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THE FORMATION OF PLANETESIMALS*

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ABSTRACT

Four stages in the accretion of planetesimals are described. The initial stage is the condensation of dust particles from the gaseous solar nebula as it cools. These dust particles settle into a thin disk which is gravitationally unstable. A first generation of planetesimals, whose radii range up to $\sim 10^{-1}$ km, form from the dust disk by direct gravitational collapse to solid densities on a time scale of the order of 1 year. The resulting disk, composed of first-generation planetesimals, is still gravitationally unstable, and the planetesimals are grouped into clusters containing approximately 10^4 members. The contraction of these clusters is controlled by the rate at which gas drag damps their internal rotational and random kinetic energies. On a time scale of a few thousand years, the clusters contract to form a second generation of planetesimals having radii of the order of 5 km. Further coalescence of planetesimals proceeds by direct collisions which seem capable of producing growth at a rate of the order of 15 cm per year at 1 a.u. The final stage of accretion during which planet-sized objects form is not considered here.

Subject headings: planets — solar system

I. INTRODUCTION

This paper reports on an investigation of the significance of gravitational instabilities in the primordial solar nebula to the planetary formation process. Of course, this subject is by no means a new one. Kuiper (1951) suggested that fragmentation of the nebula into protoplanets occurred when compression of the disk in the vertical direction due to cooling drove the density above the local Roche limit. However, he did not determine the scale of the instabilities but merely assumed that it was comparable to the planetary separations. An attempt to establish this scale was made by Urey (1966, 1972). Urey applied the dispersion relation for an infinite uniformly rotating gas to the solar nebula. This dispersion relation reads (Chandrasekhar 1955)

$$\omega^2 = k^2c^2 + 4\Omega^2 - 4\pi G\rho \,, \tag{1}$$

where ρ is the unperturbed gas density, c is the sound speed, Ω is the angular velocity, G is the gravitational constant, ω is the frequency of the disturbance, and $k = 2\pi/\lambda$ is its wavenumber. From the above dispersion relation, we see that a range of unstable wavelengths exists if $\pi G\rho > \Omega^2$. The minimum unstable wavelength is

$$\lambda_{\min} = \pi c/(\pi G\rho - \Omega^2)^{1/2} \,. \tag{2}$$

Urey assumed that $\pi G\rho$ was slightly greater than Ω^2. He then deduced that lunar-sized objects formed as the products of the collapse of regions of initial sizes $\lambda_{\min}$. One puzzling feature of Urey's work is his choice of $\lambda_{\min}$ as the dominant instability scale since Chandrasekhar's dispersion relation shows that longer-wavelength perturbations grow faster.

These attempts to attribute the planetary formation process to gravitational instabilities of the entire gaseous nebula encounter an insurmountable obstacle. The

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density required for instability is so great that either the temperature of the nebula must have been unacceptably low or else its mass must have greatly exceeded the value needed to account for the present combined masses of the planets. A brief discussion of this point is given in § III*a*.

Our contribution is to outline the process by which dust particles can accrete to form gravitationally active objects. We shall present a compelling argument that, indeed, gravitational instabilities account for the growth of objects up to several kilometers in radius. The crucial difference between our investigation and earlier analyses is that these instabilities are found to develop, not in the *gaseous* solar nebula, but in the thin disk of particulate matter that forms in the central plane during the condensation phase. While our work was in progress, two papers dealing with this problem appeared. Lyttleton (1972) pointed out that a thinning dust disk would eventually become more dense than the Roche limit. But he made no attempt to ascertain the masses of the unstable regions. Polyachenko and Fridman (1972) presented an analysis of the fragmentation of a dust disk similar to that contained in § III*a* of this paper. They solved for the density of a dust disk that would have been unstable on the scale of the present planetary separations. Not surprisingly, they found that the required mass of solid material was two orders of magnitude greater than that present in the planets. On the other hand, we use the known masses of the planets to estimate the surface density of the preplanetary dust disk and then solve for the scale of the initial instabilities.

