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Computer Simulation of the Formation of Planetary Systems

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One of the many hypotheses about the formation of the solar system postulates that the planets were formed by the aggregation of particulate matter within a cloud of dust and gas surrounding the newly-formed sun. A test of the validity of one version of this hypothesis was obtained in a computerized Monte Carlo simulation of the process.

In the model used, nuclei are "injected" into the cloud one at a time, on elliptical orbits. The dimensions of the semimajor axis and the eccentricity of the orbit of each nucleus are determined by using random numbers. As the nuclei orbit within the cloud they grow by aggregation and gradually sweep out dust-free annular lanes. If they grow larger than a specified critical mass they can begin to accumulate gas from the cloud as well. If the orbit of a planet comes inside a certain interaction distance from a planet that was formed earlier, or if the orbits cross one another, the two bodies coalesce to form a single, more massive planet which may then continue to grow by aggregation. The process of injecting nuclei is continued until all the dust has been swept from the system. At this point the run is terminated and the machine output displays the masses and orbital parameters of the planets remaining in the final configuration.

Each planetary system produced by using a different random number sequence is unique. However, all the systems so produced share the major regular features of our solar system. The orbital spacings have patterns of regularity suggestive of "Bode's law." The innermost planets are small rocky bodies; the midrange planets are large gaseous bodies; the outermost planets are generally small. The general pattern of planetary mass distribution is similar to that in our solar system with masses ranging from less than that of Mercury to greater than Jupiter's.

I. INTRODUCTION

It is hypothesized that stars and planetary systems form within cold, dark nebulae of dust and gas, specifically within objects known as "small globules," which appear to be very abundant in our galaxy. The small globules are roughly spherical in shape with masses that are estimated to embrace the mass range of individual stars, and densities of the order of 10^{-20} g cm^{-3} (Spitzer, 1968). While a complete theory of the origin of planetary systems should account for the formation of the central star as well as the planets, the scope of this paper is limited to consideration of the planetary system. The planets are hypothesized to form by aggregation of particles within the cloud of dust and gas surrounding the newly-formed star. The mechanism of the initial aggregation to form preplanetary nuclei is not specified. However once a nucleus has formed it can continue to grow by gravitational capture of particles having a low relative velocity which add to its mass and increase its ability to capture particles with slightly higher relative velocities, in a continuing process.

II. THE AGGREGATION HYPOTHESIS

Prior to the start of my computer simulation, a star of one solar mass is assumed to have formed at the center of the cloud. The star is surrounded by a spherically-symmetrical cloud of dust and gas in which the proportion of dust is of the order of 1 or 2 percent ("gas" here denoting hydrogen and helium, and "dust" denoting the remainder of the elements and their compounds). The total mass of the cloud is of the order of a few percent of that of the star. The density of the cloud decreases monotonically with distance from the star, and the density is assumed to be low enough so that the particles of which the cloud is composed may be considered to be moving about the center of mass on independent Keplerian orbits. Also some mechanism whereby particles or grains can stick together, if they touch gently enough, is postulated. Thus aggregation of particles will be occurring at random within the cloud, as governed by conditions favoring the unspecified clumping mechanism. It was recognized by Poincaré (1911) and his contemporaries that, in a cloud of particles with a nonzero net angular momentum where inelastic collisions can occur, particle orbits that are highly inclined to the invariable plane of revolution are gradually converted into lower-inclination orbits. More recently, McCrea (1960) alluded to this same process. In addition, particles on retrograde orbits are gradually eliminated from the cloud through inelastic collisions with particles moving on direct orbits; this results in a cancellation of their individual angular momenta, with both particles falling into the central star. The net result of the elimination of particles on retrograde orbits and the conversion of high-inclination orbits into low-inclination orbits is a gradual and continuing flattening of the cloud, so that, for the particulate matter, the spherical shape is lost, and the volume containing particulate orbits takes on a shape approaching that of an exocone,[1] at least in its inner regions. The term exocone, although only suggestive, is preferred to disk as being more descriptive of the general shape of the region occupied by the orbits of particles. Poincaré, in discussing a hypothesis of Du Ligondès, also showed that inelastic collisions between particles moving in the same direction tend to decrease the eccentricities of their orbits. Specifically, when two particles moving on eccentric orbits in the same direction in the same plane collide and stick together, the resultant eccentricity of the orbit of the combined bodies is generally lower than either of the original eccentricities.

At this point we can examine the formation of planets by aggregation within the cloud. The term aggregation is used instead of accretion because, in some theories of the origin of the solar system, accretion has been employed to mean the capturing by the sun of material from outside this solar system. This usually involves the sun's passing through an interstellar cloud and picking up mass from the cloud. The term accretion is avoided because of its prior use in this sense.

All during the preceding events particles of dust have been aggregating upon nuclei within the cloud. Many of these would be broken up again through collisions with one another, but here and there a nucleus would be able to grow to such size that it could begin to sweep in particles of dust and grow more rapidly as it developed an appreciable gravitational field of its own. As it orbited about it would gradually sweep out a clear dust-free lane in the exocone. If it became massive enough, it would begin to collect gas as well as dust and to grow very rapidly. In any event, at some point, when it had depleted the region from which it could sweep out dust or gas, its growth would cease.

Simultaneously other nuclei would also be growing within the exocone. As long as the orbit of the second nucleus (pretending for purposes of description that the nuclei grow one at a time, or sequentially, but realizing that many nuclei may be growing at the same time) is far removed from that of the first, it can grow independently as though the first did not exist. Similarly with other nuclei on nonintersecting paths through the cloud of dust and gas. However,

[1] Term adopted here to designate the shape produced by rotating an acute angle around an axis that passes through the vertex and is perpendicular to its bisector.

as the process continues, some nuclei, having grown to planetary size, may collide with planets that have grown earlier and merge with them to form a larger single planet. The product of the fusion process may or may not be able to grow still larger, depending on its mass and the type of growth material encountered along the new orbit that has resulted from the inelastic collision. Nuclei will continue to form and grow and merge and regrow until all the dust in the exocone has been incorporated into planetary objects. At this point the process of formation of the planetary system is complete, and there remain a number of planets in noninterfering orbits, all orbiting in the same direction around the central star, and a certain amount of leftover gas. The leftover gas, mainly hydrogen, eventually is driven entirely out of the system by the solar wind.

