CFA 2025 - 27-30 avril 2025, Paris

Influence de la dynamique des paramètres de contrôle et phénomène de basculement dans un modèle simple d'instrument à anche

Baptiste Bergeot[a], Soizic Terrien[b] and Christophe Vergez[c]

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Outline

CONTEXT. Understanding the link between **musician gesture** and resulting **regime produced by a single-reed instrument** within the framework within the framework of nonlinear dynamical systems

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Single-reed musical instruments:

Saxophones

Clarinets

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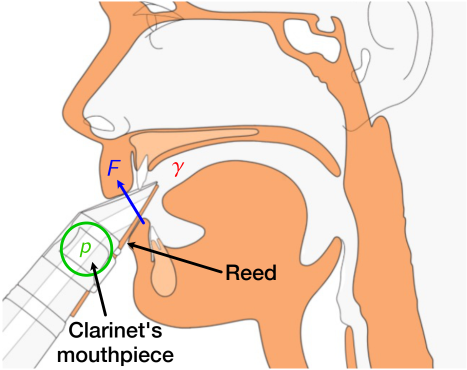
- Modeled by nonlinear dynamical systems linking control parameters (mouth pressure γ, lip force F) to output variables (acoustic pressure p inside the mouthpiece)



γ: mouth pressure

F: force applied by the lip on the reed

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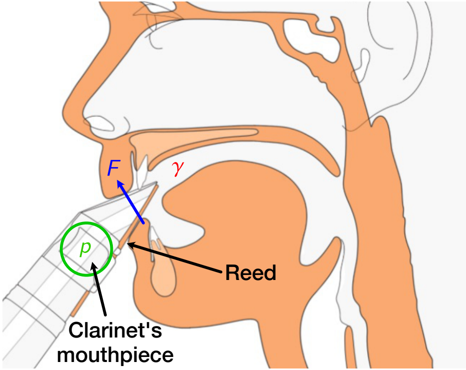

- Modeled by nonlinear dynamical systems linking control parameters (mouth pressure γ, lip force F) to output variables (acoustic pressure p inside the mouthpiece)
- Previous theoretical studies on sound production performed with control parameters constant in time show that:
  - Appearance of sound = Hopf bifurcation of the trivial equilibrium (silence, i.e., $p = 0$) to a stable periodic solution (musical note)
  - Several stable solutions coexist in general = Multistability

γ: mouth pressure

F: force applied by the lip on the reed

Observation

During transients the musician varies the control parameters in time

Research questions

- In the context of musical acoustics: during transient phases, when the control parameters vary in time:

  Do the dynamic characteristics of the control parameters impact the sound produced by the instrument? If they do, in what way?

- Open problems in nonlinear dynamics: behavior of multistable nonlinear dynamical systems with time-varying parameters

  How can Critical transition (or tipping, see e.g. (Ashwin *et al.* (2012), Philos Trans R Soc Lond, A)) be predicited?

Presented work

Predicting the global dynamic behavior (i.e., tipping or not) of a simple bistable model in the case of a attack transient, i.e., when only the mouth pressure γ increase in time.

$$\boxed{\dot{\gamma} = rg(\gamma)}$$

$g(\gamma)$: describes the shape of the time profile of γ

r: rate of growth

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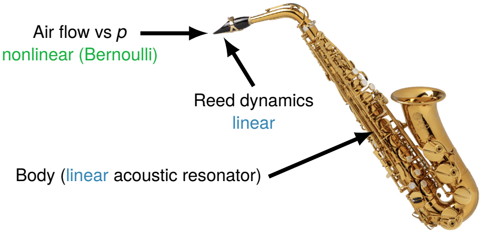
Refined physical model



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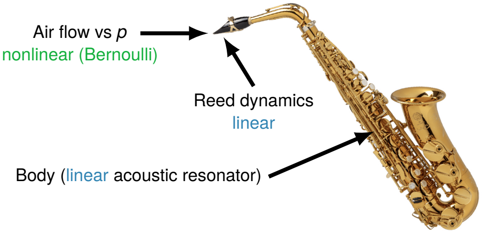

$\Longrightarrow$ System of coupled nonlinear ODEs

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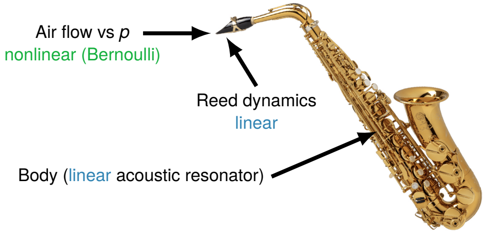

