1/18/2023

Econometrics Group Project

Neoma Business School (MSC FBD)

REYNOSO VALDES Ana Karen

MONFORT Baptiste

DURAND Valentine

Task 1

1.1 Print the names of the variables in the data set and plot the (log) time series. You can also rename the series. Save the plot on your Word document. Describe the dynamics of the series.

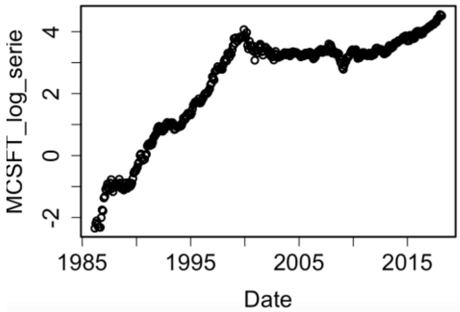


Figure 1: Microsoft log time series

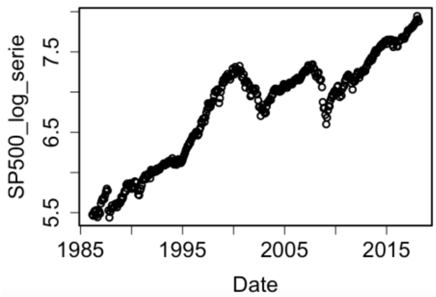


Figure 2: S&P 500 log time series

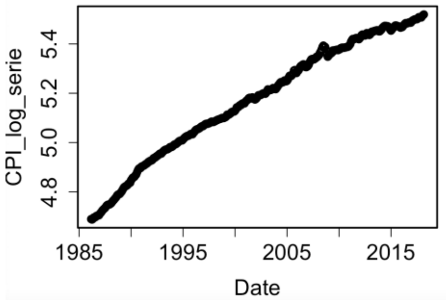


Figure 3: CPI log time series

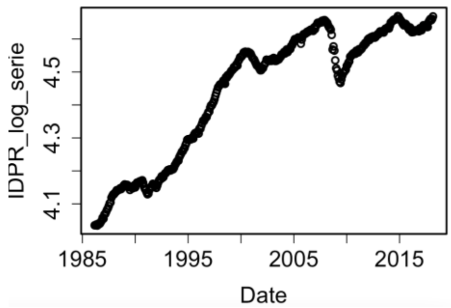
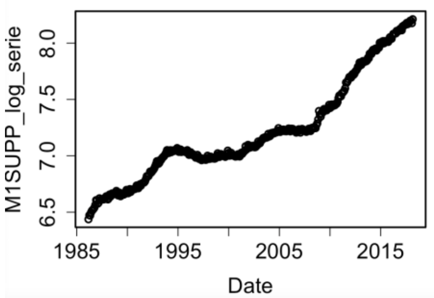


Figure 4: IDPR log time series



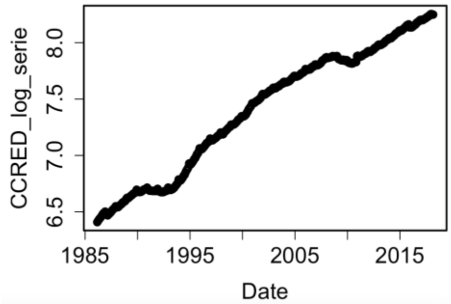


Figure 6: CCREDI log time series

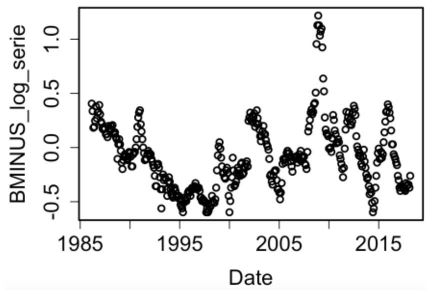


Figure 7: BMIUS log time series

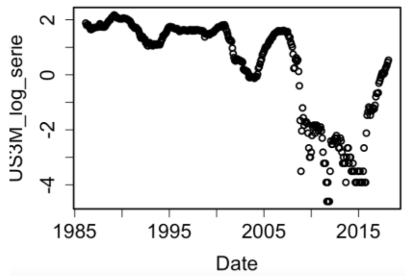


Figure 8: US3M log time series

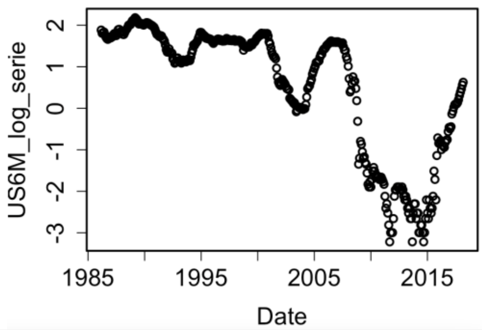


Figure 9: US6M log time series

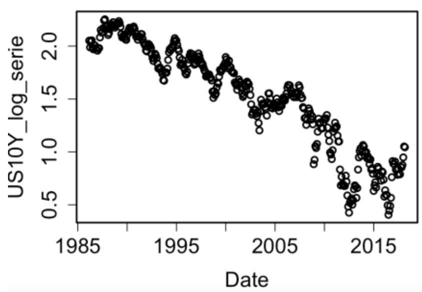
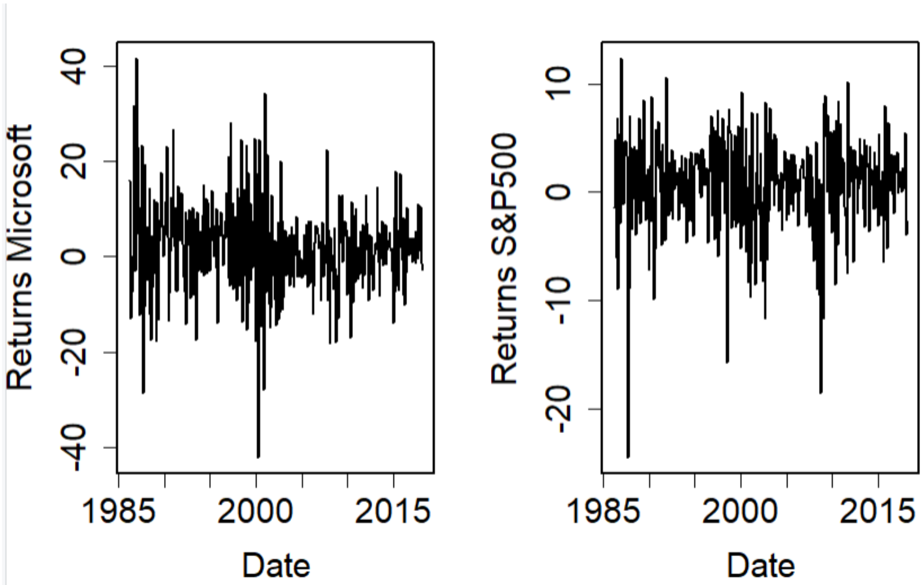


Figure 5: M1SUPPLY log time series	*Figure 10: US10Y log time series*

Most of the series have obvious trends with various degrees of variance.

We can note that by grouping by "categories" we can see that the Microsoft and S&P500 series share a global trend but Microsoft is steepest until 2000, then become more flattish later. Samely, the US3M and 6M T-bills series share exactly the same trend however the 10Y series is less flattish and we can see it has a steepest negative trend. Finally, the "Macroeconomics" series display less variance than for the bonds/equity series except for the credit spread series which is very volatile but still, remains in a constant range between –0.5 and 0.5.

1.2 Microsoft returns against S&P 500



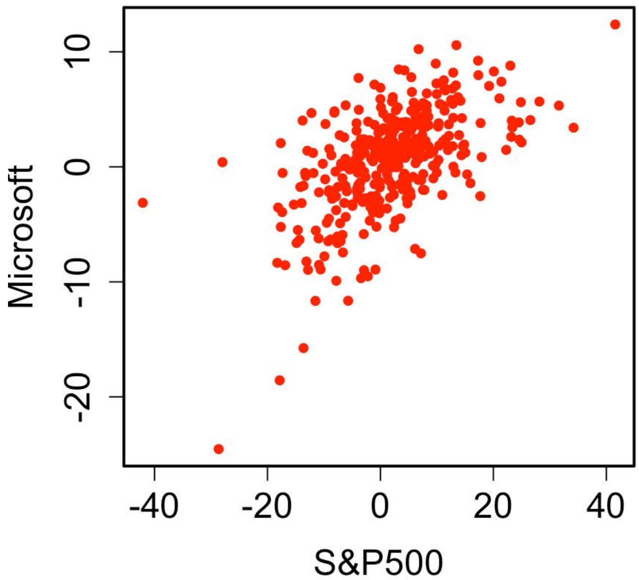


Figure 11: Microsoft returns against the S&P500 returns

We can see that they seem to have similar mean and variance, however Microsoft series display much more positive abnormal returns with a top scale of 40 whereas the S&P 500 stops at 10. More generally, Microsoft seems to have more outliers, both positive and negative than the S&P500. We can conclude that Microsoft series has more variance than the S&P500. We can also see that Microsoft variance diminishes after year 2000 whereas the S&P500 variance seems to remain the same on the whole length.

From the figure 11, we can see that the point forms a cluster with few outliers. Hence the returns of Microsoft seems to be correlated with the returns of the S&P500. Also noting the outliers don't occur at the same time for each series. A possible explanation is that the correlation is not perfect (1)

1.3 Boxplot of the Microsoft and S&P500 log-returns

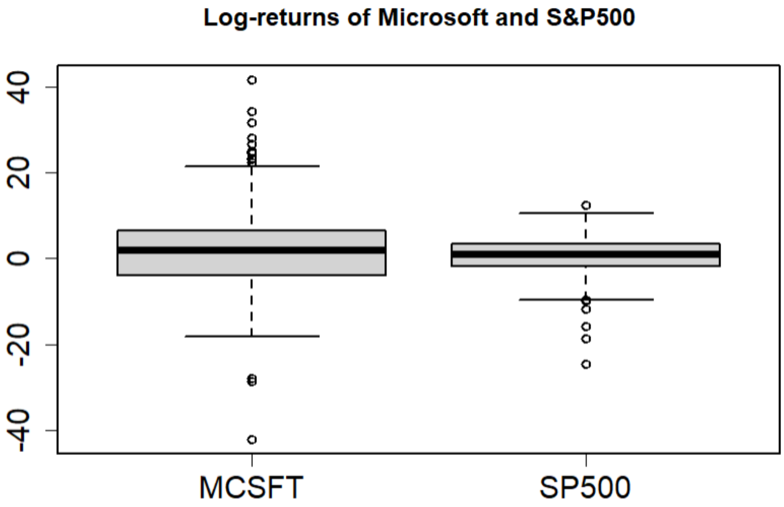


Figure 12: Boxplot of the Microsoft and S&P500 log-returns

From the box plot above we can see that these series have almost the same median, but not the same quartiles size. Microsoft series have broader 1st and 3rd quartiles highlighting more variance, as the data are spread on a broader range. Furthermore, the whisker are twice as far from the median than for the S&P500 which shows a greater variance as well. We can guess a slight asymmetry on the Microsoft series compared to the S&P500 as it seems that the 1st quartile is slightly bigger than the 3rd. To confirm this, we'll have to compare the median and the mean, and verify the skewness.

We can also see that Microsoft series display more outliers than the S&P500 series, noting they are inversely correlated.