In § II, the formation of the thin disk of condensed particles and its further evolution through gas drag will be described. The fragmentation of the disk is treated in § III, and the formation time and masses of the resulting planetesimals are calculated. It should be emphasized at the outset that at several stages of this investigation assumptions are introduced for which alternative possibilities are also quite plausible. In order to keep our exposition as lucid as possible, we shall follow one particular line of reasoning through to the end. Then in the concluding section, we shall explore the modifications which the competing assumptions would have introduced.

In order to avoid repetitious references to the numerical values of the physical parameters appropriate to the primordial solar nebula, we list here our adopted values for the vicinity of the Earth's orbit. These values will be applied in the text to all numerical calculations without further reference. The distance from the Sun is $a = 1.5 \times 10^{13}$ cm and the Keplerian mean motion is $\Omega = 2 \times 10^{-7}\ \mathrm{s}^{-1}$. The surface density of condensed matter implied by the masses of the terrestrial planets is $\sigma_p \sim 7.5$ g cm^{-2}, which represents a mass fraction $\alpha \sim 5 \times 10^{-3}$ of the entire gaseous preplanetary disk that has a surface density $\sigma_g \sim 1.5 \times 10^3$ g cm^{-2}. We use a value of $c = 7.6 \times 10^3 T^{1/2}$ cm s^{-1} for the speed of sound at temperature T. This expression is applicable to a gas composed of hydrogen molecules. Where a specific value for T is required, we use $T = 700°$ K. The half-thickness of the gas disk is $D \sim c/\Omega \sim 10^{12}$ cm, which implies a mean gas density of $\rho_g \sim 7.5 \times 10^{-10}$ g cm^{-3}. The mean free path in the gas is $l_g \sim 10$ cm, which together with the value for c, yields $\nu \sim 2 \times 10^6$ cm^2 s^{-1} for the kinematic viscosity. In the discussion of the chemical condensation of dust particles we use numerical values appropriate for iron. These are $\alpha(\mathrm{Fe}) = 1.5 \times 10^{-3}$ and $\rho_p(\mathrm{Fe}) = 7.9$ g cm^{-3}. Elsewhere, we assume $\rho_p \sim 3$ g cm^{-3} as appropriate for the mean uncompressed density of the terrestrial planets.

II. THE PARTICULATE DISK

a) *Formation*

An attractive hypothesis for the initial stage of planetary accretion has emerged from recent studies of the chemical condensation sequence of the cooling preplanetary gas. As the primordial solar nebula cools, the vapor pressure of a constituent rapidly decreases and eventually falls below its partial pressure. Presumably, at this stage the condensation of small particles ensues. This process has long been recognized to be important but has only recently been investigated in detail (Lord 1965; Larimer 1967; Larimer and Anders 1970; Lewis 1972). From the present composition of the terrestrial planets, it appears that the primary condensates in the inner solar system are iron, nickel, and iron and magnesium silicates (Larimer 1967). In the outer nebula the temperatures are lower and the bulk of the condensate is made up of water and water-ammonia ices (Lewis 1972).

Once nucleation occurs, a particle continues to grow by collecting material still in vapor phase. Its growth rate is given by (Hoyle 1946)

$$dr/dt = \alpha v_T \rho_g/\rho_p \,, \tag{3}$$

where r is the particle's radius and v_T is the thermal velocity of the constituent molecules which are still in the vapor phase. Numerically, the growth rate is on the order of centimeters per year for the more abundant minerals.

Once nucleated, a particle begins to settle through the gas toward the equatorial plane. There is a definite upper limit on the size a particle can obtain during this process. In order to calculate this limiting size, we must estimate the rate at which a particle descends to the central plane. The differential velocity between a descending particle and the local gas is set by the balance between the vertical component of solar gravity and the gas drag force. For the small particles with which we are concerned, the mean free path in the gas is long compared to the particle radius and the drag force is given by

$$F_D \sim \pi r^2 \rho_g c v_z \,, \tag{4}$$

where v_z is the vertical velocity of the particle. Setting this expression equal to the vertical force of solar gravity, we find