The merging or coalescence of colliding planets is taken as a reasonable physical process even though the problem has not yet been thoroughly explored. What would happen when two massive bodies collide would clearly depend upon the impact velocity, the angular relationships, the relative and absolute masses of the two bodies, their densities and density distributions, their material strengths, and whether they have atmospheres and the extent of their atmospheres. These factors will determine the stopping distance, the rate of conversion of kinetic energy into heat, how much mass would reach escape velocity and be ejected, the proportion of the total energy converted into rotational energy, the proportion converted into heat, and the orbital elements of the resulting coalesced body or the dispersed fragments after the collision. A statement by Hawkins (1961), which was based on his consideration of the physics of collisions of massive bodies, indicates his opinion that even rocky, atmosphereless bodies of the order of 1000 km in diameter would not lose mass when colliding with bodies of like or smaller size at low approach velocities (velocity at infinity $\leqslant 2$ km/sec). Increased mass and the presence of an atmosphere would prevent the loss of mass on collisions at even rather high approach velocities.

III. Experimental Simulation

At the start of my computer simulation the initial conditions are: a central star, of one solar mass (1 M_s), is surrounded by a cloud of dust and gas; all the retrograde particles of dust have been removed from the cloud, thus all the dust particles are moving in the same direction (the direct sense) on elliptical orbits with the center of mass at one focus; the dust particles are confined to a flattened exocone; within the exocone the ratio of dust to gas is constant, and the density of the cloud decreases monotonically with distance from the central star. In essence the computerized model simulates an experiment in which planetary nuclei are injected randomly one at a time into a cloud of dust and gas and allowed to grow by sweeping in smaller particles. When one planet has ceased to grow, another nucleus is injected. Planets are allowed to coalesce if their orbits cross or come sufficiently close to one another; growth may continue after coalescence. As the planets grow they sweep out cleared paths of an annular, washerlike shape. At first a growing planet sweeps up dust only, but if it becomes massive enough, it can begin to gather in gas as well. Nuclei are injected sequentially into the cloud until all the dust (between two arbitrary extreme boundary radii) has been swept away. At this time the experiment is over and the planetary system is considered complete.

Certain parameters must be specified to obtain quantitative results, e.g., the density distribution within the cloud; the ratio of gas to dust; a definition of critical mass, or the planetary mass above which a planet can begin to accumulate gas in addition to dust; and the orbital eccentricity of particles within the cloud. Also a few rules for the growth and coalescence of planets must be established. The primary objective of devising these experiments was to provide a realistic test of the aggregation hypothesis. Thus, emphasis was placed on finding at least one set of conditions that results in the creation of planetary systems having the general pattern of the solar system; it is not implied that these duplicate the actual set of conditions that prevailed when the solar system was formed. The main idea was to find out whether it is possible to devise a dynamic simulation of the formation of planetary systems based on the aggregation hypothesis, and whether suitable density distributions and other inputs that yield interesting results can be found.

The following conditions were assumed:

Density Distribution Within the Exocone. The overall density (ρ_2) of dust and gas within the exocone was taken to be $K\rho_1$, where ρ_1 is the density of dust, and K is a constant (the gas/dust ratio being $K - 1$). For the density distribution within the exocone, since there appeared to be no rigorous method by which a unique function might be derived, it was decided to select a generalized distribution function $[\rho_1 = f(r)]$ that yielded the following physical properties: (1) a finite density at the center ($r = 0$); (2) a monotonically decreasing density with increasing distance from the center; and (3) a finite total mass, M, within the cloud. Also a simple, continuous function rather than a complicated or discontinuous function was desired. It will be noted that any expression having properties (1), (2), and (3) will necessarily produce a distribution of mass that is a peaked function of distance, i.e., dM/dr reaches a maximum in the distance interval $(0, \infty)$. The generalized density distribution adopted was $\rho_1 = A \exp(-\alpha r^{\gamma})$, where A is the particulate density at $r = 0$, and α and γ are constants. This function possesses the desired properties. Similar functions have also been used to describe the observed spatial distribution of stars in globular clusters (Kurth, 1957); that is, somewhat similar density distributions may be found in nature.

Eccentricity of Cloud Particles. To simplify the model, all particles within the cloud were assumed to have the same orbital eccentricity, W, but the inclinations of their orbits and the orientations of the long axes of their orbits were assumed to be distributed randomly. It is not necessary to specify a maximum allowable inclination, $\theta_{\max}$, of particle orbits.

Planetary Nuclei were assumed to have an initial mass, m_0. These were injected into the cloud one at a time and allowed to grow to completion before another was injected. All planetary nuclei were injected into the invariable plane with inclination zero and direct motion but with a semimajor axis and eccentricity chosen at random, using the internal random-number generator of the IBM 7044 computer.

Sweep Radius. In addition to capturing particles that cross its orbit, a planetary nucleus was allowed to capture particles by gravitational attraction if their orbits came sufficiently close to one another, which is defined as the distance x, a function of the instantaneous mass of the nucleus m, relative to the central body (of unit mass), and its distance from the center of mass:

$$x = r[m/(1 + m)]^{1/4} = r\mu^{1/4},$$

where x may be considered the half-width of an annular ring centered upon the orbit of the gravitating body. This function is derived from an extension of the three-body problem. In the restricted three-body problem where the two massive bodies have masses $1 - \mu$ and μ, respectively (μ being much the smaller of the two), it can be shown (Dole, 1961) that particles on in-plane circular orbits about the center of mass of the system and orbiting with the same sense as the two massive bodies (i.e., direct) have unstable orbits (low velocities with respect to μ) and can be captured by μ if their orbital radii fall within an annular band approximately centered upon the orbit of μ (radius, r) and having a half-width that is dependent only upon r and μ. For small values of μ the half-width, x, can be approximated roughly by an expression of the general form: $x = r\mu^c$ where c is of the order of $1/4$. A planetary nucleus is allowed to grow iteratively by sweeping in all cloud particles that cross its orbit and all particles that come within the distance x of its orbit, gradually sweeping out, and adding to its mass, all dust particles within a flat, washerlike volume of space.

