Verifying the Mathematical Library of a UAV Autopilot with Frama-C

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Context

Formal methods

- Verification techniques based on mathematical models
- Recommended in avionics with DO-178C and DO-333 standards
- Example: abstract interpretation, deductive methods, model-checking

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The goals of this ongoing work

- Define verification processes that use formal methods,
- Apply these methods to a drone autopilot: Paparazzi.

First results

Analysis of a mathematical library of Paparazzi:

- Using Frama-C,
- Checking for the absence of runtime errors,
- Verification of some functional properties,
- Without modifying the code.

Frama-C

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Frama-C is a C code analysis tool

- Developed by CEA and Inria,
- Modular, which supports different analysis methods
  ex: static analysis with EVA or dynamic analysis with E-ACSL.

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Verification process of a C program using Frama-C:

1. Code specification with **ACSL** (*ANSI C Specification Language*),
2. Generation of the abstract syntax tree of the analyzed code,
3. Analysis of the tree by the plugins
   $\Longrightarrow$ Verify if the specification is respected.

Note: the tree analysis can be performed by several plugins.

Some Frama-C plugins

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- Interfaced with Why3 to verify goals with automatic provers (Alt-Ergo, Z3, CVC4).

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EVA (*Evolved Value Analysis*)

- Based on static analysis by abstract interpretation methods,
- Computes domains of values for each variable in the program.

Paparazzi

Presentation

Paparazzi is an autopilot for micro-drones

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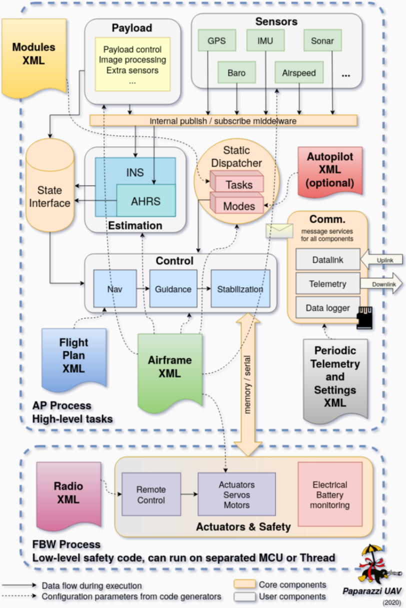
Complete drone control system:

- Offers the control software part,
- Also offers some designs of hardware components,
- Supports for ground and aerial vehicles,
- Supports for simultaneous control of several drones.

Flight system architecture



Library studied

`pprz_algebra` : mathematical algebra library coded in C ($\sim$ 4 000 loc)

The library contains:

- The definition of a representation of `vectors`,
- Different representations of vector rotations,
  rotation matrices, Euler angles, quaternions
- Elementary operations,
  ex: addition of vectors, computation of the rotation of a vector, normalization of a quaternion . . .
- Conversion functions between these different representations.

Note: Each representation/function has a fixed point (`int`) and floating-point version (for `float` and `double`).

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Process :

- Analyze the code with Frama-C using RTE and WP plugins.
- Deduce the missing information in the contract.

Analysis with Frama-C and the RTE plugin

Analysis of the instruction:

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c->x = a->x * b->x;
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- The values are not bounded.

```
/*@ assert rte: signed_overflow: -2147483648 ≤ a->x * b->x; */
/*@ assert rte: signed_overflow: a->x * b->x ≤ 2147483647; */
```

$\Longrightarrow$ Determine bounds which guarantee the absence of overflows.

Example of final contract for the function `int32_quat_comp`

```c
#define SQRT_INT_MAX4 23170 // 23170 = SQRT(INT_MAX/4)

/*@
  requires \valid(a2c);
  requires \valid_read(a2b);
  requires \valid_read(b2c);
  requires bound_Int32Quat(a2b, SQRT_INT_MAX4);
  requires bound_Int32Quat(b2c, SQRT_INT_MAX4);
  requires \separated(a2c, a2b) && \separated(a2c, b2c);
  assigns *a2c;
*/
void int32_quat_comp(struct Int32Quat *a2c,
                     struct Int32Quat *a2b,
                     struct Int32Quat *b2c)
```

Notes

EVA and WP had to be associated to verify the absence of RTE.