$$v_z \sim \Omega^2 \frac{\rho_p z}{\rho_g c} r \,. \tag{5}$$

Since the half-thickness of the gas disk is approximately c/Ω, the characteristic descent time may be expressed in terms of the surface density of the gas disk as

$$\tau_z \sim \frac{1}{\Omega} \frac{\sigma_g}{\rho_p} \frac{1}{r} \,. \tag{6}$$

From equations (3) and (6), it follows that the maximum radius, R, to which a particle can grow before reaching the central plane is

$$R \sim \frac{\alpha^{1/2} \sigma_g}{\mu^{1/4} \rho_p} \,, \tag{7}$$

where μ is the molecular weight of the condensing molecules. When applied to the condensation of iron in the terrestrial region, equations (6) and (7) yield $\tau_z \sim 10$ years and $R \sim 3$ cm.

It is obvious that the unknown number of nucleation sites is the principal uncertainty bearing on the size of the particles which settle into the central dust disk. If the number of these sites is so large that the vapor phase is significantly depleted on time scales short compared to the particle descent time, the final particle sizes will be much smaller than R given by equation (7). Because we have no reliable estimate on the actual number of nucleation sites, we shall express all future results in terms of the unknown particle radius $r \leqslant R$.

b) Gas Drag and Orbital Decay

In addition to a vertical pressure gradient, there undoubtedly exists a radial pressure gradient in the gaseous solar nebula. This affects the orbital velocity of the gas since the centripetal acceleration is produced by the difference between the inward gravitational attraction of the Sun and the outward force of the pressure gradient. Thus,

$$a\Omega_g^{\,2} = \frac{GM_\odot}{a^2} + \frac{1}{\rho_g}\frac{dp_g}{da}\,, \tag{8}$$

where Ω_g is the orbital angular velocity of the gas, $M_\odot$ is the mass of the Sun, and p_g is the gas pressure. If we make the approximation $dp_g/da \sim -c^2\rho_g/a$, then we obtain

$$\Omega_g \sim \Omega - \frac{c^2}{2\Omega a^2} \tag{9}$$

to lowest order in $(c/\Omega a)^2 \ll 1$. Here Ω is the local Keplerian velocity.

The pressure gradient has little effect on the condensed particles since their densities are many orders of magnitude higher than the gas density. However, the particles do interact with the gas through gas drag. A straightforward calculation for the rate at which a particle spirals toward the Sun yields (Whipple 1972)

$$\frac{\dot{a}}{a} \sim -\frac{2\chi\Omega}{(4\chi^2 + \Omega^2)}(\Omega - \Omega_g)\,, \tag{10}$$

where

$$\chi = \frac{3}{4}\frac{\rho_g}{\rho_p}\frac{c}{r}\,. \tag{11}$$

From equations (9), (10), and (11) it follows that the characteristic orbital decay time, $\tau_a = a/\dot{a}$, is given by

$$\tau_a \sim \frac{3\rho_g a^2}{\rho_p c r}\,, \tag{12}$$

where we have used the fact that

$$\frac{\chi}{\Omega} = \frac{3}{4}\frac{\sigma_g}{\rho_p r} > \frac{3}{4}\frac{\sigma_g}{\rho_p R} \sim \frac{3}{4}\frac{\mu^{1/4}}{\alpha^{1/2}} \gg 1\,. \tag{13}$$

The fractional decay that a particle's orbital radius suffers during the particle's descent to the central plane is

$$\Delta a/a \sim \tau_z/\tau_a \sim \tfrac{1}{3}(c/a\Omega)^2 \sim 10^{-3}\,. \tag{14}$$

The characteristic orbital decay time given by equation (12) is

$$\tau_a \sim \frac{2 \times 10^4}{r}\ \text{years} \tag{15}$$

in the terrestrial region ($a \sim 1$ a.u.), where r is to be expressed in centimeters. The estimate given above for τ_a is not applicable to particles in the dust disk unless gas molecules are able to pass freely through the disk. The ratio of the mean free path, l_p, to the thickness of the disk, d, is approximately

$$l_p/d \sim 4\rho_p r/3\sigma_p \sim 0.53r\,. \tag{16}$$

From equation (16), we see that if a substantial fraction of the particles in the disk have radii in the subcentimeter range, then $l_p/d < 1$. In this case, the gas drag is exerted on the surface of the dust disk. A short calculation of this boundary layer drag is outlined below.