Critical Mass. At first the planetary nucleus sweeps in dust particles only. However, if its mass becomes sufficiently large so that it can capture and hold gas at the temperature at its escape layer, it can also begin to sweep in gas as well as dust from the cloud. In order to retain a gas, the ratio $R^2\rho_m/T$ must be equal to or greater than $9k_1k_3{}^2/8\pi Gk_2w$, where R = radius of planet to escape layer, ρ_m = mean density of planet, T = absolute temperature of the gas at escape layer, k_1 = the Boltzmann constant, k_2 = the mass of one molecular weight, G = the gravitational constant, w = the molecular weight of the gas, and k_3 = the ratio of the escape velocity at the escape layer to the mean molecular velocity of the gas. Using Jean's criterion that k_3 must be of the order of 5 or greater for long term stability, the ratio $R^2\rho_m/T$ must be $<10^{16}\ w^{-1}$ g cm^{-1} deg^{-1} for a planet to retain a gas. It was assumed that, at critical conditions determining whether or not a growing nucleus will be able to capture and retain light gases and become a giant planet, all planets have the same atmospheric composition at the escape level, and the same albedo, and that the temperature at the escape level is determined by the received radiation from the sun at perihelion, r_p, the absorbed energy density, E, following the inverse square law, $E \sim r_p^{-2}$, and the reradiated energy following Stephan's law, $E \sim T^4$. At equilibrium $T = k_4 r_p^{-1/2}$. Assuming for simplicity that all planets have the same mean density, ρ_m, it can be shown that the critical mass may be defined as $m_c = Br_p^{-3/4}$. Depending on the values selected for w, k_4, and ρ_m, the proportionality factor B will be in the neighborhood of 1×10^{-5} to 2×10^{-5} solar mass.

Growth Above Critical Mass. Having reached critical mass, a planet can begin to capture gas as well as dust, and its ability to capture gas will increase as the ratio, m/m_c, becomes larger. Therefore it was necessary to assume a relationship between the effective density, ρ, of material being collected and the ratio, m/m_c. The function employed was $\rho = K\rho_1/[(K-1)(m_c/m)^\beta + 1]$, which has the desired characteristics that $\rho = \rho_1$ for $m = m_c$ and ρ approaches $K\rho_1$ asymptotically as m/m_c becomes large.

Rules for Coalescence. It sometimes occurs during the course of a run that two planets come within a distance x of each other. When this happens they are allowed to collide inelastically and to coalesce, forming a single planet. The body formed from the combined masses will continue to grow if conditions are suitable. To simplify the computer simulation, the processes of growth–coalescence–growth take place sequentially; that is, growth is allowed to reach completion before coalescence can take place.

Boundaries. To obtain quantitative results, it was necessary to place boundaries on the physical dimensions of the space within which the computer program was operating. An inner boundary, close to the central star, may be considered to delineate a region within which aggregation is somehow prevented by ambient conditions, such as high temperature, or from which particulate matter is removed too rapidly, by spiraling into the star as a result of aerodynamic drag, to permit the growth of nuclei. A remote outer boundary may be considered to delineate a region outside of which particulate densities are too low to permit aggregation to take place. In the computer program the boundaries were set arbitrarily at 0.3 a.u. and 50 a.u., that is, no planetary nuclei were injected with semimajor axes less than 0.3 a.u. or greater than 50 a.u.

As each planetary nucleus grows, it sweeps out a cleared lane of dust in the cloud. Some of the gas also is swept up in the vicinity of orbits of planets having masses greater than m_c. The computer program keeps an accounting of the types of bands remaining in the system at any given step: type 0 bands contain dust and gas in the original proportions; type 1 bands contain no dust but have all the original gas; type 2 bands have had some of the gas swept away as well as all of the dust. The run is terminated when there are no type 0 bands remaining between 0.3 and 50 a.u. A planetary nucleus cannot grow, of course, when it is injected into a region of space containing no dust.

In order to conduct a series of simulated syntheses of planetary systems in the ACRETE program it is necessary to input values of K, A, α, γ, W, m_0, B, and β, and to provide a series of starting numbers (X_0) for the random-number generator, a different one for each run.

TABLE I

VALUES OF A, K, AND W[a]

Set number	A	K	W
1	0.00125	100	0.15
2	0.00125	100	0.20
3	0.00150	50	0.20
4	0.00150	50	0.25
5[b]	0.00150	50	0.25

[a] A = Coefficient of density in cloud in solar masses per cubic astronomical unit; K = ratio of overall density to density of dust; W = eccentricity of particles in cloud.

[b] Set 5 involved the same parameters as Set 4. However, the initialization numbers input to the random-number generator were selected at random, as a check on the results of Set 4.

IV. Computational Results

The computer runs produced planetary systems with all the general characteristics of the solar system; small rocky planets close in, large planets composed principally of gas in the middle distance range, and small planets again in the outer orbits, the numbers of planets in each system ranged generally from 7 to 14; the orbital radii of the planets followed a pattern generally similar to that of Bode's law, i.e., a fairly uniform spacing ratio (radius of planetary orbit divided by that of next interior orbit); the total mass of the planets in each system was comparable to that of the solar system; the largest planet in each system was similar to Jupiter in mass.

A. *Planetary Systems*

In a series of preliminary runs in which the major input parameters were varied experimentally, it was established that planetary systems much like the solar system could be obtained when using the following input values: $\alpha = 5$ and $\gamma = 0.333$, in the expression $\rho_1 = A \exp(-\alpha r^{\gamma})$; $B = 1.2 \times 10^{-5}$ M_s, in the expression $m_c = Br_p^{-3/4}$; $\beta = 0.5$, in the expression for growth above critical mass; and $m_0 = 10^{-15}$ M_s, for the initial mass of an injected nucleus. Actually none of these values is highly critical; that is planetary systems still result when the parameters are varied over a fairly wide range. For example, much smaller values of m_0 could have been used without affecting the results. However, in the runs discussed below, the above

TABLE II

COMPUTER-GENERATED SYSTEMS COMPARED WITH SOLAR SYSTEM

Item	Solar system	Set 3	Set 4	Set 5
Number of planets	9			
Mean	—	10.1	9.2	9.1
Range	—	7–12	7–11	7–11
Total mass of planets ($\times 10^3$)[a]	1.34			
Geometric mean	—	1.16	1.56	1.71
Range	—	0.58–1.92	0.43–3.04	0.53–3.64
Mass of largest planet[b]	317			
Geometric mean	—	258	305	430
Range	—	90–594	63–979	89–1200
Spacing ratio				
Mean	1.86	1.73	1.84	1.86
Range	1.31–3.41	1.17–4.09	1.22–3.37	1.22–4.01

[a] Sun = 1.

[b] Earth = 1.

