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FORMATION OF THE TERRESTRIAL PLANETS

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George W. Wetherill
Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, DC 20015

1 INTRODUCTION

In attempting to describe the formation of the terrestrial planets, one immediately faces the problem of where to start. These bodies comprise less than 0.5% of the mass of the planetary system, which in turn is only ~0.1% of the mass of the Sun. Because the formation of the Sun is the principal event involved, it may seem logical to first discuss the formation of the Sun before considering such small details as terrestrial planets. On the other hand, at present the formation of stars of one solar mass ($1\ M_{\odot}$) is poorly understood (Larson 1978). For this reason perhaps it would be preferable to start with the wealth of detailed information available from observation of the Earth, Moon, and other planets. However, the problem of even approximately uniquely inverting these observations to obtain the conditions of planet formation is far beyond our present ability. Until the "forward problems" of planetary growth are much better in hand, requests for the luxury of uniqueness are premature.

For these reasons, a starting point intermediate between these alternatives has been chosen. Regardless of how stars like the Sun form, at some time matter in the form of grains or small bodies must have aggregated to form the terrestrial planets. The possible mechanisms by which this may be accomplished are

1. Gravitational collapse. Matter is aggregated into larger objects under the influence of a collective gravitational field following the development of a gravitational instability.
2. Accumulation. Bodies grow by the gradual addition of matter, following two-body collisions and coherence.

Processes 1 and 2 may occur either with or without the presence of a dynamically significant gas phase.

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The assemblage of these smaller bodies and gas in the form of a flattened rotating disk is generally referred to as the solar nebula. If the term is defined this broadly, all current theories involve a solar nebula of some kind.

In this review emphasis will be placed on the present state of our understanding of the ways in which processes 1 and 2 operate, with special attention given to their relevance to the formation of the terrestrial planets. It should not be assumed that these processes are mutually exclusive. It is just as likely that both were important at different stages of planet formation. Choice of the most probable chain of events whereby the planets were actually formed will not be possible before we understand these processes well enough to permit prediction of the properties of the resulting planets.

This approach permits starting at some intermediate stage with a collection of matter that shows some promise of being capable of evolving into the present terrestrial planets, and following the evolution of that matter by the mechanisms described above. It is necessary but not sufficient to show that this intermediate stage can indeed lead to the observed planets. It is also necessary to show that the intermediate stage arises naturally as a by-product of the formation of the Sun. Moreover, the same theory of solar formation must provide appropriate starting points for the formation of the giant planets, asteroids, comets, and satellite systems. This requirement will become more demanding as our knowledge of star formation improves.

1.1 *General Observational and Theoretical Background*

Astronomical observation (Strom, Strom & Grasdalen 1975) shows that star formation is occurring in the Galaxy in regions, such as the Orion nebula, in which hydrogen is in the molecular form, and in which the density of gas and dust is 10^3–10^6 times greater than the average interstellar density. Most observational data on newly formed stars has been obtained from the more massive O and B stars in these regions. However, pre–main-sequence stars of 1–2 $M_\odot$ exhibiting an infrared excess have been observed in the same regions. Therefore there is some observational support for the existence of "stellar nebulae" in the vicinity of young stars similar in mass to the Sun. However, few details concerning the quantity and distribution of the circumstellar dust are available. Infrared data show that it is likely that other low-mass stars are surrounded by dense, opaque dust clouds. This heavy obscuration precludes learning much about these objects by observation.

From the theoretical side, the circumstances under which the material of a molecular cloud collapses to form a single star like the Sun are also quite obscure. When rotation is included, hydrodynamic numerical simulations of such collapse appear to show that two or more gravitationally bound stars of comparable mass are usually formed, rather than a single star like the Sun (Bodenheimer & Black 1978). It is possible that single stars are formed in the more infrequent circumstance of clouds with very low angular momenta, or result from ejection of single stars from multiple star systems, or from aggregation of binary systems in an accretion disk. It is not at all clear if it is appropriate to associate the observed objects that appear to be surrounded by opaque clouds with these single stars.

In summary, although it is plausible to associate the formation of the planets with the aggregation of dust and gas in a primitive "solar nebula," neither theory nor observation of star formation provide quantitative constraints on the properties of this nebula. Since the oldest known stars of the galactic disk population appear to be about the same age as the solar system, ~ 5 billion years (Demarque & McClure 1977), it is even possible that the Sun and planets were formed during a unique epoch in galactic evolution, and that presently observed processes of star formation may not be directly applicable to the formation of the solar system.

In contrast to the rather obscure information provided by stellar astronomy, there is a fairly large body of observational data relevant to terrestrial planet formation which is obtainable from these bodies themselves, as well as from meteoritic data. Numerous examples can be given:

It is known that the relative proportions of the refractory chemical elements in the Earth, Moon, and meteorite parent bodies are very similar to those in the Sun and in average solar system matter. On the other hand, the elements that for the most part fail to form nonvolatile compounds are severely depleted in these bodies (Ganapathy & Anders 1974).

The nonzero eccentricities and inclinations of the planets and the magnitude and direction of their axial rotation vectors carry information relevant to conditions at the time of formation even though this relationship is poorly understood at present (Safronov 1969, Harris 1977).

Interpretation of radiogenic isotopic data permits inferences regarding the time scale of planet formation. For example, lunar and meteoritic data show that heat sources sufficient to produce internal igneous differentiation were present very near to the time of formation (Allègre et al. 1975, Papanastassiou & Wasserburg 1975; reviewed by Wetherill 1975; Kirsten 1978).

The observed cratering record constrains the mass and size distribution of the bodies remaining following the principal stage of planet formation (Wetherill 1977).

Even though much of these abundant data are of considerable precision, at present they fail to constrain our ideas concerning planet formation sufficiently well to allow us to make firm inferences regarding physical conditions in the solar nebula prior to planet formation. However, by an iterative process, it may be hoped that continued comparison of theory and observation will lead to a picture of terrestrial planet formation that is not only generally accepted, but has a good chance of being more or less true.

1.2 *Scope of This Review*

It is probably unnecessary to say that we are still far from this goal. Nevertheless, the volume of work both published and in progress directed toward this goal is large and increasing. The purpose of this review is to summarize recent theoretical investigations directed toward terrestrial planet formation per se, rather than to discuss general theories of star and planet formation. The views of various authors concerning the more general problems, as well as references to earlier work, can be found in two recent collections of papers (Dermott 1978, Gehrels 1978), in several books (Safronov 1969, Alfvén & Arrhenius 1976), as well as in a number of articles (Woodward 1978, Cameron 1978b, Prentice 1978, Horedt 1979). Although the evolution of the spin and orbital angular momenta of the terrestrial planets is an important aspect of the subject of this review, the present author lacks the inclination to undertake this, and defers to the forthcoming review by Harris (1981). An extensive discussion of the origin of the Earth and the Moon, with emphasis on geochemistry, has been given by Ringwood (1979).

The two general mechanisms of terrestrial planet formation, gravitational collapse and accumulation, will be discussed for the cases of both a gaseous solar nebula and a gas-free nebula. Emphasis will be placed on concepts, which will be elucidated by simplistic theoretical expressions. Readers will find at least as much complexity as they wish in the references cited.

2 GRAVITATIONAL INSTABILITY

2.1 *Formation of Giant Gaseous Protoplanets by Gravitational Instability*

Gravitational instability occurs when the density of an assemblage of dust and gas is so large that a positive perturbation in density will grow with time. Under these circumstances the mutual gravitational attraction of the material overcomes the effects of internal kinetic energy (temperature), rotation, and magnetic fields, all of which tend to inhibit gravitational instability. In the absence of rotation and magnetic fields, the critical wavelength (λ_c) of a perturbation is given by

$$\lambda_c^2 = (\pi c^2/G\rho) \tag{1}$$

where c is the sound velocity $(\gamma RT/\mu)$, ρ is the density, γ is the ratio of specific heats, T is the temperature (K), μ is the molecular weight, and G is the gravitational constant. Equation (1) is known as the Jeans criterion (Jeans 1929). Instability will result for perturbations of wavelength $\lambda > \lambda_c$, and the smallest perturbation that can grow will be given by $\lambda = \lambda_c$.

Although this criterion must be modified in the presence of rotation and magnetic fields, both of these effects oppose the development of instability, requiring even larger values of λ. Therefore Equation (1) will provide information regarding the *minimum* dimension and mass of a terrestrial planet that can form as a result of gravitational instability in a nebula of given density.

For a body of mass M, the density of a spherical body with a diameter equal to λ_c will be

$$\rho = (6M/\pi\lambda_c^3). \tag{2}$$

Substituting (2) into (1) gives

$$\lambda_c = (6MG/\pi^2c^2) = (6MG\mu/\pi^2\gamma RT). \tag{3}$$

For the case of formation of a gravitational instability in a circumsolar nebula, it is also necessary that the density be high enough to prevent tidal disruption, i.e. greater than the Roche density ρ_R:

$$\rho_R = (3M_\odot/2\pi a^3) \tag{4}$$

where $M_\odot$ is the solar mass, and a is the semimajor axis. This implies that

$$(\lambda_c/2) \leqq a(M/2M_\odot)^{1/3} \cong 1.15L \tag{5}$$

where L is the radius of the collinear Lagrangian point in the restricted 3-body problem (e.g. Blanco & McCuskey 1961, Szebehely 1967).

In the presence of nebular gas, the appropriate mass of the Earth will be the present Earth's mass (6×10^{27} g) augmented by the complement of volatile gas (mostly H_2 and He) which in material of average solar system composition is associated with the nonvolatile elements that form the Earth. Addition of this material increases the mass by a factor of ~ 375, and $M = 2.25 \cdot 10^{30}$ g. Use of Equation (3), with $T = 300$ K, $\mu = 2$, $\gamma = 1.4$ (corresponding to H_2) gives a critical wavelength

$$\lambda_c = 5.2 \times 10^{12}\ \text{cm} = 0.35\ \text{AU}.$$

The corresponding density given by Equation (2) will be

$$\rho = 3.1 \times 10^{-8}\ \text{g cm}^{-3}.$$

At the distance of the Earth from the Sun with the present value of the Sun's mass, the critical Roche density, below which the matter will be unstable with respect to tidal disruption, is 2.9×10^{-7} g cm^{-3}. Therefore an Earth "protoplanet," satisfying the Jeans criterion, will not be stable with respect to tidal disruption. Assumption of a lower value of the temperature in (3) will lead to even lower densities, and greater tidal instability.

If the plausible assumption is made that the density in the solar nebular becomes higher as the Sun is approached, the density found above for the gravitational instability leads to a total mass of the nebula within 1 AU of

$$M(<1\,\mathrm{AU}) > 1.31 \times 10^{32}\,\mathrm{g},$$

assuming a mean thickness of the nebula of 0.1AU. This is about 30 times the mass required to form the terrestrial planets. If a higher temperature, rotation, or magnetic field are invoked in order to achieve tidal stability, the quantity of excess mass becomes enormous, and no way to remove a mass so large is known.

For these reasons, formation of the terrestrial planets by gravitational instability near their present positions *after* the formation of the Sun doesn't appear to be possible. Furthermore, a fully developed planet formed in this way would resemble Jupiter, not Earth or Venus. Therefore, theories proposing that the terrestrial planets formed by gravitational instability must postulate the formation of these planets at greater heliocentric distances, the protoplanets moving sunward along with the gas that is to form the Sun. When a protoplanet is sufficiently close to the protosun, the gaseous envelope of the collapsing protoplanet can then be removed by tidal disruption leaving behind a nonvolatile core, which becomes the terrestrial planet (Donnison & Williams 1975, Cameron 1978a).