The drag force per unit area on the surface of the disk is given by

$$S = \rho_g \nu (dv_g/dz) , \tag{17}$$

where ν is the kinematic viscosity in the gas. If the boundary layer flow were laminar, the velocity gradient would be confined to an Ekman boundary layer of thickness

$$\delta \sim (\nu/\Omega)^{1/2} . \tag{18}$$

From the standard parameters we have adopted, it follows that $\delta \sim 3 \times 10^6$ cm, much smaller than the half-thickness of the gaseous disk, $D \sim 10^{12}$ cm. The tangential stress on the dust disk would be

$$S \sim \rho_g \nu a(\Omega_g - \Omega)/\delta \sim -\tfrac{1}{2}(\nu/\Omega a^2)^{1/2} \rho_g c^2 . \tag{19}$$

However, it seems quite likely that the boundary layer flow is turbulent. The Reynolds number in the Ekman layer, if the flow is assumed to be laminar, is

$$\mathscr{Re} \sim \frac{a(\Omega - \Omega_g)\delta}{\nu} \sim \frac{c^2}{2\nu^{1/2}\Omega^{3/2}a} \sim 10^4 . \tag{20}$$

Furthermore, gravitational stratification appears to be too weak to stabilize the boundary layer flow since the Richardson number (e.g., Chandrasekhar 1961) is

$$\mathscr{Ri} \sim -\frac{g_z d\rho/dz}{\rho(dv_g/dz)^2} \sim \frac{8\pi G\sigma_p \nu}{c^3}\left(\frac{\Omega a}{c}\right)^2 \sim 10^{-12} , \tag{21}$$

much smaller than the critical value of $\frac{1}{4}$.

The tangential stress due to a turbulent boundary layer is given by equation (17) but with ν replaced by the turbulent viscosity

$$\nu' \sim \Delta v_g \delta' / \mathscr{Re}^* . \tag{22}$$

Here $\Delta v_g \sim (\Omega - \Omega_g)a$ is the velocity jump across the boundary layer, δ' is the boundary layer thickness, and $\mathscr{Re}^*$ is the critical value of the Reynolds number above which the flow becomes fully turbulent. Since $dv_g/dz \sim \Delta v_g/\delta'$, the turbulent stress on the disk is simply

$$S \sim \rho_g (\Delta v_g)^2 / \mathscr{Re}^* . \tag{23}$$

From experimental data it is known that $\mathscr{Re}^* \sim 5 \times 10^2$ (Jeffreys 1959). The thickness of the turbulent boundary layer is

$$\delta' \sim \frac{\Delta v_g}{\Omega \mathscr{Re}^*} \sim \frac{1}{2}\left(\frac{c}{\Omega a}\right)^2 \frac{a}{\mathscr{Re}^*} \sim 7 \times 10^7 \text{ cm} . \tag{24}$$

The lifetime of the dust disk against orbital decay turns out to be

$$\tau_a \sim 2\alpha\left(\frac{\Omega a}{c}\right)^2 \frac{c}{(\nu\Omega)^{1/2}} \frac{1}{\Omega} \sim 1 \times 10^5 \text{ years} , \tag{25}$$

if the boundary layer flow is laminar, and

$$\tau_a \sim 4\mathscr{R}_e{}^*\alpha(\Omega a/c)^3\Omega^{-1} \sim 5 \times 10^3 \text{ years}, \tag{26}$$

in the more likely case that it is turbulent. From equations (15) and (26), it appears that the time available to initiate the next stage of the accretion process (following the condensation of small particles from the vapor phase) is about 10^3 years.

