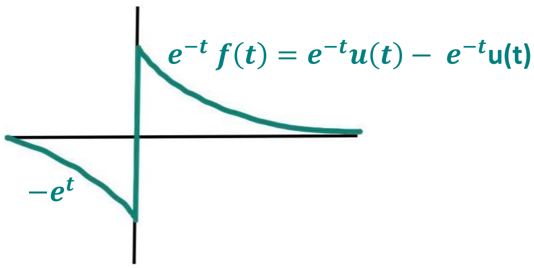
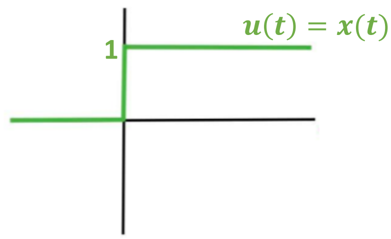
CONVOLUCIÓN

Es un tipo muy general de media móvil, como se puede observar si una de las funciones se toma como la función característica de un intervalo.

MÉTODO ANALÍTICO DE SEÑALES EN TIEMPO CONTINUO

- Se genera una señal g(t)
- Producto composición, integral de superposición, integral de Duhamel
- Método analítico





$$g(t) = f(t) * x(t) = \int_{-\infty}^{\infty} f(\partial)x(t\partial)d\partial = \int_{-\infty}^{\infty} e^{-\partial}u(\partial) - e^{-\partial}u(-\partial)u(t-\partial)d\partial =$$

$\int_{-\infty}^{\infty} e^{-\partial}u(\partial)u(t-\partial)d\partial$ - $\int_{-\infty}^{\infty} e^{\partial}u(-\partial)u(t-\partial)d\partial$

$u(\partial) \rightarrow \ \partial \geq 0$

$u(-\partial) = 1 \rightarrow \ -\partial \geq 0\,; t \geq \partial$

$t \geq \partial \geq 0$

$u(-\partial) = 1 \rightarrow \ -\partial \geq 0 \quad \partial \leq 0$

$u(-\partial) = 1 \rightarrow t - \partial \geq 0 \quad t \geq \partial$

$\begin{cases} t \leq 0 & \partial \in (-\infty, t] \\ t \geq 0 \rightarrow & \partial \in (-\infty, 0] \end{cases}$

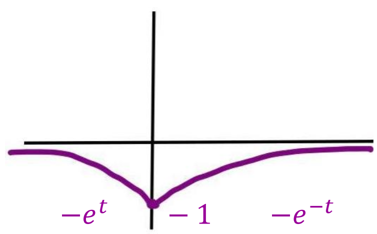
$\int_{0}^{t} e^{-\partial}d\partial u(t)$ - $[\int_{-\infty}^{0} e^{\partial}d\partial u(t) + \int_{-\infty}^{t} e^{\partial}d\,\partial u(-t)]$

$= -e^{-\partial}\Big|_{0}^{t} u(t) - [e^{\partial}\Big|_{-\infty}^{0} u(t) + e^{\partial}\Big|_{\infty}^{t} u(-\partial)]$

$= \left(-e^{-\partial} + 1\right)u(t) - [1u(t) + e^{t}u(-t)]$

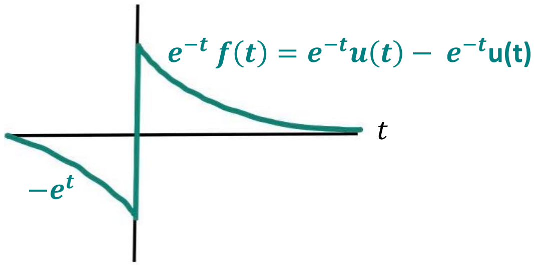
$= -e^{-t}u(t) - e^{t}u(-t)$

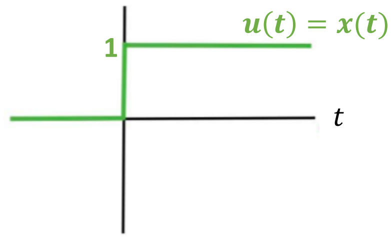
$$g(t) = \begin{cases} -e^{-t} & t \geq 0 \\ -e^{t} & t < 0 \end{cases}$$