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Simplest model having bistability

$\Longrightarrow$ One-dimensional ODE:

$$\boxed{\dot{x} = f(x, \gamma)}$$

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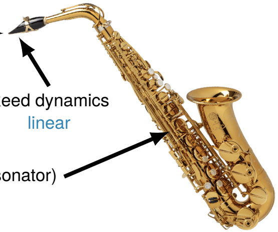

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- Silence: $\boxed{x = 0}$
- Musical note: $\boxed{x = \text{constant}}$

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Bifurcation diagram

Model with a constant γ

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- Zero (silence) or nonzero (musical note)
- **Stable** (if $\frac{\partial f(x^e, \gamma)}{\partial x} < 0$) or **unstable** (if $\frac{\partial f(x^e, \gamma)}{\partial x} > 0$)

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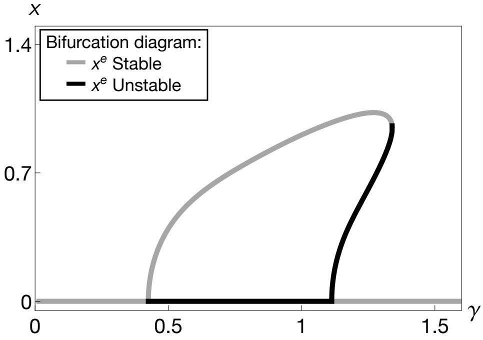

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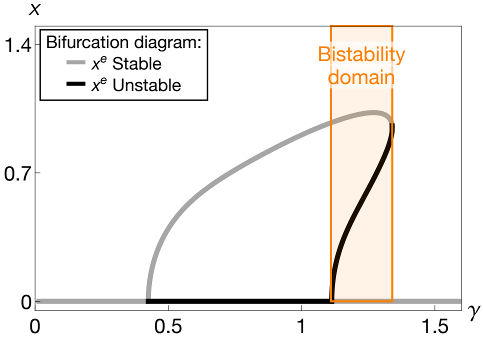


Bistability domain with coexistence:

- **Two stable equilibria** (silence/musical note)
- **One unstable equilibrium** (musical note)

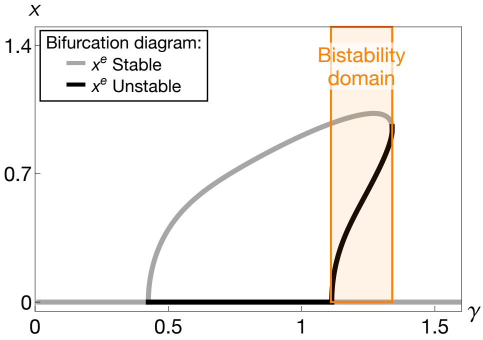
Basin of attraction

Definition (Basin of attraction)

For a **given stable equilibrium**, the basin of attraction (BA) is the set of initial conditions leading to this equilibrium.

Definition (Separatrix between 2 BAs)

Boundary in phase space separating two BAs.



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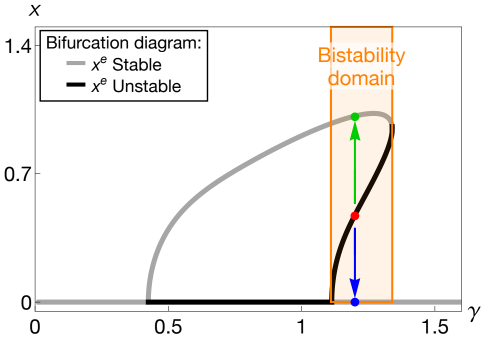

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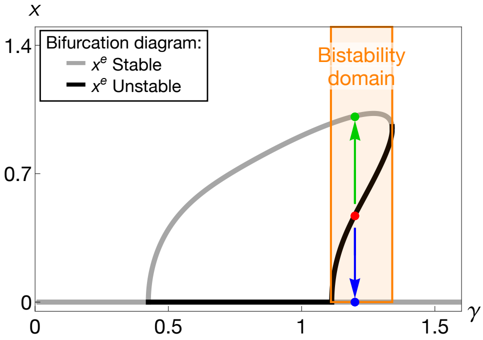

Question

Can we define a **basin of attraction** and a **separatrix** when γ increases over time to reach a **target value within the bistability domain**?