III. FRAGMENTATION AND COLLAPSE

a) *Stability*

Extensive calculations of the gravitational stability of rotating disks have been carried out in an effort to explain the spiral structure of galaxies (Toomre 1964; Goldreich and Lynden-Bell 1965*a*, *b*). We shall make use of the dispersion relation for local axisymmetric perturbations (i.e., for wavelengths $\lambda \ll a$) that reads

$$\omega^2 = k^2c^2 + \kappa^2 - 2\pi G\sigma k\,, \tag{27}$$

where $\kappa^2 = 2\Omega[\Omega + d(r\Omega)/dr]$ and c is the sound speed. The important features to note are that pressure stabilizes short disturbances and rotation stabilizes long ones. If the surface density σ is high enough, there is a range of intermediate wavelengths that are unstable. For a given c and κ, the critical value of σ above which the disk is unstable is given by

$$\sigma^* = \kappa c/\pi G\,. \tag{28}$$

This criterion is not rigorously applicable to the gaseous solar nebula since it is derived for a thin disk. Nevertheless, it does provide a good estimate of the surface density required for instability. If we substitute in equation (28) $\kappa = \Omega$, which is the appropriate expression for Keplerian motion, the critical gas surface density is

$$\sigma_g{}^* = 7.6 \times 10^3 T^{1/2} \text{ g cm}^{-2}\,. \tag{29}$$

Since the value of σ_g we obtain by augmenting the terrestrial planets up to solar composition is only 1.5×10^3 g cm^{-2}, equation (29) implies that the gaseous solar nebula is stable unless $T < 0.04°$ K. Actually, equation (29) gives an underestimate of the critical surface density for a gas disk of finite thickness by a factor of order 3. This is a consequence of basing the derivation of σ^* on a thin-disk model which overemphasizes the effects of the disk's self-gravitation for disturbances that are not much longer than its thickness. The low temperature required by equation (29) is the primary reason for rejecting theories of planetary formation that are based on the gravitational instability of the gaseous solar nebula.

We now proceed to apply the dispersion relation given by equation (27) to the stability of the dust disk that forms in the equatorial plane of the solar nebula.

The interpretation of c in this context requires some discussion. The use of a sound speed c to model the effect of the random kinetic energy of dust particles is not entirely justified because collisions between particles are inelastic. The time interval between collision, τ, is given by $\tau \sim l_p/c$, where l_p is the particle mean free path. The mean free path is $l_p \sim 4r\rho_p d/3\sigma_p$, where d is the thickness of the dust layer. The thickness is in turn a function of the dispersion velocity since it is just twice the height to which a typical particle can rise above the central plane. Thus,

$$d \sim c^2/2\pi G\sigma_p\,. \tag{30}$$

Note that in deriving equation (30) the disk's self-gravity has been used and not the vertical component of the Sun's gravity. This is because inside the disk, the gravitational acceleration due to the disk, $2\pi G\sigma_p z/d$, exceeds the vertical component of the solar gravitational acceleration, $GM_\odot z/a^3 = \Omega^2 z$, for disks which are cold enough to be unstable. If at each collision a fraction β of the impacting particles' kinetic energy is dissipated as heat, the velocity dispersion will damp on a time scale of order

$$\tau_{\text{damping}} \sim \frac{2c\rho_p r}{3\pi G\sigma_p{}^2\beta} \sim 2 \times 10^5(cr/\beta) \quad \text{s}, \tag{31}$$

where c and r are to be expressed in cgs units. From equation (28), it follows that $c < \pi G\sigma_p/\kappa$ in unstable disks. With our standard parameters, this implies $c < 8 \text{ cm s}^{-1}$ and hence, $\tau_{\text{damping}} < 1.6 \times 10^6 r/\beta$ seconds. Thus for most applications we can safely set $c = 0$. The principal exceptions arise in cases of collapse on time scales shorter than τ_{damping}.

