Feedback-like algorithm implementation in chemostat*

Algorithm for the identification of the control triggers and their application to the bioreactor in chemostat.

Algorithm 1 Implementation of Feedback-Inspired Control in Chemostat

Require: Plant model, operational constraints, initial state x^0, time step Δt, switching thresholds $\varepsilon_1, \varepsilon_2$

(A) Offline preanalysis

Initialize $T_1, C, T_3, T_4, T_f \leftarrow zeros(N+1)$

for $i = 0, \ldots, N$ **do**

$T_f(i) \leftarrow T_{\min} + \frac{i}{N} \cdot (T_{\max} - T_{\min})$

Solve OCP for horizon $T_f(i)$, returning:

$$t_1,\ x_3(t_2),\ x_7(t_2),\ t_3,\ t_4$$

$T_1(i) \leftarrow \frac{t_1}{T_f(i)}$, $C(i) \leftarrow \frac{\partial \mu}{\partial x_3}(x_3(t_2), x_7(t_2))$

$T_4(i) \leftarrow \frac{t_4}{T_f(i)}$, $T_3(i) \leftarrow \frac{t_3}{T_f(i)}$

end for

$T_3' \leftarrow T_3(\text{where } T_3 \neq 0)$, $T_f' \leftarrow T_f(\text{where } T_3 \neq 0)$

Use *Simple Linear Regression* to fit the coefficients:

$(a_1^*, b_1^*) \leftarrow SLR(T_1, T_f) \quad (a_c^*, b_c^*) \leftarrow SLR(C, T_f)$

$(a_3^*, b_3^*) \leftarrow SLR(T_3', T_f') \quad (a_4^*, b_4^*) \leftarrow SLR(T_4, T_f)$

Output (A): estimated (a_1^*, b_1^*), (a_c^*, b_c^*), (a_3^*, b_3^*), (a_4^*, b_4^*)

(B) Online observer design

Design observer *obs* such that:

$$\hat{\mathbf{x}}(k) = obs\big(u(k), y(k)\big)$$

where $\hat{\mathbf{x}}(k) = \big(\hat{x}_2(k), \hat{x}_3(k), \hat{x}_5(k), \hat{x}_7(k), \hat{x}_8(k)\big)$

Output (B): online state estimates $\hat{\mathbf{x}}(k)$

(C) Feedback controller execution

Initialize $switched \leftarrow$ false, $k \leftarrow 0$, $u(0) \leftarrow u_{\max}$

Compute $\tau_1 \leftarrow a_1^* \cdot T_f + b_1^*$, $c \leftarrow a_c^* \cdot T_f + b_c^*$

Compute $\tau_3 \leftarrow a_3^* \cdot T_f + b_3^*$, $\tau_4 \leftarrow a_4^* \cdot T_f + b_4^*$

if $\tau_3 > 0$ **then**

$v(0) \leftarrow v_{\max}$

else

$v(0) \leftarrow v_{\min}$

end if

Measure initial output $y(0)$

while $k < \lfloor T_f/\Delta t \rfloor$ **do**

while $k < \lfloor T_f/\Delta t \rfloor$ **do**

if $k \leq \tau_1 \cdot T_f/\Delta t$ **then**

// Initial flash phase

$u(k+1) \leftarrow u_{\max}$

else if *switched* $=$ false **then**

$\hat{\mathbf{x}}(k) \leftarrow obs\big(u(k), y(k)\big)$

Compute:

$$cond_1 \leftarrow \frac{\partial \mu}{\partial x_3}\big(\hat{x}_3(k), \hat{x}_7(k)\big)$$

Compute:

$$cond_2 \leftarrow \frac{d}{dt}\left(\frac{\partial \mu}{\partial x_3}\right)\big(\hat{x}_2(k), \hat{x}_3(k), \hat{x}_5(k), \hat{x}_7(k), \hat{x}_8(k)\big)$$

if $cond_1 \leq c + \varepsilon_1$ and $cond_2 \leq -\varepsilon_2$ **then**

$switched \leftarrow$ true

$u(k+1) \leftarrow u_{\max}$

else

$u(k+1) \leftarrow u_{\min}$

end if

else

// After switch phase

$u(k+1) \leftarrow u_{\max}$

end if

if $\tau_3 \cdot T_f \leq k \cdot dt \leq \tau_4 \cdot T_f$ **then**

$v(k+1) \leftarrow v_{\min}$

else

$v(k+1) \leftarrow v_{\max}$

end if

Send $u(k+1)$ and $v(k+1)$ to the plant

Measure $y(k+1)$

$k \leftarrow k+1$

end while

Output (C): control sequences $u(1), u(2), \ldots, u(\lfloor T_f/\Delta t \rfloor)$ and $v(1), v(2), \ldots, v(\lfloor T_f/\Delta t \rfloor)$

End Algorithm

Algorithm 2 $SLR(Y, X)$

Require: Y, X

$n \leftarrow dim(X)$, $\hat{X} \leftarrow mean(X)$, $\hat{Y} \leftarrow mean(Y)$

$a_n \leftarrow 0$, $a_d \leftarrow 0$

for $i = 1, \ldots, N$ **do**

$a_n \leftarrow a_n + (X(i) - \hat{X})(Y(i) - \hat{Y})$

$a_d \leftarrow a_d + (X(i) - \hat{X})^2$

end for

$a \leftarrow \frac{a_n}{a_d}$

$b \leftarrow \hat{Y} - a\hat{X}$

Output: a, b

End Algorithm =0

*B. Boerkmann, T. Bayen, M. Khammash and W. Djema, "From Optimal Control to Feedback-Inspired Optogenetic Strategies for Protein Production in Yeast," *Preprint*, 2026.