Outline

Examples illustrating the problem

$$\boxed{\begin{aligned}\dot{x} &= f(x,\gamma)\\ \dot{\gamma} &= rg(\gamma)\end{aligned}}$$

- $g(\gamma) = \gamma(1 - \gamma/\gamma_{\text{targ}})$ (tanh-like profile)
- $\lim_{t\to\infty} \gamma(t) = \gamma_{\text{targ}}$ (always the same) with $g(\gamma_{\text{targ}}) = 0$

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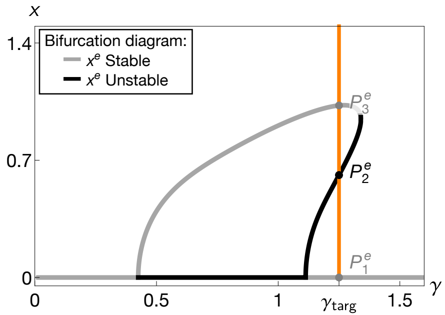

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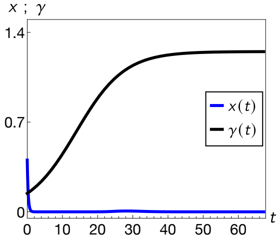
Initial condition: $N_1 = (x_{01}, \gamma_{01})$



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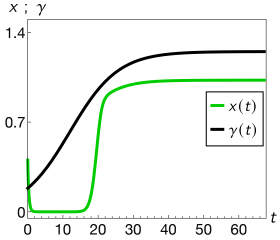

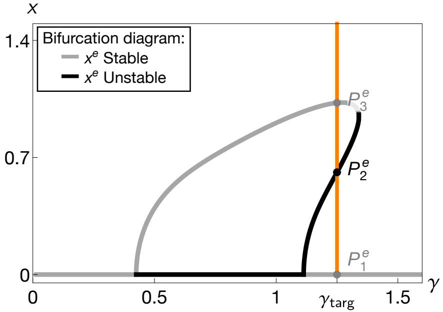

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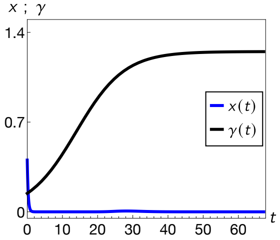

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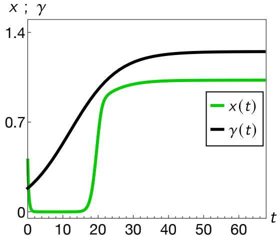

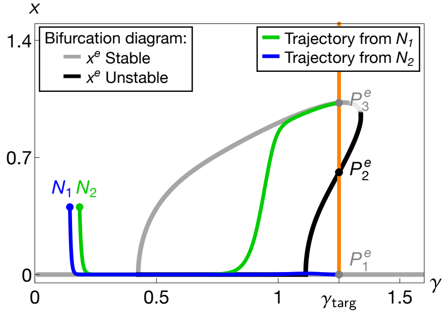

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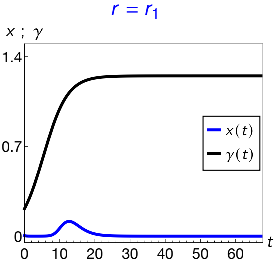

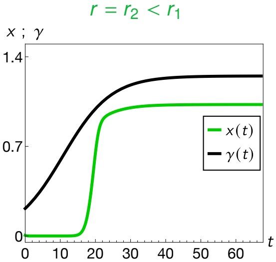

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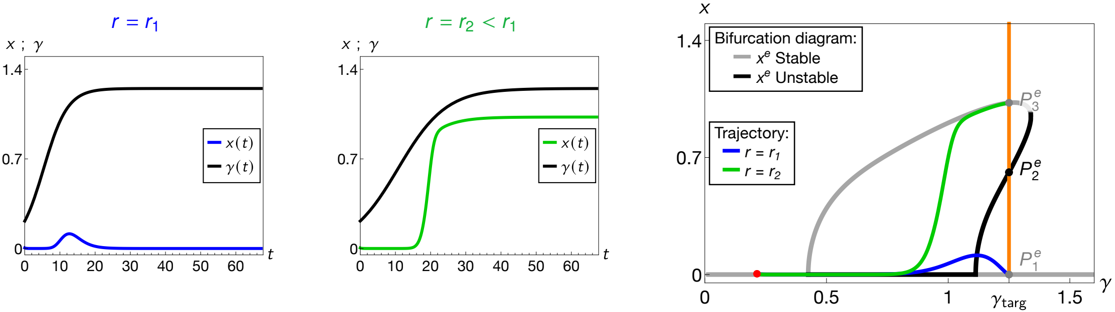

