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# On the Formation of Planets in Binary Star Systems

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**Summary.** On the assumption that planets form by accretion of Goldreich-Ward planetesimals, this paper presents a theory for the existence of regions wherein planets may grow in binary star systems. The numerical results are insensitive to assumed model parameters, particularly with respect to nebular density. The criterion for growth by accretion is that planetesimal collision velocities do not exceed a critical value, under the joint influences of nebular drag and of companion-star secular perturbations. It is found that even large binary semimajor axes ($\sim$50 AU) do not ordinarily permit planet growth. Planet growth appears possible only when the companion is very small or its orbit nearly circular, as in the binary system Sun-Jupiter; but even in this case, effects due to a solar nebula appear required for growth to proceed.

**Key words:** accretion of planetesimals — cosmogony — solar systems formation — binary stars

## 1. Introduction

Some one-half of the nearby stars are in binary systems; considering unseen companions, the number may be much higher (Abt, 1977). Hence it is of evident interest to consider whether such systems admit the presence of planets. Huang's (1960) treatment, based on the stability of orbits in the restricted three-body problem, did not consider any treatment of the mechanisms of planet formation; his conclusion, that extensive planet-populated regions may exist around the components of binaries, thus cannot be accepted a priori. Heppenheimer (1974) sought to remedy this deficiency and concluded that binary semimajor axes >30 AU are required for planet growth by accretion, even quite close to a component. However, this theory was quite model-dependent, and the selected model was biased to the optimistic side (favorable to planet formation) to meet a perceived need that the theory account for the presence of planets in the system Sun-Jupiter. In what follows, a theory is presented which is largely model-insensitive, and the problem of solar system planet formation is resolved through consideration of solar nebula effects.

The process of planet growth considered here involves accretion of planetesimals via their mutual collisions. The mean collision velocity $\overline{V_\infty}$ then must not exceed a critical value $V_c$; for $\overline{V_\infty} > V_c$, planetesimals suffer fragmentation and are not able to accrete.

We are concerned here with planetesimals of comparable mass, colliding over only a limited range of velocities. Consequently, following Greenberg et al. (1977), the most destructive collisions involve equal masses; letting their density be $\rho_p$,

$$E/V = \tfrac{1}{8}\rho_p V_c^2 \tag{1}$$

where $E/V$ is energy per volume. For a treatment favorable to planet formation, consider the planetesimals to be of rock and adopt the value $E/V = 3 \times 10^6\ \mathrm{J/m^3}$ given by Greenberg et al. Then with $\rho_p = 3000\ \mathrm{kg/m^3}$, there is the approximate value, $V_c = 100$ m/s. To the extent that planetesimals comprise material more friable than rock, this value may be conservatively high.

In what follows, we are concerned with the definition of regions where $\overline{V_\infty} \leqq V_c$. Unless otherwise specified, the adopted system of units is the normalized system of the restricted three-body problem, wherein the following quantities are set to unity: the sum of masses of the primaries; their semimajor axis; $(1/2\pi)$ times their period. We are concerned with an ensemble of planetesimals orbiting the larger primary, which has mass $m_0$; the perturbing primary mass is $m_p$. Hence Kepler's third law reads $n^2a^3 = m_0$, and the gravitational constant $G =$ unity. The binaries' semimajor axis is $a_0$; their eccentricity, $e_0$; and, following Heppenheimer (1974), we take $a/a_0 \ll 1$.

## 2. Increase in $e$: Secular Perturbations

The eccentricity and hence collision velocity are increased owing to secular perturbations from the companion star. An appropriate disturbing function, assuming coplanarity of the planetesimals with the primary, must accommodate possibly vanishingly small $e$ and be appropriate to $a/a_0 \ll 1$, while also accounting accurately for $e_0$ insofar as for typical binaries, $e_0 \sim 0.5$. Such a disturbing function $R$ is available from developments due to Kaula (1962):

$$R = m_0 a^2 \frac{1}{4}\left(\frac{1}{(1 - h_0^2 - k_0^2)^{3/2}}\{1 + \tfrac{3}{2}(h^2 + k^2)\} - \frac{15}{16}\frac{a}{a_0}\frac{hh_0 + kk_0}{(1 - h_0^2 - k_0^2)^{5/2}}\right) \tag{2}$$