1.4 Treasury bill yields at three-month and ten-year maturity

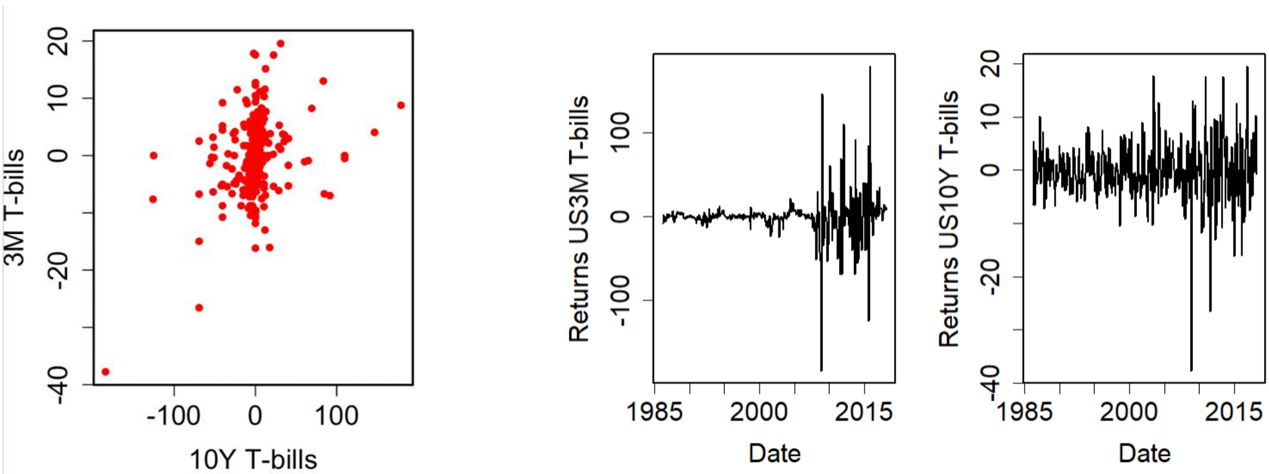


Figure 13: 3M T-bills against 10Y T-bills

From figure 13, we can see that the series have the same mean. However, when comparing the series variances, we can note that despite having less variance from 1985 until 2008, the US3M series sees a huge surge of variance up to 2018, which cause bigger outliers than for the US10Y series, which had a more flattish, yet significant, variance on the whole timestamp. To summarize, as a whole, the US10Y has an almost constant variance whereas the US3M has a lower variance up to 2008 when it surged.

We can see a cluster highlighting a possible correlation between these series. There are several outliers which in that case seem to be happening at the same time highlighting a possible correlation there too. A possible explanation is that the correlation is closer to (1) than for the other series.

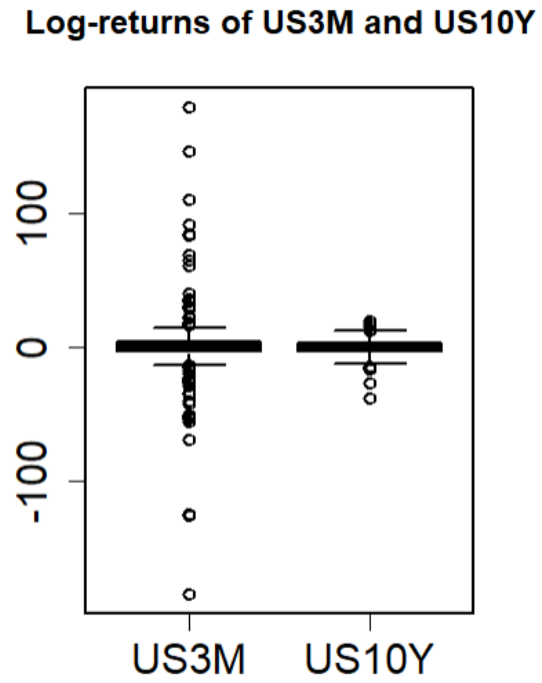


Figure 14: Boxplot of 3M T-bills and 10Y T-bills

On the other hand, for *Figure 14* we can see from the boxplot that most of the values are very close to the median, and both series share the same median and the same range of quartiles. However, there are many more outliers for the US3M series than for the US10Y series.

If we compare these last 2 figures against the results of the Microsoft and S&P500 return series, we can spot some key differences. For example, the excess return correlation seems to be negative and not perfect for Microsoft and the S&P500 whereas it seems that the excess return between the US3M and US10Y T-bills seem to appear that they are perfectly correlated because we can see as many outliers in proportion below the median and above it.

1.5 Descriptive statistics

Mean, Standard Deviation, Skewness, Kurtosis, Median

	Mean	Standard Dev.	Skewness	Kurtosis	Median
Microsoft	1.787135	9.639607	0.1244276	5.112244	1.976367
S&P 500	0.625734	4.34562	-1.086611	6.797415	1.101732
US3M	-0.3516569	26.13333	0.2747321	21.65039	0
US6M	-0.2624355	5.75709	-0.5909696	8.847052	-0.5006904

1st and 3rd quartiles

	0% (Min)	25%	50%	75%	100% (Max)
Microsoft	-42.087739	-3.839747	1.976367	6.552241	41.577184
S&P 500	-24.542804	-1.758331	1.101732	3.412809	12.378004
US3M	-184.582669	-3.285758	0	3.653	179.175947
US6M	-37.7530331	-3.4734742	-0.5006904	2.9726	19.549202

The four series do not have the same mean and median, hence they are not perfectly symmetrical. Furthermore, they all have a slight skewness. The standard deviation is significant for the Microsoft and US3M series which translates the high variance they share.

The normal distribution is characterized by a 0 skewness and a kurtosis of 3.

Regarding our data, none of our series can be normally distributed.

Task 2

2.1 ACF and PACF

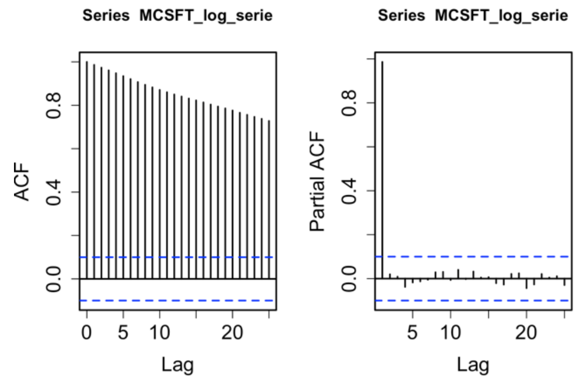


Figure 15: Microsoft ACF and PACF

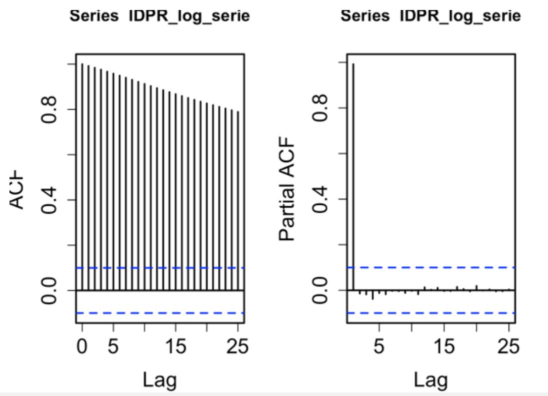


Figure 20: IDPR ACF and PACF

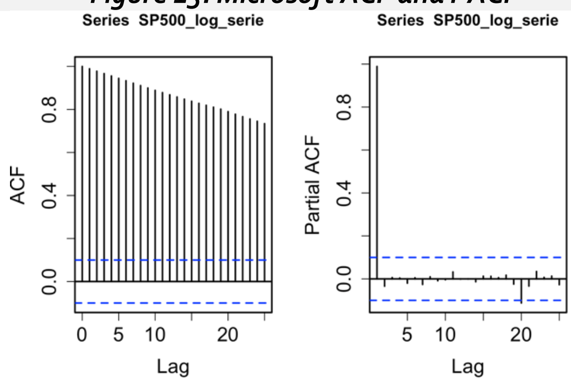


Figure 16:S&P 500 ACF and PACF

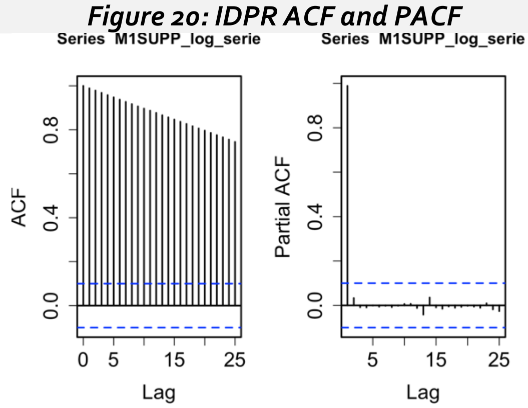
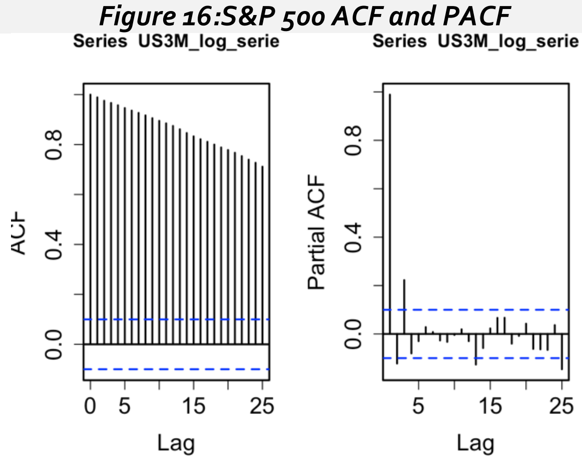


Figure 21: M1SUPPLY ACF and PACF



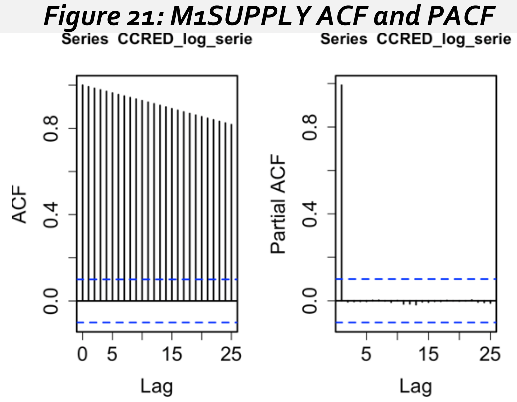
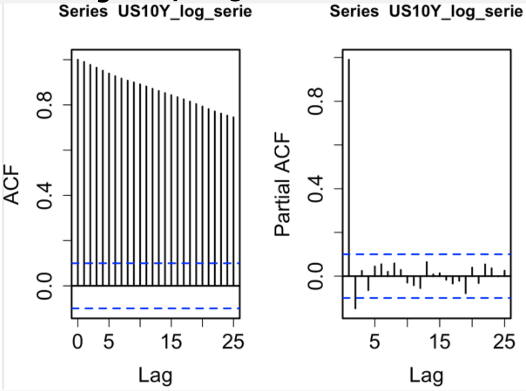


Figure 17:US3M ACF and PACF

Figure 22: CCREDI ACF and PACF



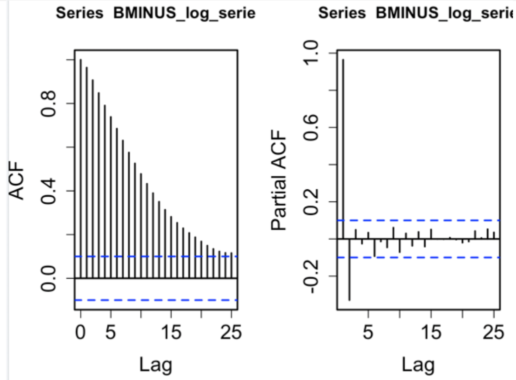
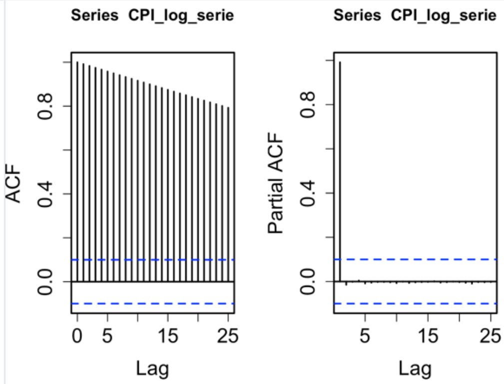


Figure 18:US10Y ACF and PACF

Figure 23: BMIUS ACF and PACF



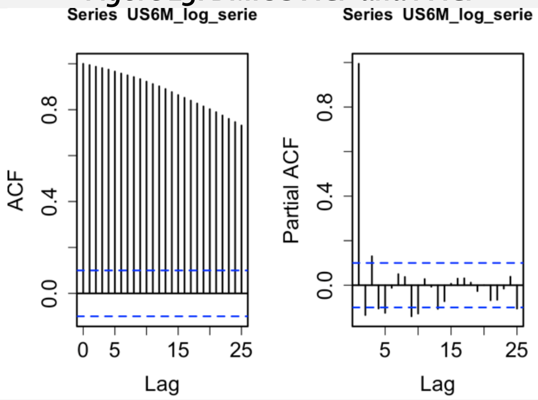


Figure 19: CPI ACF and PACF

Figure 24: US6MI ACF and PACF

For all the series we can see that the ACF is decreasing extremely slowly.

Whereas for the partial autocorrelations we see a faster decrease. For CPI and CCREDUT the range is respected, and the lines don't go over it, this means they're a White Noise

The PACF of US6M and US3M is exponentially decaying but it's not too significant.

From these figures we could guess that we are working with an autoregressive model of order one which gives a single exponential model.

