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ON THE ORIGIN OF THE SOLAR SYSTEM

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Statement of the Problem.—A satisfactory theory of the origin of the solar system must account for the presence and the properties of the planets and the smaller bodies surrounding the sun, and preferably, but not necessarily, for the dynamical properties of the sun also. This means that we shall be concerned with the following bodies and properties:

(*1*) *Nine Planets:* The orbits around the sun are nearly coplanar and nearly circular; the largest inclinations and eccentricities are those of the outermost and innermost planets, Pluto and Mercury. The motions around the sun are all in the same sense (direct, by definition). The rotation of the planets is also direct, with obliquities less than 30°, except for Uranus where the angle has the exceptionally high value of 97°. The distances of the planets from the sun exhibit some degree of regularity (Bode's law). The masses of the four inner planets are roughly $10^{-6}\odot$ and the densities 4.1–5.5 cgs.; while the four Jovian planets have masses some hundred times larger and densities between 0.7 and 2.5 cgs.

(*2*) *Thirty Known Satellites:* The satellite systems vary from the beautifully regular case of Uranus to a completely irregular system like Neptune and the abnormal Earth-Moon system. Partly regular and partly irregular systems are those of Jupiter and Saturn. "Regularity" is measured by low inclinations with respect to the planetary equator; small orbital eccentricities, e; direct motion with respect to the planetary rotation; and some degree of regularity in the mean distances, a, to the planet.

(*3*) *The Asteroids and Meteorites:* The a-values of the 1168 largest asteroids[1] are roughly distributed according to $a = 2.89 \pm 0.24$ (*p. e.*) astr. units, though irregularities and fine structure occur in the distribution (Kirkwood Gaps) caused by the perturbations of Jupiter. Some 30,000 asteroids are accessible to telescopic observation and the indications are that their a distribution is not very different from that of the largest members. Meteorites strike the Earth after having arrived in orbits showing a distinct similarity to those of asteroids ($i \simeq 10°$, $e \simeq 0.2$).

(4) *The Comets, Both Periodic and Non-periodic, with Their Associated Meteor Showers:* The i-values are nearly random except for the short-period comets and meteors which show a preference for small i and direct motion. The e-values are moderate for the short-period comets and almost unity for the remainder. Since the long-period comets with $e < 0.9$ cannot approach the Earth, we have no direct knowledge of them.

The sun has an angular momentum of only about 0.3% of that possessed by the entire planetary system. The smallness of this ratio requires an explanation, but the process controlling the origin of the planetary system is not necessarily the sole cause of it.

Analogy with Binary Stars.—The solar system is not the only multiple system known to astronomy. Simple calculation shows that planetary systems other than our own would be inaccessible to observation. But companions massive or luminous enough to be observable are found in great numbers. A detailed study of the nearest stars shows that at least half of these are in fact double or multiple systems.

The *mean separations* a in binaries are roughly given by[2] an error curve in $\log a$ having a dispersion in ${}^{10}\log a$ of $\sqrt{2}$:

$$F(\log a) = Ce^{-[(\log a - 1.27)/2]^2} \qquad (1)$$

The median value of a is therefore about 18 astr. units, a value remarkably close to the position of the most massive planets in the solar system.

The *mass ratios* in binary systems of approximately the solar mass have a distribution roughly given by[3]

$$f\left(\frac{m_2}{m_1}\right) = \text{constant}\left(\text{valid for } 0.3 < \frac{m_2}{m_1} < 1.0\right) \qquad (2)$$

The data for $0 < \dfrac{m_2}{m_1} < 0.3$ are still too sporadic to give a trustworthy check on equation (2) in that range. But several mass ratios down to about 0.02 have been found and equation (2) appears to give the correct order of magnitude of the relative frequency of such cases. The absolute frequency of binaries with large mass ratios is found by putting the constant in equation (2) at about 0.7, i.e., a quantity of the order of 1. We shall, therefore, obtain a tentative estimate of the occurrence of small companions by using