$$h = e \sin \tilde{\omega}; \qquad h_0 = e_0 \sin \tilde{\omega}_0$$

$$k = e \cos \tilde{\omega}; \qquad k_0 = e_0 \cos \tilde{\omega}_0$$

where $\tilde{\omega}$, $\tilde{\omega}_0$ are longitudes of periastron. The Lagrange planetary equations are given (Brouwer and Clemence, 1961):

$$\frac{dh}{dt} = \frac{1}{na^2}\frac{\partial R}{\partial k}; \qquad \frac{dk}{dt} = -\frac{1}{na^2}\frac{\partial R}{\partial h};$$

$$dh_0/dt = dk_0/dt = 0 \tag{3}$$

and the last two of Equations (3) reflect the fact that the planetesimals do not perturb the companion star. Hence it is possible to fix $\tilde{\omega}_0$: $\tilde{\omega}_0 = 0$, by definition. Equations (3) then are solved for the case of a planetesimal initially with $e = 0$, i.e. $h = k = 0$:

$$\tan \tilde{\omega} = -\frac{\sin u(t - t_0)}{1 - \cos u(t - t_0)}; \qquad u = \frac{3}{4}\frac{m_p}{n}\frac{1}{(1 - e_0^2)^{3/2}}$$

$$e = \frac{5}{2}\frac{a}{a_0}\frac{e_0}{1 - e_0^2}\left|\sin \frac{u}{2}(t - t_0)\right| \tag{4}$$

A naive interpretation of Equations (4) is that the orbits are coelliptic and coapsidal; neighboring orbits then do not intersect and secular perturbations cannot then be invoked to delimit a planet-forming region. However, not all planetesimals will simultaneously begin to "feel" the perturbations and hence there is a range of $t_0$. An estimate for this range is available following the planetesimal-formation scenario of Goldreich and Ward (1973), which takes place within a planet-forming circumstellar nebula.

Thus, one considers that initially, dust particles form within the nebula by direct condensation from vapor, as the nebula cools. These settle to the nebular midplane. They are typically of centimeter size and form a dust disk at the midplane. Due to its self-gravity, the disk is unstable and suffers direct gravitational collapse, yielding a population of first-generation planetesimals with diameters of the order of a kilometer. This population is still gravitationally unstable, and its members group into clusters of $\sim 10^4$ members. Within such clusters, internal planetesimal relative motions are damped by nebular drag; the clusters thus contract to form a second generation of planetesimals, with diameters of the order of 10 km.

Under this scenario, the planetesimals would feel the secular perturbations upon the initial formation of first-generation bodies by direct gravitational contraction, since these bodies would represent the earliest appearance of planetesimals capable of following Keplerian orbits. The range in $t_0$ then is the range in formation time for such bodies. The actual contraction, within the dust disk, proceeds over $\sim$0.1 yr or less. The controlling factor then is the maximum descent time for dust particles, condensed within the nebula, in their motion toward the nebular midplane. This is (see Equations 6 and 7 of Goldreich and Ward, 1973):

$$\tau_z = \frac{1}{n}\frac{\mu^{1/4}}{\alpha^{1/2}} \tag{5}$$

$\alpha$ = mass fraction of condensibles in the nebula, taken as 0.005; $\mu$ = molecular weight of the condensibles. For typical materials such as iron and iron or magnesium silicates, $\mu^{1/4} = 4$ to 5. Also, in Equation (4), the time scale for secular perturbations is $\tau_p = n/m_p$. Consequently, for the planetesimals' orbits to be initially nearly coapsidal, $\tau_z \ll \tau_p$ or, approximately, $m_p a^3 \ll 0.015$. This is nearly always the case, so one cannot appeal to an initial spread in $t_0$ to yield any major departure from coapsidality.

A more significant effect is due to the spread in $\tilde{\omega}$ which develops, for adjacent orbits, over many cycles of $u$. Let two planetesimals be at $a$, $a + \Delta a$; the time scale for differential regression of the apsidal lines then is

$$\tau_r = \frac{a}{\Delta a}\frac{n}{m_p} \tag{6}$$

For $\Delta a/a = 0.01$ and $a = 1$ AU, $\tau_r \sim 10^5$ years for typical binaries, as well as for the system Sun-Jupiter when one accounts for the initial presence of the solar nebula. By contrast, let $\tau_g$ be the time scale for planet growth by accretion; according to Weidenschilling (1974), $\tau_g \sim 10^7$ years. Consequently, for $a \gtrsim 0.1$ AU, one may regard $\tilde{\omega}$ as randomized. But for $a \ll 1$ AU it is possible that $\tau_r > \tau_g$; then the apsidal lines are coincident to within a range $\Delta\tilde{\omega} = \tau_g/\tau_r$. In general, one approximates $\overline{V_\infty}$ by

$$\overline{V_\infty} = naek; \qquad k = \min(1, \tau_g/\tau_r) \tag{7}$$

and in what follows, $k$ will be carried as a model-dependent parameter.