The physics of this chain of events is very complex. No secure calculations of the expected number, position, and size distribution of the gaseous protoplanets have been presented. The detailed nature of the redistribution of mass and angular momentum during the collapse of the solar nebula has not been worked out. Calculations have been presented regarding the evolution and differentiation of the protoplanets (DeCampli & Cameron 1979, Cameron 1979). When one considers the complexity of this problem, it is hard to be at all certain that all the required events will occur on schedule. For example, the timing of solar and protoplanetary growth must be so well coordinated that the observed number of terrestrial planets will enter the region of tidal instability after the formation of a nonvolatile core but before the collapse of a significant fraction of their gaseous envelopes. In order to produce the gravitational instabilities necessary to form the terrestrial planets, the solar nebula must contain an excess mass of about 1 $M_{\odot}$ which will have to be removed by rather uncertain mechanisms.

Because of these quantitative uncertainties, much work remains to be done before this can be regarded as a probable mechanism for the formation of terrestrial planets. Even if it is granted that Jupiter and Saturn formed in this manner, it seems just as likely that the inner solar system would be populated by a large mass of solid debris with smaller dimensions: a mixture of solid material which aggregated in the absence of gravitational instability, protoplanets which were tidally disrupted at an early stage of their evolution, and fragments of protoplanetary cores disrupted by close approaches to one another or by collisions. If this is the case, the formation of the terrestrial planets will consist of the accumulation of this debris, along the lines of the accumulation theories to be discussed later.

If the terrestrial planets are indeed the fully collapsed cores of giant gaseous protoplanets, the time scale of planetary formation is rather well fixed at $\sim 10^5$ years, because of the required simultaneity of the growth of the Sun and planets. The time scale is determined by the time required for the Sun to complete its growth following the stage in which the nebular density becomes sufficiently large to produce gravitational instabilities. This time scale is shorter than the predicted time scale for terrestrial planet formation by accumulation by a factor of 100 to 1000.

2.2 *Gravitational Instability in a Central Dust Layer*

Although it is likely that initially a protosolar cloud would be highly turbulent, energetic considerations preclude maintenance of much turbulence beyond the stage at which the Sun approached its final mass. Unless terrestrial planets formed by the growth and stripping of giant gaseous protoplanets, as discussed in Section 2.1, the planets were formed from a nonturbulent nebula. It is also energetically impossible for a flat solar nebula of dimensions comparable to the solar system to remain at temperatures above the condensation temperature of silicates for times longer than about 10^3 years (Ter Haar 1950).

Under these conditions solid grains will spiral down through the gas toward the central plane of the nebula, and form a flat central dust layer. The time scale for the formation of the dust layer will be short: $\sim 10^5$ yr for 1 μm particles, 10 yr for particles that have grown to 1 cm diameter (Safronov & Ruskol 1957, McCrea 1960, Weidenschilling 1977a, 1979, Kusaka, Nakano & Hayashi 1970). As Edgeworth (1949) and Gurevich & Lebedinskii (1950) pointed out, if the dust layer is sufficiently thin, gravitational instabilities will develop within the dust layer itself, even though the nebula as a whole is gravitationally stable. The formation and evolution of these dust layer instabilities have been discussed by several authors (Safronov 1960, 1969, 1975, Lyttleton 1961, 1972, Goldreich & Ward 1973, Polyachenko & Fridman 1972). Following the discussion by Goldreich & Lynden-Bell (1965) the Jeans criterion for the gravitational stability of a thin rotating layer is given by a dispersion relationship for the propagation of a sound wave

$$p = p_0 \exp \{2\pi i(x-vt)/\lambda] \tag{6}$$

where p is the perturbation in pressure, λ the wavelength of the perturbation, and v the velocity of propagation of a perturbation of wavelength λ. The dispersion of this velocity is given by

$$v^2 = c^2 - G\sigma\lambda + (\omega^2\lambda^2/4\pi^2) \tag{7}$$

where σ is the density per unit area of the layer, ω is the angular velocity, and c is the sound speed, which for the dust layer is taken to be comparable to the velocity associated with the random kinetic energy of the dust particles. When v^2 is negative, a disturbance will grow exponentially. v^2 will not be negative for any value of λ unless

$$c < (\pi G\sigma/\omega). \tag{8}$$

This is a necessary condition for an instability to develop.

For a "minimum solar nebula," i.e. one containing no more dust than that required to form the observed terrestrial planets, $\sigma \sim 10\,\mathrm{g\,cm^{-2}}$. Use of Equation (8) then implies a random velocity of the dust particles $c \lesssim 10\,\mathrm{cm\,s^{-1}}$. This velocity is quite low, but in the absence of turbulence or significant gravitational perturbations, there is no compelling reason that velocities could not fall below this value. The approximate thickness of this dust layer will be given by the condition that c must be less than that required for a particle at the midplane to rise to the surface against the gravitational field of the layer, given by

$$d = (c^2/2\pi G\sigma) \cong 240\,\mathrm{km}. \tag{9}$$

At the Earth's distance the corresponding density is $\sim 4 \cdot 10^{-7}\,\mathrm{g\,cm^{-2}}$, slightly above the Roche limit for tidal disruption by the Sun.

The size range of the dust layer instabilities will correspond to those values of λ for $v^2 < 0$ in Equation (7). At the critical value of the velocity c given by Equation (8)

$$\lambda_c = (2\pi c/\omega) \sim 3 \cdot 10^8\,\mathrm{cm} \tag{10}$$

which will be associated with a mass

$$m_c = \tfrac{4}{3}\pi(\sigma/d)(\lambda_c/2)^3 = 2.4 \times 10^{18}\,\mathrm{g}. \tag{11}$$

About 5×10^9 dust condensations of this kind are required to form the terrestrial planets.

At first these dust condensations will be very tenuous assemblages. The subsequent evolution of these low density condensations into solid bodies is complex, and has been discussed by Safronov (1969) and Goldreich & Ward (1973). Following their formation, the dust condensations will contract until an equilibrium state is reached determined by the threshold for rotational instability. This equilibrium will correspond to solid densities of ~ 3 g cm^{-3} only for small initial condensations $m \sim 10^{-4}\, m_c$. Goldreich & Ward consider the evolution of a cluster of these smaller bodies derived from a single condensation of mass m_c. Their relative velocity is reduced by gas drag and they coalesce to form solid bodies of mass m_c ($\sim 2 \times 10^{18}$ g) on a time scale of ~ 100 yr. Safronov does not consider gas drag, but rather evaluates the averaging of rotational spin vectors which occur during the growth of bodies of mass m_c by dissipative collisions, and concludes that this effect will decrease the rotational velocity enough to permit the dust condensations to contract to solid densities in $\sim 10^4$ yr at the Earth's distance from the sun, by which time the bodies will have grown to masses of $\simeq 10^{20}$ g. Although these arguments are quite different, it seems likely that at least one of these processes will operate and that the initial diffuse dust condensations will evolve into solid bodies of $\lesssim 10$ km diameter on time scales of $\lesssim 10^3$ yr. Following the formation of these solid bodies it is unlikely that further gravitational instabilities can occur in the terrestrial planet region, and the subsequent growth of planets must occur by a series of accretional collisions whereby smaller bodies are swept up by larger ones, either in the presence or absence of a gaseous resisting medium.

The possibility of building ~ 10 km bodies on a short time scale eliminates the often-mentioned problem of understanding how bodies could stick to one another and grow, prior to being sufficiently large for gravitational binding to become important. This seems pleasing because our ignorance of the physical and chemical nature of these primitive bodies causes hypothetical sticking mechanisms to appear ad hoc and be, therefore, suspect. However, as Weidenschilling (1977a) has pointed out, quite low sticking efficiencies suffice to produce ~ 1 km bodies prior to their settling into an extremely thin dust layer. In this case the effect of differential gas drag, collisions, and the mutual gravitational perturbations of the bodies would result in velocities sufficiently high to preclude the onset of gravitational instability. Therefore it would be a mistake to conclude that the solar system *must* have developed dust-layer instabilities simply because this does not require specification of sticking processes that are poorly understood, but that quite possibly may have occurred anyway.

3 ACCUMULATION OF PLANETESIMALS BY GRADUAL SWEEPING UP OF MATTER

The collective gravitational forces of the solar nebula may have been insufficient to cause its material to aggregate into larger bodies. In this case growth can still occur by the collision and cohesion of individual bodies. Cohesion can result from gravitational attraction or by chemical or other physical attractive forces such as magnetism. The effects of nongravitational cohesion ("sticking") are not well understood because of lack of detailed knowledge of the physical conditions that prevailed in the solar nebula. However, it is plausible that for small objects (e.g. $\lesssim 1$ cm diameter) cohesion will primarily depend on nongravitational forces, whereas for bodies $\gtrsim 1$ km diameter, gravitational attraction will dominate. In the intermediate size range, both of these forces will be relatively weak, and it is difficult to say which will be the more important.

The minimum collision velocity of two bodies in heliocentric orbit is their gravitational escape velocity:

$$v_e = [2G(m_1+m_2)/(r_1+r_2)]^{1/2}. \tag{12}$$

If two bodies impact at this velocity they will adhere to one another because collisions will never be perfectly elastic, and if any energy is lost in the collision the bodies will be gravitationally bound. At higher collision velocities it is energetically possible for the bodies to bounce apart without cohering, and growth can occur only if the collision is sufficiently dissipative. At impact velocities above ~ 5 m s^{-1} (~ 10 miles h^{-1}) even fairly strong objects, for example automobile fenders, will deform and break, and collisions will tend to be fairly dissipative. This velocity corresponds to the escape velocity of a solid body about 7 km in diameter, i.e. about the size of the bodies that may result from gravitational instabilities in a central dust layer. Under these circumstances impacts at 1.5 times the escape velocity or even higher may still permit cohesion.

For bodies with relative velocity v prior to a close encounter, the impact velocity v_i will be

$$v_i = (v^2+v_e^2)^{1/2}. \tag{13}$$

Safronov (1969) makes considerable use of a useful dimensionless parameter θ (which I have termed the "Safronov number"),

$$\theta = (v_e^2/2v^2). \tag{14}$$

Safronov (1969, Equation 7.12) defines θ by an equation of the form of Equation (14), but defines v to be the relative velocity of the body with respect to a body moving in a circular orbit with semimajor axis corresponding to the instantaneous heliocentric distance of the encounter. When the accumulation of the terrestrial planets is calculated (Safronov's Equation 9.12) the "planetary embryos" are assumed to be moving in circular orbits, and his definition is then equivalent to the relative velocity between the two bodies, given by (14). Kaula (1979a) apparently defines the reference orbits in a still different way. In this article, θ will be used in a general way to represent equations of the form of (14). When appropriate, the exact way in which the velocity is defined will be indicated by subscripts on θ. It should be noted these distinctions may be significant (cf Section 3.1.2).

For $v_i = 1.5v_e$, the value of θ calculated from (14) is $\theta = 0.4$. Lower values of v correspond to higher values of θ. Although the actual value of θ for which cohesion is possible obviously depends on the unknown physical properties of the colliding bodies, it is likely that values of $\theta \gtrsim 1$ will result in cohesion, whereas values of $\theta \lesssim 0.2$ will result in fragmentation without cohesion. Experimental data relevant to this matter have been provided by Hartmann (1978).