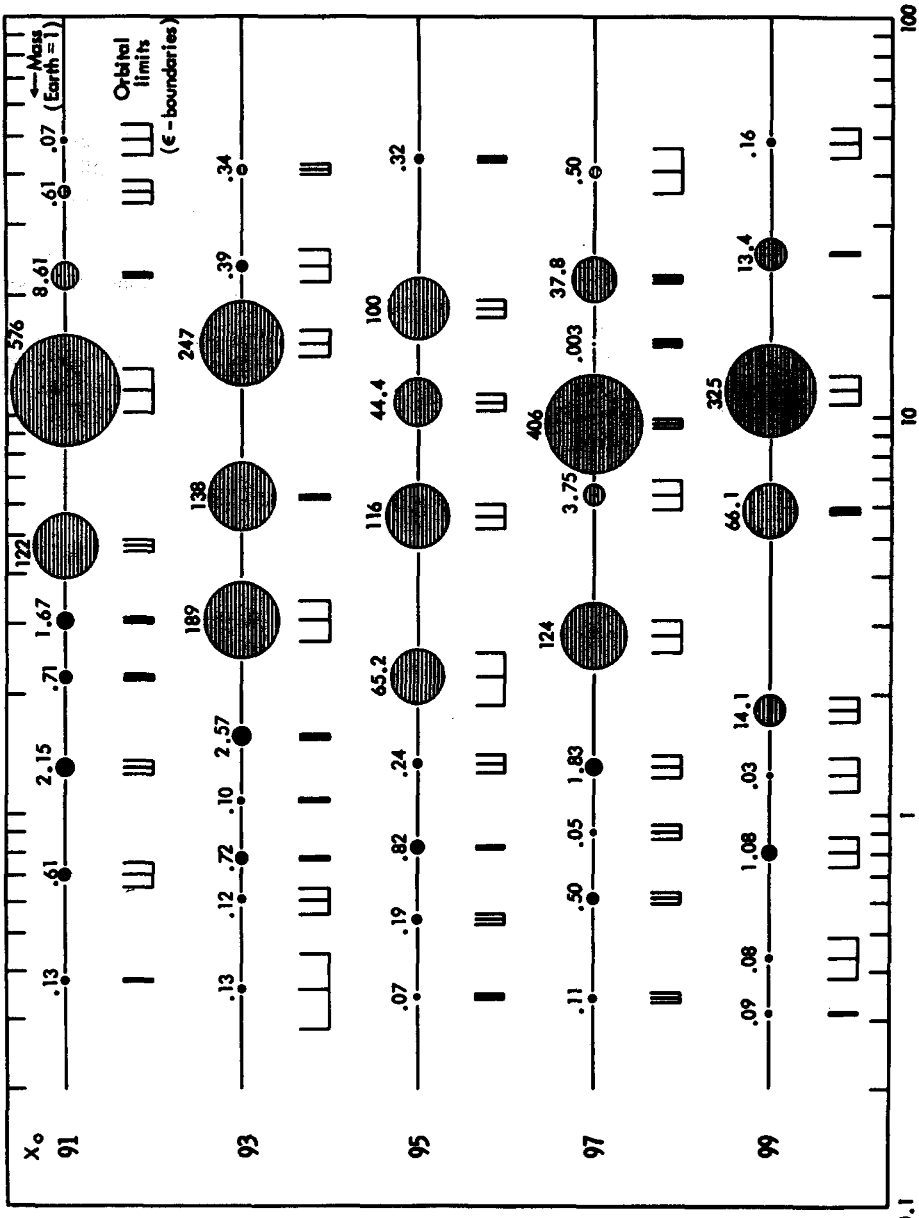

FIG. 1. Examples of planetary systems generated in computer synthesis. X_0 is initiating number for random number series used in program. Positions of circles along line indicate mean orbital radius; numbers above circles and sizes of circles indicate planetary masses (radius of circle ~ planetary mass$^{1/3}$); solid circles are terrestrial bodies; horizontal shading designates gas giants.