It may be shown (following e.g. Heppenheimer, 1978) that nebular drag cannot give rise to an aerodynamic effect promoting randomization of $\tilde{\omega}$ since such drag produces no secular change in $\tilde{\omega}$.

In the second of Equations (4), $\bar{e} = (2/\pi)e_{\max}$. Consequently, one defines a maximum value $a = a_1$ within which, from secular perturbations alone, $\overline{V_\infty} \leqq V_c$:

$$a_1^{1/2} = \frac{\pi}{5} m_0^{-1/2} \frac{(1 - e_0^2)}{e_0} \frac{V_c}{k} \tag{8}$$

Now define $(a_0)_{\mathrm{AU}} = a_0$ in AU; let $m_\odot$ = solar mass. Then

$$na = 2.9777 \times 10^4 (m_0/m_\odot (a_0)_{\mathrm{AU}})^{1/2}\ \mathrm{m/s} \tag{9}$$

and, taking $V_c = 100$ m/s and expressing $a_1$ in AU,

$$a_1 k^2 = 4.4524 \times 10^{-6} (m_\odot/m_0) \frac{(1 - e_0^2)^2}{e_0^2} (a_0)^2_{\mathrm{AU}} \tag{10}$$

Consider now the feasibility of using data for $m_0$, $m_p$ $(a_0)_{\mathrm{AU}}$, $e_0$ as given in standard sources such as Van de Kamp (1969, 1971) in dealing with the remote epochs wherein planet formation occurred. As noted, Weidenschilling (1974) finds $\tau_g \sim 10^7$ years for terrestrial planets. The observed dynamical state of binaries with $a_0 > 1$ AU may be explained in terms of the initial existence of a multi-body association of gravitationally bound protostars; numerical results due to Szebehely and Peters (1967) can be interpreted as indicating that such an association will eject members, on a time scale of a few Kepler periods, till a stable binary is achieved. Hence the presently observed values of $e_0$ indeed appear pertinent to virtually the whole of the planet-forming time.

But $m_0$, $m_p$, $(a_0)_{\mathrm{AU}}$ require modification to account for T Tauri mass loss and associated binary dynamical evolution. Kuhi (1964) estimates that T Tauri mass loss accounts for 30–40% of a star's initial mass. Letting $K$ = ratio of stellar mass, pre- to post-loss, the numerical factor in Equation (9) must be multiplied by $K^{-2}$ if data such as Van de Kamp's are to be used. It thus appears reasonable to take $K = 1.5$ so that, in Equation (9), a numerical factor $2.0 \times 10^{-6}$ is appropriate.

## 3. Reduction in $e$: Nebular Drag

The estimate $a_1$ is certainly a lower bound for the limit of the planet-forming region, since $e$ is damped owing to nebular drag. The nebula under discussion is the "cocoon nebula" surrounding a forming star. The nebula does not rotate with locally Keplerian velocity but is partially supported by the radial pressure gradient. Hence the local rotation velocity is reduced from the Keplerian by a factor $V_\theta$ (Goldreich and Ward, 1973):

$$V_\theta = -c^2/2na$$

where $c$ = sound speed. On assumption that the nebular drag on the planetesimals is Newtonian (proportional to the square of the relative velocity), the drag-associated quantities $da/dt$, $de/dt$ may be derived; this has been done elsewhere (Heppenheimer, 1978). Goldreich and Ward (1973) have noted that $da/dt$ is never sufficient to produce inward spiralling into the primary; hence $da/dt$ will not be further treated. As for $e$, two drag laws arise. For $c^2 > n^2a^2e$, the planetesimal always is faster than the nebula, and, to the lowest order in $e$,

$$\frac{de}{dt} = -\frac{1}{2} \frac{\rho_g \beta}{na} c^2 e \tag{11}$$

For $c^2 < n^2a^2e$, the planetesimal is alternately faster and slower than the nebula, and, neglecting terms in $c^4$ and $e^4$,

$$\frac{de}{dt} = -\frac{2}{\pi} \rho_g \beta n a e^2 \tag{12}$$

$\rho_g$ = nebular density, $\beta$ = ballistic parameter. Associated with the two decay laws will be two estimates for the upper bound of the limit for the planet-forming region, respectively $a_2$ and $a_3$, associated respectively with Equations (11) and (12).