$$f\left(\frac{m_2}{m_1}\right) = 1. \qquad (2')$$

Anticipating the conclusion made below that the formation of the planetary system was a special case of the very general process of binary formation, we may estimate the probability of this event by means of equation (2′). As the appropriate mass m_2 we may use either the mass of Jupiter, $10^{-3}\odot$, or the hypothetical mass of the entire solar nebula from which all the planets formed, $10^{-1}\odot$. The use of the second quantity requires a further estimate of the fraction of cases in which a secondary nebula of mass $10^{-1}\odot$ was spread out and formed several small bodies instead of a single stellar companion. This fraction, obviously < 1, may as to order of magnitude be put at 10^{-1}. The probability that a main-sequence star like the sun is attended by a planetary system may therefore be estimated to be of the order of 10^{-2} or 10^{-3}. On the basis of these developments the event would be enormously more probable than on the basis of the Tidal Theory or any other theory requiring a stellar collision or near collision; the latter would make the probability per star of the order of $10^{-10^{1/2}}$, or 10^8 times smaller than according to the process envisaged here.

The problem of the origin of binaries is considered in a separate article;[4] in it a provisional interpretation is given of equations (1) and (2). It is shown that virtually all stars are likely to have originated as components of double and multiple systems. This follows from the result that the probability of a proto-star having a small-enough angular momentum to allow the formation of a single, rotationally stable, star is less than 10^{-3}. Many of the original double and multiple stars must have dissolved later due to internal gravitational perturbations and effects of passing stars, so that at present the fraction of apparently single stars is substantial, roughly $^1/_4$ to $^1/_3$.

The analogy between binaries and solar system, suggested by the similarity of internal dimensions, will be complete if it can be shown that a small pre-stellar cloud moving about the primitive sun may form either a single stellar companion or a family of planets, depending on the mass distribution within this secondary cloud.

The Solar Nebula.—The composition of the planets, particularly of the terrestrial planets, differs appreciably from that of the sun. It is now realized that the composition of the sun is typical for that of stars and even interstellar matter; it may therefore be called the *cosmic* composition. The cloud from which the planets formed will almost certainly have been of cosmic composition. A comparison of the planetary composition with the cosmic composition will therefore give a lower limit to the mass of the nebula from which the planets condensed and will contain many clues to the condensation process itself. In this manner the mass of the solar nebula may be estimated to be in excess of $0.01\odot$ and probably more nearly $0.1\odot$. The realization that the pre-planetary cloud (or *solar nebula*) had this comparatively large mass has caused a revision in our concept of the possible processes by which planetary condensation may have taken place.

It appears now that gravitational action of the nebula on itself can have been important. It is seen below that precisely this property has made the formation of planets possible at all.

We shall now sketch the principal processes leading up to the formation of the planets, their masses, orbits, rotations and satellites. A more complete account is given in two papers now in press.[5]

Celestial mechanics indicates that the planets have not altered their distances from the sun by large factors.[6] (Later we shall find that the alterations were, in fact, remarkably small.) The solar nebula must therefore have extended at least from Mercury to Pluto. It appears that the density distribution in this nebula need not be assumed, but can actually be derived from the present planetary distances; but as a first step we may simply assume that the total mass of the nebula is $0.1 \odot$, as is suggested by the present planetary masses and compositions. This assumption had already been made by v. Weizsäcker[7] who found that turbulent dissipation will cause the solar nebula to dissolve with a half-life of about 10^7 years. The dissipation is caused by the different angular velocities of the different parts of the nebula. The time available for the formation of condensation products is therefore limited.

Another process, working in the opposite sense, is the slow collapse of the nebula parallel to the resultant axis of rotation (the z-coordinate). The initial stages of the collapse are fast, everywhere comparable to the orbital period of rotation around the sun. The later stages are much slower, 10^4–10^5 years in duration, and governed by the rate at which excess heat, caused by contraction, can be radiated away. Therefore, a *maximum of density* will result, some 10^4–10^5 years after the solar nebula got first in place; it is expected that the planets segregated close to or somewhat before that epoch.