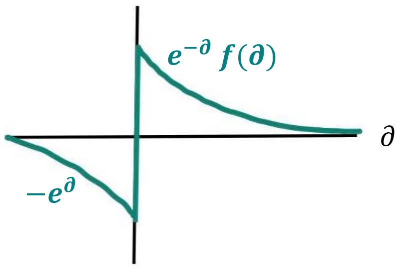
MÉTODO GRÁFICO DE SEÑALES EN TIEMPO CONTINUO

1. Seleccionar una de las dos señales para graficar con el argumento **T**
2. La segunda señal se grafica con el argumento **–T**
3. E la segunda señal para cada valor horizontal de cambio de geometría se suma la variable **t**
4. Se traslada la segunda señal mediante la variable t$\in \mathbb{R}$ hasta que $f(\boldsymbol{T})g(\boldsymbol{t}-\boldsymbol{T})$ no sea cero
5. Mediante el paso anterior se eligen los valores de **T** para realizar la integral
6. Se anota el intervalo de **t** para los cuales los pasos 4 y 5 son validos
7. Se repite los pasos del 4 al 6 hasta que **t** tome todos los valores reales

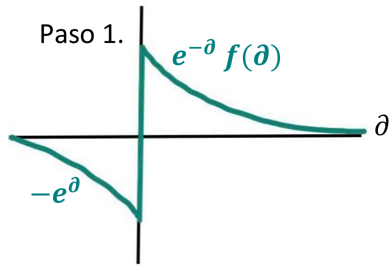


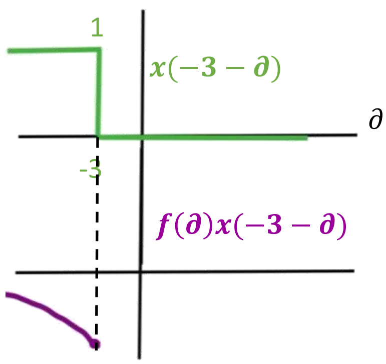


$$g(t) = \int_{-\infty}^{\infty} f(\partial)x(t-\partial);\;\; g(-3) = \int_{-\infty}^{\infty} f(\partial)x(-3-\partial)d\partial$$

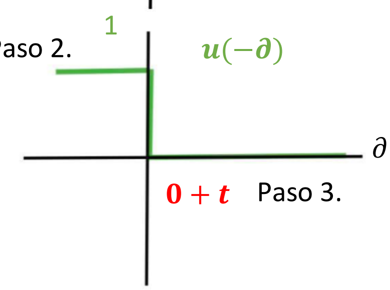


Paso 1.





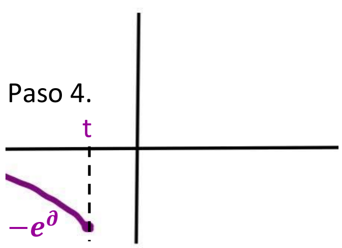
Paso 2.



Paso 3.

$$g(-3) = \int_{-\infty}^{-3} -e^{\partial}d\partial = -e^{\partial}\Big|_{-\infty}^{-3} = -e^{-3}$$

Paso 4.



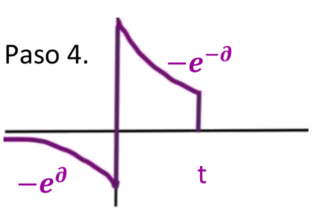
Paso 5.

$\int_{-\infty}^{t} e^{\partial}d\partial = -e^{t}$

Paso 6.

$t \leq 0$

Paso 4.



Paso 5.

$\int_{-\infty}^{0} e^{\partial}d\partial + \int_{0}^{t} e^{-\partial}d\partial$

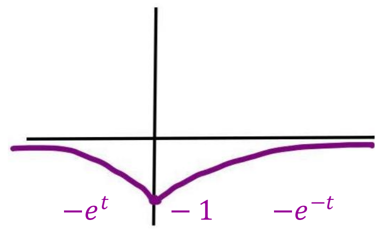
=-1-$(e^{-t} - 1)$

=-e^{-t}

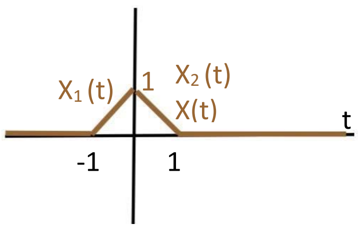
Paso 6.