2.2 Testing if series are: stationary, serially correlated, homoscedastic, and normally distributed.

For the following tests we consider a 95% level of confidence.

Non stationarity (Augmented Dicky Fuller Test)

For testing non stationarity we use the augmented dicky fuller, where:

Ho: non-stationarity

H1: stationarity.

Log Series Name	P-value	Interpretation
Microsoft	0.5373	we accept Ho: non stationarity
SP&500	0.4428	we accept Ho: non stationarity
CPI	0.3111	we accept Ho: non stationarity
IDPR	0.8341	we accept Ho: non stationarity
M1SUPPY	0.6335	we accept Ho: non stationarity
CCREDIT	0.7234	we accept Ho: non stationarity
BMINUSA	0.2627	we accept Ho: non stationarity
USTB3M	0.2801	we accept Ho: non stationarity
USTB6M	0.1648	we accept Ho: non stationarity

USTB10Y	0.1683	we accept Ho: non stationarity

Non autocorrelation (Breusch Godfrey)

Log Series Name	P-value
Microsoft	2.2e-16
SP&500	2.2e-16
CPI	2.2e-16
IDPR	2.2e-16
M1SUPPY	2.2e-16
CCREDIT	2.2e-16
BMINUSA	2.2e-16
USTB3M	2.2e-16
USTB6	2.2e-16
USTB10Y	2.2e-16

For testing the non-autocorrelation, we use the Breusch Godfrey test where:

Ho: There's no autocorrelation

H1: There's autocorrelation

After testing each log series, we conclude that in all of them we **reject** Ho, meaning there is autocorrelation based on the p-value.

Homoskedasticity

For testing homoscedasticity, we use the Garch test where:

Ho: Homoscedasticity

H1: Heteroscedastic

<table>
<tr><th>Log Series Name</th><th>Garch results</th><th>Interpretation</th></tr>
<tr><td>Microsoft</td><td><pre>Optimal Parameters

        Estimate Std. Error   t value Pr(>|t|)
mu   -2.348777   0.266421   -8.8160 0.000000
ar1   1.000000   0.000868 1152.6103 0.000000
omega   0.000281   0.000131   2.1455 0.031916
alpha1 0.102878   0.028146   3.6552 0.000257
beta1   0.863997   0.032692   26.4286 0.000000

Robust Standard Errors:
        Estimate Std. Error   t value Pr(>|t|)
mu   -2.348777   0.026116 -89.9348 0.000000
ar1   1.000000   0.000931 1074.5709 0.000000
omega   0.000281   0.000132   2.1332 0.032911
alpha1 0.102878   0.025265   4.0720 0.000047
beta1   0.863997   0.024295   35.5625 0.000000</pre></td><td>All the parameters are statistically significant at a level of 5%, we accept Ho meaning there is homoscedasticity</td></tr>
<tr><td>SP&500</td><td><pre>Optimal Parameters

        Estimate Std. Error t value Pr(>|t|)
mu   5.476045   0.058998 92.8173 0.000000
ar1   1.000000   0.001306 765.8973 0.000000
omega   0.000042   0.000029   1.4605 0.144154
alpha1 0.141866   0.038909   3.6461 0.000266
beta1   0.848379   0.032020 26.4954 0.000000

Robust Standard Errors:
        Estimate Std. Error t value Pr(>|t|)
mu   5.476045   0.005918 925.2693 0.000000
ar1   1.000000   0.001714 583.5029 0.000000
omega   0.000042   0.000030   1.4153 0.156970
alpha1 0.141866   0.046621   3.0430 0.002343
beta1   0.848379   0.039912 21.2564 0.000000</pre></td><td>All the parameters are statistically significant except for omega, where we accept Ho meaning there is homoscedasticity</td></tr>
</table>

CPI

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      4.689511    0.004351 1077.7782 0.000000
ar1     1.000000    0.000521 1920.6419 0.000000
omega   0.000007    0.000000  328.8356 0.000000
alpha1  0.435838    0.101840    4.2796 0.000019
beta1   0.086600    0.027196    3.1843 0.001451

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      4.689511    0.000848 5532.7775 0.000000
ar1     1.000000    0.000735 1361.4279 0.000000
omega   0.000007    0.000000  270.3484 0.000000
alpha1  0.435838    0.101838    4.2797 0.000019
beta1   0.086600    0.029797    2.9063 0.003657
```

All the parameters are statistically significant, we reject Ho, meaning it is heteroskedastic

IDPR

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      4.034973    0.006061  665.6788        0
ar1     1.000000    0.000647 1544.5570        0
omega   0.000014    0.000002    6.9958        0
alpha1  0.278516    0.043504    6.4021        0
beta1   0.355834    0.039257    9.0643        0

Robust Standard Errors:
        Estimate  Std. Error    t value Pr(>|t|)
mu      4.034973    0.000233 17341.9944 0.000000
ar1     1.000000    0.001155   866.0174 0.000000
omega   0.000014    0.000006     2.3062 0.021098
alpha1  0.278516    0.087667     3.1770 0.001488
beta1   0.355834    0.270596     1.3150 0.188509
```

All the parameters are statistically significant, we reject Ho, meaning it is heteroskedastic

M1SUPPY

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      7.016065    0.006073 1155.2566 0.000000
ma1     0.837947    0.018884   44.3725 0.000000
omega   0.000039    0.000023    1.7424 0.081434
alpha1  0.350367    0.082811    4.2309 0.000023
beta1   0.642793    0.081279    7.9085 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      7.016065    0.021630 324.3730 0.000000
ma1     0.837947    0.018086  46.3317 0.000000
omega   0.000039    0.000015   2.6050 0.009187
alpha1  0.350367    0.048784   7.1821 0.000000
beta1   0.642793    0.053747  11.9595 0.000000
```

All the parameters are statistically significant except for omega, where we accept Ho meaning there is homoscedasticity

CCREDIT

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      6.408198    0.009322  687.4079        0
ar1     1.000000    0.000461 2168.2919        0
omega   0.000005    0.000000  113.0452        0
alpha1  0.017000    0.002034    8.3595        0
beta1   0.921331    0.007796  118.1747        0

Robust Standard Errors:
        Estimate  Std. Error    t value Pr(>|t|)
mu      6.408198    0.000633 10125.3574        0
ar1     1.000000    0.000644  1553.2674        0
omega   0.000005    0.000000    35.8862        0
alpha1  0.017000    0.002263     7.5125        0
beta1   0.921331    0.012763    72.1889        0
```

All the parameters are statistically significant, we reject Ho, meaning it is heteroskedastic

BMINUSA

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.395011    0.084251   4.6885 0.000003
ar1     0.995540    0.006424 154.9715 0.000000
omega   0.000359    0.000216   1.6616 0.096594
alpha1  0.067352    0.027522   2.4473 0.014395
beta1   0.877451    0.046045  19.0563 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.395011    0.018256  21.6371  0.00000
ar1     0.995540    0.006898 144.3196  0.00000
omega   0.000359    0.000338   1.0616  0.28844
alpha1  0.067352    0.047629   1.4141  0.15733
beta1   0.877451    0.079891  10.9831  0.00000
```

In the robust error the constant omega and alpha 1 have a p-value greater than our level of confidence, for that reason for these variables we accept Ho meaning there is homoscedasticity

USTB3M

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      1.810637    0.078594  23.0377 0.000000
ar1     0.970066    0.006067 159.8843 0.000000
omega   0.000187    0.000056   3.3159 0.000914
alpha1  0.408976    0.048659   8.4049 0.000000
beta1   0.590024    0.037496  15.7355 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      1.810637    0.104301  17.3597  0.00000
ar1     0.970066    0.014704  65.9715  0.00000
omega   0.000187    0.000075   2.4912  0.01273
alpha1  0.408976    0.054291   7.5330  0.00000
beta1   0.590024    0.049385  11.9475  0.00000
```

All the parameters are statistically significant, we reject Ho, meaning it is heteroskedastic

USTB6

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      1.857129    0.084067  22.0910 0.000000
ar1     0.984117    0.005642 174.4157 0.000000
omega   0.000226    0.000070   3.2304 0.001236
alpha1  0.392739    0.058302   6.7363 0.000000
beta1   0.606261    0.047239  12.8338 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      1.857129    0.063333  29.3233 0.000000
ar1     0.984117    0.014018  70.2028 0.000000
omega   0.000226    0.000121   1.8663 0.062004
alpha1  0.392739    0.076638   5.1246 0.000000
beta1   0.606261    0.073786   8.2165 0.000000
```

All the parameters are statistically significant except for omega, where we accept Ho meaning there is homoscedasticity

USTB10Y

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      2.046247    0.051712  39.5702  0.00000
ar1     0.995950    0.005412 184.0429  0.00000
omega   0.000045    0.000036   1.2489  0.21171
alpha1  0.103848    0.036611   2.8365  0.00456
beta1   0.895148    0.035336  25.3324  0.00000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      2.046247    0.011346 180.3510  0.00000
ar1     0.995950    0.007431 134.0291  0.00000
omega   0.000045    0.000045   1.0073  0.31381
alpha1  0.103848    0.074521   1.3935  0.16345
beta1   0.895148    0.062111  14.4121  0.00000
```

In the robust error the constant omega and alpha 1 have a p-value greater than our level of confidence, for that reason for these variables we accept Ho meaning there is homoscedasticity

Normality

For testing normality, we use the Jacque Bera test, in which we accept the null hypothesis (Ho) when there is normality, whereas as H1 is accepted when normality is not present.

Log Series Name	Pvalue	Interpretation
Microsoft	2.2e-16	Ho is **rejected** and H1 accepted, there is **not** normality
SP&500	6.468e-07	Ho is **rejected** and H1 accepted, there is **not** normality
CPI	7.757e-06	Ho is **rejected** and H1 accepted, there is **not** normality
IDPR	4.384e-11	Ho is **rejected** and H1 accepted, there is **not** normality
M1SUPPY	1.071e-07	Ho is **rejected** and H1 accepted, there is **not** normality
CCREDIT	6.344e-08	Ho is **rejected** and H1 accepted, there is **not** normality
BMINUSA	2.2e-16	Ho is **rejected** and H1 accepted, there is **not** normality
USTB3M	2.2e-16	Ho is **rejected** and H1 accepted, there is **not** normality
USTB6	1.021e-14	Ho is **rejected** and H1 accepted, there is **not** normality
USTB10Y	2.091e-06	Ho is **rejected** and H1 accepted, there is **not** normality

2.3 Presence of multicollinearity

2.3.1 Fit of model

```
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)      -35.25441    2.51302 -14.029  < 2e-16 ***
SP500_log_serie    2.09699    0.15164  13.829  < 2e-16 ***
CPI_log_serie      7.61907    0.55234  13.794  < 2e-16 ***
IDPR_log_serie     2.86478    0.74629   3.839 0.000145 ***
M1SUPP_log_serie  -1.01319    0.22368  -4.530 7.95e-06 ***
CCRED_log_serie   -2.90876    0.32627  -8.915  < 2e-16 ***
BMINUS_log_serie   0.11877    0.10482   1.133 0.257896
US3M_log_serie     0.04381    0.08371   0.523 0.601087
US6M_log_serie    -0.02032    0.10398  -0.195 0.845191
US10Y_log_serie    0.08133    0.15668   0.519 0.604022
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.346 on 375 degrees of freedom
Multiple R-squared:  0.9602,    Adjusted R-squared:  0.9592
F-statistic:  1005 on 9 and 375 DF,  p-value: < 2.2e-16
```

Figure 25: Fitness of the model

For testing the fitness of the model one measure to analyze is the R squared, because it represents the relation with explained variation and the total variation of the model.

Overall, we can conclude that the model is fit if the R squared is near one (1). After performing a linear model test, we can observe that in fact we have an adjusted R squared and multiple R squared values really close to 1, in this case both rounding to 0.96.

We conclude that it has a good goodness of fit because the model explains approximately the 96% of the data variability.

2.3.2 Diagnostic testing on the residuals of the model

<u>Testing OLS assumptions:</u>

1) *E(Res)=0*

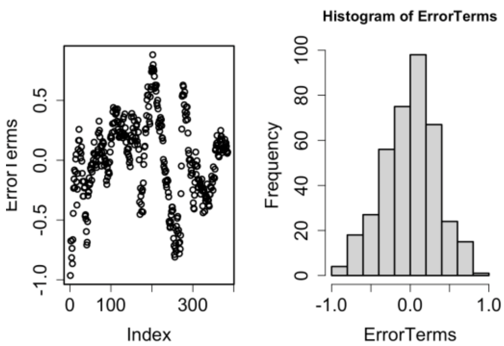


Figure 26: Error Terms plot and histogram

The first assumption is that the mean of the residuals will always be zero if there's a **constant term in the regression** in our model the error terms have a mean of 2.923347e-17 and a standard deviation of 0.3419632.