Condensation products will form wherever the vapor is supersaturated.[8] Unless some large-scale coalescing property within the solar nebula can be found we must anticipate the formation of enormous numbers of small condensations (a process somewhat comparable with the formation of raindrops in a cloud), each growing slowly with time. The coalescing property needed to account for a small number of large planets was assumed by Weizsäcker to be a regular turbulence pattern in the solar nebula. Weizsäcker assumed a geometrical pattern of primary vortices, separated by rings of secondary "roller-bearing" vortices; he further assumed that condensation would progress most rapidly along the roller-bearing circles. This was assumed to lead to masses of sufficient prominence to insure their stability against subsequent destruction and hence their subsequent growth by accretion of neighboring material. Ter Haar[9, 10] elaborated Weizsäcker's picture and refined the discussion of the condensation process. Chandrasekhar and Ter Haar[10] showed that the regular turbulence pattern envisaged by Weizsäcker would not arise but should be replaced by the Kolmogoroff spectrum of turbulence previously derived for non-rotating media. The writer thereupon showed[5] that secondary eddies were less suited to foster condensation than the primary eddies of the Kolmogoroff spectrum, because any "roller-bearing" eddies that may form when large clouds collide would be short-lived and of distinctly higher temperature, but only slightly higher density, than the primary eddies. He further showed that even the primary vortices had comparatively short lifetimes, of the order of 10^2 years rather than 10^6 years as is necessary for the formation of appreciable condensations. The repeated formation and dissolution of primary vortices in more or less random positions woud lead to innumerable small condensations throughout the solar nebula instead of a few large ones. This obstacle can be overcome only if it is assumed that gravitational action intervenes and keeps some of the large vortices together long enough for massive condensations to form. This requires that these vortices have a density large enough to protect them from tidal disruption by the sun, i.e., their density must exceed the critical value, ρ_R, at which the internal attraction within the cloud and the tidal force by the sun are equal. This critical density is called the *Roche* density; it is intimately related to the well-known Roche limit, used, e.g., in the discussion of the stability of satellites and the Ring of Saturn. The Roche density varies inversely as the cube of the distance from the sun and is given by

$$\rho_R = \left(\frac{2\beta R_\odot}{a}\right)^3 \rho_\odot = \frac{6\beta^3 M_\odot}{\pi a^3} = \frac{24\pi\beta^3}{GP^2}, \qquad (3)$$

in which β is a quantity of the order of unity, a the distance to the sun and P the corresponding period of revolution around the sun; $R_\odot$ and $M_\odot$ are the radius and mass of the sun. The value of ρ_R is about 10^{-5} cgs. near Mercury, 10^{-6} near the Earth, 10^{-8} near Jupiter and 10^{-10} near Neptune. (The precise values depend not only on the mass and the distance to the sun but also on the molecular and turbulence velocities;[5] the range of uncertainty caused by the latter is covered by the approximate nature of β.) Condensation of masses of planetary size could not have occurred unless the mean density of at least some vortices or clouds in the solar nebula exceeded ρ_R. On the other hand, the mean density of these clouds could not have exceeded ρ_R by a large factor either. Such an excess in density would have caused the domain of the cloud to extend beyond its initial boundary and therefore the adjacent parts of the solar nebula would automatically have become part of the cloud until the mean density was lowered to a value of the order of ρ_R itself.

The conclusion to which we have come, that the segregation of pre-planetary masses took place essentially at the Roche density, appears to be capable of empirical verification. It can be shown[5] that the stability of two small spherical masses in contact, each at the Roche density, is not appreciably affected by the presence of the remainder of the solar nebula except near its outer edge. Therefore if m and r are the masses and the radii of the small spheres, we find from (3):

$$\frac{m}{M_{\odot}} = \left(\frac{2r\beta}{d}\right)^3. \tag{4}$$

If we introduce $\Delta = 2r = a_2 - a_1$, the distance between the centers of the masses whose mean distance from the sun is a, we have

$$\frac{m}{M_{\odot}} = \left(\frac{\beta\Delta}{a}\right)^3. \tag{4'}$$

If Δ should in some cases be as large as $a/3$ or $a/2$, as is indicated by the present planetary separations, we must allow for the fact that the flat disk-shaped solar nebula would not have filled such large spheres except for an equatorial slice occupying the fraction f of the order of perhaps 10^{-1}. Further, a comparison of the cosmic abundances with the present chemical composition of the planets shows that no more than a fraction g of the original pre-planetary clouds has been retained, g being about 10^{-1} for Jupiter and 10^{-3} for the Earth. The geometric mean of g for all planets may be roughly 10^{-2}. Therefore, if M_P represents the present planetary masses, we should expect

$$\frac{M_P}{M_{\odot}} = fg\left(\frac{\beta\Delta}{a}\right)^3, \tag{4''}$$

while empirically we find approximately[5, 11]

$$\frac{M_P}{M_{\odot}} = 10^{-4}\left(\frac{\Delta}{a}\right)^3. \tag{5}$$

It may be shown[5] that f is between 10^{-1} and 10^{-2}. The remarkable agreement between the predicted relation (4″) and the empirical relation (5) *shows that the segregation of planetary masses did take place essentially at the Roche density.* The closeness of the agreement is appreciated by remembering that the Roche density varies by the factor 500,000 between Mercury and Neptune and the M_P values are proportional to this quantity.