In the absence of random motions ($c = 0$), the dust disk is unstable to all axisymmetric perturbations of wavelength shorter than the critical wavelength

$$\lambda_c = 4\pi^2 G\sigma_p/\Omega^2\,. \tag{32}$$

For uniformly rotating disks, the stability criterion for nonaxisymmetric perturbations is the same as that for the axisymmetric ones. The situation is more complicated for differentially rotating disks. The shear associated with the differential rotation converts an arbitrary nonaxisymmetric disturbance into an approximately axisymmetric one in a time of the order of a few rotation periods. Fortunately, as equation (27) shows, the growth time for perturbations having wavelengths shorter than λ_c is less than an orbital period. (Actually, eq. [27] only shows this for axisymmetric disturbances, but the same result also holds for more general perturbations.) Thus, for perturbations which are somewhat smaller than λ_c, we can forget the distinction between axisymmetric and more general perturbations and just use equation (27) to get an estimate of the exponential growth rate. The exact value of $\lambda < \lambda_c$, below which equation (27) may be used for nonaxisymmetric perturbations, is not well defined. It depends upon the magnitude of the initial perturbation which determines how fast growth into the nonlinear regime is achieved. We shall express all future results in terms of $\xi\lambda_c$, where ξ is on the order of, but less than, unity.

b) *Fragmentation*

The largest fragments that form when the unstable disk breaks up have masses of order $m \sim \sigma_p\xi^2\lambda_c{}^2$. Numerically, $\lambda_c \sim 5 \times 10^8$ cm and $m \sim 2 \times 10^{18}\,\xi^2$ g. Note that $\lambda_c \ll a$, so that our application of the dispersion relation given in equation (27) is justified.

Regions containing total masses as large as m cannot collapse unimpeded. As a fragment contracts, its gravitational binding energy, U', increases as $U' \sim U\lambda/\lambda'$. However, as a consequence of the conservation of internal angular momentum, its rotational energy E_R' increases as $E_R' \sim E_R(\lambda/\lambda')^2$. Furthermore, if the contraction time scale is shorter than the damping time for random motions, the random kinetic energy of the particles, T', increases as $T' \sim T(\lambda/\lambda')^2$. If the release of gravitational binding energy exceeds the demands of the rising rotational and internal kinetic energies, the excess energy will appear in the form of a bulk contraction velocity. On the other hand, if during the collapse the required rate of increase of rotational and random kinetic energies cannot be met by the release of gravitational potential energy, the contraction velocity will decay and reexpansion will ensue. In this latter case, the dimension of the fragment will eventually begin to oscillate about an equilibrium size that is determined by

$$\frac{d}{d\lambda'}(E_R' + T' - U') = 0\,, \tag{33}$$

or, in differentiated form, by

$$2E_R' + 2T' - U' = 0\,. \tag{34}$$

The condition for marginal stability ($\omega^2 = 0$) given by the dispersion relation (eq. [27]) is of this form. In terms of the wavelength λ, it yields

$$\lambda^2\Omega^2 + 4\pi^2c^2 - 4\pi^2G\sigma_p\lambda = 0 \tag{35}$$

for the marginally stable mode. We may identify each of the three terms in equation (34) with the corresponding term in equation (35).[1] Consider next an unstable linear perturbation of the dust disk (i.e., one for which $\omega^2 < 0$) of wavelength λ. In the absence of interactions with other contracting fragments, this perturbed region would initially collapse and then oscillate about an equilibrium size λ' given by

$$(\lambda^2\Omega^2 + 4\pi^2c^2)(\lambda/\lambda')^2 - 4\pi^2G\sigma_p\lambda(\lambda/\lambda') = 0 \tag{36}$$

or

$$\lambda' = \frac{\lambda^2\Omega^2 + 4\pi^2c^2}{4\pi^2G\sigma_p}\,. \tag{37}$$

As written, equations (36) and (37) do not include the effects of damping on the random motions of the particles. However, it seems likely that damping is always fast enough to make the random motions unimportant to the collapse dynamics. To appreciate this, note that the damping time as given in equation (31) depends inversely on σ_p^2 and thus decreases as $(\lambda'/\lambda)^4$ as the collapse proceeds. The characteristic time scale for collapse is never shorter than $\sim(\lambda'/G\sigma_p)^{1/2}$ and thus, decreases more slowly as $(\lambda'/\lambda)^{3/2}$. From the numerical estimate of the damping time given by equation (31), it appears that the damping of the random kinetic energy is sufficiently rapid to render pressure unimportant in the initial stages of the contraction. The preceding argument then implies that the effects of pressure remain small for all later stages of collapse.

