lesson Name + Knowledge Points + Methods of Lesson Summarization + Output Format Question:
Lesson Summary	Now, please integrate the previously generated contents from the "Introduction", "Explanation of New Knowledge", and "Reinforcement and Improvement" sections to analyze and design the "Class Summary" teaching activity for the "Stratified Random Sampling" lesson in the "Statistics" unit. The new knowledge taught in this lesson includes: stratified random sampling or stratified sampling, and methods of stratified sampling. The design forms for the "Class Summary" can refer to, but are not limited to: teacher's summary, student's summary speech, group summary discussion, mind maps, etc. Please design the "Class Summary" activity in a detailed lesson plan format and outline the knowledge content of this lesson point by point. Please do not exceed the scope of the knowledge content in your design. Here is the content format:

[Teacher Activity 1.1]:

[Teacher Activity 1.2]:

[Summary of Knowledge from this Lesson]:

Answer:

[Teacher Activity 1.1]: Invite several students to share their understanding and feelings about what they have learned in today's class, including the concept of stratified random sampling, its implementation steps, and application scenarios. Encourage students to summarize and share in the form of mind maps.

[Teacher Activity 1.2]: Based on the students' sharing, the teacher will supplement and clarify, emphasizing the importance and application of stratified random sampling. The teacher can take this opportunity to answer questions students might have had during the learning process and provide feedback on students' strengths and areas for improvement demonstrated in the activity.

[Summary of Knowledge from this Lesson]:

- (1) Stratified Random Sampling: Stratified random sampling is a sampling method where the population is divided into several non-overlapping sub-populations based on certain characteristics. Samples are then randomly selected from each sub-population (i.e., stratum), and all samples from the strata are combined as the final sample.
- (2) Methods of Stratified Sampling: The main methods of stratified sampling are equalsize stratified sampling and proportional stratified sampling. Equal-size stratified sampling refers to drawing the same number of samples from each stratum, while proportional stratified sampling means that the number of samples drawn from each stratum is proportional to that stratum's size in the overall population.
- (3) Application of Stratified Random Sampling: Stratified random sampling can better reflect the characteristics of the overall population, improving the efficiency and accuracy of sampling. As a result, it is widely used in many fields such as social surveys, market research, and educational evaluations.
- (4) Operational Steps of Stratified Random Sampling: Determine the basis for stratification -> Divide into strata -> Decide the number of samples for each stratum -> Randomly sample within each stratum -> Combine the samples from all strata.