For smaller bodies in the meter size range the situation is more complex. Gravitational forces are weak and quite low velocities are sufficient to exceed the escape velocity. On the other hand high impact velocities (very small values of θ) may facilitate sticking. For example, rifle bullets may remain imbedded in a target at velocities $\sim 1\ \mathrm{km\,s^{-1}}$, whereas if they are simply thrown at the target at $\sim 5\ \mathrm{m\,s^{-1}}$ they will merely bounce off. In such circumstances there will be a range of θ extending from ~ 1 to some very low value, depending on the nature of the bodies, for which coherence does not take place. For values of θ above or below this region coherence can occur.

Whether or not growth of terrestrial planets will occur by continued two-body collisions will therefore depend on whether or not

1. their relative velocity remains sufficiently low ($\theta \gtrsim 1$),
2. a sufficient number of bodies remain in intersecting orbits.

Accumulation of solid bodies may take place in the presence of a gas phase, or under conditions in which gas drag forces are negligible. Gas drag will lead to lower relative velocities, which in itself will promote growth. However, gas drag will lead to circularization of orbits, which can reduce the number of bodies in intersecting orbits, thus inhibiting growth.

If it is assumed that the terrestrial planets accumulated by a gradual accumulation process, it is not certain whether this accumulation took place prior to the loss of residual gas from the interplanetary medium, or afterward. It is sometimes written that accumulation of planets requires the presence of a gas phase to slow down the colliding bodies, and that this can be used as a constraint on the timing of the growth of the Earth and the dissipation of nebular gas. As discussed above, this is not true, provided $\theta \gtrsim 1$ for the largest bodies throughout the course of their growth. Subsequent discussion will show that the opposite problem, too low velocities even in the absence of gas, may prove a more serious obstacle. Because of their hydrogen-rich composition, it is clear that Jupiter and Saturn must have formed in the presence of a gas phase, regardless of whether they were formed by a massive gravitational instability or by an accumulation process. As will be discussed later in this section, the time scale for the formation of Jupiter and Saturn by an accumulative process will be longer than that for the Earth and Venus. Therefore if these gas-rich planets formed by *accumulation* in a gaseous nebula, it is likely that the terrestrial planets also formed in the presence of gas inasmuch as they would be expected to form sooner.

Although they are not compelling, there are some good reasons why this conclusion may be unattractive. If the Earth grew in the presence of nebular gas it might be expected that it would capture much of the gas into its primitive atmosphere. Hydrogen and helium could subsequently be lost by thermal escape or by photoreactions with solar ultraviolet radiation. This is not obviously possible for heavier gases such as the other inert gases, CO_2 and N_2, and polymerized hydrocarbons. However, the Earth is strongly depleted (by factors of 10^4–10^{11}) in these gases, relative to their expected abundance in a solar nebula. Sekiya, Nakazawa & Hayashi (1979) have presented arguments to support the view that these heavier gases may be swept along by hydrogen as it escapes. Their theory requires assumptions regarding the mechanism for hydrogen loss that are not necessarily valid. The loss of these gases is sometimes attributed to a T-Tauri solar "hurricane" before the Sun reached the main sequence (Horedt 1978), although it is not known if such a process could really remove a massive terrestrial atmosphere. It would take place $\sim 10^6$ yr after the formation of the Sun, and the accumulation calculations to be discussed later imply time scales of $\sim 10^8$ yr for the formation of the solid core of Jupiter, during which time the hydrogen must still be present.

Another reason for believing Jupiter formed before the terrestrial planets is that this provides an explanation for the gross mass depletion of the asteroid region and the small mass of Mars. During the final stages of Jupiter's growth residual planetesimals from the Jupiter region would be perturbed by Jupiter into the region between 4.8 and 1.0 AU and the planetesimals may have destroyed or ejected most of the material originally present. This event may also be responsible, in some rather vague way, for the present high relative velocities of the asteroids (Safronov 1969, Wetherill 1972, Kaula & Bigeleisen 1975, Weidenschilling 1975, 1977b).

For these reasons it is worthwhile to retain in one's mind the possibility that the growth mechanisms for Jupiter and Saturn were quite different from those responsible for the growth of the terrestrial planets. For example, in the theory of Cameron (1978b) Jupiter may have been a giant gaseous protoplanet, whereas stable objects of this kind may have failed to develop in the vicinity of the Earth. Although the formation of the giant planets is beyond the scope of this review, it should be pointed out that the complex history of growth and stripping required for the formation of the terrestrial planets by massive gravitational instabilities is less complex for Jupiter and Saturn.

3.1 *Accumulation of Planetesimals from a Gas-Free Circumsolar Swarm of Bodies*

During the earliest stage of solar system history, large amounts of gas must have been present, and therefore in some sense the beginning of planetary growth must have involved a gas phase. For example, if the original ~ 5 km diameter solid bodies resulted from dust-layer gravitational instabilities (see Section 2.2), these bodies were formed in a gaseous nebula. The first stages in their growth must involve the effects of gas drag, as discussed by Goldreich & Ward (1973). However, it is quite possible that this gas was removed by solar UV or corpuscular radiation on a time scale (i.e. 10^4–10^6 yr) which is less than that required for these bodies to grow by mutual collisions into terrestrial planets. If this is so, then the significant body of theory that has been developed for gas-free accumulation can be used to describe the growth of the terrestrial planets. The most complete quantitative discussion of this theory is given in the book by Safronov (1969).

The basic idea of gas-free planetary accumulation is quite simple. As a body moves through the solar nebula it will collide with other bodies. As discussed earlier in this section, if the relative velocities of the bodies are sufficiently low ($\theta \gtrsim 1$), the bodies will cohere to one another and grow. This process will continue until all the material has accumulated into bodies moving in orbits that are sufficiently isolated from one another to preclude further collision and growth.

The theoretical problem that must be addressed is whether or not some plausible initial assemblage of bodies will spontaneously collide and grow into large objects with masses and orbits similar to those of the present terrestrial planets. If such growth is possible, an adequate theory should also quantitatively describe the time scale required for accumulation, as well as the size, velocity, and orbital distribution during the course of accumulation. More detailed work has been carried out concerning this mode of planetary accumulation than for any of the alternatives, but there are still a number of questions that must be resolved more clearly before we can say the theory is reasonably complete, or if indeed it is even possible that gas-free accumulation could have played a major role in the formation of the terrestrial planets.

The initial state usually assumed in the gas-free accumulation problem is a swarm of small (e.g. 1 km diameter) solid bodies moving about the Sun in Keplerian orbits that are nearly coplanar and circular. The total mass and semimajor axis distribution of these bodies is taken to be similar to that of the terrestrial planets into which they are to evolve.

3.1.1 EARLY STAGES OF ACCUMULATION During the early stages of the accumulation process the bodies will be "closely packed" and close encounters can occur even though the eccentricities are very low. For purposes of illustration, consider that all the mass of the terrestrial planets (1.2×10^{28} g) were spread out in a plane disk of width 1 AU consisting of a large number of bodies of mass m' initially in circular orbits. If $m' \leqq 2 \times 10^{14}$ g the bodies will physically touch when overtaking one another because of their Keplerian motion, even if their gravitational attraction is neglected. As long as $m' \lesssim 4 \cdot 10^{24}$ gm bodies in circular orbits will pass within their spheres of influence as defined in the restricted three-body problem (Tisserand 1896). Very low eccentricities ($\sim 10^{-3}$) are sufficient to permit these ~ 1000 km diameter bodies to collide with one another simply on geometrical grounds.

Numerical simulations of this early stage of planetary growth have been presented by Greenberg et al. (1978). Because of the close packing referred to above, their assumption that the separation of bodies, associated with differences in their semimajor axis and heliocentric distance, can be ignored is reasonable. The dynamical evolution was treated as if the bodies were simply particles confined to a three-dimensional space or "box," as is assumed in the kinetic theory of gases.

In this work it was assumed that the initial state consisted of 10^{12} bodies one kilometer in diameter, moving in low inclination orbits with $e = 0.0001$. As a result of low velocity collisions these bodies grew, at higher velocities fragmentation was assumed to occur, based on the experimental work of Hartmann (1978). Close encounters, which are "near misses" rather than collisions, cause perturbations in the eccentricity of the bodies as a consequence of the mutual gravitational attraction of the bodies. On the average, the effect of these perturbations will be to increase the eccentricity of the bodies, whereas that of collisions will be to circularize the orbits. The evolution of the swarm was followed in successive time steps. The bodies were sorted into size "bins" following each fragmentation or accumulation event and the average eccentricity and inclination for each size bin recalculated. By thus treating all bodies in the same size range as having the same eccentricity and inclination, it was possible to handle this very large number of bodies.

A typical result of Greenberg et al. (1978) is shown in Figure 1. At first the size distribution is sharply peaked near the assumed initial size. Somewhat later, smaller bodies are produced by fragmentation and some larger bodies by accumulation. As time goes on, the region < 1 km begins to fill in and in the 1–10 km range a size distribution develops that is very roughly similar to the power law

$$(dN/dr) \propto r^{-3.4}; \quad (dN/dm) \propto m^{-0.8} \tag{15}$$

predicted by theoretical discussions (Zvyagina, Pechernikova & Safronov 1973, Pechernikova 1974, Pechernikova, Safronov & Zvyagina 1976). During this time the mean velocity remains $\sim 1\ \mathrm{m\,s^{-1}}$, about the escape velocity of the bodies in which most of the mass is concentrated. However, even at this early stage of accumulation, a few bodies in the size range ~400 km are produced. In the calculations it was assumed that the

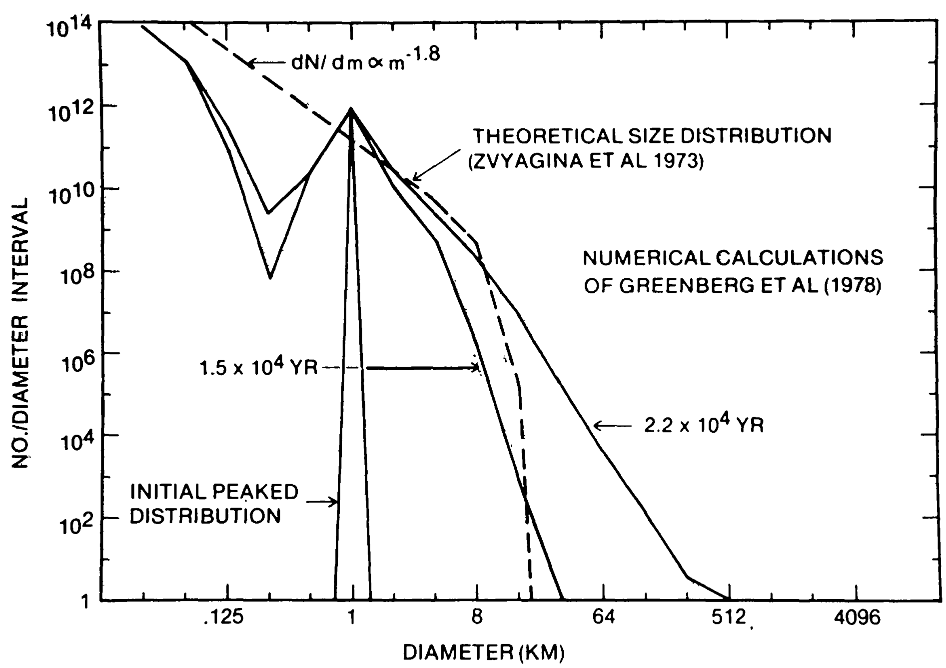


Figure 1 Numerical accumulation calculation by Greenberg et al. (1978). After $\sim 10^4$ years the initial distribution of 10^{12} 1 km diameter bodies has evolved into a distribution containing objects as large as 512 km. The abundance of large bodies relative to smaller objects is in disagreement with theoretical models based on coagulation theory, and arise as a consequence of the gravitational cross section (Equation 16) causing the kernel in the coagulation equation to be nonlinear.

probability of collision was proportional to the "gravitational cross section" of the body. This is larger than the geometrical section by a factor

$$f_g = 1+(v_e^2/v^2) = 1+2\theta. \tag{16}$$

The enhanced cross section is a consequence of "gravitational focusing," whereby the distance of closest approach of the two bodies is less than the distance between their initial trajectories. This is equivalent to the difference between the asymptotic separation and the perigee distance in the hyperbolic two-body problem (e.g. Wood 1961). For the largest bodies of Figure 1, $v_e \sim 1\,\mathrm{km\,s^{-1}}$ and $f_g \sim 10^6$. Thus the larger bodies are able to sweep up smaller objects much more efficiently, and this effect is responsible for the early appearance of the $\sim$400 km bodies.