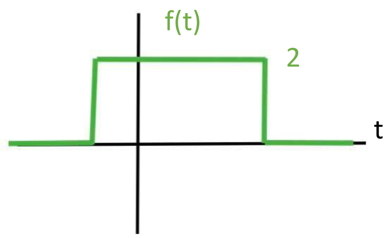
$t \geq 0$

$$g(t) = \begin{cases} -e^{t} & t \leq 0 \\ -e^{-t} & t \geq 0 \end{cases}$$

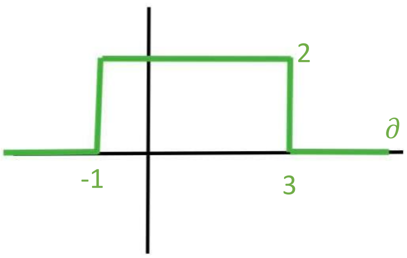


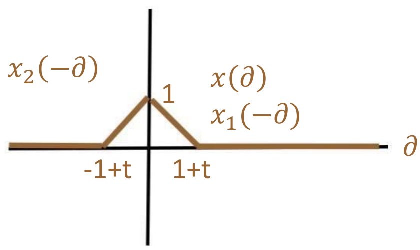
EJEMPLO: g(t) = f (t) * x (t):





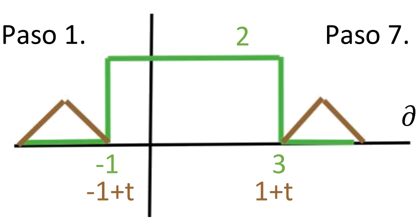
$$x(t) = \begin{cases} x_1(t) & -1 \leq t \leq 0 \\ x_2(t) & 0 \leq t \leq 1 \end{cases} = \begin{cases} t+1 & -1 \leq t \leq 0 \\ -t+1 & 0 \leq t \leq 1 \end{cases}$$



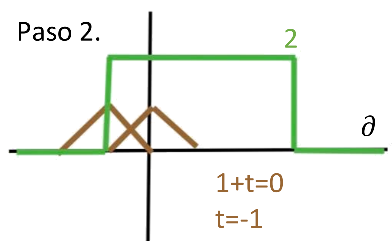


Paso 1.

Paso 7.



Paso 2.



$1 + t \leq -1 \qquad t \leq -2 \rightarrow g(t) = 0$

$-1 + t <\geq 3 \qquad t \geq 4 \rightarrow g(t) = 0$

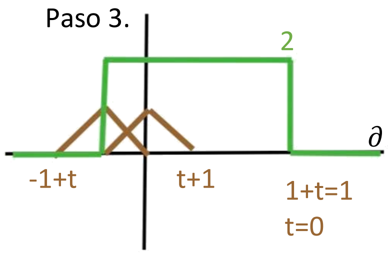
$$\int_{-1}^{1+t} 2(t - \partial + 1)d\partial =$$

$$2(t+1)\int_{-1}^{1+t} d\partial - 2\int_{-1}^{1+t} \partial d\partial =$$

$$= 2(t+1)[1+t+1] - [(1+t)^2 - 1] =$$

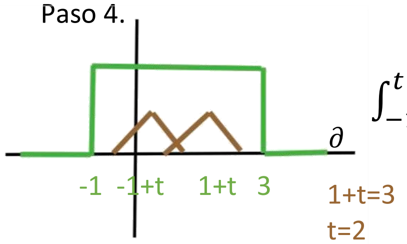
$$2(t+1)(t+2) - [t^2 + 2t] = (t+2)^2$$

Paso 3.



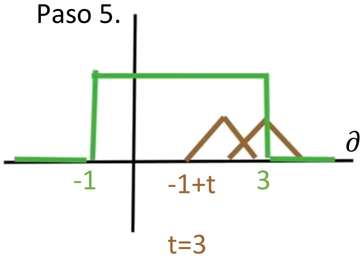
$$\int_{-1}^{t} 2(-((t - 2(+1)d\partial + \int_{t}^{t+1} 2(t - \partial + 1)d\partial$$

Paso 4.



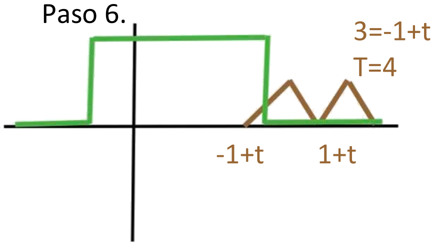
$$\int_{-1+t}^{t} 2(-t-\partial)+1)d\partial + \int_{t}^{1+t} 2(t-\partial+1)d\partial$$

Paso 5.