It appears that the comparison between equations (4″) and (5) may even be refined to take account of the variation with M_P of g in (4″). For the pair Jupiter-Saturn the value of f is about $10^{-1^1/_2}$, while g is roughly 10^{-1}. If $\beta = 1$ we should for this pair expect the coefficient in (5) to be $10^{-2^1/_2}$. This is precisely what is found.[11] For the pair Earth-Venus, $f = 10^{-1}$, $g = 10^{-3}$, roughly, and hence the expected coefficient in (5), 10^{-4}. This, again, is confirmed.[11] It is significant that the empirical relationship (5) holds for both planets and satellites; we return to this comparison later.

These examples show that the systematic deviation[11] from equation (5) is caused by the systematic change of g with M_P. A second result,[11] that pairs with very unequal masses are systematically somewhat displaced with respect to pairs of nearly equal masses, has not yet been interpreted.

With the density in the plane of symmetry determined, the density distribution at right angles to this plane (z-direction) may be computed. Weizsäcker and Ter Haar in their discussions of the hydrodynamics of the solar nebula assumed that the z-component of the force of gravity is given by the z-component of the solar attraction. With the equatorial density as high as ρ_R the z-force near the plane of symmetry will in fact be predominantly caused by the attraction of the nebula itself, in much the same way as the z-force in the galaxy near the galactic plane is determined by the

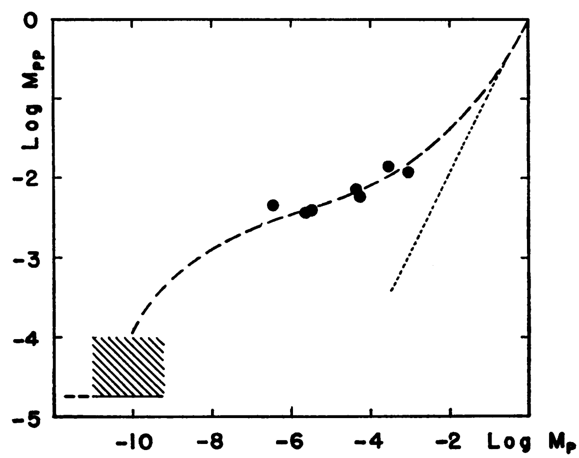

FIGURE 1

Relation of computed mass of proto-planet, M_{PP}, to present observed mass of planet, M_P. Unit of mass is Sun.

density distribution of the surrounding parts of the galaxy and not by the galactic nucleus. The ratio of the two z-forces, nebula/sun, is $24\beta^3$, or about 25. The solar nebula is flatter than considered by previous investigators,[7, 9, 10] by this same factor. The resulting large density increase in the new model allows the operation of *gravitational instability*. It is this mechanism which amplifies initial density fluctuations caused by turbulence[5] and is responsible for the formation of large clouds (proto-planets) which are gravitationally stable. These stable clouds exist long enough for the condensation processes to run their course and for large masses to form instead of billions of small, unrelated drops or flakes.

The density distribution in the z-coordinate may be computed from an equation resembling the barometric formula, the field of force being known and also, approximately, the molecular velocities. The latter follow from the distribution in temperature with distance from the sun, which in turn may be estimated when allowance is made for the opacity of the nebula.