It is not at all certain that the proposed enhancement in the rate of growth of the large bodies is physically realistic. The use of Equation (16) and the treatment of the encounter between two bodies in Keplerian orbit about a third body (the Sun) are valid approximations when the approach velocity is high enough (Öpik 1951, Cox, Lewis & Lecar 1978). However, when the velocity is so low that the encounter time is a significant fraction of the orbital periods of the encountering bodies, this is not the case (Dole 1962, Giuli 1968, Cox 1978). Under such circumstances the approximations introduced by breaking the actual three-body problem into separate two-body problems fail. If the effect of this failure is to reduce f_g significantly below that given by (16) it is likely that the high mass "tail" of the size distributions shown in Figure 1 will be steeper, and approximate establishment of the power law size distribution (15) will extend to higher masses. If (16) is more or less valid, it is likely that runaway growth of the largest bodies will take place, and the end result of this stage of growth will be the formation of about 1000 $\sim 10^{25}$ g bodies in nearly circular, low inclination, noncrossing orbits. It is conceivable that longer range perturbations by protoplanets in the outer solar system, or resonant perturbations by terrestrial planetesimals, could destabilize the situation, produce crossing orbits, and permit further evolution toward the present system of terrestrial planets. But this has not been demonstrated as yet.

3.1.2 THE SAFRONOV STEADY STATE VELOCITY In the event that runaway growth and isolation do not occur and size distribution with most of the mass in the largest bodies, e.g. (15), is established, the analytical results principally developed by Safronov and his co-workers will become applicable. The most important result of these studies is that the competition between mutual collisional damping and gravitational acceleration by the members of the swarm results in a steady state velocity distribution, with the mean velocity being comparable with the escape velocity of the largest body, i.e. $\theta \approx 1$. Relative velocities will then remain low enough to permit the largest bodies to grow, and will "keep step" with the growth of these bodies, increasing with the radius of the largest bodies and never deviating far from their escape velocity. Under such conditions, approximate analytical expressions based on coagulation theory show that a power law of the form (15) represents a quasi-steady state for the size distribution for all but the largest members of the swarm.

The fundamental reason the velocity of the swarm keeps step with the growth of the bodies is that the change in relative velocity accompanying a close gravitational encounter between two bodies of the same mass is given by

$$(\Delta v)_g \sim (2GM/Dv) \tag{17}$$

where M is the mass of the bodies and D is the distance of closest approach (Öpik 1951). The change in kinetic energy (E_g) following a single encounter will be

$$\begin{aligned}\Delta E_g &= \tfrac{1}{2}M(v+\Delta v)^2 - \tfrac{1}{2}Mv^2 \\ &= \tfrac{1}{2}M[2\Delta v - (\Delta v)^2].\end{aligned} \tag{18}$$

After a large number (n) of gravitation encounters the first term on the right of Equation (18) will average to zero and

$$n\overline{\Delta E_g} = \Sigma\Delta E_g = \Sigma\tfrac{1}{2}M(\Delta v)^2. \tag{19}$$

The contribution to $\Sigma\Delta E_g$ in the time interval Δt from those bodies passing at a distance between D and $D+\Delta D$ will be

$$(\Delta E_g)_D = \pi\rho_s vD\Delta D(\Delta v)^2\Delta t = \Delta t(4\pi\rho_s G^2M^2/vD)\Delta D \tag{20}$$

where ρ_s is the mass density of the swarm. Integrating the encounter distance out to the edge of the sphere of influence

$$D = R_i = 2^{1/5}(M/M_\odot)^{2/5}a \tag{21}$$

we obtain

$$\Sigma\Delta E_g = 4\pi\rho_s G^2M^2v^{-1}\,[\ln(R_1/2R)]\Delta t. \tag{22}$$

This increase in kinetic energy will be offset by a loss of energy caused by inelastic collisions

$$\Sigma\Delta E_c = \pi(2R)^2\varepsilon\rho_s v^3\Delta t, \tag{23}$$

where ε is a dimensionless factor representing the fractional loss of energy in collisions. In the steady state, the gain and loss will be equal. Equating (22) to (23) gives

$$v^2 = (GM/R)[\varepsilon^{-1} \ln (R_i/2R)]^{1/2} = 0.5v_e^2 [\varepsilon^{-1} \ln (R_i/2R)]^{1/2}. \quad (24)$$

From Equation (21) it may be seen that $(R_i/R) \propto R^{0.2}$, and thus the ratio $\ln (R_i/2R)$ varies only by a factor of ~ 3 as R grows from 10 km to 5000 km. For $\varepsilon = 1$ and $R = 1000$ km, $v^2 = 0.98\, v_e^2$, $\theta = 0.5$ ($\sim$constant), and $v \approx v_e$. Because v_e is proportionate to R, v will be so also, i.e. v will increase linearly with the radius. This result is not sensitive to the use of the geometrical cross section instead of the gravitational cross section (Equation 16) because gravitational focusing increases both the collision rate and the close encounter rate.

If the size distribution follows a power law, rather than all the masses being equal, the situation is more complex. For power laws of the form

$$(dN/dm) \propto m^{-q} \quad (25)$$

and with $q < 2$, most of the mass will be in the largest bodies. These largest bodies will be responsible for most of the perturbations, and therefore for the increase in energy in the appropriate expression analogous to Equation (22). However, if $q > 1.67$ as well, most of the integrated area will be in the small bodies, and mutual collisions between small bodies will be the dominant mode for energy loss. Moreover, in this case the velocity distribution will be size-dependent. The largest bodies will tend to be in orbits of low eccentricity and inclination because only rare close encounters with other large bodies will increase their velocity. For the smallest bodies, the large ratio of cross section to mass will also result in relatively circular orbits, and maximum velocity will occur at some intermediate value of the mass. Both the numerical investigations and analytical theories referred to earlier suggest that $1.67 < q < 2.0$. Although much work remains to be done on this problem of the coupled mass and velocity distribution, it is quite likely that, except for a few of the largest mass bodies, most of the objects fall in this range where the mass is concentrated in the largest bodies, and the area in the smaller ones.

Detailed analytical theories of the velocity of an accumulating planetesimal swarm with unequal masses have been given by Safronov (1969) and Kaula (1979a). All this work is in turn based on the stellar dynamical methods of Chandrasekhar (1942). In these theories the heliocentric motion of the swarm is not explicitly introduced. However, the velocity gradient resulting from the differential Keplerian velocity is an intrinsic aspect of the problem. Furthermore, the perturbations of the semimajor axes of the bodies are as important as changes in eccentricity in their effect on the velocity distribution.

The results of these analytical theories are in fair agreement (i.e. a factor of ~2) with one another, and with the simplified discussion given above, in predicting values of θ in the range from 1–5. They differ primarily as a consequence of different approaches to the calculation of the mutual gravitational acceleration, and to some extent to differences in the way the circular reference orbits are defined. As mentioned above, there are conceptual problems in treating this problem, which involves Keplerian heliocentric motion in an essential way, by the Chandrasekhar approach, which ignores heliocentric motion.

Numerical calculations for the case of a nonaccumulating swarm of bodies of equal mass have been presented, in which the heliocentric motion is explicitly introduced (Wetherill 1979). These calculations show (Figure 2) that the average velocity approaches an approximate equilibrium value regardless of whether the initial velocities are chosen to be above or below the steady state value. The steady state velocities increase linearly with radius (Figure 3) as expected, and the numerical values of θ are in the range predicted by analytical theories. The numerical calculations also verify that the velocity distribution is nearly Maxwellian, as assumed in the analytical theory. It is also found that mutual perturbations cause a marked diffusion of semimajor axis (Figure 4). This result has no analytical counterpart because differences in semimajor axes are not included in these theories.

It appears that, for a given set of assumptions regarding size distribution and degree of inelasticity of the bodies, the theoretical results can be

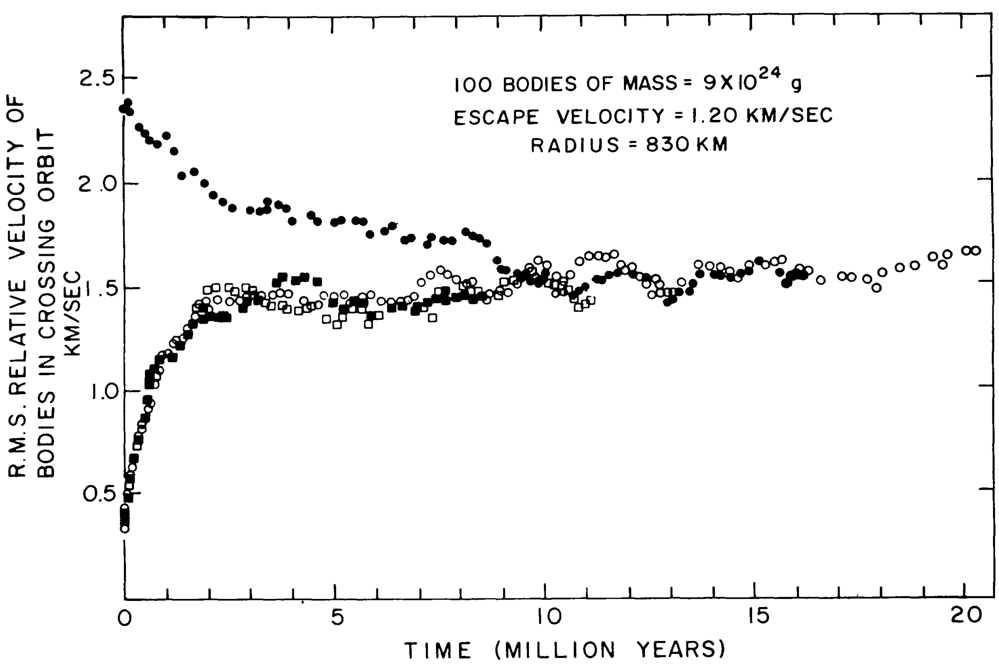


Figure 2 Numerical calculations of the approach to a steady state mean velocity of a nonaccreting swarm of bodies of equal mass. The equilibrium velocity is nearly independent of the initial velocity distribution (Wetherill 1979).

calculated to within a factor of ~ 2. Further precision, and detailed understanding of the variation of the velocity and size distributions with time and with one another, will require extension and improvement of both the analytical and numerical theories.