According to the output of our plots, histograms, and statistics we can say the first assumption of the OLS is respected.

2) Var(Res)=sigma^2 (constant & finite)

   For this assumption we use the following tests: White's test and ARCH test, where both have null hypothesis states that $\delta_1 = \delta_2 = 0$ (homoskedasticity).

   The p-value result for both was 2e-16, so we reject H0 and accept H1 (heteroskedasticity)

   Also, after performing the White's correction we get the following results:

```
                   Pr(>|t|)
(Intercept)       < 2.2e-16 ***
SP500_log_serie   < 2.2e-16 ***
CPI_log_serie     < 2.2e-16 ***
IDPR_log_serie     0.000381 ***
M1SUPP_log_serie  1.924e-06 ***
CCRED_log_serie   < 2.2e-16 ***
BMINUS_log_serie   0.255511
US3M_log_serie     0.604094
US6M_log_serie     0.850398
US10Y_log_serie    0.582071
```

Figure 27: White's correction results

Most of the values are heteroskedastic except for BMINUSA, USTB3M, USTB6M and USTB10Y.
The second condition is not present in all the variables of the model meaning dispersion is high at the start of the sample and then reduces.
If we compare our results of the White's Test and our Garch test, we see that in fact the series has an overall heteroskedastic behavior.

3) Residuals not autocorrelated

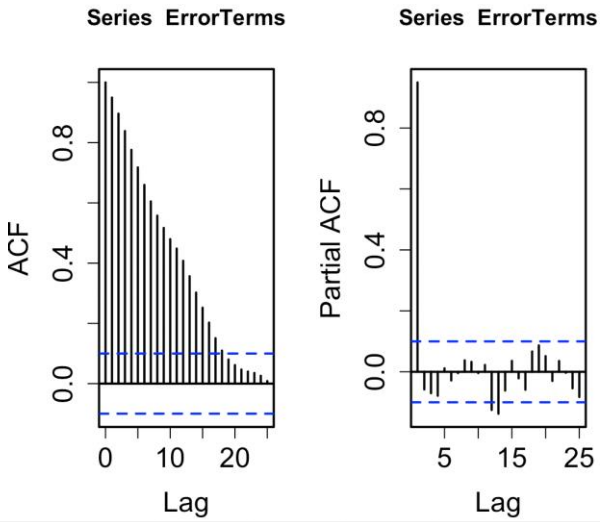


Figure 28: Error Terms ACF and PACF

We see that the ACF or Error Terms are steadily decreasing and regarding the PACF of the Errors we see that it sometimes decays, and it goes outside the range, nevertheless it seems to remain stationary
In addition, we also used the Box Pierce test also known as the Ljung-Box test where we use the following hypotheses:
Ho: The residuals are independently distributed.
H1: The residuals are not independently distributed; they exhibit serial correlation
For this test, we obtain a p-value of 2.2e-16, meaning the null hypothesis is rejected, in other words, the error terms are independently distributed, and they exhibit serial correlation.
Once again, this assumption of OLS is not respected.

4) X(t) and e(t) non-correlated
When testing this assumption, we observe that p-value is 8.101e-05 , hence X(t) and e(t) exhibit serial correlation since the null hypothesis is rejected.

5) Residuals are normally distributed
For testing normality, we perform the Bera-Jarque test where the null hypothesis is accepted when we're working with normality. Our p-value = 0.2104 so, we accept the null hypothesis, meaning that our residuals are normally distributed.

2.3.3 Presence of possible structural breaks in the coefficients

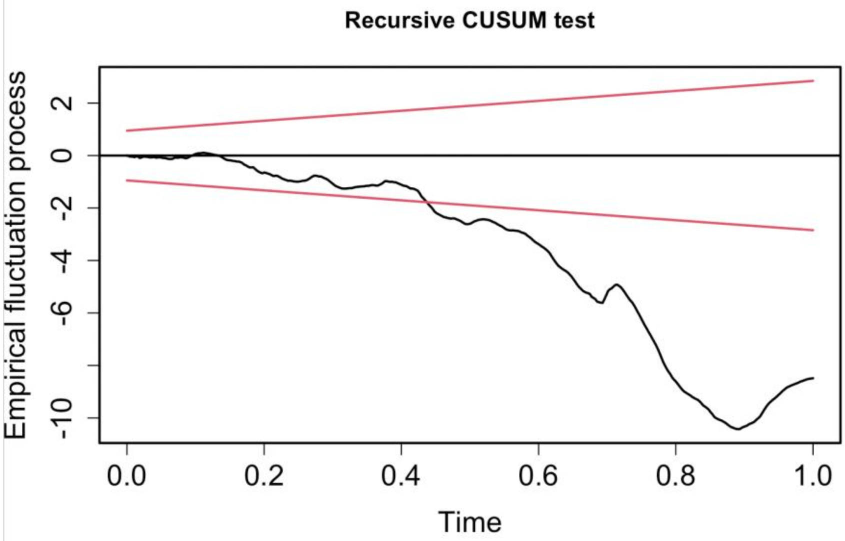


Figure 29: Recursive CUSUM test

For the structural breaks we use the Chow Test where we test the null hypothesis of no structural breaks.

We also use the CUSM test, where we plot and test the null hypothesis of no structural breaks.

The p-value of the Chow test is 1.257e-11.

We reject the null hypothesis of those structural breaks since the p-value of the Chow test is less than our level of confidence and also because of the behavior of the plot of the CUSUM test

2.4 eGarch test

eGARCH models volatility as a function of past squared residuals and past volatility. Specifically, the e-GARCH model uses an exponential function to model the volatility, which allows it to capture both positive and negative shocks to the volatility

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu       1.40037    0.018431 75.97873 0.000000
omega   -0.41984    0.186680 -2.24901 0.024512
alpha1   0.07151    0.094234  0.75886 0.447937
beta1    0.95178    0.063222 15.05470 0.000000
gamma1   2.18071    0.790699  2.75796 0.005816

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu       1.40037    0.024727  56.6329 0.000000
omega   -0.41984    0.194455  -2.1591 0.030844
alpha1   0.07151    0.026288   2.7203 0.006523
beta1    0.95178    0.036966  25.7473 0.000000
gamma1   2.18071    0.849004   2.5686 0.010212
```

Figure 30: eGarch Test

To test the asymmetric effects, we must focus on results of Gamma1, in this case it is 0.067, greater than our level of confidence of level. So, we rejected our null hypothesis, and we conclude that there are not asymmetric effects.

2.5 GARCH-DCC test

GARCH-DCC model allows for the volatility of multiple time series to be affected by the past values of the volatilities of all the time series

```
Optimal Parameters
------------------------------------
                              Estimate  Std. Error  t value Pr(>|t|)
[MCSFT_log_serie].mu          3.284127    0.010753 305.4154 0.000000
[MCSFT_log_serie].omega       0.002532    0.000972   2.6052 0.009182
[MCSFT_log_serie].alpha1      0.763916    0.141403   5.4024 0.000000
[MCSFT_log_serie].beta1       0.217398    0.113995   1.9071 0.056510
[SP500_log_serie].mu          7.145071    0.022078 323.6233 0.000000
[SP500_log_serie].omega       0.001422    0.000691   2.0575 0.039639
[SP500_log_serie].alpha1      0.999000    0.442071   2.2598 0.023832
[SP500_log_serie].beta1       0.000000    0.417103   0.0000 1.000000
[Joint]dcca1                  0.225138    0.044381   5.0728 0.000000
[Joint]dccb1                  0.774544    0.044524  17.3961 0.000000
```

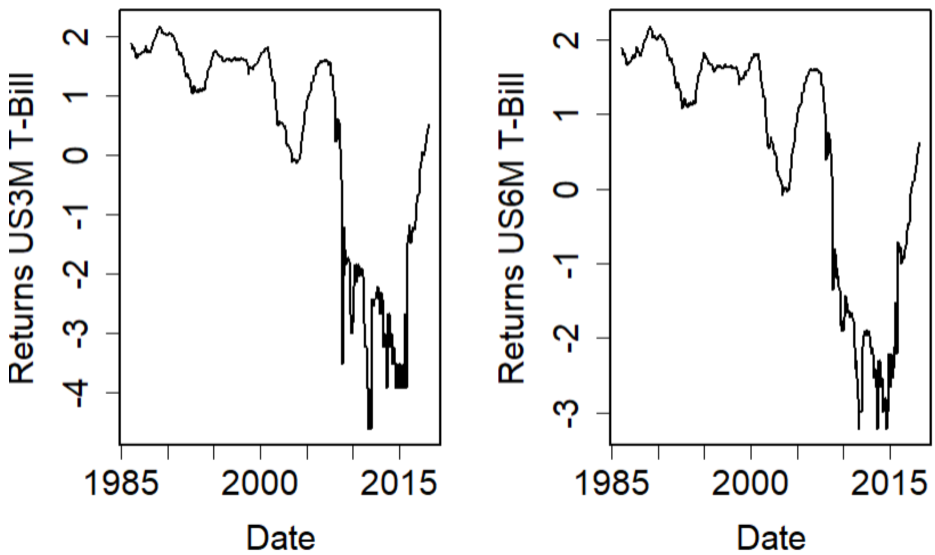
Figure 31: Garch Test

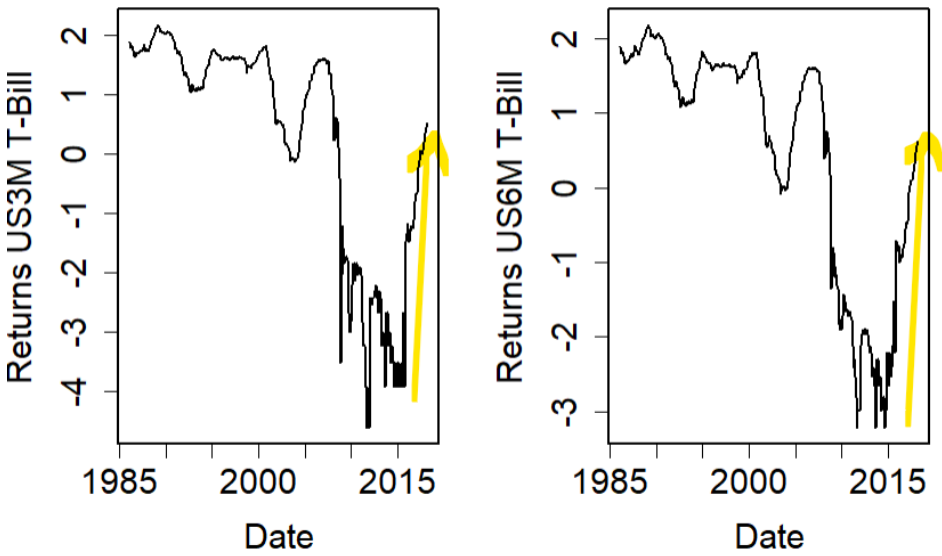
For this test they null hypothesis is stated by "Ho: non serial correlation"

After analyzing our output, we can see that all values are significant because we reject the null hypothesis except for beta1 of Microsoft (reject 95%) and the S&P coefficient.

Task 3

3.1 Observe the plot of the three-month and six-month maturity yields in Task 1.1. Could the series be cointegrated?





For the series to be cointegrated means that no matter what differences we have between them in the short term, in the long term they are bound, and we observe an equilibrium, in other words, they converge.

Here the series seems to share the exact same mean and variance up to 2008. After that shock, we can see some differences in the charts, however, they seem to converge again later. Hence, we can guess a long-term equilibrium, the series could be cointegrated.

3.2 Regress the six-month yields on the three-month yields, save the output and comment the results. Test for the presence of a cointegrating relationship between the two yield series. Can we conclude that the series of the sixmonth and three-month yield are actually cointegtated? If "yes", what does it mean? Is the OLS estimator a valid estimator in this case? Motivate your answer

Before doing the regression and cointegration test we should first establish two properties of our series:

- 1 : Are they stationary? (There is no point to check cointegration if they are stationary)

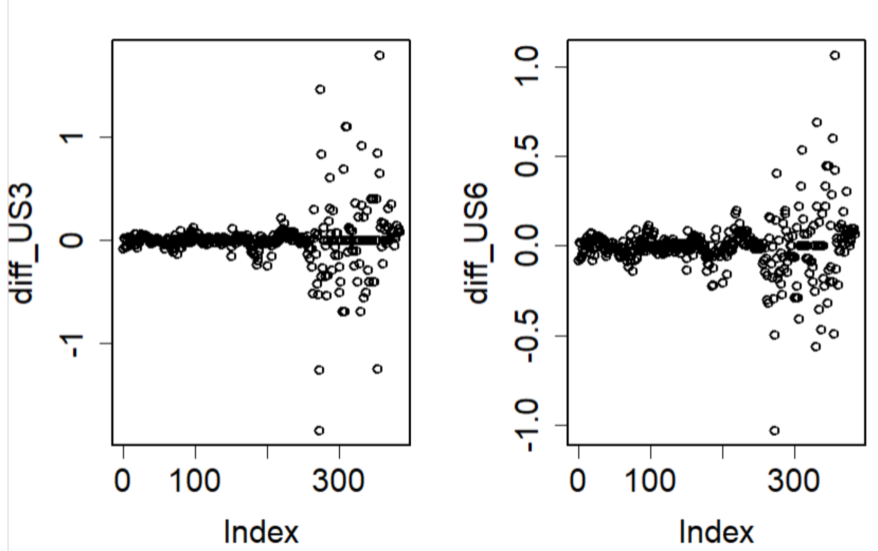
- 2: Are they integrated of the same order? (If they are not the same order, we can't find at least one linear combination which is stationary)

We established earlier that our series are non-stationary.

Log Series Name	P-value	Interpretation
Microsoft	0.5373	we accept Ho: non stationarity
SP&500	0.4428	we accept Ho: non stationarity
CPI	0.3111	we accept Ho: non stationarity
IDPR	0.8341	we accept Ho: non stationarity
M1SUPPY	0.6335	we accept Ho: non stationarity
CCREDIT	0.7234	we accept Ho: non stationarity
BMINUSA	0.2627	we accept Ho: non stationarity
USTB3M	0.2801	we accept Ho: non stationarity
USTB6M	0.1648	we accept Ho: non stationarity
USTB10Y	0.1683	we accept Ho: non stationarity

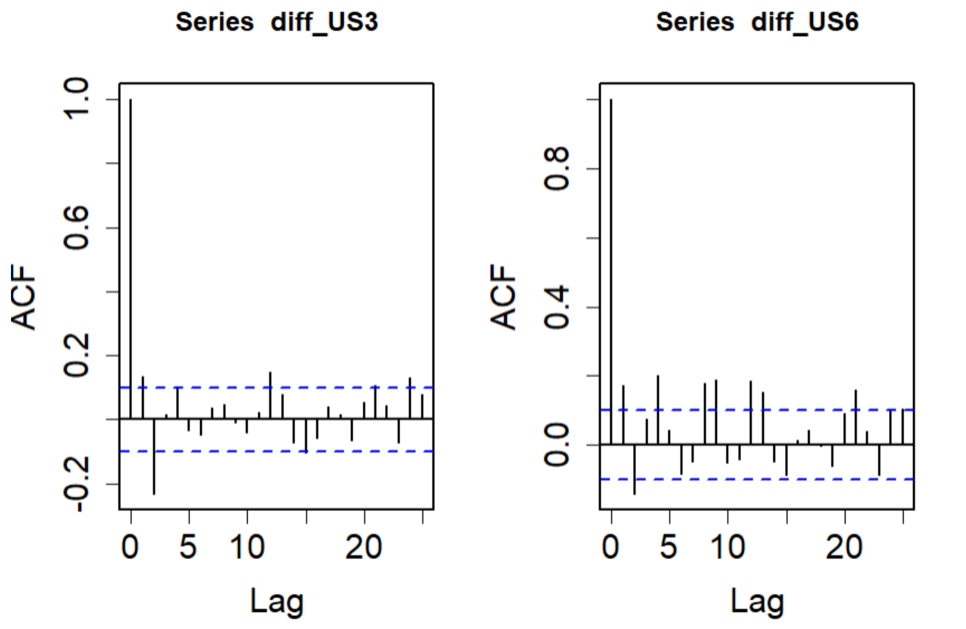
Now let's study if they are both integrated of the same order.

If we differentiate the series one time and plot the series, we can see that they are smoother around the mean.



testing if the series differentiated once becomes stationary of US3M

The ACF plots show us that after differentiating one time the series, their errors die out very quickly, which is a good sign for stationarity.



ACF test for differentiation test of US3M and US6M

Now we perform an ADF test on the differentiated series to check whether they are stationary or not.

Null hypothesis: No-stationarity

Alternative hypothesis: Stationarity