The condition that the proto-planets segregated at the Roche density, together with the density gradients in z just found, fixes completely the mass distribution for each proto-planet, both in the xy plane and the z-direction. We need only to add the dimensions in the xy plane for a complete description of the proto-planets, including their masses. The diameters of the proto-planets, at their maximum extent, are found by assuming that the proto-planets grew until they touched their neighbors on each side and by the further assumption that the planets formed at their present

TABLE 1

PLANET	LOG a	LOG a BOUNDARY	Δ LOG a	a BOUNDARY	MAX. $2R(PP)$	MAX. $M(PP)$	LOG $\frac{M(PP)}{M(P)}$
Mercury	−0.330		....		....	....	....
(aph.)		−0.24		0.58			
Venus	−0.141		0.17		0.27	0.0037	3.18
		−0.07		0.85			
Earth	0.000		0.16		0.38	0.0038	3.10
		+0.09		1.23			
Mars	+0.183		0.18:		0.63:	0.0045:	4.14:
		+0.27:		1.86:			
Asteroids	+0.45:		0.28:		....	....	....
		+0.55:		3.55:			
Jupiter	+0.716		0.30:		3.5	0.012	1. 0
		+0.85		7.1			
Saturn	+0.979		0.30		7	0.014	1.69
		+1.15		14			
Uranus	+1.283		0.23		10	0.0072	2.22
		+1.38		24			
Neptune	+1.478		0.20:		14	0.0057:	2.04
		+1.58:		38			
Pluto	+1.597		....		....	....	....

NOTE: Unit of a is a. u. PP = proto-planet; P = planet. Cf. figure 1.

distances to the sun. The resulting partitioning of space is shown in table 1; it was done in a logarithmic plot because of the approximate validity of Bode's law and the fact that all relevant functions have an exponential run with a. The hypothesis that the proto-planets at their maximum size touched each other, accounts for the fact that at that point further growth was arrested; nearly all of the solar nebula would thus have been "swept clean." At the same time a mechanism is provided for the explanation of retrograde satellites in the outer regions of the "sphere of action" of such planets as did not lose thereafter too great a fraction of their original mass (Jupiter and Saturn).

The preceding arguments fix the masses of the proto-planets, M_{PP}, uniquely. They are found in table 1 and plotted against the present planetary masses, M_P, in figure 1. The closeness of the relation is very remarkable. The largest deviation is that of Mars, amounting to $10^{0.2}$ or a factor 1.6 in M_{PP}. This may be interpreted to mean that the diameter of proto-Mars was in fact only 0.8 of that assumed; this difference is not serious since the outer boundary of the Mars zone is ill-defined.

The curve relating M_{PP} to M_P has been tentatively extended beyond the region of the planets, as follows. For *any* value of M_P we must have $M_P \leq M_{PP}$. The limiting line so defined is dotted in figure 1; very probably this line is an asymptote. Further, it has been shown[5] that there is a theoretical lower limit to M_{PP}, given by $\log M_{PP} = -4.8$. All points must necessarily lie above this limit. It is assumed that Ceres and a few other asteroids are true planetary condensations. Their M_P values are about $10^{-10}\odot$. The shaded region in figure 1 shows the most probable location of these objects.

The closeness of the relation in figure 1 shows two things: (*a*) The position of Uranus and Neptune is intermediate between that of the Earth and Venus on the one hand and that of Jupiter and Saturn on the other. This shows that the relation in figure 1 is not strongly affected by the distance to the sun, though an extra factor of two or three in the present masses of the Jovian planets would hardly be discovered in this manner. This means, e.g., that the Earth is different from Jupiter *not* because it originated closer to the sun but primarily because proto-Earth was less massive than proto-Jupiter. The existence of a planet like Pluto outside Neptune becomes at once "acceptable." (*b*) The distances of the planets to the sun cannot have greatly changed during the last 3.10^9 years, at least not in an irregular fashion. If they had, the relation in figure 1 could not possibly have been close since M_{PP} depends in a sensitive way on the present planetary distances. This conclusion goes much beyond the powers of celestial mechanics which for such extremely long intervals has made only the most general predictions.[6]

The masses of the proto-planets are about as large (within 20 or 30%) as might have been expected from the present planetary compositions. This conclusion is an important check on the theory presented. It is the first time that planetary masses have been predicted with any degree of accuracy.

By allowing 0.003 $M_\odot$ for each of the proto-planets which formed Mercury, the asteroids and Pluto we find from table 1 the total mass of that part of the solar nebula that was responsible for the formation of the planets. It is found to be

$$\sum M_{PP} = 0.060\, M_\odot. \qquad (6)$$

The total mass of the solar nebula must have been somewhat (but probably not appreciably) larger and may be put in round figures at $10^{-1}\ M_\odot$.