Consider now the model-dependent quantities $\rho_g$, $\beta$, $c$. We have

$$\beta = AC_d/2m = 3C_d/4\rho_p D \tag{13}$$

$A$ = cross-sectional area, $C_d$ = drag coefficient = 0.44 (Whipple, 1972), $m$ = planetesimal mass, $\rho_p$ = its density, $D$ = its diameter. Evidently one may remove the model-dependence of $\rho_g\beta$ if $D \propto \rho_g$. This is nearly true for Goldreich-Ward (1973) planetesimals, which form through gravitational instability of a disk of condensed particles at the nebular midplane. Then

$$D = \frac{4\pi\alpha\rho_g c}{n^2} \left(\frac{3G^2}{2\rho_p n}\right)^{1/3} \tag{14}$$

$G$ = unity as noted, $\alpha$ = mass fraction of condensibles in the nebula, taken as 0.005.

Equation (14) is correct only for a low-mass nebula, in which the vertical force of gravity is due to the attraction of the primary. However, Cameron (1978) has argued that a high-mass nebula, of mass $m_{\mathrm{N}} \sim m_0$, would be turbulent and would present an environment wherein the Goldreich-Ward mechanism would be prevented from proceeding. Cameron has also presented a mechanism whereby this turbulence would lead to rapid nebular mass-loss, converting it into a low-mass nebula, in which (as he argues) the Goldreich-Ward mechanism could indeed proceed.

There remains for consideration $c$: $c = 76T^{1/2}$ m/s in a gas of hydrogen molecules; $T$ = Kelvin temperature, and from consideration of nebular adiabats, $T \propto \rho_g^{2/3}$. However, it is found that for reasonable choices of adiabat, e.g. that of Larson (1969), it is approximately true that $c = 0.1na$.

The treatment which follows thus involves the uncertain quantities $K$, $\rho_p$, $\alpha$, $V_c$, $D$, $C_d$, $c/na$, or, if $c$ is to be defined through an adiabat, $\rho_g$. It will be shown that the theoretical results, however, are quite insensitive even to order-of-magnitude uncertainties in these quantities. The resulting theory can be described as model-insensitive.

Evidently $a_1$ is a lower bound. An upper bound can be defined as the limit of the region where drag is significant, and this may be given by

$$((de/dt)_{\text{sec.pert's}})_{\max} = (de/dt(e_{\max}))_{\text{drag}} \tag{15}$$

where $(de/dt)_{\text{sec.pert's}}$ is given by Equation (4), as is $e_{\max}$, and $(de/dt)_{\text{drag}}$ = Equation (11) or (12); the right-hand side of Equation (15) involves a functional dependence on $e_{\max}$. Thus, for $a_2$, using Equation (11),

$$\frac{15}{16}\frac{m_p e_0}{m_0^{1/2}}\left(\frac{a_2}{1-e_0^2}\right)^{5/2} = \frac{\rho_g \beta}{2na_2}\, c^2 e_{\max},$$

$$e_{\max} = \frac{5}{2}\, a_2 \frac{e_0}{1-e_0^2}, \qquad c = 0.1na_2 \tag{16}$$

and eliminating $\beta$ via Equations (13, 14) gives

$$a_2^5 = \frac{0.03C_d}{1.6\pi\alpha\rho_p^{2/3}}\left(\frac{2}{3}\right)^{1/3}\cdot\frac{4(1-e_0^2)^{3/2}}{3m_p m_0^{-5/3}} \tag{17}$$

To express $\rho_p$ in normalized units, unit density = $(m_0 + m_p)a_0^{-3}$, or, taking $\rho_p = (\rho_p)_0$ g/cm$^3$,

$$\rho_p = (\rho_p)_0 \cdot 1.69 \times 10^6 (a_0)^3_{\text{AU}}(m_\odot/(m_0 + m_p)) \tag{18}$$