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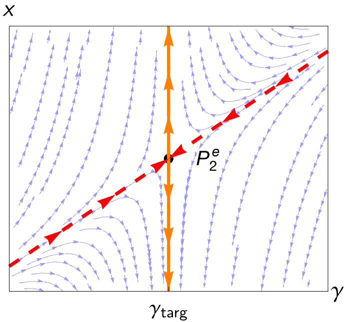
Predicting tipping

Tipping separatrix

P_2^e is a saddle equilibrium point with:

- Unstable manifold
- Stable manifold



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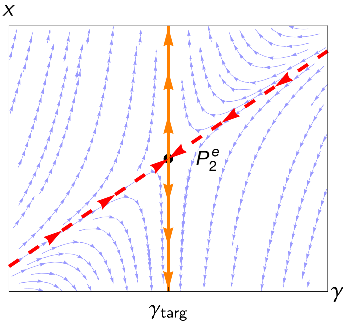

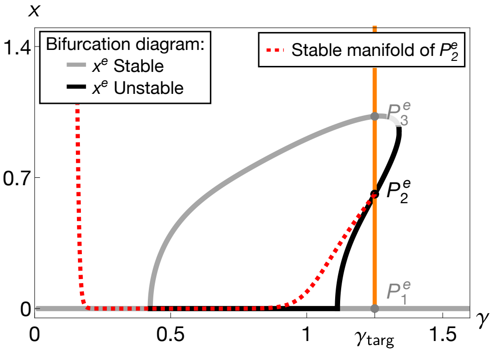

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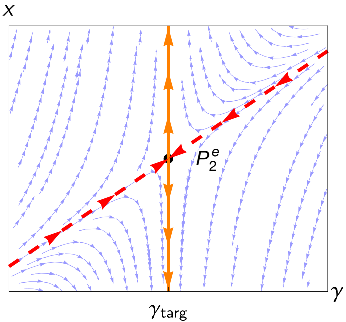

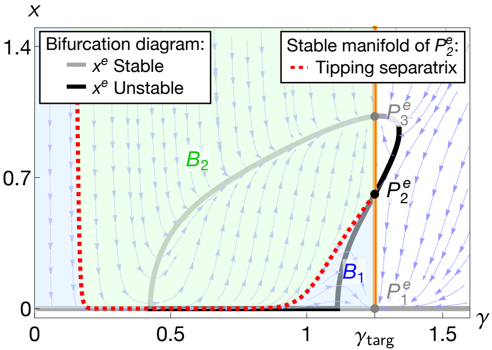

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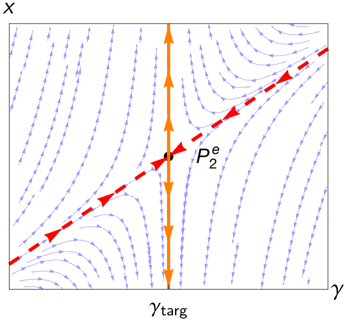

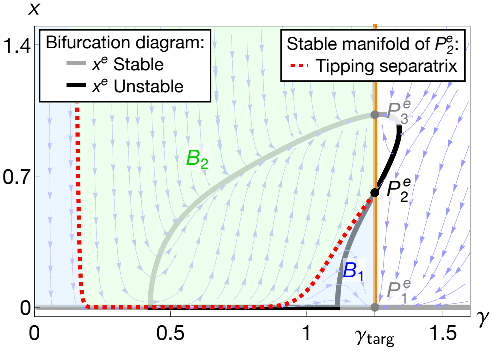

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$\Longrightarrow$ Stable manifold of P_2^e = **TIPPING SEPARATRIX**

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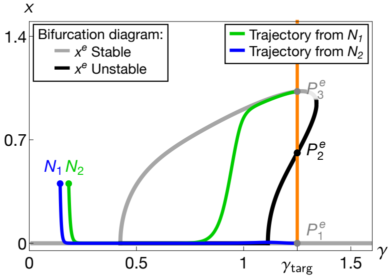