The Bode Law. Planets and Binaries.—We must now consider the factors that determine the relative spacings, Δ/a, of the planets. Why did nine planets form and not 10^2 smaller ones?

The mode of formation of the proto-planets gives a clue. If the model of one proto-planet per ring, valid for the large planets, is extended to smaller and smaller bodies, we note that a smaller and smaller fraction of the ring is occupied to the Roche density. It follows that the spacing of the planets at the distance a from the sun is determined by the ratio of the density in the solar nebula to the local Roche density.

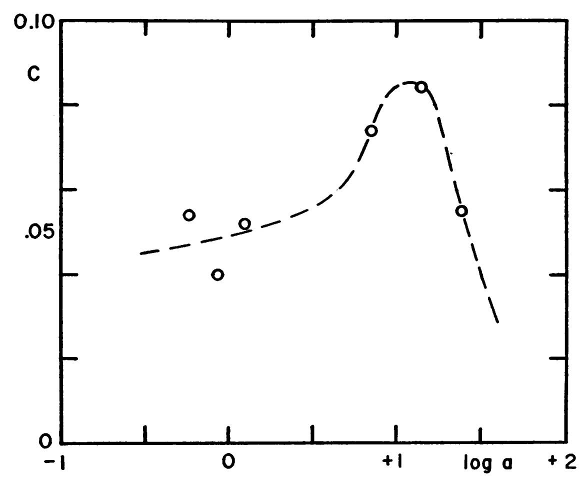

FIGURE 2

Mass distribution in solar nebula prior to formation of proto-planets (tentative).

In this manner the density distribution is found which existed in the solar nebula prior to the break-up into proto-planets. In order to express the result we take as reference a nebula having everywhere the Roche density in the xy plane. Such a nebula will have a given "surface density" of matter when projected on the xy plane. In fact, it is one in which the amount of matter per unit interval of log a is constant. In terms of this standard surface distribution, the pre-planetary nebula had the density 0.05 from about $^1/_2$ to 2 astr. units, rising to 0.09 at 10 astr. units, and falling off to below 0.05 at 30 astr. units. It is illustrated in figure 2.

From the theory of gravitational instability it follows that a lower limit exists below which stable proto-planets cannot form. This limit in the units of figure 2 is found to be

$$C_0 = 0.05\alpha^{-1}\beta^{-3/2}, \tag{7}$$

in which both α and β are quantities of the order of unity. It appears therefore that the solar nebula had only just enough surface density to cause formation of planets. A reduction by a factor of two or three (or more) would have been fatal to this process; billions of comet-like bodies would have formed instead. This result may also be interpreted to mean that the hydrogen content of the solar nebula could not have been much less than supposed; a reduction by a factor three over the adopted cosmic composition would have caused a nearly equal reduction in mass, and would again have prevented planets to form. It is possible that the hydrogen content was actually somewhat larger than assumed.

If, on the other hand, the surface density of matter over the xy plane had been larger than found in figure 2 by a factor of three or more, no ordinary planets would have formed either but instead one or more stellar companions. This results from the circumstance that a rather small increase in C (see Fig. 2) causes a large increase in the mass of the proto-planet, which in turn causes a still greater increase in the mass of the final body (Fig. 1).

Three conclusions follow: (a) the so-called Bode law is no real law but a consequence of the density distribution in a which the solar nebula happened to have; the larger the density, in terms of the local Roche density, the larger the ratio between the radii of two consecutive orbits and the larger the masses of the proto-planets (and, finally, the larger the masses of the planets); (b) the process of formation of a planetary system is a special case of the almost universal process of binary-star formation; the crucial parameter is the surface density of the secondary nebula in terms of the local Roche density; (c) inspection of figure 1 suggests that the subsequent evolution of the proto-planets is not greatly influenced by the distance to the sun, but instead largely determined by the mass of the proto-planet itself (once formed, the object is "on its own"). Space does not permit to discuss the theoretical interpretation of this conclusion; it appears to be connected with the rate at which the lighter gases can escape.[5]