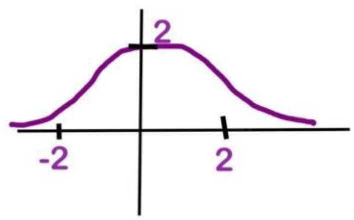
$$\int_{-1+t}^{t} 2(-(t-\partial)+1)d\partial + \int_{t}^{3} 2(t-\partial+1)d\partial = 1-(t-1)(t-3)$$

Paso 6.



$$\int_{-1+t}^{3} 2(-(t-\partial)+1)d\partial$$

$$g(t) = \begin{cases} 0 & t \leq -2 \\ (t+2)^2 & -2 < t \leq -1 \\ 2-t^2 & -1 < t \leq 0 \\ 2 & 0 < t \leq 2 \\ -t^2+4t-2 & 2 < t \leq 3 \\ (t-4)^2 & 3 < t \leq 4 \\ 0 & t > 4 \end{cases}$$



CAUSALES DE LONGITUD INFINITA DE SEÑALES CONTINUAS

Si las señales son causales y tienen duración infinita, entonces la convolución es:

$$g(t) = \begin{cases} \int_{0}^{t} f(\partial)x(t-\partial)d\partial & si\ 0 \leq t \\ 0 & t < 0 \end{cases}$$

$$g(t) = \int_{-\infty}^{\infty} f(\partial)u(\partial)x(t-\partial)u(t-\partial)d\partial = \int_{0}^{t} f(\partial)x(t-\partial)d\partial\ u(t)$$

$\partial \geq 0\ ; \quad t-\partial \geq 0 \quad \rightarrow \quad t \geq \partial$

$Ahora\ t \geq 0$

2) $\int_{-\infty}^{\infty} e^{\lambda\partial}u(\partial)u(t-\partial)d\partial = \int_{0}^{t} e^{\lambda\partial}d\partial \big|_{0}^{t} = \frac{1}{\lambda}\left(e^{\lambda t}-1\right) = \frac{e^{\lambda t}-1}{\lambda}$

4) $\int_0^t e^{\lambda_1 \partial} e^{\lambda_2 (t-\partial)} d\partial = e^{\lambda_2 t} \int_0^t e^{(\lambda_1-\lambda_2)\partial} d\partial = \frac{e^{\lambda_2 t}}{\lambda_1-\lambda_2} e^{(\lambda_1-\lambda_2)\partial} |_0^t =$
$\frac{e^{\lambda_2 t}}{\lambda_1-\lambda_2} (e^{(\lambda_1-\lambda_2)t} - 1) = \frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_1-\lambda_2} \; para \; t \geq 0$

5) $\int_0^t e^{\lambda \partial} e^{\lambda (t-\partial)} d\partial = e^{\lambda t} \int_0^t d\partial = e^{\lambda t}(t-0) = t e^{\lambda t} \; para \; t \geq 0$

CORRELACIÓN

Es una medida de la similitud entre dos señales, frecuentemente usada para encontrar características relevantes en una señal desconocida por medio de la comparación con otra que sí se conoce.

CRUZADA MEDIANTE CONVOLUCIÓN DE SEÑALES CONTINUAS

$$\boldsymbol{T_{xy}(t) = \int_{-\infty}^{\infty} x(t-\partial) y(\partial) d\partial \ldots (1)}$$

$$u = t + \partial \;; \quad \frac{du}{d\partial} = 1; \quad \partial = u - t$$

$$T_{xy}(t) = \int_{-\infty}^{\infty} x(u) y(u-t) du$$

$$\boldsymbol{T_{xy}(t) \int_{-\infty}^{\infty} x(\partial) y(\partial - t) d\partial \ldots . (2)}$$

$$T_{xy}(t) \int_{-\infty}^{\infty} x(\partial) y(\partial - t) d\partial = \int_{-\infty}^{\infty} x(\partial) y\big(-(t-\partial)\big) d\partial = \int_{-\infty}^{\infty} x(t) \left| \begin{matrix} y(-t) \\ t \to \partial \end{matrix} \right| \begin{matrix} d\partial \\ t \to t - \partial \end{matrix}$$

$$= \boldsymbol{x(t) * y(-t) \ldots (3)}$$

OBSERVACIONES:

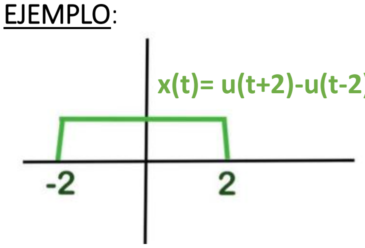
- $Correlación \; (probabilidad)$
- $T_{XX}(t) función \; de \; autocorrelación$
- $T_{XY}(t) \neq T_{YX(t)}$
- $la \; autocorrelación \; no \; siempre \; existe, por \; ejemplos \; la \; autocorrelacion \; de \; u(t)$
- $La \; correlación \; cruzada \; presenta \; problemas \; en \; señales \; periódicas$
- $Una \; definición \; general \; de \; correlación:$

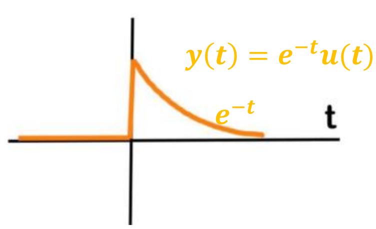
  $\int_{-\infty}^{\infty} x(\partial) y^*(\partial - t) d\partial \qquad y^* complejo \; conjugado$

  $T_{XX}(t) = \int_{-\infty}^{\infty} x(\partial) x^x(\partial - t) d\partial$

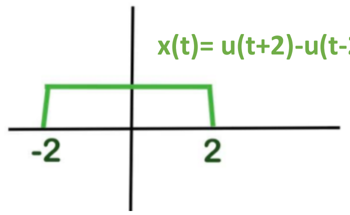
  $T_{XX}(0) = \int_{-\infty}^{\infty} x(\partial) x^*(\partial) d\partial = \int_{-\infty}^{\infty} ||x(\partial)||^2 d\partial = E_x$

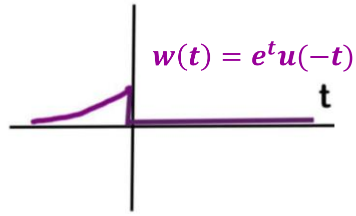
EJEMPLO:





$Realizar\ T_{xy}(t), por\ (3)$

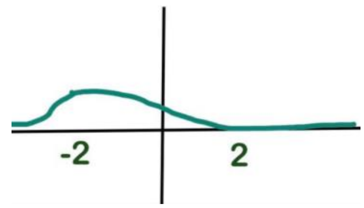




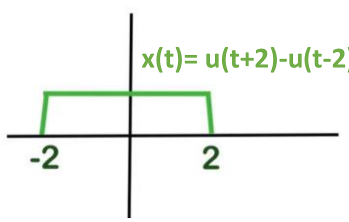
$$T_{xy}(t) = x(t) * y(-t) = x(t) * w(t)$$

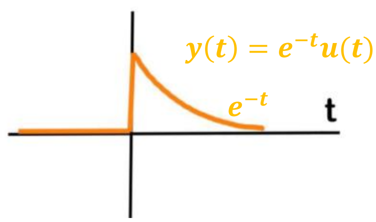
$$CONVOLUCIÓN \begin{cases} MÉTODO \\ GRÁFICO \end{cases}$$

$$T_{xy}(t) = \begin{cases} e^{t+2} - e^{t-2} & t \leq -2 \\ 1 - e^{t-2} & -2 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases}$$



CRUZADA MEDIANTE MÉTODO ANALÍTICO DE SEÑALES CONTINUAS





$$\boldsymbol{x(t-\partial)y(\partial) = \big(u(t+2+2) - u(t+\partial-2)\big)e^{-\partial}u(\partial)}$$

$$\boldsymbol{T_{xy}(t) = \int_{-\infty}^{\infty} e^{-\partial}u(t+\partial+2)u(\partial) - \int_{-\infty}^{\infty} e^{-\partial}u(t+\partial-2)u(\partial)d\partial}$$

$$\boldsymbol{1.1\quad \int_{-t-2}^{\infty} e^{-\partial}d\partial = e^{t+2} \qquad t \leq -2 \quad u(t+\partial+2) \geq 0; u(\partial) \geq 1 \qquad \partial \geq -t \geq -2 \quad \partial \geq 0}$$

$$\boldsymbol{1.2\quad \int_{0}^{\infty} e^{-\partial}d\partial = 1}$$

$$\boldsymbol{2.1\quad \int_{0}^{\infty} e^{-\partial}d\partial = 1 \rightarrow t \geq 2}$$

$$\boldsymbol{2.2\quad \int_{2-t}^{\infty} e^{-\partial}d\partial = e^{t-2} \rightarrow t \leq 2}$$

$$\boldsymbol{T_{xy}(t) = \begin{cases} e^{t+2} - e^{t-2} & t \leq -2 \\ 1 - e^{t-2} & -2 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases}}$$