3.1.3 TIME SCALE FOR ACCUMULATION If it is assumed that the velocity distribution given by theory is correct, the time scale for gas-free accumulation of the terrestrial planets is constrained to be in the range 10^7–10^8 yr. This can be seen approximately as follows.

In the "kinetic theory of gas" approach used in the analytical theories the rate of sweep up mass by a planet of mass M from a swarm of density ρ_s with velocity v will be

$$(dM/dt) = \pi R^2 \rho_s v(1+2\theta) \tag{26}$$

where the last factor on the right represents the gravitational enhancement of the radius (16). In terms of the radius (R) this is equivalent to

$$(dR/dt) = (\rho_s v/4\rho_p)(1+2\theta) \tag{27}$$

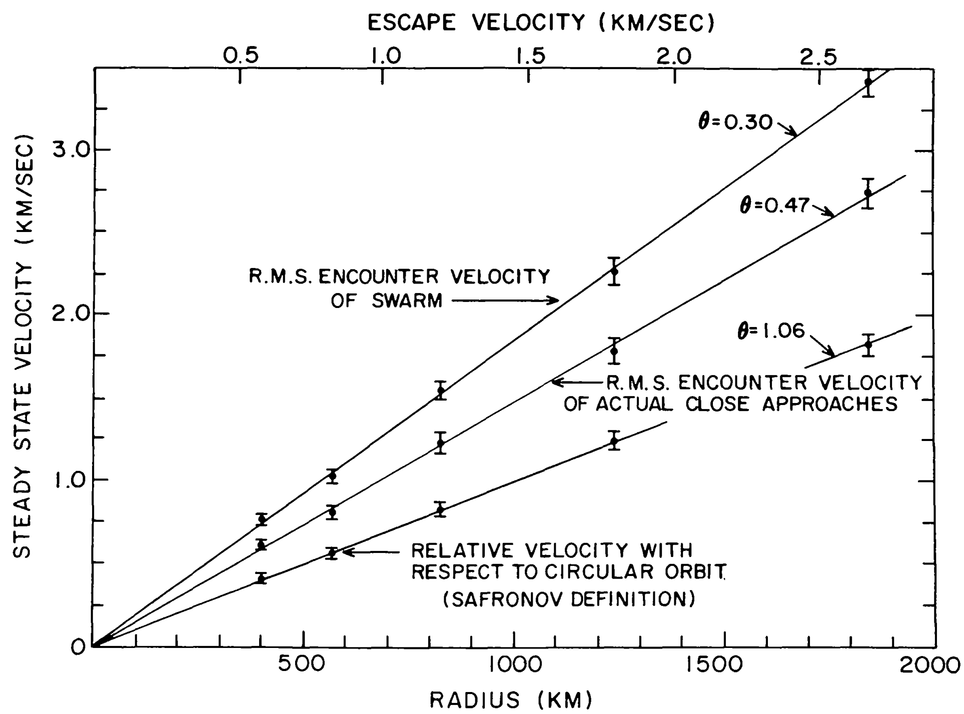


Figure 3 Steady state velocities found by numerical calculations of nonaccreting swarms of bodies of the same size, as a function of radius. The uppermost straight line is the root mean square relative velocity of all the bodies that can encounter one another; the middle line represents the relative velocity weighted according to the probability of encounter; the lowermost line is the mean relative velocity with respect to a circular orbit with semimajor axis equal to the heliocentric distance of the encounter. All of these velocities are linear functions of radius and escape velocity (Wetherill 1979).

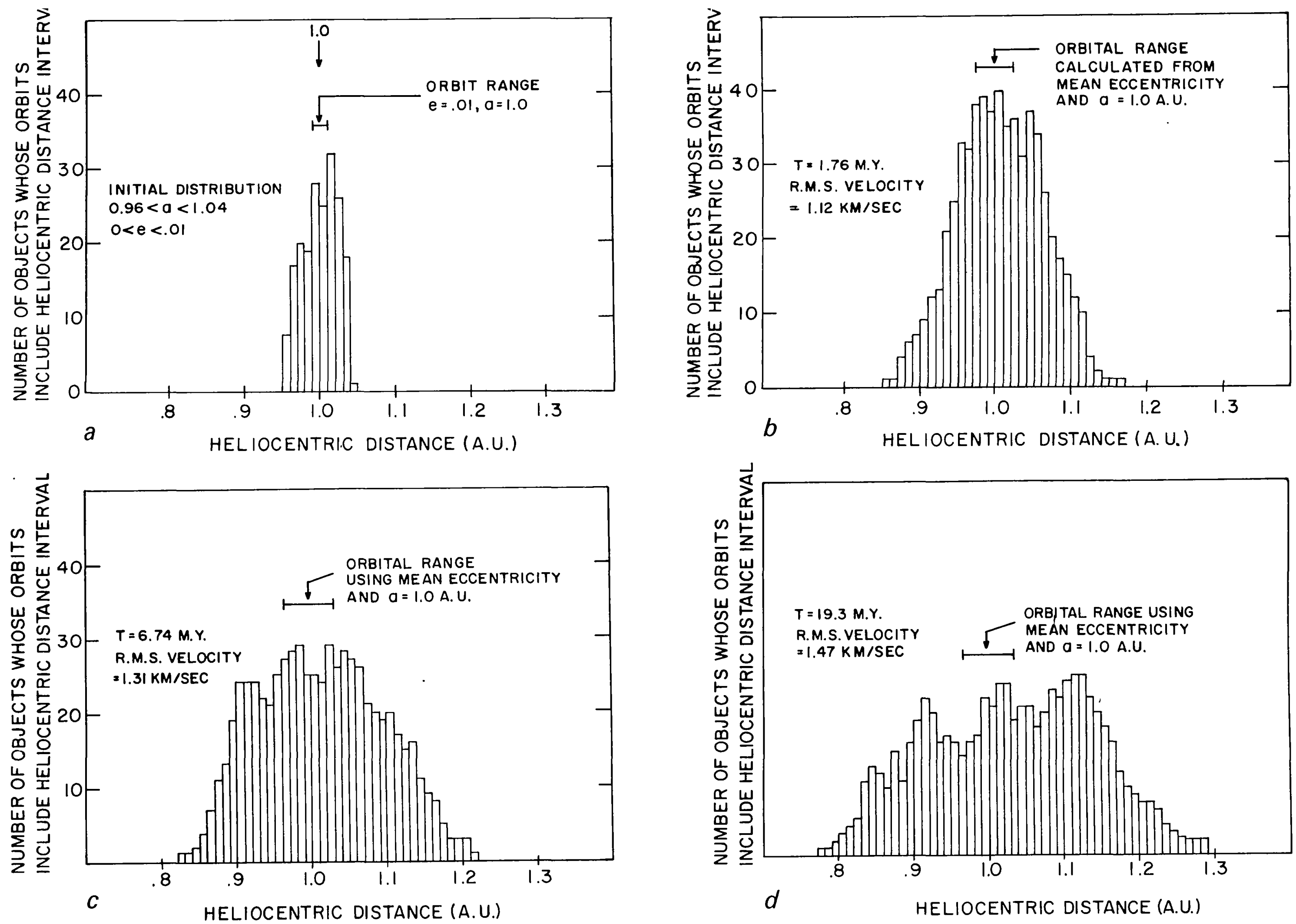


Figure 4 Radial distribution of a swarm of nonaccreting bodies of equal size (9×10^{24} g) as a function of time. The bracket labeled "orbit range" is the range of distance that would be swept out if the radial distribution resulted from all bodies having the same semimajor axis and the mean value of their eccentricity (Wetherill 1979). (*a*) Initial state. (*b*) After 1.76 m.y. Mutual gravitational encounters have caused significant "radial diffusion" even though the velocity has not yet reached the steady state value of 1.45 km/s. (*c*) After 6.74 m.y. The velocity is approaching the steady state value. In a real swarm significant accretion would have taken place. (*d*) After 19.3 m.y. Radial diffusion continues to spread the swarm, as a consequence of conservation of angular momentum in a swarm losing energy by collisions. This case is not physically significant because 19 m.y. is comparable to the accumulation time.

where ρ_p is the density of the solid bodies. The density ρ_s can be estimated by assuming the total mass that is to become a terrestrial planet of mass M_p is distributed over a volume

$$\Omega = 4\pi a \Delta a \Delta z \tag{28}$$

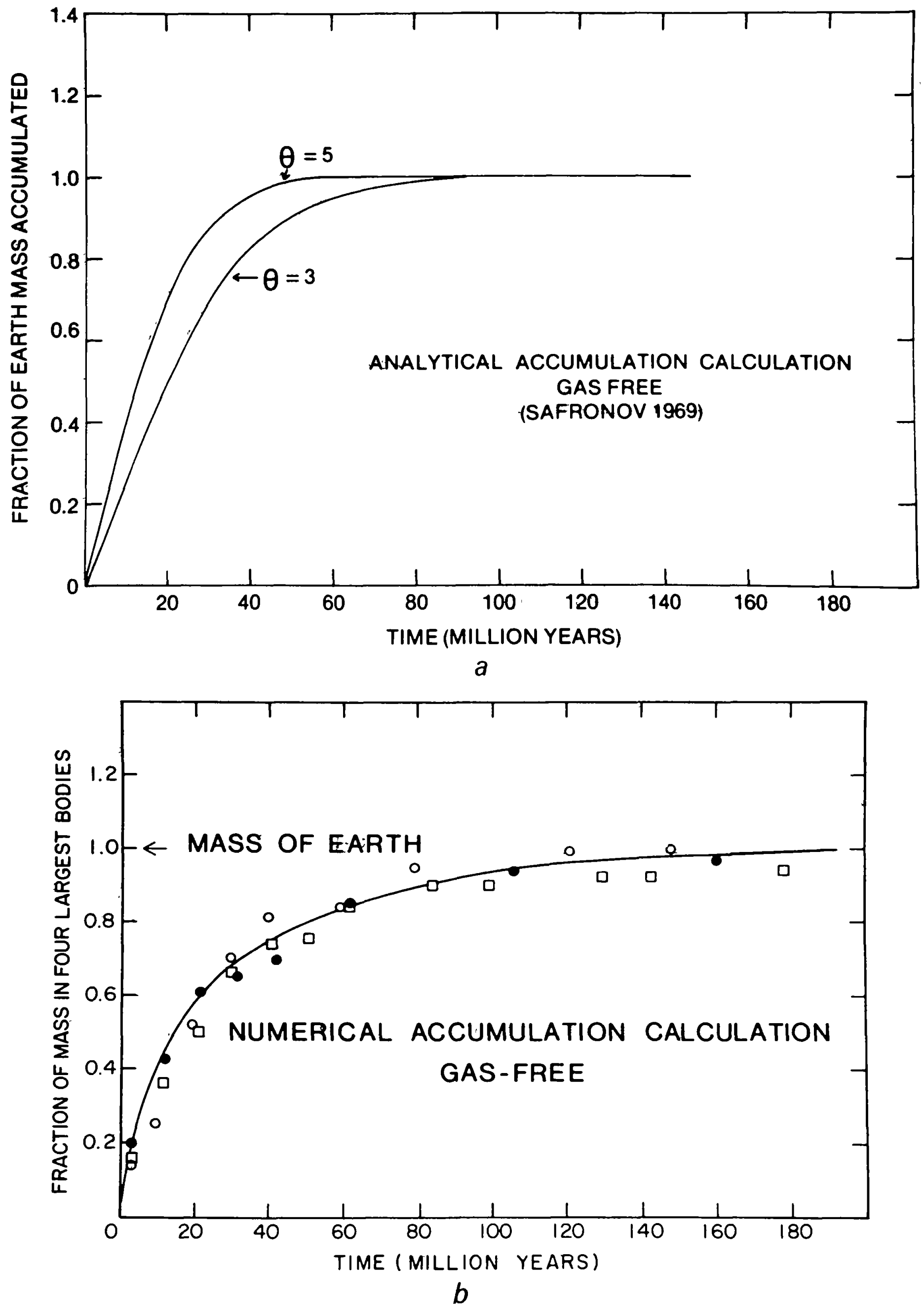


Figure 5a, b Analytical and numerical calculations of accumulation of the Earth as a function of time. (*a*) Analytical accumulation for two values of the Safronov number θ (Equation 14; Safronov 1969). (*b*) Three-dimensional numerical calculations of gas-free accumulation (Wetherill 1979).