Conclusions.—We may now turn to the problems listed on page 1 and list the solutions now at hand or indicated. The common direction of revolution and the low relative orbital inclinations are accounted for by the flatness of the solar nebula. The internal viscosity of the nebula accounts for the near-circular orbits. The fact that both Mercury and Pluto are exceptions in their inclinations and eccentricities may be attributed to the absence of constraining action on the proto-planets formed on the fringes of the solar nebula. The direct rotation of the planets is attributed[5] to solar tidal friction on the proto-planets. The solar tidal force nearly equals the self-attraction for each of the proto-planets at their maximum extension; i.e., in the proper units Neptune is no farther away from the sun than Mercury and the tidal effects are equally large in both cases. Regardless of any initial rotational motion a direct rotation will be forced upon the proto-planet, with a period equal to the orbital period. As is readily verified, this leads to an amount of angular momentum per unit mass some 10^4 times greater than found on the present planets. Part of this is lost during the evaporation process of the proto-planets (the ejected molecules carry off more than the average amount of angular momentum per unit mass); while part of it is lost by continued solar tidal friction during the contraction process. The latter cause has a secular effect on the obliquities; it has been shown[5] that they will increase some three or fivefold, from initial obliquities of the order of 3° (expected from the turbulent solar nebula, and consistent with the relative orbital inclinations) to the present values. The largest obliquity to which this process can lead is 90°; retrograde rotation cannot arise by the processes considered. It is not clear why Uranus has passed the upper limit by 7°; possibly some extraneous object has moved through the solar system. The present periods of rotation have not yet been accounted for quantitatively. This appears to be a very complex problem, with physics, chemistry and dynamics all playing a role. We have here perhaps the most important potential source of information still unused in the reconstruction of the planetary condensation processes.

The regular satellites may be explained in a manner analogous to that found for the planets themselves.[5] Little progress has been made so far with their condensation processes, which should prove very instructive in view of the large density differences known to exist among the satellites. The retrograde satellites of Jupiter and Saturn have been interpreted[5] as having been caused by glancing collisions between the corresponding proto-planets. They were assumed to have been retained by these large planets only because these planets lost a much smaller fraction of their initial mass. It is possible, however, that capture has played a role instead. This requires further investigation. The asteroids were not formed in a region of low density in the solar nebula. In such a region no planets of any kind could have formed. Rather we must assume that the density was well above C_0 of equation (7), but that the formation of a normal-size proto-planet was prevented by proto-Jupiter (mass = $0.012\odot$). It can be shown that in the presence of strong perturbations a small proto-planet, of a given density close to the local Roche density, is more stable than a large one of the same density. The total number of small proto-planets estimated to have formed in the region between Mars and Jupiter is between 5 and 10. They formed small planets, like Ceres (cf. figure 1 and accompanying discussion). It is assumed that two of these collided sometime during the last 3.10^9 years, an event having a sufficiently large probability. Thereafter secondary collisions became increasingly frequent. The most recent of these collisions account for the Hirayama families. In this manner thousands of asteroids were formed, being the largest of the fragments, as well as billions of meteorites.[12]

The outermost region of the solar nebula, from 38 to 50 astr. units (i.e., just outside proto-Neptune), must have had a surface density below the limit set by equation (7). The temperature must have been about 5–10°K. when the solar nebula was still in existence (before the proto-planets were full grown), and about 40°K. thereafter. Condensation products (ices of H_2O, NH_3, CH_4, etc.) must have formed, and the flakes must have slowly collected and formed larger aggregates, estimated to range up to 1 km. or more in size. The total condensable mass is about 10^{29} g., but not all of this could be collected. These condensations appear to account for the comets, in size,[13] number[13] and composition.[14]