```
                Augmented Dickey-Fuller Test

data:  diff_US3
Dickey-Fuller = -11.457, Lag order = 2, p-value = 0.01
alternative hypothesis: stationary
```

```
	Augmented Dickey-Fuller Test

data:  diff_US6
Dickey-Fuller = -10.234, Lag order = 2, p-value = 0.01
alternative hypothesis: stationary
```

Here we inputted "Lag order = 2" because based on what we saw on the ACF plots that it takes 2 lags for the errors to die out.

Our p-values are less than 0.05, in fact they are so small that R only display 0.01 but for this test we will be ok with a 95% level.

We reject the null hypothesis, the series differentiated one time are stationary. US3M and US6M series are integrated of the first order I (1).

We can now test for the cointegration relationship.

The regression gives us the coefficients below.

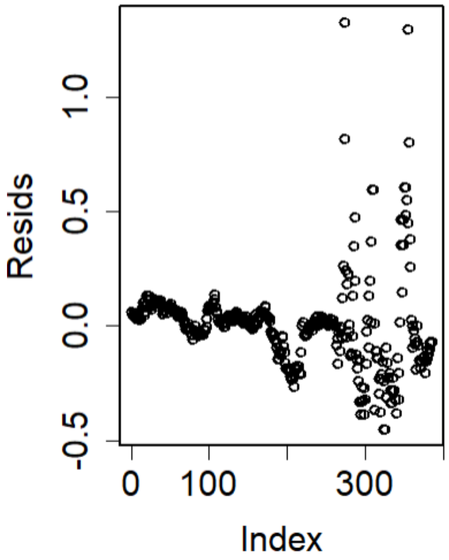
```
Call:
lm(formula = US6M_log_serie ~ US3M_log_serie)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.45202 -0.09022  0.01171  0.05639  1.32593 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)    0.253362   0.009951   25.46   <2e-16 ***
US3M_log_serie 0.834542   0.005326  156.68   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1928 on 383 degrees of freedom
Multiple R-squared:  0.9846,	Adjusted R-squared:  0.9846 
F-statistic: 2.455e+04 on 1 and 383 DF,  p-value: < 2.2e-16
```

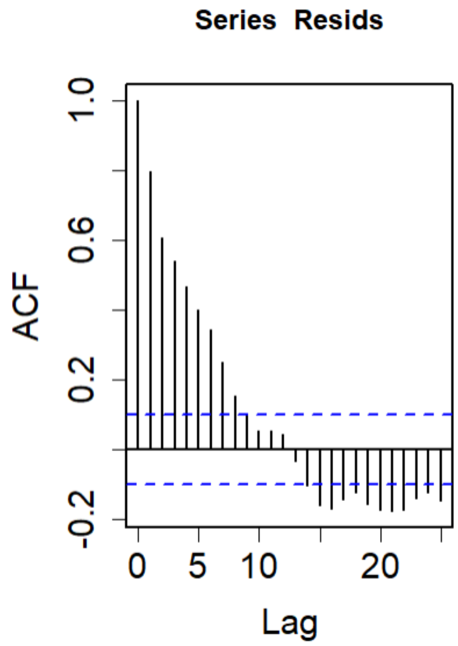
The intercept of the linear regression is 0.25 and the slope coefficient is 0.83.

If we test the statistical significance of these coefficients, we can state that the p-values being considerably small, since they are far below 0.05 or even 0.01..

Hence, we reject the null hypothesis that the intercept or the slope coefficient are statistically not different from 0 at 1% significance level.

We also saved the residuals from the linear regression in the below left chart. We also plot an ACF on the residuals.





To test the cointegration we must do a unit root test on the residuals to assess stationarity (Engle-Granger approach). Here we can first see that the errors die out relatively quickly on the ACF chart. Which again is a good hint for stationarity.

For the unit root test, we will set the lag to 10 as per the ACF chart. We also input "nc" in the parameter for "no trend, no constant" according to what we can see in the residuals plot above.

Null hypothesis: No-stationarity

Alternative hypothesis: Stationarity

```
Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 10
  STATISTIC:
    DF: -4.2467
  P VALUE:
    t: 2.669e-05
    n: 0.1562
```

With our p-value of **2.669e-05** we reject the null hypothesis at 99% confidence interval, our residuals are stationary.

Because our residuals are stationary, I(0), we can reject the null hypothesis of the Engle-Granger test, equivalently, we reject that there is no co-integration relationship between our series.

Hence, we know that the series are bound on the long term, furthermore, we have a linear combination of 2 non-stationary series which is stationary. Using the OLS estimator on non-stationary series can lead to spurious regression. However, cointegration implies that the OLS will converge to the true value.

Hence we could do forecast with the OLS estimator and have good faith in its ability to predict reality.

3.3 Based on your results and conclusion in 3.2, can we express the relationship between the six-month and three-month yield through an error correction model (ECM)? Motivate your answer and if affirmative, estimate the appropriate ECM model. Provide the economic interpretation of the coefficients

We proved that the series were cointegrated, meaning that in long term they converge to the same values. However, in the short term we still have differences. To describe the series in the short term we must make some corrections, hence doing an Error Correction Model (ECM).

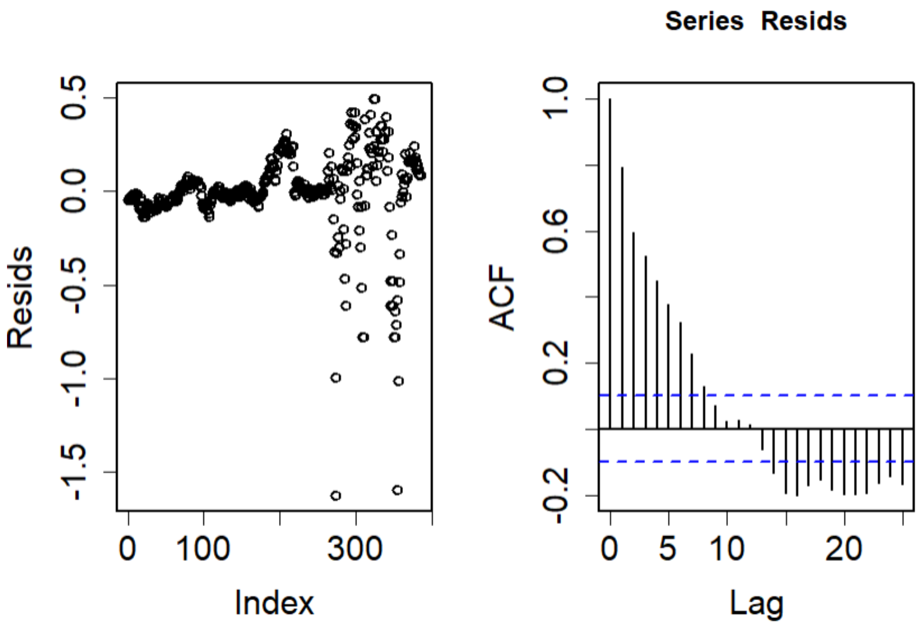
```
Call:
lm(formula = diff_US6 ~ diff_US3 + ResidsAdj)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.45501 -0.02547  0.00069  0.02380  0.47272 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.001496   0.004033  -0.371    0.711    
diff_US3     0.500002   0.015847  31.551  < 2e-16 ***
ResidsAdj   -0.100734   0.021489  -4.688 3.85e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.07902 on 381 degrees of freedom
Multiple R-squared:  0.7244,    Adjusted R-squared:  0.7229 
F-statistic: 500.6 on 2 and 381 DF,  p-value: < 2.2e-16
```

The intercept of the model is –0.001496, the 0.500002 represents the short-term adjustment of the US6M series to a change in the US3M series, the ResidsAdj of –0.100734 represents the long-term adjustment to the equilibrium.

Furthermore, we can see that apart from our intercept, our coefficients are statistically different from 0 considering their p-values, at a level of 99% significance. The intercept not being the most important coefficient here, we believe it is not damaging to the model ability to perform accurate regression.

3.4 Regress the three-month yields on the 6-month yields. Do the results in tasks 3.2 and 3.4 change? Provide an economic explanation of your answer.

We established that our series were non-stationary and integrated of order 1, so let's have a look directly at the residual of the regression.



We can spot an inverse pattern for the residuals plot compared to the regression of US6M over US3M series.

However, the ACF remains the same. We perform again an ADF test for stationarity among the residuals.

Null hypothesis: No-stationarity

Alternative hypothesis: Stationarity

```
Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 10
  STATISTIC:
    DF: -4.3785
  P VALUE:
    t: 1.53e-05
    n: 0.1498
```

With a different p-value, yet enough to reject the null hypothesis, we can say the residuals of US3M over US6M are stationary. Hence our series are cointegrated based on the Engle-Granger approach.

We have the same relationship between our series that above, hence let's jump to the ECM model estimation.