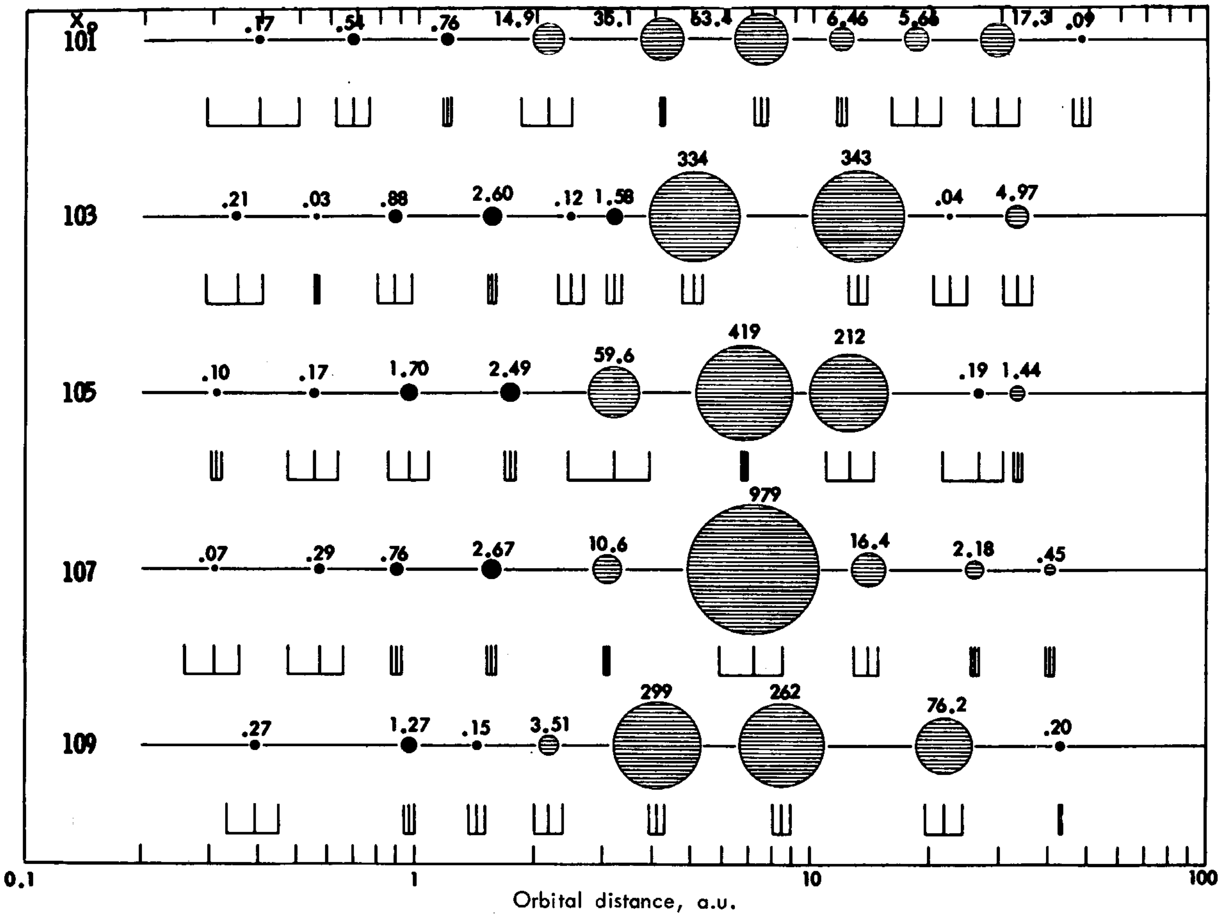

FIG. 2. Examples of planetary systems generated in computer synthesis. X_0 is initiating number of random number series used in program. Positions of circles along line indicate mean orbital radius; numbers above circles and sizes of circles indicate planetary masses (radius of circle ~ planetary mass$^{1/3}$); solid circles are terrestrial bodies; horizontal shading designates gas giants.

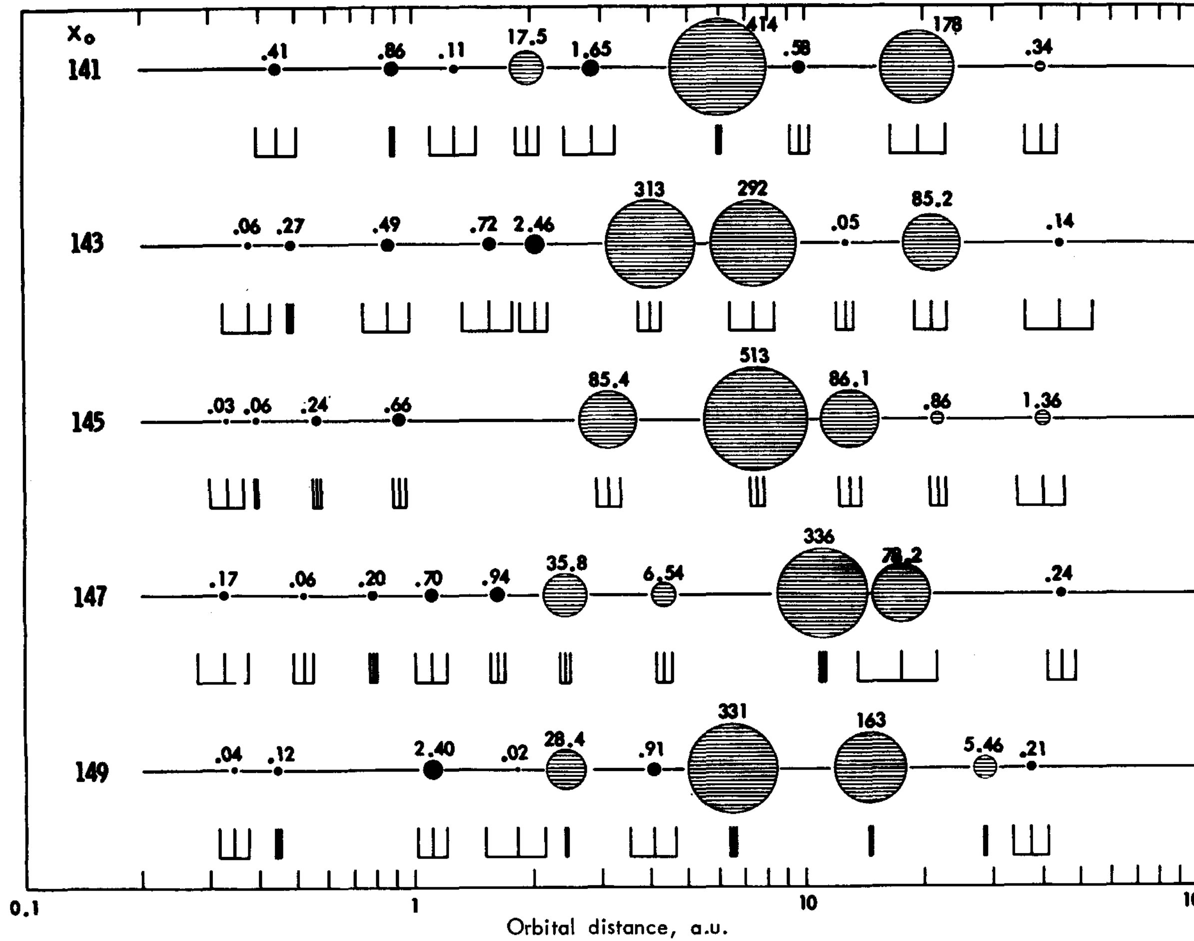

FIG. 3. Examples of planetary systems generated in computer synthesis. X_0 is initiating number for random number series used in program. Positions of circles along line indicate mean orbital radius; numbers above circles and sizes of circles indicate planetary masses (radius of circle ~ planetary mass$^{1/3}$); solid circles are terrestrial bodies; horizontal shading designates gas giants.