Taking $(\rho_p)_0 = 3$, and expressing $a_2$ in AU,

$$a_2 = 0.11573\, \frac{(1-e_0^2)^{0.3}}{m_p^{0.2} m_0^{-1/3}}\left(\frac{m_0+m_p}{m_\odot}\right)^{2/15}(a_0)^{0.6}_{\text{AU}}\cdot K^{8/15}\ \text{AU} \tag{19}$$

where the correction for T Tauri effects has been applied. Similarly, using Equation (12) one derives

$$a_3 = 0.28511\, \frac{(1-e_0^2)^{1/8}}{m_0^{-5/12}}\left(\frac{e_0}{m_p}\right)^{1/4}\left(\frac{m_0+m_p}{m_\odot}\right)^{1/6} \times (a_0)^{1/2}_{\text{AU}}\cdot K^{2/3}\ \text{AU} \tag{20}$$

The model-insensitivity of $a_2$, $a_3$ then results from the uncertain parameters being raised to low powers. In particular,

$$a_2 \propto K^{8/15}\rho_p^{-2/15}\alpha^{-1/5}C_d^{1/5}(c/na)^{1/5} \tag{21}$$

$$a_3 \propto K^{2/3}\rho_p^{-1/6}\alpha^{-1/4}C_d^{1/4}(c/na)^{-1/4}$$

and if $c$ is defined through an adiabat then $a_2 \propto \rho_g^{1/15}$, $a_3 \propto \rho_g^{-1/12}$. On the other hand, Cameron (1978) has criticized the Goldreich-Ward (1973) scenario for planetesimal formation, which calls for rapid effectiveness of the gravitational instabilities in a disk of condensed particles, the disk being formed immediately after condensation of the particles. Cameron proposes that nebular turbulence would inhibit formation of this particulate disk, until the nebular density was reduced. To the extent that this effect may have been important, let $C$ be the factor by which $\rho_g$ must be multiplied to yield tha nebular state within which a particulate disk can form. On the assumption that no condensed particles are lost during this nebular evolution, the cited values for $a_2$, $a_3$ are overestimated by factors $C^{-1/5}$, $C^{-1/4}$ respectively. On the other hand, it is possible that condensibles are depleted along with the rest of the nebula, so that a fraction $(1 - C)$ of the condensed matter is lost. Then, provided that $c$ is defined through an adiabat, the correction factors for $a_2$, $a_3$ are respectively $C^{1/15}$, $C^{-1/12}$. The correction factors are respectively $\Delta(c/na)^{1/5}$, $\Delta(c/na)^{-1/4}$ where $\Delta(c/na)$ is the fractional change in $(c/na)$ assumed (if any indeed is), if $c$ is not defined through an adiabat.

## 4. Numerical Examples

To indicate the nature of the results, consider three typical binaries, numerical parameters for which are taken from Van de Kamp (1969, 1971). $K = 1.5$ is assumed, and the $a_i$ are in AU.

$\alpha$ Centauri A: $m_0/m_\odot = 1.06$, $m_p/m_\odot = 0.87$, $a_0 = 23.1$, $e_0 = 0.521$, $a_1k^2 = 1.969 \times 10^{-3}$, $a_2 = 0.901$, $a_3 = 2.046$.

40 Eridani B: $m_0/m_\odot = 0.42$, $m_p/m_\odot = 0.20$, $a_0 = 33.6$, $e_0 = 0.402$, $a_1k^2 = 2.338 \times 10^{-2}$, $a_2 = 1.160$, $a_3 = 1.440$.

$\eta$ Cassiopeiae A: $m_0/m_\odot = 0.85$, $m_p/m_\odot = 0.52$, $a_0 = 67.6$, $e_0 = 0.495$, $a_1k^2 = 2.501 \times 10^{-2}$, $a_2 = 1.786$, $a_3 = 3.121$.

It is evidently of interest whether one may derive an estimate for the limit of the planet-forming zone, intermediate between $a_1$ and $a_2$, $a_3$, and which overcomes the sensitivity of $a_1$ to $V_c/k$ insofar as $a_1 \propto V_c^2/k^2$. The answer is in the affirmative. Such an intermediate estimate is available from consideration of the consequences of planet growth.

Thus, let planetesimal diameter $D$ increase by a factor $U$, other parameters remaining unchanged. Then $a_2$, $a_3$ change respectively by factors of $U^{-1/5}$, $U^{-1/4}$. To determine the effect on $a_1$, we consider that $V_c$ is increased due to the body self-gravitation.