```
Call:
lm(formula = diff_US3 ~ diff_US6 + ResidsAdj)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.07666 -0.02499  0.00070  0.03203  0.94297 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.001165   0.006854   0.170    0.865    
diff_US6     1.443920   0.045828  31.507  < 2e-16 ***
ResidsAdj   -0.194456   0.030021  -6.477 2.89e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1343 on 381 degrees of freedom
Multiple R-squared:  0.7374,	Adjusted R-squared:  0.736 
F-statistic: 534.9 on 2 and 381 DF,  p-value: < 2.2e-16
```

The intercept of the model is 0.001165, the 1.443920 represents the short-term adjustment of the US3M series to a change in the US6M series, the ResidsAdj of –0.194456 represents the long-term adjustment to the equilibrium.

Furthermore, we can see that apart from our intercept, our coefficient is statistically different from 0 considering their p-values, at a level of 99% significance.

So far, the results of 3.2 seems unaffected by the order of the regression. Meaning that no matter if we regress the US3M series of the US6M series or the opposite, we still come across the cointegration relationship at the end.

However, the results of 3.3 are different. We estimate the ECM which gives us coefficient that are different.

We have for instance two different short-term adjustment values: [0.500002; 1.443920].

That means that the regression order has an impact on the correction strength applied to our series and the way (negative/positive).

Task 4

4.1 Estimate a VAR model for the three-treasury bill yields for the following order: three-month maturity, six-month maturity, and ten-year maturity. Use the information criteria to determine the appropriate lag length [Hint: Use lag.max=5]. Save the estimation output of your selected VAR on the Word document and comment the results.

A Vector Autoregressive Model (VAR) is a system of more than one variable. This model allows the value of a variable to not depend only on its own lags and errors terms.

We have to select the appropriate lag length. We should pick the lag length which minimizes our loss of information. Equivalently, the information criteria is a measure of loss of information by adding lags. In that case we will select a maximum lag of 5, beyond 5 lags, let's say up to 20, it is an overfitting of the model, which is useless.

We can see from the below results that with the AIC we will take 5 lags, but only 1 if we pick the SC.

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     5      2      1      5

$criteria
                   1             2             3             4             5
AIC(n) -1.373550e+01 -1.380783e+01 -1.379161e+01 -1.383178e+01 -1.387412e+01
HQ(n)  -1.368602e+01 -1.372125e+01 -1.366793e+01 -1.367098e+01 -1.367622e+01
SC(n)  -1.361083e+01 -1.358965e+01 -1.347993e+01 -1.342660e+01 -1.337543e+01
FPE(n)  1.083304e-06  1.007726e-06  1.024223e-06  9.839436e-07  9.432196e-07
```

Assessing the SC information criteria for 1 lag.

With 1 lag, we have the VAR results displayed below for our series.

US3M T-bill:

```
Estimation results for equation diff_US3:
=========================================
diff_US3 = diff_US3.l1 + diff_US6.l1 + diff_US10.l1 + const

              Estimate Std. Error t value Pr(>|t|)    
diff_US3.l1   0.411908   0.092735   4.442 1.17e-05 ***
diff_US6.l1  -0.574934   0.164673  -3.491 0.000537 ***
diff_US10.l1 -0.015896   0.243632  -0.065 0.948011    
const        -0.003808   0.013099  -0.291 0.771415    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.2561 on 379 degrees of freedom
Multiple R-Squared: 0.0497,	Adjusted R-squared: 0.04218 
F-statistic: 6.607 on 3 and 379 DF,  p-value: 0.0002318 
```

The coefficients US3.l1 and US6.l1 are statistically significant at a level of 99%, meaning we can reject the null hypothesis of these coefficients being not statistically significantly different from 0.

Rejecting that null hypothesis means that our values lie close to the real value. Hence, we can trust these and their impact on the series.

However, this is not the case for the coefficients US10.l1 and const, which are not statistically significant at a level of 10%.

Economically speaking, our above results mean that at a 1% level, US3m and US6M lag 1 has a significant impact (positive and negative respectively) on the US3M T-bill series whereas at a 10% level the series US10Y and the const lag 1 do not have a significant impact on our US3M T-bill series.

US6M T-bill:

```
Estimation results for equation diff_US6:
=========================================
diff_US6 = diff_US3.l1 + diff_US6.l1 + diff_US10.l1 + const

              Estimate Std. Error t value Pr(>|t|)    
diff_US3.l1   0.321303   0.051136   6.283 9.11e-10 ***
diff_US6.l1  -0.316970   0.090804  -3.491 0.000538 ***
diff_US10.l1  0.136948   0.134343   1.019 0.308668    
const        -0.002616   0.007223  -0.362 0.717428    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.1412 on 379 degrees of freedom
Multiple R-Squared: 0.1238,	Adjusted R-squared: 0.1169 
F-statistic: 17.86 on 3 and 379 DF,  p-value: 7.347e-11
```

The US3M and Us6M lag 1 coefficient are significant at a level of 1% meaning that they have a significant impact (positive and negative respectively) on the US6M T-bill series.

Furthermore, the coefficients US10Y and const lag 1 are both statistically significant and do not have a significant impact on the series.

US10Y T-bill:

```
Estimation results for equation diff_US10:
==========================================
diff_US10 = diff_US3.l1 + diff_US6.l1 + diff_US10.l1 + const

              Estimate Std. Error t value Pr(>|t|)
diff_US3.l1   0.058156   0.020149   2.886  0.00412 **
diff_US6.l1  -0.098743   0.035780  -2.760  0.00607 **
diff_US10.l1  0.238109   0.052936   4.498 9.13e-06 ***
const        -0.001967   0.002846  -0.691  0.48998
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.05564 on 379 degrees of freedom
Multiple R-Squared: 0.07304,    Adjusted R-squared: 0.0657
F-statistic: 9.954 on 3 and 379 DF,  p-value: 2.488e-06
```

Here we can see a change. All our lags 1 are statistically significant at a level of 1% except for the const. But that means that for our 3 lags 1, US3M, US6M and US10Y, they have a significant impact on the US10Y T-bill series.

Correlation matric of Residuals:

```
Correlation matrix of residuals:
          diff_US3 diff_US6 diff_US10
diff_US3    1.0000   0.8460    0.2933
diff_US6    0.8460   1.0000    0.3281
diff_US10   0.2933   0.3281    1.0000
```

We can see that our 10Y series is more and more correlated to short term series, the more they get close to 10Y maturity, until it reaches 1 when we regress a 10Y on a 10y series. That probably highlight an underlying relationship.

Assessing the information criteria for AIC with 3 lags.

With 3 lags, we have the VAR results displayed below for our series.

US3M T-bill:

```
Estimation results for equation diff_US3:
=========================================
diff_US3 = diff_US3.l1 + diff_US6.l1 + diff_US10.l1 + diff_US3.l2 + diff_US6.l2 + diff_US10.l2 + di
ff_US3.l3 + diff_US6.l3 + diff_US10.l3 + const

              Estimate Std. Error t value Pr(>|t|)
diff_US3.l1   0.330354   0.095635   3.454 0.000615 ***
diff_US6.l1  -0.288168   0.172263  -1.673 0.095203 .
diff_US10.l1  0.104653   0.243439   0.430 0.667523
diff_US3.l2  -0.510712   0.096653  -5.284 2.16e-07 ***
diff_US6.l2   0.620854   0.171610   3.618 0.000338 ***
diff_US10.l2 -0.204503   0.250800  -0.815 0.415365
diff_US3.l3  -0.031950   0.098797  -0.323 0.746584
diff_US6.l3   0.072248   0.167271   0.432 0.666050
diff_US10.l3  0.355826   0.244704   1.454 0.146762
const        -0.001979   0.012687  -0.156 0.876133
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.2468 on 371 degrees of freedom
Multiple R-Squared: 0.1356,     Adjusted R-squared: 0.1147
F-statistic: 6.468 on 9 and 371 DF,  p-value: 1.468e-08
```

We can notice here that at a level of 10% only the US3 lags 1 and 2 and US6M lags 1 and 2 have a significant impact on the Us3M T-bill series.

US6M T-bill:

```
Estimation results for equation diff_US6:
=========================================
diff_US6 = diff_US3.l1 + diff_US6.l1 + diff_US10.l1 + diff_US3.l2 + diff_US6.l2 + diff_US10.l2 + di
ff_US3.l3 + diff_US6.l3 + diff_US10.l3 + const

              Estimate Std. Error t value Pr(>|t|)
diff_US3.l1   0.271927   0.053550   5.078 6.05e-07 ***
diff_US6.l1  -0.161875   0.096457  -1.678  0.09415 .
diff_US10.l1  0.189877   0.136311   1.393  0.16446
diff_US3.l2  -0.175616   0.054120  -3.245  0.00128 **
diff_US6.l2   0.123407   0.096091   1.284  0.19985
diff_US10.l2 -0.066835   0.140432  -0.476  0.63441
diff_US3.l3  -0.002645   0.055320  -0.048  0.96188
diff_US6.l3   0.076027   0.093661   0.812  0.41747
diff_US10.l3  0.174774   0.137019   1.276  0.20292
const        -0.001836   0.007104  -0.259  0.79615
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.1382 on 371 degrees of freedom
Multiple R-Squared: 0.1782,	Adjusted R-squared: 0.1582
F-statistic: 8.938 on 9 and 371 DF,  p-value: 3.154e-12
```

Here we see that only the lags of US3M has a significant impact on the US6M series at a level of 1%

US10Y T-bill:

```
Estimation results for equation diff_US10:
==========================================
diff_US10 = diff_US3.l1 + diff_US6.l1 + diff_US10.l1 + diff_US3.l2 + diff_US6.l2 + diff_US10.l2 + d
iff_US3.l3 + diff_US6.l3 + diff_US10.l3 + const

              Estimate Std. Error t value Pr(>|t|)
diff_US3.l1   0.048805   0.021381   2.283   0.0230 *
diff_US6.l1  -0.069650   0.038513  -1.808   0.0713 .
diff_US10.l1  0.276919   0.054425   5.088 5.76e-07 ***
diff_US3.l2  -0.031817   0.021609  -1.472   0.1418
diff_US6.l2   0.024636   0.038367   0.642   0.5212
diff_US10.l2 -0.128800   0.056071  -2.297   0.0222 *
diff_US3.l3   0.017478   0.022088   0.791   0.4293
diff_US6.l3  -0.042866   0.037397  -1.146   0.2524
diff_US10.l3  0.102066   0.054708   1.866   0.0629 .
const        -0.002154   0.002836  -0.759   0.4482
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.05518 on 371 degrees of freedom
Multiple R-Squared: 0.1049,	Adjusted R-squared: 0.08317
F-statistic:  4.83 on 9 and 371 DF,  p-value: 4.072e-06
```

There we see that at a level of 10%, all our lag 1 have a significant impact, furthermore only the lags 2 and 3 of the US10Y series have a significant impact on the US10Y T-bill series.

Conclusion:

By comparing the SC and AIC information criteria, we can see that first, not all coefficients have a significant impact on the series whether it is the US3M,6M or 10Y. We spotted that US3M series is more impacted by its own lag than by the other series' lag. Then, when we get a broader maturity (3M to 6M) the series is affected by its own lag but also by the shorter maturity series lags (see the US6M series coefficients significance).

Finally, the US10Y series is impacted significantly by the lags of all series and its own.