Fragments collapse directly to form solid bodies provided that their equilibrium contraction corresponds to spatial densities at least as great as that of the solid material. This condition on the equilibrium contraction is approximately

$$\lambda/\lambda' \gtrsim (\pi\rho_p\lambda/6\sigma_p)^{1/3}\,. \tag{38}$$

If equation (37) is substituted into the above inequality (and the initial random kinetic energy is set to zero), we obtain

$$\lambda^4 \lesssim \frac{384\pi^5G^3\sigma_p^4}{\Omega^6\rho_p} = 384\pi^5\left(\frac{\Omega^2}{G\rho_p}\right)\left(\frac{G\sigma_p}{\Omega^2}\right)^4\,, \tag{39}$$

$$\lambda \lesssim 5 \times 10^6\ \text{cm}\,. \tag{40}$$

[1] The model of a contracting region as a rotating subdisk is imperfect for two reasons: (1) particles in a disk with differential rotation do not execute circular motions when observed in a corotating inertial frame centered on the contracting region; and (2) although tidal forces are important in determining λ_c, they are not important in later stages of collapse and our λ^{-2} scaling law does not take this into account. However, since we are interested only in establishing the size of the planetesimals to within an order of magnitude, these deficiencies do not alter our results.

The mass and diameter of the solid bodies that form from fragments of initial size λ are $m \lesssim 2 \times 10^{14}$ g and $\lambda' \lesssim 0.5$ km.

c) *Further Growth*

We have shown how dust particles coalesce into planetesimals with radii of a few tenths of a kilometer. Because $\lambda_c/\lambda \sim 10^2$, these first-generation planetesimals will be grouped into rotating disklike associations containing $\sim 10^4\xi^2$ members. Although each cluster is stable against collapse on the gravitational free-fall time scale, it does contract slowly as gas drag reduces its internal rotational and random kinetic energy. Since the Reynolds number for the gas flow about bodies the size of the planetesimals is large, the drag force arises from the formation of a turbulent wake and is

$$\boldsymbol{F}_D = C_D \pi r^2 \rho_g |\boldsymbol{v}_g - \boldsymbol{v}_p| (\boldsymbol{v}_g - \boldsymbol{v}_p) , \tag{41}$$

where C_D is the dimensionless drag coefficient typically on the order of a few tenths. The contraction time for the associations is then given by

$$\tau \sim \frac{4}{3C_D} \left(\frac{\rho_p}{\rho_g}\right) \left(\frac{a\Omega}{c}\right) \frac{r}{c} \sim 1.3 \times 10^2 r_4/C_D \text{ years} , \tag{42}$$

where r_4 is the radius in units of tenths of kilometers. It is easily shown that the cluster contraction time is comparable to the relaxation time due to binary encounters between planetesimals. Thus, the contraction of a cluster should proceed without a significant loss of members by evaporation. The masses and radii of the second generation of planetesimals range up to

$$m \sim \lambda_c^2 \xi^2 \sigma_p \sim 2 \times 10^{18} \xi^2 \text{ g} \tag{43}$$

and

$$r \sim 5\xi^{2/3} \text{ km} . \tag{44}$$

Beyond this stage it appears unlikely that further growth proceeds by means of collective gravitational instabilities such as we have been describing. The frictional effect of gas drag does destabilize axisymmetric perturbations for wavelengths larger than λ_c. We cannot be certain that the axisymmetric perturbations by themselves are not a significant feature in producing a limited further growth of planetesimals. However, it is possible to prove that they are too slow to be responsible for the accumulation of material over interplanetary distances.