where Δz is the thickness of the disc, given by

$$\Delta z \approx a \sin i \approx (av/\sqrt{3}v_c \qquad (29)$$

where v_c is the circular Keplerian velocity $(GM_\odot/d)^{1/2}$ and equipartion is assumed between the vertical, tangential, and radial components of the velocity v. Use of (28) and (29) in (27) gives

$$(dR/dt) = [\sqrt{3}v_c(1+2\theta)M_p]/(16\pi a^2 \Delta a \rho_p) \sim \text{constant.} \qquad (30)$$

Substitution of $M_p = 6 \times 10^{27}$ g (Earth mass), $a = 1$ AU, $\Delta a = 0.5$ AU, $v_c = 30 \text{ km s}^{-1}$, $\rho_p = 5.5 \text{ g cm}^{-3}$, and $\theta = 3$ gives a radial growth rate

$$(dR/dt) \cong 15 \text{ cm yr}^{-1}. \qquad (31)$$

At this rate the growth of the Earth will require 40 million years. The actual time will be longer because the density of the swarm will decrease as the mass is swept up, e.g. when the Earth has grown to 80% of its final radius, ρ_s and (dR/dt) will be a factor of 2 smaller. More detailed analytical calculations of growth time taking this and other factors into consideration have been given by Safronov (1969) under various assumptions and the result is always $\sim 10^8$ yr (Figure 5). The same result is found by numerical calculations (Wetherill 1979; see Figure 5b). One hundred million years seems to be the time scale required for this mode of planet formation to go to completion, under a wide range of circumstances. Equation (30) shows that gas-free growth of a 10 Earth mass Jovian core would require ~ 5 times longer.

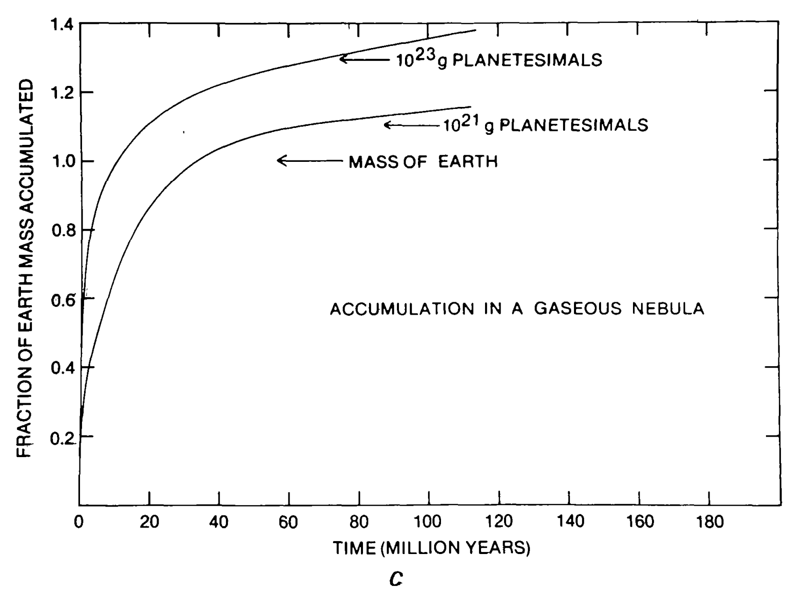


Figure 5c Analytical and numerical calculations of accumulation of the Earth as a function of time. (*c*) Accumulation in a gaseous nebula, assuming two different sizes for the planetesimals (Hayashi et al. 1977).

Some authors have formally used (26) to infer much more rapid growth rates for the Earth by choosing a value of the velocity that is arbitrarily low, leading to arbitrarily large values of θ. In addition to the problem that v is not a free parameter, but is determined by the mass distribution and physical properties of the swarm, this formal use of (26) is invalid for another reason. When v is too low the orbit of the growing planetary embryo will not intersect all the nearly circular orbits of the bodies in the swarm, and its growth cannot proceed to completion. Weidenschilling (1974) has also shown that for a distribution of semimajor axes the integrated contribution of individual bodies with low values of v converges, and that the singularity as $v \to 0$ in Equation (16) has no physical significance.

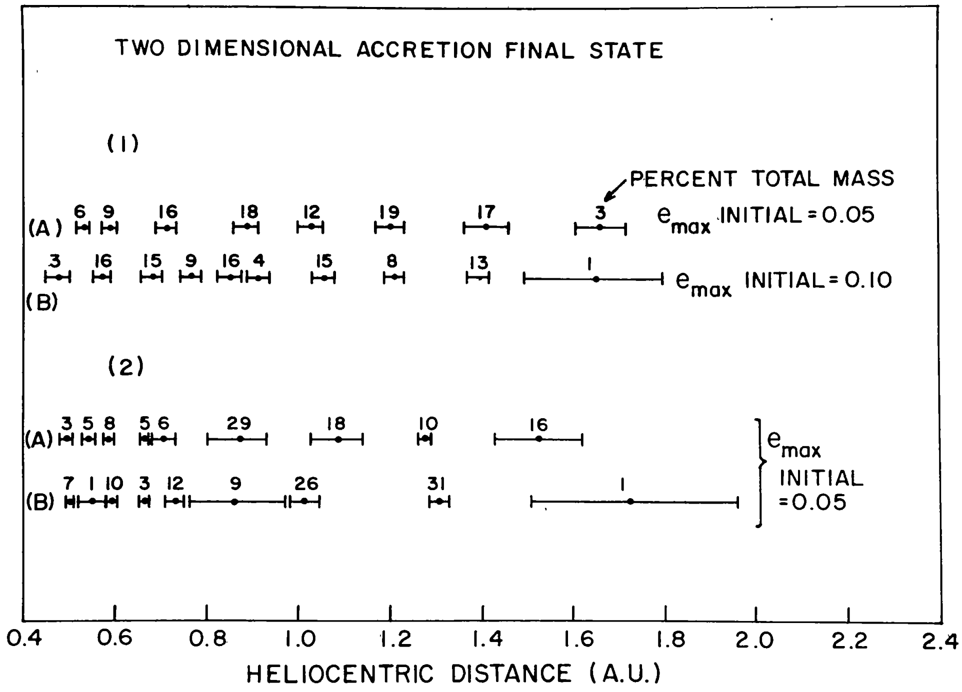
3.1.4 SIMULTANEOUS GAS-FREE ACCUMULATION OF SEVERAL TERRESTRIAL PLANETS In the real solar system the accumulation of the terrestrial planets proceeded more or less simultaneously. Present analytical theories describe only the formation of a single planet because multiplanet growth involves differences in heliocentric distance, which are not introduced in these theories. Older numerical simulations of multiplanet growth are of little relevance because they neglect other essential factors, as discussed by Wetherill (1978). For example, the calculations of Dole (1970) and Isaacman & Sagan (1977) ignore mutual gravitational perturbations, and simply assume velocities that are impossibly high in the gaseous interplanetary medium used in their model. In contrast, Hills (1970) includes gravitational perturbations but ignores collisions.

To my knowledge Cox (1978) is the first work that includes both of these essential phenomena. In his work an assemblage of 100 bodies of equal mass, with total mass equal to that of the present terrestrial planets, is initially confined to the region of the terrestrial planets. The eccentricities are chosen as randomly distributed between zero and a maximum value e_{max}. The inclinations are zero, both initially and throughout the calculation, i.e. the problem is treated two dimensionally. The dynamical evolution of this system is followed. Mutual gravitational perturbations are calculated by a method that has been shown to be an accurate approximation to straightforward numerical integration (Cox, Lewis & Lecar 1978). Coherence and accumulation occur when two bodies collide at sufficiently low energy, and the kinetic energy of their relative motion is dissipated in the collision.

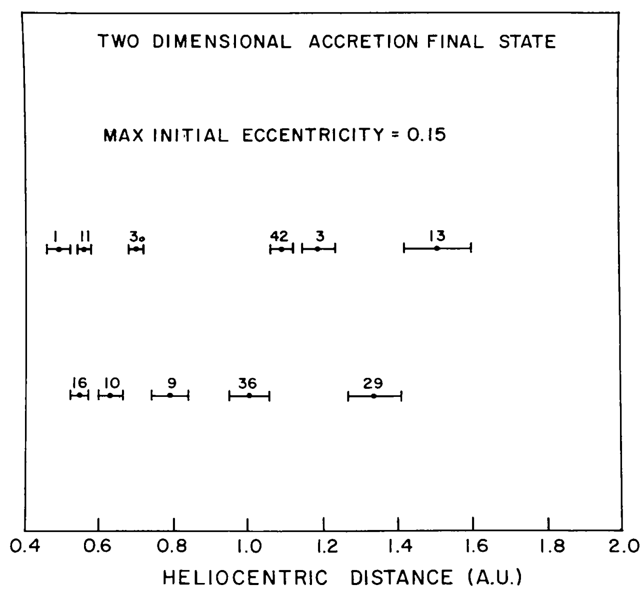
Cox's calculations illustrate clearly a matter of central importance. When the initial eccentricity is chosen to be $\lesssim 0.10$, the final result of the calculation is an assemblage of planets that is too numerous, too small, and too closely spaced (Figure 6*a*). This is a consequence of averaging of orbits of cohering bodies (Ziglina 1976, Ziglina & Safronov 1976), which causes a number of bodies to evolve into circular orbits as they grow, rather than remain in more eccentric orbits that cross one another. In order for the observed system of terrestrial planets to form, it is necessary that this "isolation of dominant planetary embryos" be delayed until only two large embryos (corresponding to Earth and Venus) and two small embryos (corresponding to Mercury and Mars) remain. The tendency toward isolation must be offset by some process, e.g. higher initial eccentricity (Figure 6*b*) which opposes the tendency toward circularization.

Extension of Cox's calculations to three dimensions (Wetherill 1979) shows that in the actual problem studied by Cox, the premature embryos were an artifact of the two-dimensional calculations, which underestimate the effect of perturbations caused by encounters, relative to collisions (Figure 6*c*). There are real effects, such as more collisional damping, which lead to the same problem. In this two-dimensional calculation, collisional damping was probably significantly less than in a real accumulating system. In a real system the 100 large bodies assumed in the calculations would be accompanied by a swarm of small bodies which would cause the entire system to become more dissipative. In the three-dimensional case, it is found that increasing the dissipation by a factor of about three again leads to premature embryos (Wetherill 1979). It is also found that if the four required embryos are placed in nearly circular orbits too early, then unwanted extra embryos will emerge between them (Wetherill 1978 and unpublished work). This effect is related to that of Horedt (1979) who obtained too many small planets with an analytical theory, assuming the feeding zone to be determined solely by the eccentricity of the bodies. Since radial diffusion will occur (Figure 4) Horedt's constraint errs in the direction of being too severe.