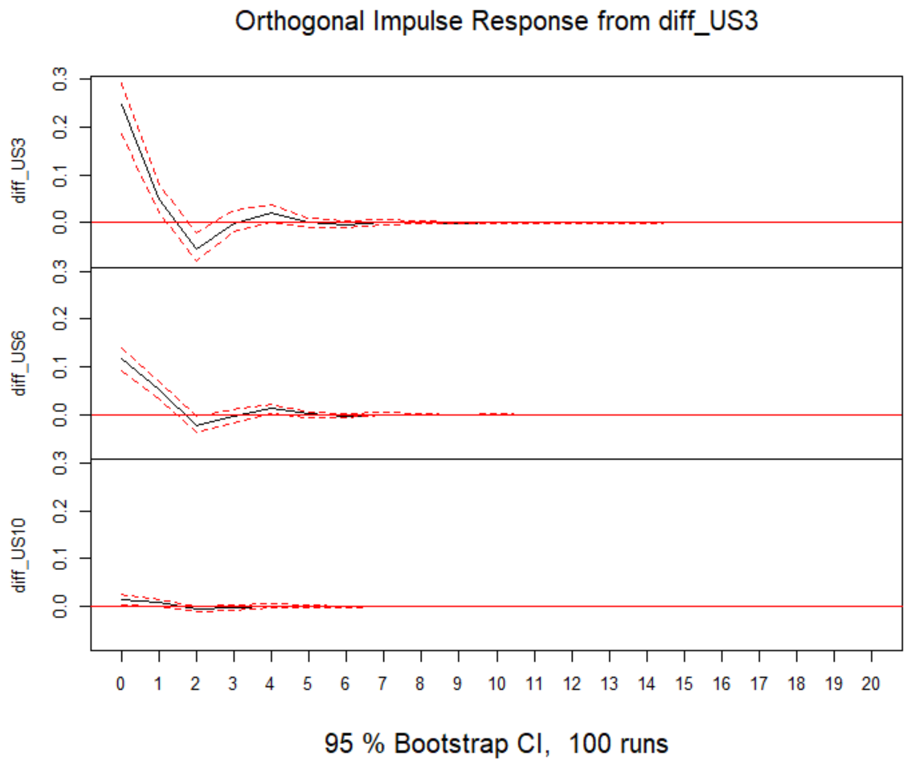
We can thus observe a gradation of the explicative power of coefficients on one series the more we get to higher maturity.

4.2 Obtain the impulse responses and the forecast error variance decomposition (FEVD) of the estimated VAR, save plots and results on your Word document and comment them. Provide an economic interpretation of your results.

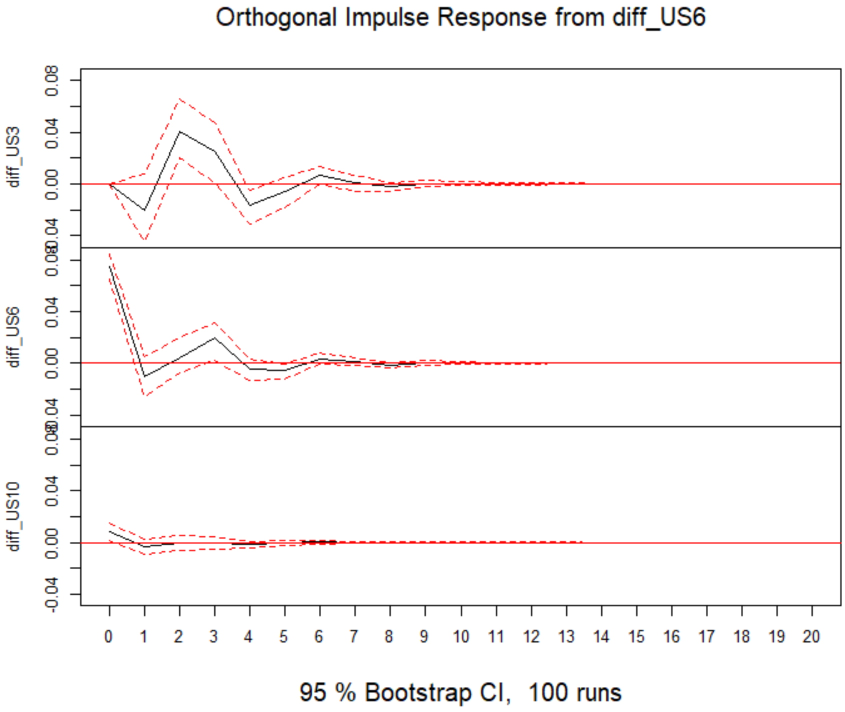
The impulse response tracks the responsiveness of the dependent variable in the VAR model to shocks to the error term. Hence, we can see how long a shock persists and in what degree.

In the below case, the impulse response from US3M T-bill to the variable itself and US6M and Us10Y.

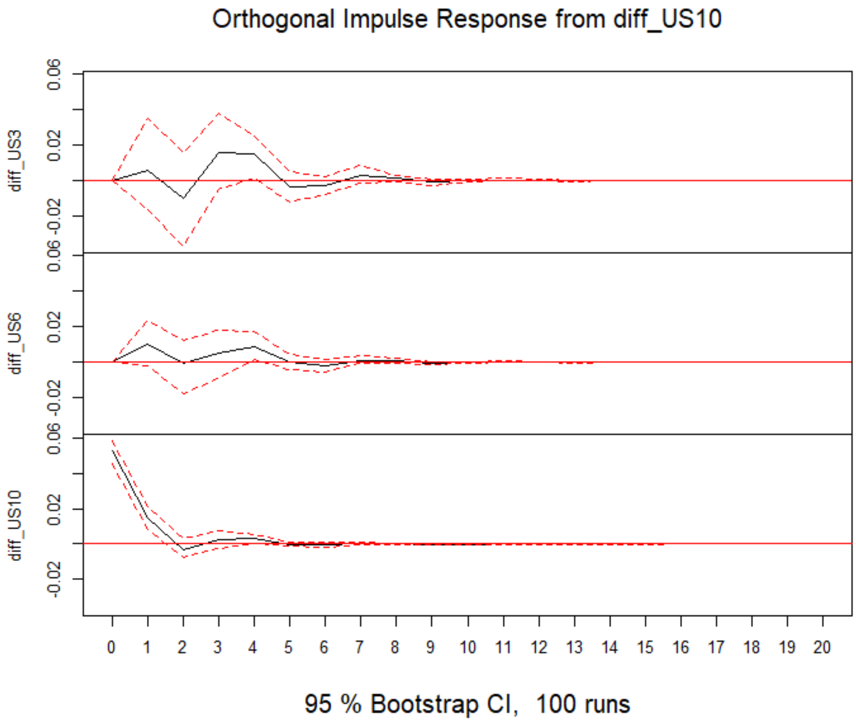
We can see that the shock doesn't have the same duration for all 3 variables of the VAR model. The US3Mseries is very responsive to a shock in its variable and to a lesser extent to a shock in the US6M variable. The US3M is almost irresponsive to a shock in the US10Y variable. Also noting that the responsiveness is of the same order (negative/positive) either for shocks to US3M and US6M variable.



Samely, we can see that the US6M series is very responsive to shocks on the US3M variable but also in the same degree to a shock in the variable itself. Finally, it seems irresponsive too to a shock in the US10Y variable. Also noting that here, the responsiveness of the series is of the same order (negative/positive) either for shocks to US3M and US6M variable.



Finally, the US10Y series is also very responsive to a shock in US3M and US6M variables but also to itself. Nothing that the responsiveness of the series to shocks in itself is the opposite of shocks in US3M and US6M. Which could describe an inverse relationship. Taking into account that further lags than 1 are not always statistically significant (observed in previous question) we base our hypothesis of inverse relationship solely on the lag 1 observation.



From an economic viewpoint, we can say that:

- US3M series is very impacted by itself, and to a lesser extent by US6M and not impacted by US10Y
- US6M series is very impacted by US3M and itself in almost the same degree and not impacted by US10Y
- US10Y series is very impacted by US3M, US6M and itself.

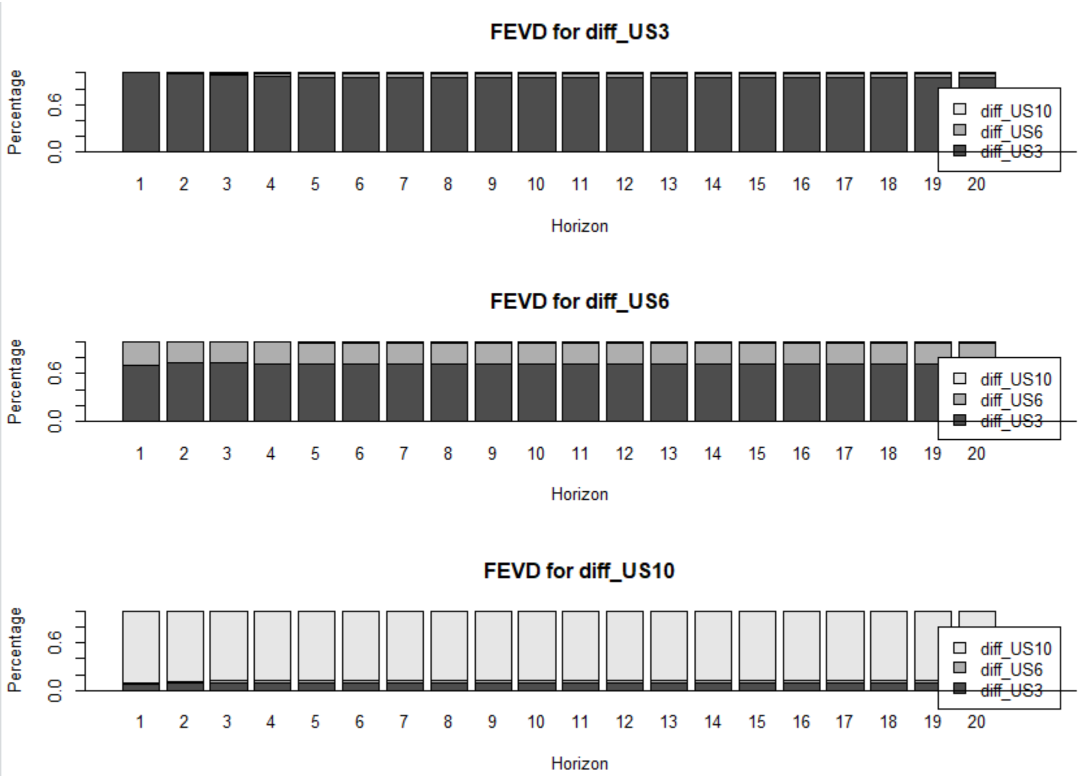
- There is an inverse relationship between the responsiveness to shocks on the US10Y series when we shock the short-term maturity and the long term one. When short term series moves up, our long-term series moves down.

The forecast error variance decomposition is another way to review the VAR system's dynamics. It gives the proportion of the movements in the dependent variables that are due to their "own" shocks, opposed to shocks in other variables. When forecasting, it indicates how much variability of this forecast depends on the others variables.

It shows the contribution of each variable to the forecast errors.

We can see from the below that not all the variation in US3M T-bill series that is due only from shocks to US3M T-bill series. The US6M has a small part of explanation (less than 10%). Samely, not all the variation in the US10Y T-bill series is due to shocks in itself but a small part is caused by the US3M series, in a bigger proportion than US6M by the way (almost 15%).

Finally, the variation in the US6M series is due to shocks in itself and in US3M (almost 40%)



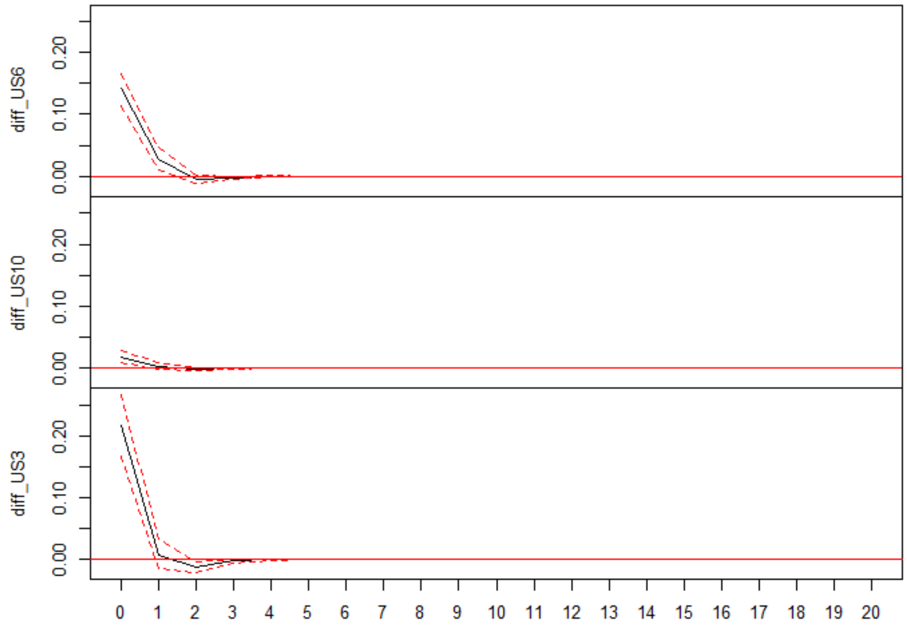
4.3 Verify whether results in Task 4.2 are sensitive to the variable ordering. Discuss the economic implications of your results when compared to results in Task 4.2.