Direct particle-particle collisions are probably the dominant accretion process following the formation of the second-generation planetesimals. It seems plausible to expect near encounters between planetesimals to build up a dispersion velocity on the order of the escape velocity from the surface of a typical object $v_e = (2GM/r)^{1/2}$. The corresponding disk thickness is $d \sim v_e/\Omega$. The growth rate produced by direct impacts is simply

$$dr/dt \sim \Omega\sigma_p/\rho_p \sim 15 \text{ cm year}^{-1} . \tag{45}$$

d) *Survival Time*

As long as the gaseous solar nebula is present, gas drag will produce a slow inward drift of the planetesimals toward the Sun. It is essential that at each stage of accretion, the particle growth time be short compared to the orbital lifetime set by gas drag. We have already verified that this condition is well satisfied through the formation of the second generation of planetesimals. Past this stage, the orbital lifetime, as determined from equations (9) and (41), becomes

$$\tau \sim \frac{8}{3C_D}\left(\frac{\rho_p}{\rho_g}\right)\left(\frac{a\Omega}{c}\right)^3 \frac{r}{c} \sim 6 \times 10^5\, r_5/C_D \text{ years}, \tag{46}$$

where the particle radius r_5 is expressed in units of kilometers. Comparison of equations (45) and (46) reveals that at each stage of accretion the survival time exceeds the growth time by at least two orders of magnitude.

IV. DISCUSSION

There are two related uncertainties which plague any attempt to present a detailed account of the early stages of planetary accretion. The first of these, to which we have previously referred, is the question of the number of nucleation sites that form during the condensation of minerals from the gaseous phase. If the number of independent nucleation sites is so small that the vapor phase is not seriously depleted in the time it takes the particles to gravitationally settle into the equatorial plane, the resulting particles have radii several centimeters in size and the settling time is about 10 years. On the other hand, if the number of nucleation sites is so large that the vapor phase is exhausted before appreciable settling takes place, the resulting dust particles may be very much smaller and the corresponding settling time much longer. A related question is the relative rate of cooling of the preplanetary nebula to the settling time of the dust particles. If the cooling time is short compared to the settling time, the chemical composition of the dust disk reflects typical cosmic abundances. However, if the cooling time is longer than the settling time, chemical fractionation occurs at the earliest stage of planetary accretion.

Of the two problems discussed above, the question of the number of nucleation sites is the more fundamental. Because it directly bears on the size and hence the settling time of the dust particles, it sets the time scale to which the cooling time must be compared. It also indirectly affects the cooling time itself, since the opacity of the nebula depends upon the number and size distribution of the dust grains.

Another problem worth mentioning is that gravitational instabilities will begin to grow in the dust disk as soon as its vertical thickness is less than λ_c. Thus, our discussion in terms of the dispersion relation for a thin disk is an oversimplification of the true situation. Fortunately, the time scale for growth of these instabilities is identical to that for the thinning of the disk. The major new feature introduced by the finite thickness of the disk is just a decrease in the growth rate of the first-generation planetesimals from $2\pi(G\sigma_p/\lambda)^{1/2}$ to $2\pi(G\sigma_p/\lambda_c)^{1/2}$. The calculated masses of the objects are not affected by the finite thickness of the disk.

The main contribution of this investigation is the demonstration that sizable planetesimals can accrete directly from dust grains by means of gravitational instabilities. Thus, the fate of planetary accretion no longer appears to hinge on the stickiness of the surfaces of dust particles. Although we have dismissed the sticking of dust grains as unnecessary to the planetary accretion process, there is a more fundamental reason for disregarding it altogether. That is, even if the dust grains tended to stick together upon impact, the growth of solid bodies by this process would be much slower than by the gravitational instabilities we have described.

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Note added in proof.—Recently, a Russian to English translation of a book entitled *Evolution of the Protoplanetary Cloud and Formation of Earth and the Planets* has appeared, which contains a discussion of the fragmentation of a primordial dust layer which is in some ways similar to that presented here. Safronov credits Gurevich and Lebedinskii (1950) as being the first to obtain the critical wavelength and mass.