These numerical studies of multiplanet accumulation can be thought about in a more general way. Theories of multiplanet accumulation have as their initial state a large number of relatively small bodies in orbits of low eccentricity and inclination, and with a distribution of semimajor axes. They have as their final state an assemblage consisting of a smaller number of larger bodies, also in more or less circular and coplanar orbits. For a theory to be successful the latter assemblage must have at least a high statistical probability of corresponding to something like the observed terrestrial planets in number, size, and semimajor axes. It is significant that the real solar system stopped short of growing the minimum number of terrestrial planets. The numerical studies cited show that for a given initial state there are evolutionary trajectories leading to final states with a range of sizes and number of final planets. The principal parameter defining these trajectories is the ratio of velocity increments caused by gravitational



a



b

Figure 6a, b Numerical calculations of multiplanet accumulation final states. (*a*) Two-dimensional calculations: (*1*), Cox (1978) and (*2*), Wetherill (1979). For the initial eccentricities chosen, an excessive number of terrestrial planets result. (*b*) Two-dimensional calculations by the same authors showing the smaller number of planets formed if quite large values of initial eccentricity are used.

perturbations to the velocity decrements caused by energy dissipation in collisions, i.e. the value of θ. If this ratio is too low, an excessive number of planets is produced; if the ratio is too high, too few are produced. The actual value of the ratio is determined by the physical properties of the bodies, and to some extent by choice of the initial state. The size distribution of the bodies will be an important, but not an independent, variable, because it will be determined by the physical properties and the initial state.

We can therefore expect that anything that tends to increase this ratio, e.g. more elastic collisions, size distributions weighted toward large bodies, and long range perturbations, will lead to a small number of final planets. Anything that tends to decrease this ratio, e.g. more dissipative collisions or concentration of the mass in small bodies, will lead to a large and possibly excessive number of final planets. The role of gas drag is more complex. Although gas drag tends to damp velocity increments, it also can lead to changes in semimajor axis, which can promote growth by bringing nearly circular orbits into intersection. Gas drag will be discussed further in Section 3.2.

Several authors have recently discussed a model in which the gravitational perturbations are primarily dominated by a single embryo (Levin 1978a,b, Greenberg 1979, Safronov 1979). Safronov (1969) states that the effect will be to "pass control" of the gravitationally induced velocity increments over to the second largest body of the swarm. This is a

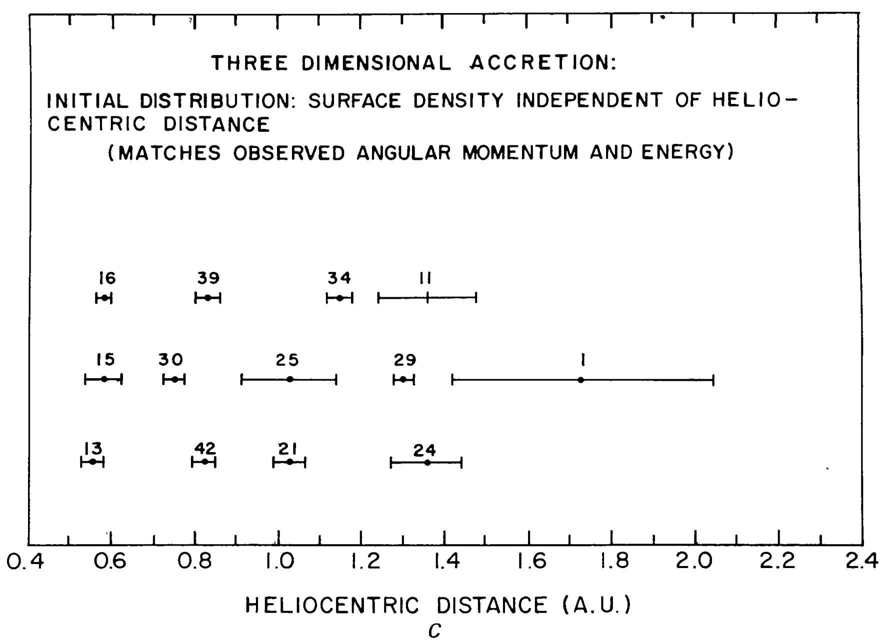


Figure 6c Numerical calculations of multiplanet accumulation final states. (*c*) Three-dimensional calculations with low initial eccentricity ($e_{max} = 0.05$), leading to a small number of planets (Wetherill 1979). In a real solar system greater energy dissipation is probable, which may invalidate this result.

consequence of the Jacobi parameter of the restricted three-body problem being equivalent to the relative velocity of the swarm body and the embryo. Both these quantities tend to be conserved when perturbations from only one embryo become dominant, and the mean velocity increments will be smaller. The effect will be to increase the number of final planets. Because the production of as few as four planets already appears marginal, the conclusion of Levin and Greenberg that low velocities imply shorter time scales for planetary growth is probably incorrect. More likely, the time scales become infinite because needed material is stranded in noncrossing orbits. It is probable that some minimum velocity needs to be maintained during most of the growth following the stage ($\sim 10^{25}$ g bodies) at which the close packing of bodies breaks down. Whether or not this velocity need be as high as that found by merger of the accumulation zones of Earth and Venus (Wetherill 1976) is not known.

If the foregoing discussion is correct, it has the somewhat ironic consequence that the *low* velocities and runaway accretion found by Greenberg et al. (1978) could be necessary during the early stages of accumulation. Such a low velocity may be required in order to remove small bodies from the swarm, and thereby permit *high* velocities to be generated by the relatively unhindered mutual gravitation perturbations of the remaining 10^{25}–10^{26} g bodies during the later stages of growth.

The concept of "jet streams" introduced by Alfvén (1969) may be relevant here. The term envisions an assemblage of colliding bodies gravitationally evolving into a few larger bodies. In some sense those multiplanet accumulation trajectories that lead to a few large planets could be thought of as these jet streams. However, jet streams are said to result when gravitational interactions are negligible, and evolution results from collisional dissipation. This corresponds to an extreme case of zero for the velocity increment/decrement ratio. Numerical studies by Ip (1977), confirmed by unpublished work of the present author, show that insofar as these calculations are relevant to the question, the necessary focusing of semimajor axes does not occur in the absence of mutual perturbations. However, these calculations make the assumption, introduced by Öpik (1951), that the longitudes of perihelion and node are uncorrelated, whereas for the "jet streams" discussed by Alfvén (1969) this correlation is an essential aspect of the concept. The only existing calculations including gravitational perturbations that could bear on the question of whether jet streams can develop are those of Cox (1978), who doesn't discuss the question, and whose results are limited by being two-dimensional. For this reason it is hard to rule out the possibility that in some sense failure of these longitudes to be randomized might be significant. The matter should be investigated further, even though some of the arguments advanced by Alfvén (1971) have been shown to be erroneous (Henon 1978).

During the later stages of growth the kinetic energy accompanying high velocity impact of large bodies with the embryos would be an important source of initial heat (Barrell 1918). If the planets actually formed by gas-free accumulation along the lines discussed in this section, the initial temperatures of the Earth and Venus would be high even though $\sim 10^8$ yr was required for accumulation (Safronov 1969, 1978, Kaula 1979c). Many conventional thermal history discussions (e.g. Hanks & Anderson 1969, Mizutani, Matsui & Takeuchi 1972, Wetherill 1972) incorrectly imply that rapid accumulation ($\sim 10^4$ yr) is required if the initial temperature of the Earth is to be high. This is not the case if terrestrial planets formed in the manner described in this section or, for that matter, in any other way (Section 3.2). It is not clear whether or not smaller bodies (Moon, Mars, Mercury) could be extensively melted by the impacts predicted by this theory (Kaula 1979b,c, Wetherill 1976). If the melting of the Moon was limited to a relatively shallow "magma ocean" (Solomon & Chaiken 1976) the actual melting of the Moon would have been only marginal. Therefore, a correct theory would only predict marginal melting, and it will be difficult to define the physical parameters required in the theory sufficiently to know if the predicted melting is too much or too little.

After the growth of the terrestrial planets is nearly complete, the theory predicts a residual population of large bodies. Some of these will evolve into orbits with lifetimes of 10^8–10^9 yr and will contribute to, perhaps dominate, the cratering history of the Moon and planets for the first $\sim 10^9$ yr of their history. The question of whether or not the observed cratering of the Moon is consistent with gas-free accumulation theory has been discussed (Wetherill 1977), as well as the role this residual swarm may have played in the formation of the asteroidal parent bodies of the differentiated meteorites (Wetherill & Williams 1979, Wasson & Wetherill 1979). Progress on this question will require much more understanding of the importance of tidal disruption of the residual planetesimals by the planets during the first 10^9 yr of solar system history. Existing treatments of tidal disruption consider only the static stability of the body while it is within the Roche limit of the planet (Aggarwal & Oberbeck 1974, Ziglina 1978). However, the $\lesssim 1$ h duration of the passage within the Roche limit may be insufficient to actually fragment and disperse the body.

3.2 *Accumulation in a Gaseous Medium*

If anything like the usual picture of solar system formation from a solar nebula is correct, it must be supposed that at least the earliest stages of growth took place in a gas-rich interplanetary medium, if only to explain the composition of the Sun, Jupiter, and Saturn. It is very uncertain when the interplanetary medium first became essentially gas-free. Presumably this took place after the formation of Jupiter and Saturn, but the time scale for the formation of those planets depends on the way in which they were formed, and the time of gas loss could have been anywhere in the range $\sim 10^5$–$\gtrsim 10^8$ yr. Most authors (with the exception of Alfvén & Arrhenius 1976) have assumed that the early ~ 1 km size planetesimals were formed in the presence of at least as much gas as that required to complement the observed nonvolatile mass of the planets ($\sim 0.01\ M_\odot$) and usually enough more (0.03–0.1 $M_\odot$) to permit a nonsolar composition for Jupiter and Saturn and some weakly bound material in the Uranus-Neptune region.