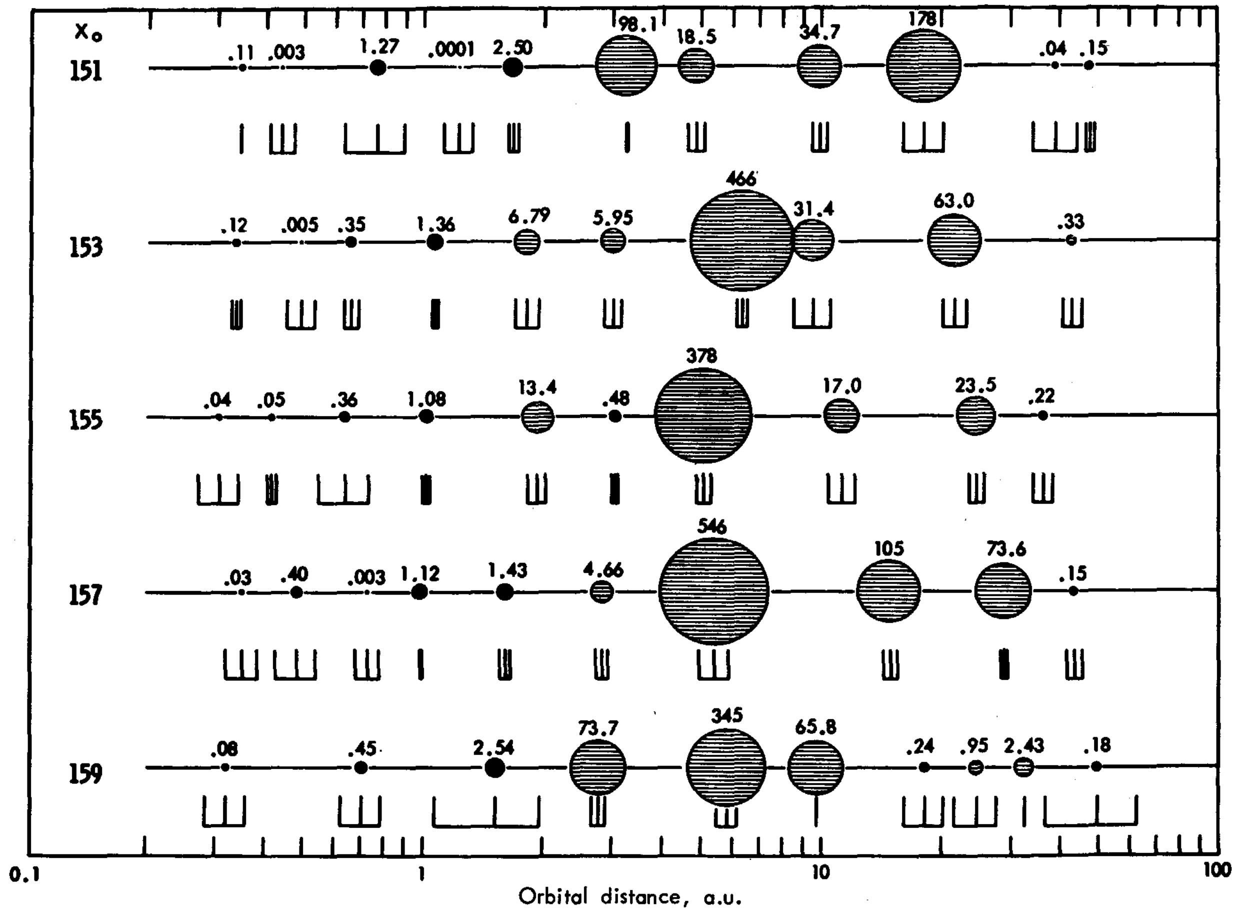

FIG. 4. Examples of planetary systems generated in computer synthesis. X_0 is initiating number for random number series used in program. Positions of circles along line indicate mean orbital radius; numbers above circles and sizes of circles indicate planetary masses (radius of circle $\sim$ planetary mass$^{1/3}$); solid circles are terrestrial bodies; horizontal shading designates gas giants.

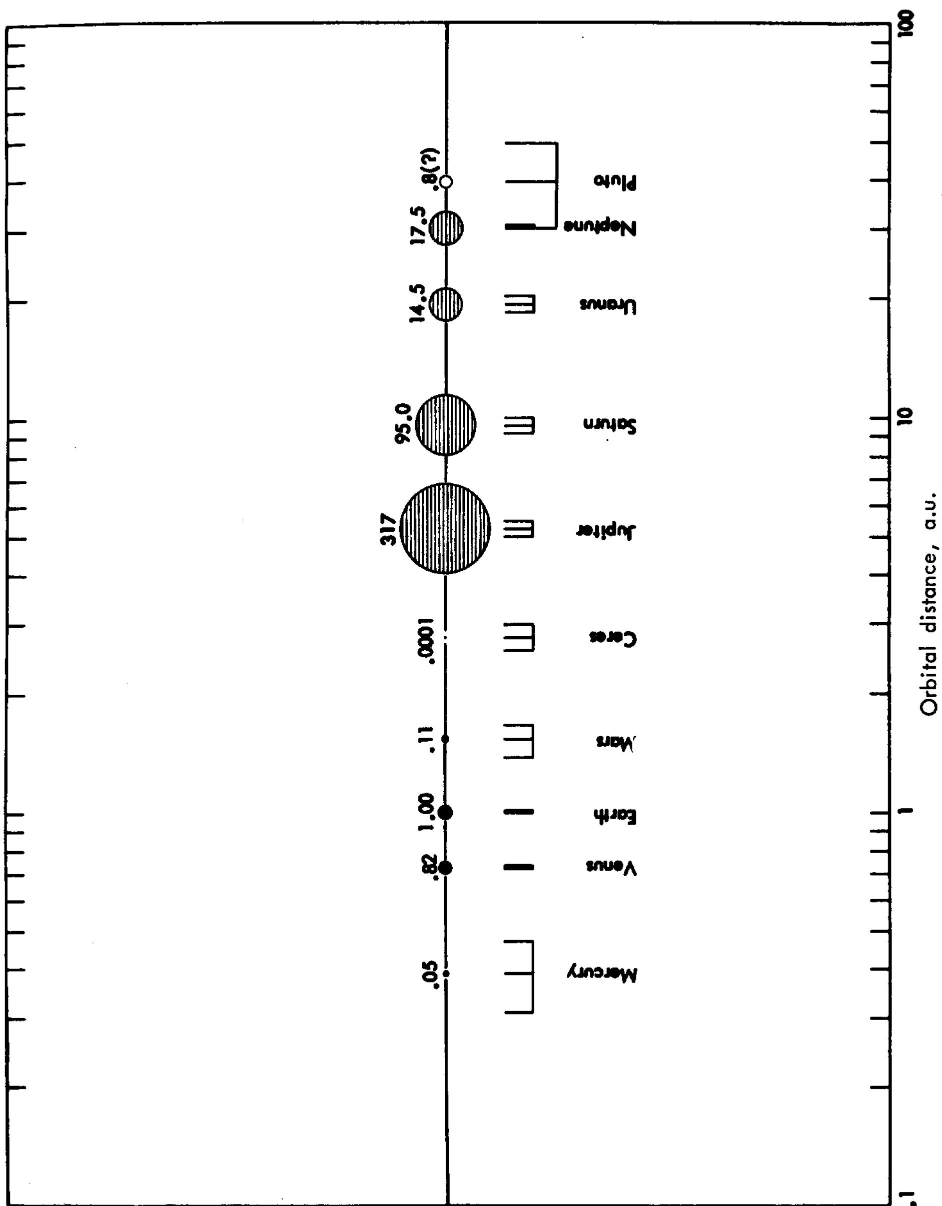

FIG. 5. The solar system represented in same manner as Figs. 1–4.