EJEMPLO

1.1 $\displaystyle\int_{t}^{\infty} \boldsymbol{e^{t}\, e^{-\partial} d\partial = 1 \qquad t \geq -2}$

1.2 $\displaystyle\int_{-2}^{\infty} e^{t} e^{-\partial} d\partial = e^{t+2} \qquad t \leq -2$

2.1 $\displaystyle\int_{t}^{\infty} e^{t} e^{-\partial} = 1 \qquad t \geq 2$

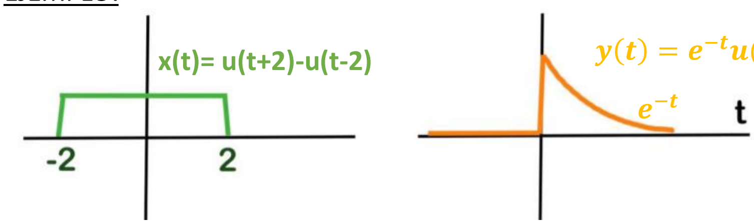
2.2 $\displaystyle\int_{2}^{\infty} e^{t} e^{-\partial} d\partial = e^{t+2} \qquad t \leq 2$

$$T_{xy}(t) = \begin{cases} e^{t+2} - e^{t-2} & t \leq -2 \\ 1 - e^{t-2} & -2 < t \leq 2 \\ 0 & t \geq 2 \end{cases}$$

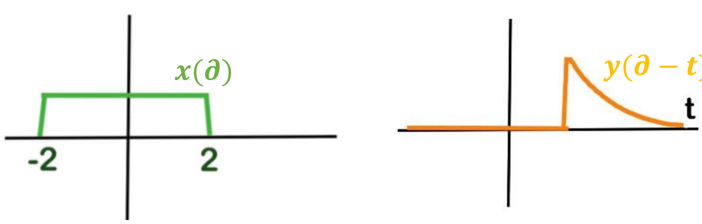
CRUZADA MEDIANTE MÉTODO GRÁFICO DE SEÑALES CONTINUAS

1. *Dibujar* $x(\partial)$
2. *Dibujar* $y(\partial)$
3. *Agregar "t" en cada cambio de fórmula de* $y(\partial)$
4. *Se traslada la señal* $y(\partial)$ *de tal manera que se obtenga* $x(\partial)y(\partial - t)$
5. *De la geometría del paso anterior encontrar los valores de* ∂
6. *Se encuentran los valores de t de tal manera que el paso anterior sea valido*
7. *Se repiten los pasos del 4 al 6 para todos los valores de t reales*

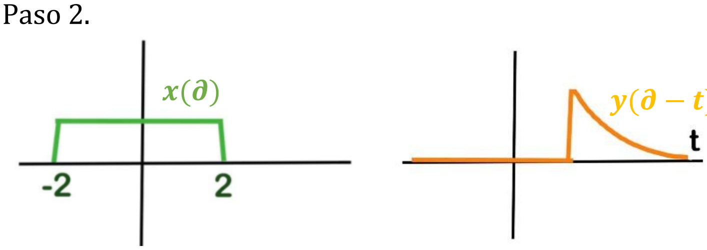
EJEMPLO:



Paso 1.
$t \geq 2$

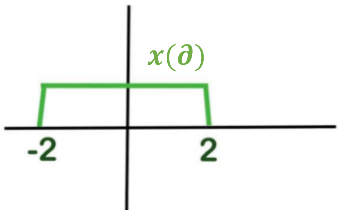


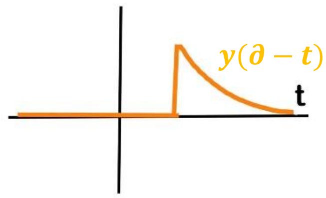
Paso 2.



$$\int_t^2 e^{-(\partial - t)} d\partial = 1 - e^{t-2} \qquad -2 \le t \le 2$$

Paso 3.





$$\int_{-2}^2 e^{-(\partial - t)} d\partial = e^{t+2} - e^{t-2} \qquad t \le -2$$

$$T_{xy}(t) = \begin{cases} 0 & 2 \le t \\ 1 - e^{t-2} & -2 \le t \le 2 \\ e^{1+t} - e^{1-2} & t \le -2 \end{cases}$$