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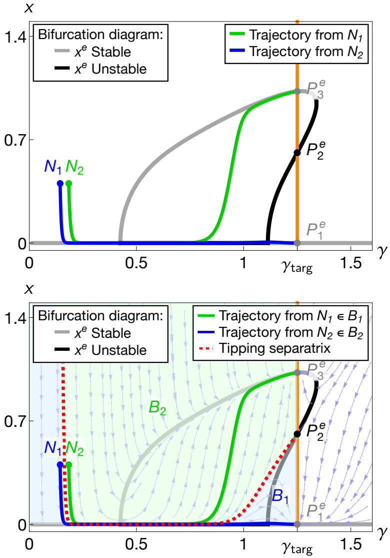

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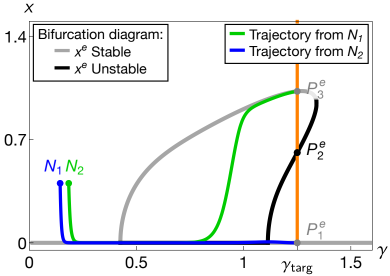

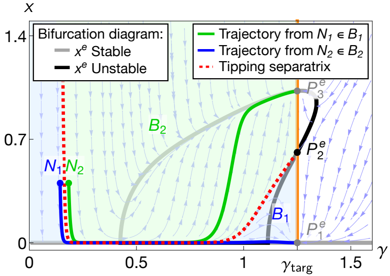

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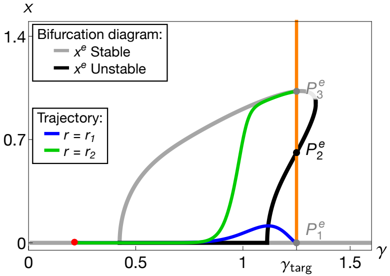

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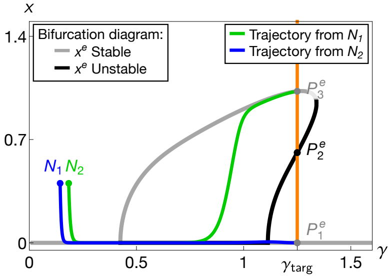

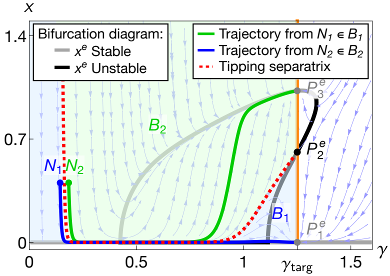

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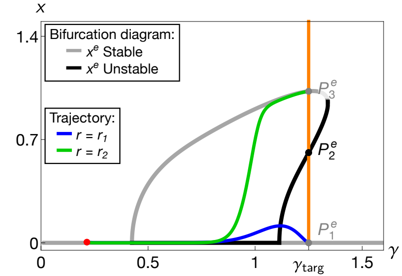

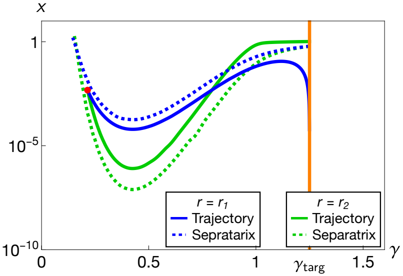

Outline

Conclusion

Results from [Bergeot *et al.* (2024), *Chaos: An Interdisciplinary Journal of Nonlinear Science*]

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Predicting transient dynamics in a model of reed musical instrument with slowly time-varying control parameter

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AFFILIATIONS
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- The case where the **bistability domain is crossed** without any saturation occurring within it is also considered:

  A **more complex mathematical framework** must be used (*Geometric Singular Perturbation Theory*) to define the tipping separatrix

- In both cases:

  The **knowledge of the single solution** (the **tipping sepratrix**) can be used to describe the **global behavior of the model**

- **Final objective:** to understand and predict the content of the transient when multiple multistability domains are crossed before saturation

Conclusion

Results from [Bergeot *et al.* (2024), *Chaos: An Interdisciplinary Journal of Nonlinear Science*]

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Predicting transient dynamics in a model of reed musical instrument with slowly time-varying control parameter

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Perspectives

- **Multistability in more refined model of reed instruments**
  - Equivalent of the tipping separatrix in the case of a bistability between musical notes or tristability
  - Compute separatrices using advanced numerical methods like continuation, machine learning (talk of S. Terrien)
- The tipping separatrix can correspond to very small amplitude values for some ranges of the mouth pressure:
  - **The effect of noise must be taken into account**

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Thank you for your attention

Questions?