TABLE III

COMPUTER-GENERATED SYSTEMS COMPARED WITH SOLAR SYSTEM

	Mass range								
	10^{-10}	10^{-9}	10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}
	Number of planets in indicated mass range								
Solar system	1	0	0	2	3	2	2	0	
Set 3 average	0.03	0.03	0.53	3.2	3.4	1.3	1.9	0.33	
Set 4 average	0.03	0.08	0.40	2.9	2.8	1.1	1.7	0.65	
Set 5 average	0	0.02	0.18	2.6	2.8	1.6	1.3	0.68	

parameters were fixed at the values noted, and parameters: A, central cloud density in solar masses per cubic astronomical unit (a.u.); K, ratio of total cloud density to density of dust; and W, mean eccentricity of orbits of particles in the cloud, were varied to find combinations of conditions that produced planetary systems most similar to the solar system in general character. (It is postulated that the solar system may be taken as a representative example of a planetary system.) Similarity to the solar system was judged on the criteria of number of planets, N, and the total mass of the planets, $\sum m$. (For the solar system $N = 9$ and $\sum m = 1.345 \times 10^{-3}$ M_s.[2]) Five sets of runs (of 40 runs each) have been carried out, in which A, K, and W were assigned the values in Table I.

Samples of the results of the Set 4 runs are illustrated herein, and the results of the runs of Sets 3, 4, and 5, in which planetary systems closely similar to the solar system were obtained, are summarized in Table II. The similarities to the solar system in spacing of orbits and sizes of individual planets may be seen in Figs. 1 through 4, in which the orbital radii are depicted on a logarithmic scale, and the planetary masses are indicated by the sizes of the circles (radius of circle proportional to $m^{1/3}$). Terrestrial bodies are shown as solid circles, gas giants by horizontal shading. Figure 5 shows the solar system represented in the same manner for comparison.

From the schematic diagrams, as well as from the summary data, it may be seen that these planetary systems bear many marked resemblances to the solar system. It is not to be expected that any of the systems so produced would be identical to the solar system in all respects; this is far too much to expect from so small a sample. Yet the solar system could be intermingled with the others and not be recognized as not being a member of the same set.

The total masses of the planets in the Set 4 systems, for example, are similar to that of the solar system, averaging $1.56 \times 10^{-3} M_s$ versus $1.34 \times 10^{-3} M_s$ for the solar system. The masses of the largest bodies in these systems average $0.92 \times 10^{-3} M_s$ versus $0.96 \times 10^{-3} M_s$ for Jupiter. The orbital spacing ratios are also very similar to those in the solar system, averaging 1.77 (1.84) versus 1.69 (1.86) for the solar system, the figures in parentheses being the averages when all bodies of mass less than $10^{-7} M_s$ are excluded. Some systems contain spacing ratios smaller than any in the solar system; some contain spacing ratios comparable to that between Mars and Jupiter. Similar results were obtained in Set 5.

As shown in Table III, the mass distributions also have general similarities to that of the solar system planets, with most of the bodies falling into the mass range $10^{-7} M_s$ to $10^{-3} M_s$ and being distributed rather evenly within this range.

B. Multiple Star Systems

The effects of changing A (coefficient of density in the cloud) were investigated in a separate series of computer runs. Four

[2] Or $N = 10$ if Ceres is counted as a planet.