- WP is overloaded when accessing values by reference,
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The `real` **arithmetic model** (real in the mathematical sense) has been used to verify floating-point version of the functions.

The `real` model guarantees :

- The absence of division by 0,
- The lack of dereference of invalid pointers.

But the absence of overflows is not verified.

Functional verification

Functional verification

Offer guarantees on the behavior or the result of a function.

Example: *Functional properties for square root function*

```
/*@
  requires x >= 0;
  ensures \result >= 0;
  ensures \result * \result == \old(x);
  assigns \nothing;
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The functional verification was only done for some floating-point functions.

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First, it is necessary to define:

- **Types**,
  ex : `RealVect3`, `RealRMat`, `RealQuat`.
- **Elementary functions**,
  ex: addition of vectors, rotation of a vector...
- **Conversion functions** between certain representations,
  ex: Definition of the function `rmat_of_quat` : $\mathbb{H} \rightarrow M_{3,3}(\mathbb{R})$,

```
/*@
    logic RealRMat l_RMat_of_FloatQuat(struct FloatQuat *q) =
    [...]
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- **Lemmas...**

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$$\forall q \in \mathbb{H},\ \forall v \in \mathbb{R}^3,\quad q(0,v)q^* = (0, \texttt{rmat\_of\_quat}(q).v)$$

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Finally, the functional properties are expressed in the form of predicates:

- M is a rotation matrix: $M.M^t = I$
- ...

Example

Spécification of the function `float_rmat_of_quat`.

```
/*@
    requires \valid(rm);
    requires \valid_read(q) && finite_FloatQuat(q);
    requires unitary_quaternion(q);
    requires \separated(rm, q);
    ensures rotation_matrix(l_RMat_of_FloatRMat(rm));
    ensures special_orthogonal(l_RMat_of_FloatRMat(rm));
    ensures l_RMat_of_FloatRMat(rm) ==  l_RMat_of_FloatQuat(q);
    assigns *rm;
*/
void float_rmat_of_quat(struct FloatRMat *rm,
                        struct FloatQuat *q)
```

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$\Longrightarrow$ Extend the contracts.

ex: Extension of the contract for the function $sinf$.

```
/*@ requires finite_arg: \is_finite(x);
    assigns \result \from x;
    ensures finite_result: \is_finite(\result);
    ensures result_domain: -1. <= \result <= 1.;
    ensures result_value: \result == \sin(x);
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$\Longrightarrow$ Enable interactive mode of Frama-C to use Coq.

Summary of the functional verification

Functional verification offers guarantees on the behavior of a function.

- Functional properties are expressed using ACSL.
- The verification can be done automatically or interactively.

Using the `real` model:

- Offers no functional guarantee during execution.
- Used to verify that the code is correct in a mathematical sense,

Conclusion

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Summary :

- Verification of the absence of runtime errors in the library,
- Verification of functional properties on some floating-point functions.

$\Longrightarrow$ Approximately 3,500 lines of annotation.

`gitlab.isae-supaero.fr/b.pollien/paparazzi-frama-c`

Perspectives:

- Verification of calls to library functions,
- Verifying the floating-point library without the `real` model,
- Verifying the Paparazzi flight plan generator.

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Thank you

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Examples of lemmas proved with Coq

- Lemma to verify the correctness of the function `quat_of_rmat`

$$\forall R \in SO_3(\mathbb{R}),\ \forall q \in \mathbb{H},$$
$$||q|| > 0 \ \wedge\ Tr(R) > 0$$
$$\rightarrow (R = \texttt{rmat\_of\_quat}(q) \leftrightarrow q = \texttt{quat\_of\_rmat}(R))$$

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$$\rightarrow (R = \texttt{rmat\_of\_quat}(q) \leftrightarrow q = \texttt{quat\_of\_rmat}(R))$$

- Lemma used to verify that `rmat_of_euler` compute rotation matrix:

$$\forall a, b, c \in \mathbb{R},$$
$$\sin(a)^2 \cos(b)^2$$
$$+ (\sin(a)\sin(b)\cos(c) - \sin(c)\cos(a))^2$$
$$+ (\cos(c)\cos(a) + \sin(a)\sin(b)\sin(c))^2 = 1$$