Hence, following Equation (1), we again consider collisions involving bodies of equal mass and regard the binding energy as given by

$$E/V = \tfrac{1}{8}\rho_p V_c^2 = (E/V)_0 + \rho_p V_e^2 \tag{22}$$

and $(E/V)_0$ is $(E/V)$ in Equation (1). The second term on the right-hand side is the per-unit-volume energy required to produce escape to infinity, against the combined gravities of both colliding bodies. For a spherical body of diameter $D$ in kilometers, $V_e = 0.0167 D\sqrt{\rho_p}$. Taking once again $\rho_p = 3000$ kg/m$^3$,

$$V_c^2 = V_{c0}^2 + 6.693D^2 \tag{23}$$

where we take $V_{c0} = 100$ m/s as before.

It is now necessary to define a value for $D$; from the Goldreich-Ward theory, $D \sim 1$ km. To make the computations simpler we take $D = 1.222$ km. Then, for example,

$$\begin{aligned} a_1 \frac{(V_{c0}^2 + 10U^2)/k^2(aI_2)}{V_{c0}^2/k^2(a_1)} &= aI_2 = U^{-1/5}a_2 \\ a_1 \frac{(V_{c0}^2 + 10U^2)/k^2(aI_3)}{V_{c0}^2/k^2(a_1)} &= aI_3 = U^{-1/4}a_3 \end{aligned} \tag{24}$$

where we write $k(aI_2)$, $k(aI_3)$, $k(a_1)$ to express the fact that $k = k(a)$ and is model-dependent. It will be seen that, typically, $aI_2, aI_3 > 0.1$ AU so that, in accordance with the discussion preceding Equation (7), $k(aI_2) = k(aI_3) = 1$. Then the left-hand sides of Equations (24) include the term, $a_1k^2(a_1)$; but this is the well-defined quantity given by Equation (10) and tabulated for the test cases. Consequently,

$$\begin{aligned} a_1(1 + 10^{-3}U^2) &= aI_2 = U^{-1/5}a_2 \\ a_1(1 + 10^{-3}U^2) &= aI_3 = U^{-1/4}a_3 \end{aligned} \tag{25}$$

For the three stars discussed, we have, with $aI_2$, $aI_3$ in AU,

α Centauri A: $aI_2 = 0.276$, $aI_3 = 0.439$
40 Eridani B: $aI_2 = 0.436$, $aI_3 = 0.425$
η Cassiopeiae A: $aI_2 = 0.649$, $aI_3 = 0.850$

and these computed quantities change by only $\sim 0.01$ when $K = 1.0$. To indicate further the model-insensitivity of these results, let initially $V_c = 1000$ m/s. Then, for example,

$$100a_1(1 + 10^{-5}U^2) = aI_2 = U^{-1/5}a_2 \tag{26}$$

and, taking for example α Centauri A,

$$aI_2 = 0.303 \qquad aI_3 = 0.466$$

On the other hand, let initially $V_c = 100$ m/s but let $D = 0.1222$ km. Then, for example, and again for α Centauri A,

$$\begin{aligned} &a_1(1 + 10^{-5}U^2) = aI_2 = U^{-1/5}a_2; \\ &aI_2 = 0.181, \qquad aI_3 = 0.263 \end{aligned} \tag{27}$$

In the solar system, Mercury lies at $a = 0.387$ AU. If we assume that $aI_2$ or $aI_3$ must have at least this value for a planet to form, then only for η Cassiopeiae can one plausibly propose the existence of planets. More generally, a value $(a_0)_{\mathrm{AU}} > 50$ may be necessary to form even such a "Mercury", or close planet.

## 5. Stars with Small Companions

Consider now the system Sun-Jupiter, along with two other systems involving companions of comparable size; Equation (25) is used.

Sun: $m_0/m_\odot = 1.00$, $m_p/m_\odot = 0.001$, $a_0 = 5.2$, $e_0 = 0.05$, $a_1k^2 = 2.152 \times 10^{-2}$, $a_2 = 1.537$, $a_3 = 2.265$, $aI_2 = 0.558$, $aI_3 = 0.629$.

61 Cygni: $m_0/m_\odot = 0.70$, $m_p/m_\odot = 0.008$, $a_0 = 2.54$, $e_0 = 0.7$, $a_1k^2 = 9.785 \times 10^{-6}$, $a_2 = 0.479$, $a_3 = 1.249$, $aI_2 = 0.096$, $aI_3 = 0.157$.