In a solar nebula containing gas and solid particles, the gas phase will be partially supported by a gas pressure gradient effectively changing the gravitational attraction of the Sun. This will cause the velocity v_g of the gas to differ slightly from the Kepler velocity v_k:

$$v_g^2 = v_k^2 + \frac{r}{\rho_g}\frac{dP_g}{dr} \tag{32}$$

where P_g and ρ_g are the pressure and density of the gas, as discussed by Whipple (1973) and Kusaka, Nakana & Hayashi (1970). The gas density and pressure probably decreased with heliocentric distance, and for plausible values of the physical parameters, v_g will be less than v_k by $\sim 0.1\%$. In the absence of gas, solid bodies will move at velocity v_k. Because $v_g < v_k$ they will experience a gas drag force. This drag will cause very small particles to move essentially at the gas velocity. The drag force will have

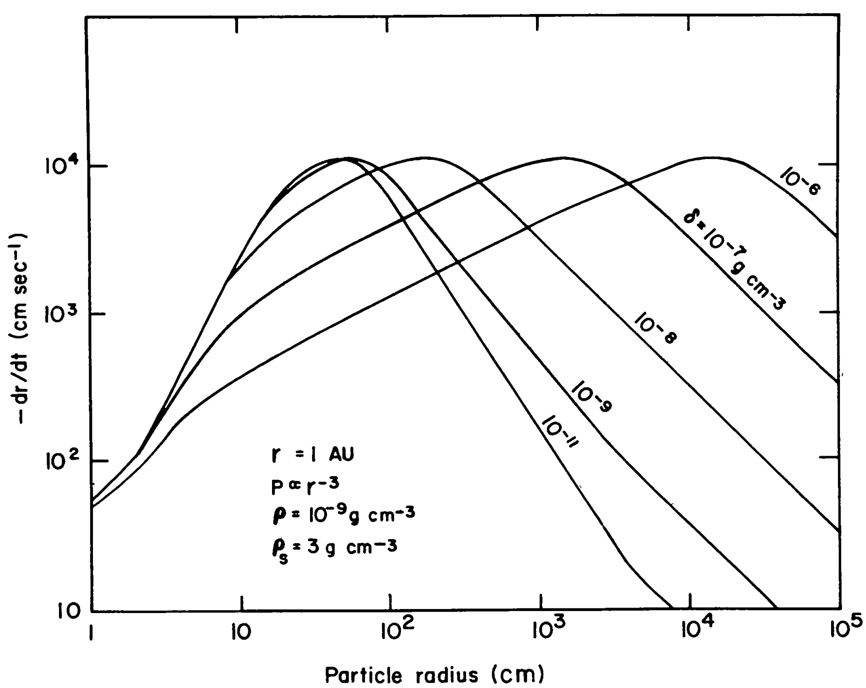


Figure 7 Radial velocity of small bodies, subject to the effects of gas drag and particle-particle collisions (Weidenschilling 1977a,c). δ is the space density of solid particles.

little effect on the motion of very large bodies. Intermediate size bodies will tend to spiral sunward. The resulting radial velocity can be as high as $\sim 0.1 \text{ km s}^{-1}$ for ~ 1 m size bodies using plausible values of the gas and dust density (Figure 7; Weidenschilling 1977a). In the early stages of planetary growth this size-dependent radial motion could have caused objects to spiral into the Sun, collide with one another, and could have precluded the low dust velocity required for dust layer gravitational instability (Section 2.2). It is possible that such effects may have caused radial separation of iron and silicate particles, and influenced the composition of Mercury (Weidenschilling 1977a).

Assuming that $\sim 10^{18}$ g bodies are formed either by dust layer instability or by sticking processes, the gravitational accumulation of these objects into $\sim 10^{25}$ g objects has been described by Nakagawa (1978). For objects this large, spiraling sunward by gas drag is unimportant. In contrast to the analytical theories discussed in the gas-free case, this analytical theory introduces the radial distribution of the bodies by use of a Fokker-Planck equation (e.g. Richards 1959) for the distribution of the bodies in space and time. Similar use of the Fokker-Planck equation for the gas-free case would be of interest in considering analytically the radial dispersion found numerically (Figure 4). Because this is essentially a diffusion approximation it permits calculation of a characteristic diffusion time for the bodies in phase space and demonstration that the mean square values of the random components of velocity represent a balance between the effect of gravitation encounters and gas drag. The resulting distribution of velocity (rms velocity $\sim 16 \text{ m s}^{-1}$) is then used to calculate a growth rate as a function of mass by use of the gravitational cross section (Equation 16) together with Equation (26). It is found that $\sim 10^{22}$ g bodies are formed on a time scale of $\sim 10^4$ yr. As discussed in Section 3.1.1 the bodies are "closely packed" during this stage of growth and the assumption that the accumulation process does not greatly diminish the number of bodies in intersecting orbits is probably a reasonable one.

In many ways the problem calculated analytically by Nakagawa resembles the gas-free numerical calculation of Greenberg et al. (1978), except that in Nakagawa's case gas drag limits the growth of mean velocity, rather than collisional dissipation. Nakagawa calculates a velocity distribution such that for a swarm of equal mass bodies, the root mean square velocity increases with $m^{\sqrt{2}/3}$, rather than with $m^{1/3}$ as found by Safronov (1969) for the equivalent collisional case. (See discussion in Section 3.1.2.) However, Greenberg et al. (1978) find that the velocities of the largest bodies in the accumulating swarm remain very small, as long as most of the mass continues to reside in the smallest bodies. Nakagawa's assumption that the velocity of the larger bodies will be given by the result he found for large bodies of equal mass is not likely to be correct if the result of Greenberg et al. is correct. As long as the assumption of "close packing" remains valid, the effect of the resulting lower velocities of the large bodies will be to cause them to grow even faster than found by Greenberg et al.

Hayashi, Nakazawa & Adachi (1977) consider a later stage of accumulation. It is assumed that, as a result of the process described by Nakagawa, a few $\sim 10^{25}$ g embryos have been formed near the position of the present planets and that the remaining mass is in the form of 10^{21} g bodies. As noted above, because of the difficulty of simultaneously treating by analytical methods the coupled mass and velocity distribution, this distribution may not actually be achieved. It could be that most of the mass will reside in $\sim 10^{25}$ g bodies in nonintersecting orbits, in which case the evolution to be described seems unlikely to occur.

In the presence of gas, capture of the $\sim 10^{21}$ g "planetesimals" by the $\sim 10^{25}$ g embryos will occur if as a result of gas drag and mutual perturbations the body enters the "Hill sphere" of the protoplanet. The Hill sphere is a volume surrounding the embryo bounded radially by the inner and outer collinear Lagrangian points of the restricted 3-body problem (see, for example, Blanco & McCuskey 1961, Szebehely 1967). Its dimensions are of order

$$d \sim \left(\frac{m}{M_0}\right)^{1/3} a \tag{33}$$

and are comparable in size to the "sphere of influence" defined by Equation (21). The probability is high that a body with slightly positive total energy will lose sufficient energy within the Hill sphere to be captured into circum-embryo orbit. This orbit will decay rapidly and the planetesimal will be accumulated by the embryo (Pollack, Burns & Tauber 1979).

As Hayashi, Nakazawa & Adachi point out, the rate of growth of the embryo will be limited by the rate at which planetesimals are supplied to its Hill sphere. For 10^{21} g planetesimals the rate of supply will result from the eccentricity and inclinations being "pumped up" to steady state values $\sim 10^{-3}$ representing a balance between mutual encounters and gas drag. The resulting velocity difference between the planetesimal and the gas will cause it to spiral inward on a time scale of $\sim 10^7$ years at the Earth's distance and $\sim 10^8$ yr at the distance of Jupiter. The increase in Earth mass as a function of time is shown in Figure 5*c*. Comparison with the gas-free results (Figure 5*a,b*) indicates that somewhat more rapid accumulation is predicted. However, the result is dependent on the initial size of the planetesimals. If, instead of 10^{21} g objects, the earlier stage of growth considered by Nakagawa resulted in larger bodies in isolated orbits, then the necessary pumping up of eccentricity and inclination would not occur. When the effect of the resulting lower relative velocity between planetesimal and gas is combined with the lower radial velocity characteristic of large bodies because of their small area-to-mass ratio, it is possible that the embryo may not grow much at all. Intermediate conditions would correspond to arbitrarily long accumulation times of $\gtrsim 10^7$ yr. Hayashi, Nakagawa & Adachi calculate more *rapid* accumulation times for large planetesimals. However, this depends on their assumption that these large bodies will have high enough eccentricities for their orbits to remain crossing as they spiral inwards. As discussed in Section 3.1.4, avoidance of premature isolation of embryos is a difficult enough problem in the gas-free case, and the tendency for orbits of very large bodies to become circular and noncrossing will be greater when gas drag is present. As before, long range perturbations might conceivably provide some way out of this problem, but again this may require the prior formation of large planets.

If the Earth formed by accumulation of solid bodies from a gaseous nebula, then a large quantity of gas would also have accumulated. Hayashi, Nakazawa & Mizuno (1979) calculate this to be $\sim 3 \cdot 10^{26}$ g, i.e. 5% of the present Earth mass and 6×10^4 times more massive than the present atmosphere. These workers calculate that this optically opaque atmosphere could lead to surface temperatures as high as 4000 K, and that early melting would occur even though the time scale for accumulation is $\sim 10^7$ yr. Thus all the models of terrestrial planet formation considered, giant gaseous protoplanets, gas-free accumulation, and accumulation in a gaseous nebula, lead to an initial Earth in which very extensive melting and differentiation are predicted. The time scale for accumulation doesn't make any difference in this regard. The situation for the Moon is less clear. The theory of Hayashi, Nakazawa & Mizuno would lead to low lunar initial temperatures because the primordial atmosphere of such a small planet would be very small.

The massive primordial atmosphere would consist principally of H_2 and He and could escape (Hunten & Donahue 1976). It is not clear what would be expected for the other volatile species expected in the solar nebula, e.g. H_2O, CO_2, Ne, Xe. Mizuno, Nakazawa & Hayashi (1979) consider the question of the dissolution of primordial rare gases in the melted Earth, and conclude that quantities of neon 2–400 times the present atmospheric inventory should have dissolved and been trapped in the Earth's interior. A much greater quantity should have been present in the primordial atmosphere. Sekiya, Nakazawa & Hayashi (1979) argue that Kr and Xe may be carried along with H_2 and He if the rate of outflow is sufficiently large. When this problem is fully understood, it may be that the present chemical and isotopic composition of the Earth's atmosphere and interior may provide evidence for or against accumulation in a gaseous solar nebula.

To the author's knowledge, no results of multiplanet accumulation in the presence of a gas phase have been presented as yet. It is hard to guess how this would affect the discussion in Section 3.1.4. During the later stages of accumulation the gas drag forces may permit the removal of smaller bodies from the swarm and allow the velocities of the large bodies to be "pumped up" more readily. On the other hand, the increased dissipation resulting from gas drag would work in the opposite direction and it isn't clear which effect would be more important. Another problem requiring investigation is the effect of small quantities of gas, such as might be released by collisions of planetesimals containing volatile material in concentrations similar to those of carbonaceous meteorites.

4 CONCLUDING REMARKS

Initial states intermediate in the evolution of the solar nebula have been described which are at least consistent with our very imperfect knowledge of star formation. The possible role of the two growth mechanisms, gravitational instability and gradual accumulation, in transforming these initial states into the observed planets has been discussed. In the course of this discussion it should have been obvious that there are many uncertain "leaps of faith" that must be taken. The elimination of some of these uncertainties is straightforward and simply requires some hard work, e.g. the clarification of radial diffusion in multiplanet accumulation. Other questions, such as the size distribution of fragments from tidally disrupted 10^{25} g bodies, will probably be very intractable.

Also it should not be imagined that the initial states discussed are the only ones possible. It does not take very much imagination to think of others and thereby get your name on a theory. However, until we understand how to use the basic mechanisms to go from initial states to prediction of their observable consequences, these speculations are of limited usefulness.

In view of this situation, it seems premature to become very vehement in the advocacy of any particular one of these "theories." This is not a time for "great debates" regarding solar system origin, but rather for the patient solving of difficult theoretical problems, gathering of data, and searching for ways to confront theoretical predictions with observable data.

Not long ago alternative theories of solar system origin were not much more than a source of entertainment to most working scientists. To some extent this view is still prevalent. However, today implicit and explicit ideas concerning solar system formation influence the context and significance of a large fraction of current planetological and geological research. It may be expected that many more able workers will soon be carrying out serious work on problems of solar system origin. It is important that this does not lead to the Tower of Babel, but rather that they try to understand one another and to explain their own work to others as well as they are able. If this happens, the increased interest in this field could lead to remarkably rapid progress on this ancient and fundamental scientific question.

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