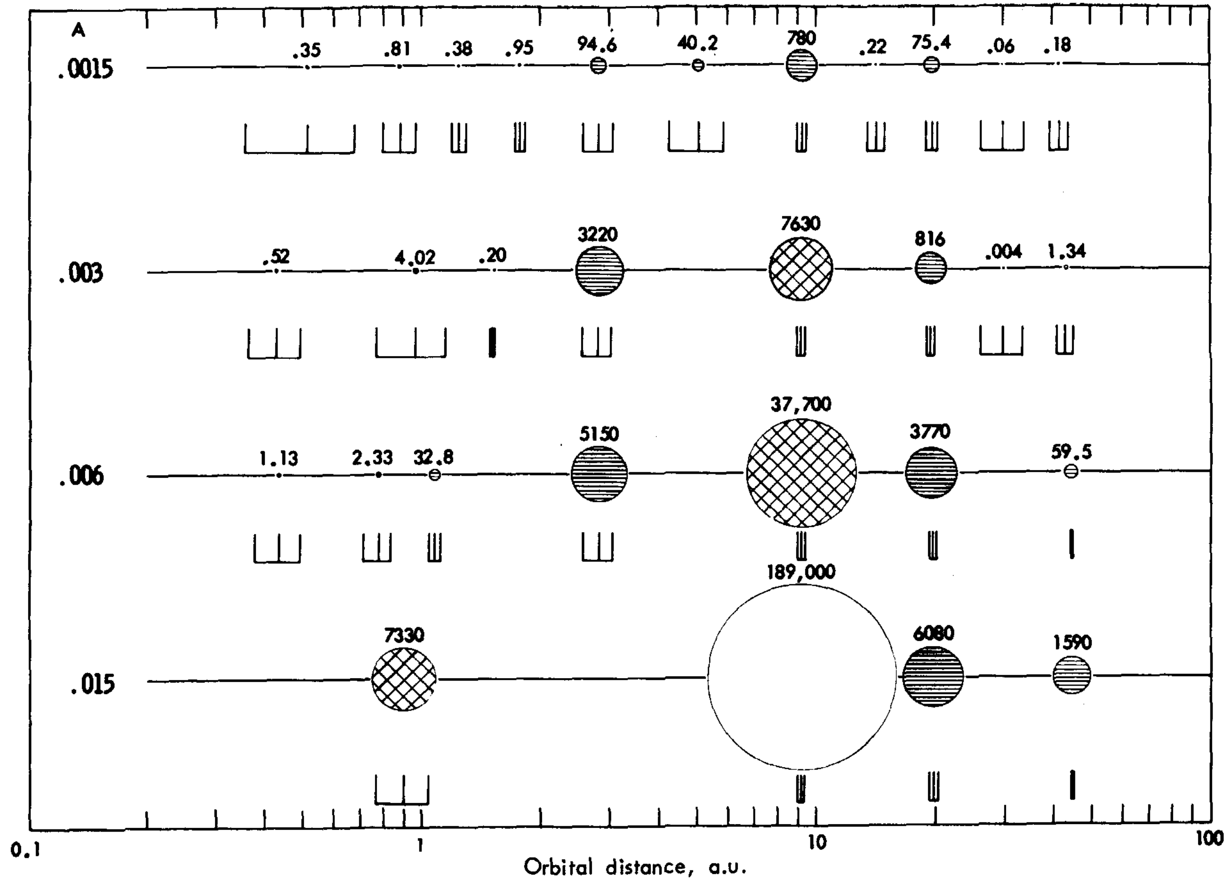

Fig. 6. Examples of binary and multiple star systems generated in computer synthesis. Starting number, X_0, for random number series is 25 in all cases. The effect of increasing A, the coefficient of density in the original cloud, is shown. Scale of diameters not the same as in Figs. 1–5. Solid circles are terrestrial, horizontal shading designates gas giants, cross-hatching designates red dwarf stars, the open circle represents an orange dwarf star.

runs were made at each condition, employing the same starting random numbers to reduce the effects of changing too many variables at the same time.

Parameters held constant were $K = 100$, $W = 0.15$. Parameter A was varied from 0.0015 to 0.015. Some of the results are illustrated in Fig. 6, where red dwarf stars are identified by cross-hatchings; the open circle represents an orange dwarf star.

Even small increases in A result in large increases in the total mass of the systems produced; increasing A also decreases the average number of planets per system. As may be seen in Fig. 6, for $A = 0.003$ and 0.006 the planetary system has become a binary star system, the body near 9 a.u. having grown large enough to be considered a red dwarf star. Observationally, the two stars of smallest mass now known are members of a binary system designated L726-8; each star has a mass estimated at about 0.04 M_s (about 40 times the mass of Jupiter) or 13,000 times the mass of the earth. The lower limit on stellar mass is believed to be near 0.02 M_s. It will be noticed that the binary star systems still contain numerous planetary bodies. As A is increased still more the systems become multiple-star systems and the number of planetary companions diminishes. Actually, the results at the higher values of A should be considered only suggestive of the general trend since the total mass of the "planetary" bodies is now becoming fairly high with respect to that of the central body, so that the original simplifying assumptions, which were adequate when the total planetary mass was well below 0.01 M_s, no longer apply so satisfactorily. The gravitational attractions of the several large masses for each other can no longer be considered to have negligible effects on the secular stability of the systems. This is pushing the program somewhat beyond its original intent (to create planetary systems similar to the solar system). However, it would be readily possible to modify the program slightly to provide more rigorously for cases in which some of the planetary bodies grow to stellar mass. In any event, the general trend is clear. Simply increasing the value assigned to one parameter makes it possible to generate widely spaced binary and multiple-star systems.

V. Conclusions

The theory of the formation of planetary systems presented here depends upon aggregation, a mechanism by which small particles can cling together to form preplanetary nuclei. If such processes did take place during the early days of the solar system, then the major properties of the solar system emerge. The present theory accounts for the distribution of mass among the planets, the differences in composition between the close-in terrestrial planets and the giant planets farther out and the near-constancy of spacing ratio for orbital distances. [The fact that the freely rotating (not tidally braked) bodies rotate with the same sense that they revolve about the sun has already been explained qualitatively by Guili (1968) who showed that the impacting of many small bodies from elliptical prograde orbits onto a planetary body results in a net prograde rotation of the body.]

The theory also provides an explanation for the fact that the planetary axes of rotation are tilted out of normal to the invariable plane, the result of chance inelastic collisions and coalescences with a few rather massive bodies in addition to the many small bodies being collected. While the computer program does not make provisions for the generation of satellites, it is implied by the aggregation theory that the nuclei of the large satellites (including the Moon) formed in the vicinity of their primary planets very early in the process and that they grew by aggregation at the same time their primary planets were growing but at a slower rate, since they were initially smaller and competing for mass. It is implied they were originally in orbits more remote from their primary planets and that the orbital distances decreased as the primaries grew in mass. In other words, the large satellites have accompanied their primaries "since the beginning."

The present theory is compatible with the fact that the earth (presumably each of the other planets too) is still sweeping up mass in the form of meteorites and micrometeorites at a rate of several thousand tons per day. The rate must be diminishing as the system becomes cleaner with the passage of time. Thus the rates of mass aggregation must have been much greater in the past than they are now and all the planets were subjected to a heavy bombardment of infalling material, as evinced by the pock-marked face of the Moon, the craters of Mars, and the ancient large meteorite craters of the earth (now much weathered for the most part).

The theory is also compatible with the presence of comets in the solar system, which may be considered primordial matter that has not yet been swept up by the planets.

It is, of course, recognized that numerous assumptions had to be made in producing a quantitative model of the formation of planetary systems (this is unavoidable) and that some of the results depend heavily upon these assumptions. For example, the fact that the largest planetary bodies in each system are found in the midrange, roughly between 5 and 20 a.u., is a result of using a density distribution of the type selected. However, many of the general results are not critically dependent upon the specific forms of the relationships that were used, or the values of the parameters involved.

It is concluded that the aggregation theory for the formation of planetary systems can account for many of the intriguing regularities and irregularities of the solar system, and is compatible also with the observed properties of many of the known, widely-separated binary star systems.

No attempt has been made to place a time scale on the aggregation process. However, since the capture cross section of any gravitating body is extremely large for particles having very low relative velocities, if such particles are available and if a body can retain those particles that impinge upon it, it should very rapidly, within a small number of revolutions, collect many particles with very small relative velocity and grow in mass, continuously increasing its ability to attract and capture particles from a widening region around its orbit until the volume from which it can grow has been depleted of material.

The simulation program was deliberately simplified for exploratory purposes; for example, by injecting all preplanetary nuclei into the cloud with zero inclination (orbits lie in the invariable plane), by omitting provisions for generating satellites, and by treating all planets as having the same bulk density in spite of differences in composition. I do not believe, however, that the overall results, as presented, would be changed significantly by making the model more complicated.

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