ε Eridani: $m_0/m_\odot = 0.8$, $m_p/m_\odot = 0.006$ to 0.05, $a_0 = 7.9$, $e_0 = 0.5$, $a_1k^2 = 3.511 \times 10^{-4}$, $a_2 = 1.176$ to 0.770, $a_3 = 2.549$ to 1.534, $aI_2 = 0.300$ to 0.204, $aI_3 = 0.441$ to 0.280.

Data for 61 Cygni are from Strand (1942); for ε Eridani from Van de Kamp (1972). These three systems thus resemble closely the three systems of Section 4. In particular, it is evidently difficult to reconcile the present theory with the existence of terrestrial planets.

But in these systems, the mass of the companion is sufficiently small that in the early stages of planet formation, it is not correct to regard the system as consisting only of the cited components. Instead, one must additionally consider the gravitational effects of the nebula. Unlike a binary companion, a rotating nebula should not be regarded as possessing intrinsic eccentricity, since in such a rotating fluid mass, any motions having the effect of eccentricity tend quickly to damp out through hydrodynamic effects. Thus, one must consider the perturbations on a planetesimal's $e$ due jointly to a nebula and to a companion star.

The pertinent theory (Heppenheimer, 1978) corresponds to the Brouwer-Clemence (1961) theory for the secular perturbations of an ensemble of planets upon an asteroid. That theory involves coefficients obtained by summation over the perturbing planets; for the present theory, such terms are replaced by corresponding coefficients obtained by integration over the nebular disk. It is found that $(de/dt)_{\mathrm{sec.pert's}}$, from Equation (4), is essentially unchanged but that $e_{\max}$ is given by

$$e_{\max} = \frac{5}{2}\frac{a}{a_0}\frac{e_0}{1 - e_0^2}\frac{m_p}{m_N}T \tag{28}$$

where $m_N$ = mass of nebula and $T$ is a term of order unity associated with the assumed nebular structure. Equation (28) holds as written for $m_N \gg m_p$. An exact expression corresponding to Equation (28), valid for all $m_N$, has been derived (Heppenheimer, 1978) for the case of an augmented-mass nebula of the type studied by Weidenschilling (1977): a planar structure with surface density falling off as $a^{-3/2}$. This expression is found for $e_0 \ll 1$:

$$e_{\max} = e_0 \frac{b_{3/2}^{(2)}}{b_{3/2}^{(1)} + 0.5233n(a - 1)m_N/m_p} \tag{29}$$

where the $b_{3/2}^{(j)}$ are Laplace coefficients. The physical meaning of equations such as (28, 29) is that the nebula behaves qualitatively as if it were an additional perturbing planet having zero eccentricity. Hence $d\tilde{\omega}/dt$ in Equation (4) is increased, and $e_{\max}$ is decreased, by an "averaging" factor of order $m_N/m_p$.

Under Equation (28), $a_2$ is unchanged, $a_3$ decreases by a term of order $(m_p/m_N)^{1/4}$; but $a_1$ increases by a term of order $(m_N/m_p)^2$. This is significant since for $a \leqq a_0$, planet formation can proceed without drag.

The value of $m_N$ is known only crudely. One may mention low-mass models (Hartmann, 1972; Schatzman, 1972) for which $m_N = 0.01$ to $0.1\ m_\odot$. Weidenschilling (1977) gives $m_N/m_\odot = 0.01$ to 0.07. However, all such low-mass models attenuate Jupiter's perturbations sufficiently for the terrestrial planets to grow, at least during the time the nebula is in existence. One need not invoke the ad hoc assumption of a very low initial Jupiter $e_0$ (Heppenheimer, 1974). On the other hand, the larger values of $m_0$ and particularly $e_0$, for 61 Cygni and $\varepsilon$ Eridani, appear to render difficult any proposal that planets may have formed there. Thus, if we wish $a_1 = 1$ AU for $\varepsilon$ Eridani, then $m_N/m_p \sim 100$, which is a high-mass nebula in which turbulence could have inhibited formation of Goldreich-Ward planetesimals (Cameron, 1978). For stars such as $\alpha$ Centauri, no plausible nebular models can substantially change the results given here.

The case of $\varepsilon$ Eridani is of interest since Huang (1960) proposed it as a star likely to have Earthlike planets. But from the foregoing, it is evident that such a suggestion can be supported only if one regards that star's companion as having $m_p$ near the lower limit of the range of values proposed by Van de Kamp. Even then, one also requires the nebula to have been of several times 0.1 solar mass, when the planets were forming.

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