

Feedback-like algorithm implementation in chemostat*

Algorithm for the identification of the control triggers and their application to the bioreactor in chemostat.

Algorithm 1 Implementation of Feedback-Inspired Control in Chemostat

Require: Plant model, operational constraints, initial state x^0 , time step Δt , switching thresholds $\varepsilon_1, \varepsilon_2$

(A) Offline preanalysis

Initialize $T_1, C, T_3, T_4, T_f \leftarrow \text{zeros}(N + 1)$

for $i = 0, \dots, N$ **do**

$$T_f(i) \leftarrow T_{\min} + \frac{i}{N} \cdot (T_{\max} - T_{\min})$$

Solve OCP for horizon $T_f(i)$, returning:

$$t_1, x_3(t_2), x_7(t_2), t_3, t_4$$

$$T_1(i) \leftarrow \frac{t_1}{T_f(i)}, C(i) \leftarrow \frac{\partial \mu}{\partial x_3}(x_3(t_2), x_7(t_2))$$

$$T_4(i) \leftarrow \frac{t_4}{T_f(i)}, T_3(i) \leftarrow \frac{t_3}{T_f(i)}$$

end for

$T'_3 \leftarrow T_3$ (where $T_3 \neq 0$), $T'_f \leftarrow T_f$ (where $T_3 \neq 0$)

Use *Simple Linear Regression* to fit the coefficients:

$$(a_1^*, b_1^*) \leftarrow SLR(T_1, T_f) \quad (a_c^*, b_c^*) \leftarrow SLR(C, T_f)$$

$$(a_3^*, b_3^*) \leftarrow SLR(T'_3, T'_f) \quad (a_4^*, b_4^*) \leftarrow SLR(T_4, T_f)$$

Output (A): estimated (a_1^*, b_1^*) , (a_c^*, b_c^*) , (a_3^*, b_3^*) , (a_4^*, b_4^*)

(B) Online observer design

Design observer obs such that:

$$\hat{\mathbf{x}}(k) = obs(u(k), y(k))$$

where $\hat{\mathbf{x}}(k) = (\hat{x}_2(k), \hat{x}_3(k), \hat{x}_5(k), \hat{x}_7(k), \hat{x}_8(k))$

Output (B): online state estimates $\hat{\mathbf{x}}(k)$

(C) Feedback controller execution

Initialize $switched \leftarrow \text{false}$, $k \leftarrow 0$, $u(0) \leftarrow u_{\max}$

Compute $\tau_1 \leftarrow a_1^* \cdot T_f + b_1^*$, $c \leftarrow a_c^* \cdot T_f + b_c^*$

Compute $\tau_3 \leftarrow a_3^* \cdot T_f + b_3^*$, $\tau_4 \leftarrow a_4^* \cdot T_f + b_4^*$

if $\tau_3 > 0$ **then**

$$v(0) \leftarrow v_{\max}$$

else

$$v(0) \leftarrow v_{\min}$$

end if

Measure initial output $y(0)$

while $k < \lfloor T_f / \Delta t \rfloor$ **do**

 Compute $u(k+1)$ as in the batch implementation, using τ_1 and c

if $\tau_3 \cdot T_f \leq k \cdot dt \leq \tau_4 \cdot T_f$ **then**

$$v(k+1) \leftarrow v_{\min}$$

else

$$v(k+1) \leftarrow v_{\max}$$

end if

Send $u(k+1)$ and $v(k+1)$ to the plant

Measure $y(k+1)$

$$k \leftarrow k + 1$$

end while

Output (C): control sequences $u(1), u(2), \dots, u(\lfloor T_f / \Delta t \rfloor)$ and $v(1), v(2), \dots, v(\lfloor T_f / \Delta t \rfloor)$

End Algorithm

Algorithm 2 SLR(Y, X)

Require: Y, X

$$n \leftarrow \dim(X), \hat{X} \leftarrow \text{mean}(X), \hat{Y} \leftarrow \text{mean}(Y)$$

$$a_n \leftarrow 0, a_d \leftarrow 0$$

for $i = 1, \dots, N$ **do**

$$a_n \leftarrow a_n + (X(i) - \hat{X})(Y(i) - \hat{Y})$$

$$a_d \leftarrow a_d + (X(i) - \hat{X})^2$$

end for

$$a \leftarrow \frac{a_n}{a_d}$$

$$b \leftarrow \hat{Y} - a\hat{X}$$

Output: a, b

End Algorithm

*B. Boerkmann, T. Bayen, M. Khammash and W. Djema, "From Optimal Control to Feedback-Inspired Optogenetic Strategies for Protein Production in Yeast," *Preprint*, 2026.