When we order the variables differently, let's change the order to US6M, US10Y, US3M. We get the following impulse response dans forecast error variance decomposition.

For comparison, for the impulse response from US6M to shocks in US3M we see that shock dies out in period 4 instead of 9. In general, we get a totally different plot for the responsiveness of US6M to shocks in US3M or US6M. There are less up-and-down. However, shocks in US10Y for these series keep the same pattern.

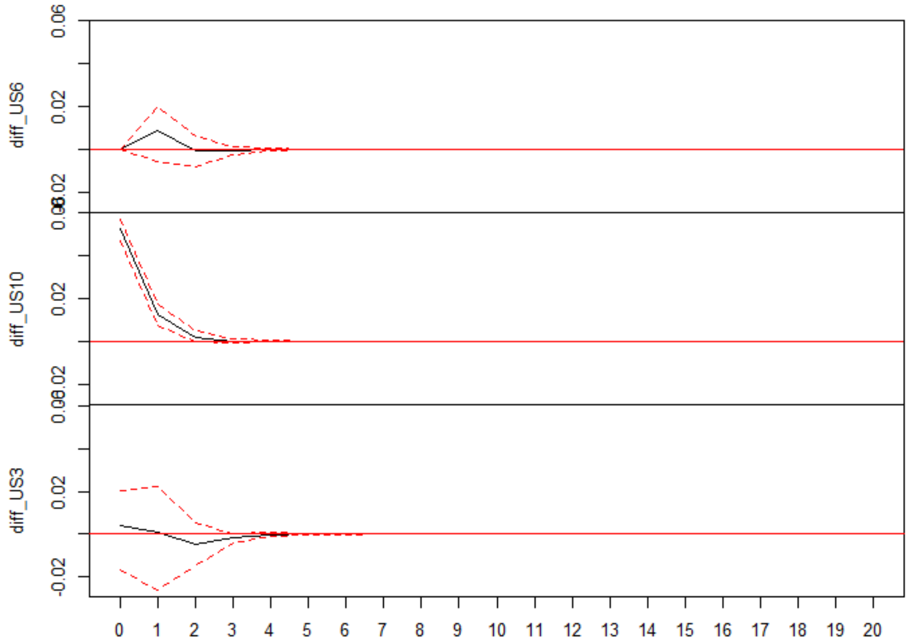
Finally, for the FEVD, we can see roughly the same proportions, this change has not significantly changed this.

Orthogonal Impulse Response from diff_US6

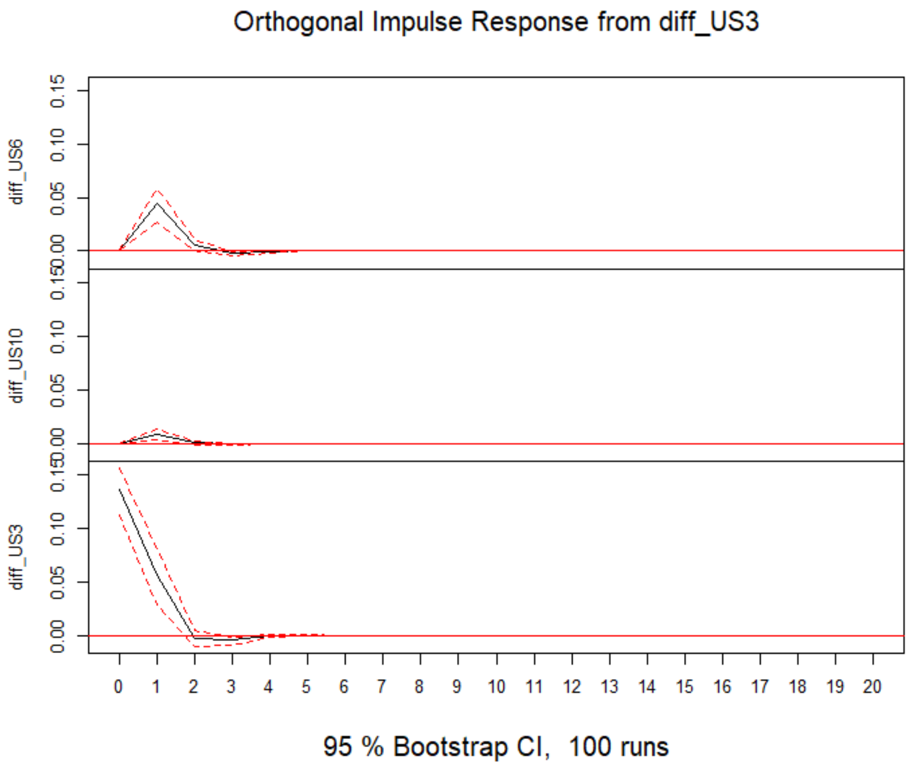


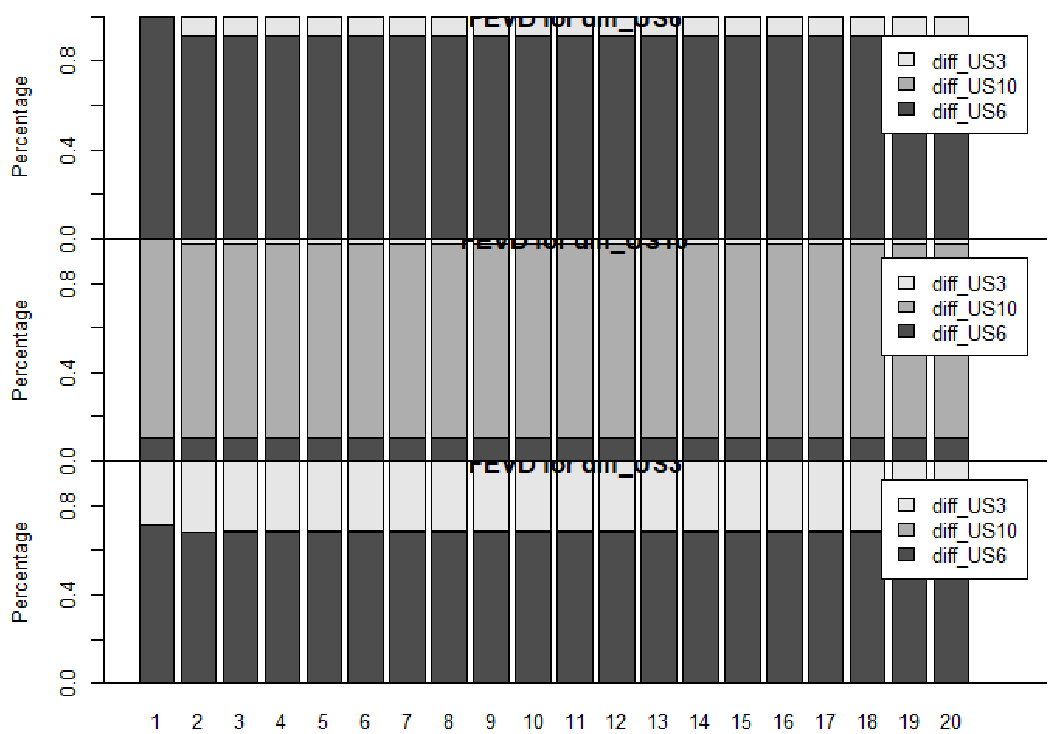
95 % Bootstrap CI, 100 runs

Orthogonal Impulse Response from diff_US10



95 % Bootstrap CI, 100 runs





Overall we see that changing the order of the VAR system can imply a huge reduction of time it takes for shock to die out, however it doesn't apply to all series. And the FEVD seems to remain stable.

Economically speaking it means that our VAR model ability to do predictions is affected by the order of the variables. There is a vast difference between a series whose shocks die out quickly and one whose shocks persist longer. Even if the contribution of each variable to the model errors doesn't change that much, the persistance of shocks on many lags can imply spurious forecasts because the series whose forecasted won't be stationary.

4.4 Use the Johansen approach to model the system of yields and to test for cointegration between them. Discuss your results in light of the empirical evidence in Shea (1992)

The Johansen test is used to test the cointegration of the series, where the null hypothesis means that there is no cointegration between series and the alternative hypothesis means that there is a cointegrating relationship between two or possibly more time series.

We first determine the number of lags to minimize our AIC to fill this argument in the Johansen R function.

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     5      2      1      5

$criteria
                   1             2             3             4             5
AIC(n) -1.373550e+01 -1.380783e+01 -1.379161e+01 -1.383178e+01 -1.387412e+01
HQ(n)  -1.368602e+01 -1.372125e+01 -1.366793e+01 -1.367098e+01 -1.367622e+01
SC(n)  -1.361083e+01 -1.358965e+01 -1.347993e+01 -1.342660e+01 -1.337543e+01
FPE(n)  1.083304e-06  1.007726e-06  1.024223e-06  9.839436e-07  9.432196e-07
```

In this case it is 5 lags. Which gives us the below values for the Johansen approach.

```
######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend in cointegration

Eigenvalues (lambda):
[1] 2.560666e-01 1.978450e-01 9.609388e-02 5.551115e-17

Values of teststatistic and critical values of test:

           test 10pct  5pct  1pct
r <= 2 |  38.29 10.49 12.25 16.26
r <= 1 | 121.84 22.76 25.32 30.45
r = 0  | 233.95 39.06 42.44 48.45

Eigenvectors, normalised to first column:
(These are the cointegration relations)

               diff_US3.l5   diff_US6.l5 diff_US10.l5    trend.l5
diff_US3.l5   1.000000e+00  1.000000e+00   1.00000000  1.00000000
diff_US6.l5  -1.204870e+00 -1.554797e+00  -3.46732020 -3.01663307
diff_US10.l5  5.574345e-02 -1.822634e+01   2.03750761 -0.98411193
trend.l5     -1.236213e-06 -1.914639e-05   0.00013974 -0.09015213

Weights W:
(This is the loading matrix)

            diff_US3.l5 diff_US6.l5 diff_US10.l5     trend.l5
diff_US3.d  -2.26466833  0.04827857   0.26695858 3.115601e-19
diff_US6.d  -0.57457813  0.02521705   0.25520660 4.225535e-20
diff_US10.d  0.01469935  0.04905684   0.01642531 3.893877e-21
```

We are interested in the section on the values of the test statistic which test 3 cases.

r=0 null hypothesis is that there is no cointegration relationship between our series. r <= 1 null hypothesis is that there is less than or equal to 1 cointegration relationship between our series. r <= 2 null hypothesis is that there are less than or equal to 2 cointegration relationship between our series.

Here we compare our test statistic with our critical values at a level of 95%.

For r=0 we can see that our test statistic is above the critical value, hence we can reject the null hypothesis of no cointegration.

Furthermore, we can even reject the null hypothesis of having less than or equal to 2 cointegration relationships.

Our series are cointegrated meaning that there exists at least one linear combination of non-stationary variables that is stationary.

Based on empirical evidence in Shea (1992), there may be sections of the yield-to-maturity curve that are cointegrated, but the cointegration may be more complex than it would be under the "expectation hypothesis". In other words, we may accept the hypothesis that short-term yields are cointegrated with the rest of the term structure, but it is more difficult to expand this hypothesis to other points of the yield curve.

On our side we demonstrated that our yield series| (US3M, 6M, 10Y) are cointegrated. These results from Shea 1992 would imply that this property has limited usefulness. It appears that the relationship between our series on short term vs long term yields is much more complex than our model allows us to do accurate forecasts.

Hence, we believe that we can conclude that the cointegration we demonstrated can be useful to do forecasts on short terms series. However, when trying to add long term yield to the model, this would make our forecasts spurious especially if we try to interpolate between our US3M series and US10Y series as the other points of the yield curve won't be necessarily cointegrated with the 3M yield series and 10Y yield series.

References.

QuantStart. (n.d.). Johansen test for cointegrating time series analysis in R. QuantStart.
https://www.quantstart.com/articles/Johansen-Test-for-Cointegrating-Time-Series-Analysis-in-R/