The planet Pluto, which sweeps through the whole zone from 30 to 50 astr. units, is held responsible for having started the scattering of the comets throughout the solar system. Pluto's perturbations will have caused initial, near-circular, cometary orbits to become moderately elliptical; thereupon stronger perturbations by Neptune and the other major planets will have scattered them even more broadly. As Oort[13] and others have shown, the quantity which is spread nearly uniformly in both directions is the quantity a^{-1}, the reciprocal of the semimajor axis (which is related to the energy of the object). A certain fraction of the comets will be scattered in the region of very small a^{-1} values, i.e., in the outer regions of the "sphere of action" of the sun. As Oort[13] has shown, stellar perturbations will redistribute the orbital elements there, and in particular make the motion around the sun one of random orientation. Oort[13] shows that the dynamical half-life of a comet in this outer region is about 10^{10} years. The comets which we observe today were sent back to the inner regions of the solar system by small random stellar perturbations. The above views are an adaptation of Oort's[13] dynamical analysis; but we differ in our hypothesis as to the region where the comets originated. Oort[13] assumes that they were formed between Mars and Jupiter, in association with the origin of asteroids. The composition of the comets indicates condensation at a very much lower temperature, around 10°K., consistent with the region of origin proposed here. The evaporation and subsequent complete disintegration of comets into the minute particles which cause meteors and the Zodiacal Light is also understandable from their formation outside Neptune. Asteroidal bodies would be expected to remain intact or possibly break up into a few large fragments.

The theory described here does not depend on any specific *ad hoc* assumptions. Certain assumptions which were made at the outset, e.g., that the planetary distances have not changed appreciably or that the solar nebula was approximately of cosmic composition, appeared capable of verification afterwards. One assumption, that the sun was already formed as a star and of a luminosity approximately equal to that found today, requires further study.[15] Certain investigations on the contraction and condensation process of the proto-planets need still be made, including the analysis of solar tidal friction on these composite structures. Finally, the cause of the small solar rotation must be cleared up; it is undoubtedly connected with the larger problem of why nearly all *G*-type dwarf stars, in single and in binary systems, have such slow rotations. It is felt, therefore, that this problem is not necessarily a part of a theory on the origin of the solar system.

The probability of a star being attended by a planetary system was estimated to be between 10^{-2} and 10^{-3}. The total mass of the galaxy is about $2.10^{11}\odot$; while the average stellar mass is about $0.5\odot$. From these figures the total number of planetary systems in the galaxy is estimated to be of the order of 10^9. One can only speculate on the possible forms of life which may have developed on these many unknown worlds.

[1] Those with $g \leq 10.9$ in the 1947 Catalogue of Minor Planets. The quantity g is the magnitude of the asteroid reduced to unit distance from the Earth and unit distance from the Sun; it depends chiefly on the *size* of the body since the albedos are probably not very different.

[2] Kuiper, G. P., *Publ. Astr. Soc. Pac.*, **47**, 138 (1935).

[3] *Ibid.*, p. 147.

[4] Kuiper, G. P., *Astrophys. J.*, in press, (1951).

[5] Kuiper, G. P., "On the Origin of the Solar System," Chap. 8 of *Astrophysics*, edited by J. A. Hynek, McGraw-Hill Book Co. (in press). (This paper was submitted for publication November, 1949; it was given a limited distribution in February, 1950.) Also, *Ibid., Astrophys. J.*, 1951.

[6] Brown, E. W., *Publ. A.S.P.*, **44**, 37 (1932).

[7] Weizsäcker, C. F. v., *Zeitschr. f. Astrophys.*, **22**, 319 (1944).

[8] Becker, R., and Doering, W., *Ann. Phys.*, **24**, 719 (1935). Ter Haar, D., *Bull. Astr. Inst. Neth.*, **10**, 3 (1943).

[9] Ter Haar, D., "Studies on the Origin of the Solar System," *Kgl. Danske Videnskab. Selskab, Mat-Fys. Med.*, **25**, No. 3 (1948). A summary appeared in Science, **107**, 405 (1948)

[10] Ter Haar, D., *Astrophys. J.*, **111**, 179 (1950). Chandrasekhar, S., and Ter Haar, D., *Ibid.*, p. 187.

[11] Kuiper, G. P., "The Law of Planetary and Satellite Distances," *Ibid.*, **109**, 308 (1949).

[12] Kuiper, G. P., "On the Origin of Asteroids," *Astron. J.*, **55**, 164 (1950).

[13] Oort, J. H., *Bull. Abstr. Inst. Neth.*, **11**, 91 (1950).

[14] Whipple, F. L., *Astrophys. J.*, **111**, 375 (1950).

[15] Actually, it is more probable that the Sun had not completed its contraction and that its radius was very substantially larger than at present (by a factor in excess of 10 and possibly as large as 50). This will have caused a reduction in the solar luminosity by the square root of this ratio, say 5. This reduction in solar temperature facilitates the processes considered in the text.