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Operations and Algebraic Thinking for 2nd graders

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Educational Goals, Instruction & Assessment

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■ Educational Design Focus

- Learning challenges & domains

I noted learning challenges in math education: students are often not interested and have difficulties understanding the materials.

They sometimes lack confidence regarding their abilities in mathematics and reject math education considering themselves as not "math people".

These difficulties in understanding and confidence may come from early bad experiences in math (see *Why a Colorado Researcher Believes Preschoolers Should Learn--and Play with--Math, July 14, 2017, Chalkbeat).* Some students have trouble understanding numbers and operations, they suffer from this difficulty when building on new math content. They would often keep struggling during later studies.

Considering the importance of basic mathematics abilities for one's daily life, and the impact that may have early mathematics experiences, I therefore decided to design a math course for second graders and address core math competences such as counting and basic operations.

- Educational standards & project emphasis

I would like to target the learning goals of counting & algebraic thinking (see CCSSI Math standards).

I think that this is the basic knowledge to feel comfortable with Math in one's future studies, and it is possible to find real-world connections to students' life, to foster motivation and interests toward learning.

The project being a 10-hour unit project only, its focus was narrowed down to counting/basic algebra.

The targeted school level is 2nd grade. The standards for second graders from CCSSI Math Standards are (in the algebraic thinking / counting domain):

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Based on these standards, I would like to focus on the three following components as they seem a strong basis for Math skills and understanding:

Operations and Algebraic Thinking

- **Represent and solve problems involving addition and subtraction.**
- **Understand and apply properties of operations and the relationship between addition and subtraction.**
- **Work with equal groups of objects to gain foundations for multiplication.**

- Learners' grade level & knowledge, skills, dispositions

I already participated in a math online learning project for 7th graders in France, so I would like to see how to teach to other educational levels and focus particularly on Math 2nd graders, because this is the time when the algebraic and operational thinking is built.

The skills central to what I plan to teach are: understanding the place value of numbers, how to add & subtract little numbers, even understanding the basis of multiplication.

Type of students:

2nd graders

No disabilities or mental disorders

Community:

An American school. The demographics of the class will be 100% average SES.

The cultural background of the class and school are mostly American-like with 30% of other close occidental cultural background (latino etc.) still involving the use of common daily objects such as watches, matches, roman numbers…

Parents do not prevent the children from learning math (let's say parents do not have a negative image on math and value work and homework). However, those parents do not have PhD in Math or are math teachers, so they just give common sense and basic help for the students for their homework.

Where:

The school is in Pittsburgh, in an average SES neighborhood.

I will teach them in class. There will be 20 students in the class.

When/Relation to other learning objectives:

The specific learning objectives I chose usually come at the beginning of the year as stated in the standards because they are kind of basic knowledge.

I think it would be worth taught during the entire school year, mixing with other subjects (geometry, etc.). This is Spiraling through the year, refresh the skills and take a step up to the next level.

I will therefore have an introduction and a continuous curriculum with short activities each week (maybe 45min) until I get to the 10h. This process will provide me time to adjust the class objectives to students' progress during the year.

- Importance of the subject: Real-world connection

It is important for children to learn those basis as it builds their conception of Math and their relationships with it.

What they will be able to DO with this knowledge in their own life:

Time notions:

- Learn time reading: reading time on digital clocks or watches.
- Add and subtract days, minutes, or seconds to wait for an upcoming event, or know how long to wait for other students / parents / break time between classes.
- Reading calendars, agendas, monthly programs etc.
- https://dreme.stanford.edu/news/reimagine-calendar-activities-early-childhood-classrooms

Counting money and objects, adding / subtracting, sense of intrinsic value

- Count their money with bills and coins adding up to a few dollars.
- Add and subtract money to pay something or to give money to another student.
- Count objects at home: if they need to collect 15 nuts for cooking.
- Learn that money is not about the number of bills or coins you have but about the sum of their value.

Cooking / Measuring

- Cooking is full of counting examples: for a cake, counting the ml / oz / lb, etc.
- Adding or subtracting or even multiplying the recipe quantities by the number of people eating.
- Make use of appropriate tools: Measuring a distance in their garden, the length of a paper sheet for drawing, the size of an object, etc.

Hobbies, outdoor activities

- When reading a book: counting the pages in the book and adding some pages like when they need to go 10 pages forward for example.
- Outdoor activities and sport: they need to count for sport performances as they start practicing team sports: how many goals / points in a basketball / football / soccer game, how many people in each team, why is the team with more players advantaged over the other one, etc.

Temperature

- Know that numbers can have a concrete meaning in temperatures for example
- Associate temperature differences and changes to numbers
- Know that there are different temperature scales

Numbers in real life:

- Ask children where they see numbers in their real life
- When they need to add or subtract numbers in life

The main real-world concepts I will include in my educational design are:

Cooking

Time measurements

Counting money and objects

Numbers in real life

Many people give up Math as they are afraid of it, I wonder if it is caused by unmotivating and harsh starts with the discipline. If they have these appropriate basic knowledge and skills, they may feel like math is intuitive and easy.

The standard practices in math for 2nd grade are:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

I chose to focus on the practices most aligned with the learning standards I targeted above. These learning standards also help target the dispositional abilities I mentioned above that are helpful for students in their real-world environment:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
8. **Look for and express regularity in repeated reasoning.**

These practices are real-world connected because they structure how to use basic math in logical thinking and the use of tools to explicit the strategies employed. (see above examples). The contents on themselves are important for everyday life, and unfortunately many people still live without these basic skills.
The standards and their connections to students' reality:

• Represent and solve problems involving addition and subtraction:
Solving common counting problems: adding up task durations to know the global duration when planning an activity, additions and subtractions to follow cooking recipes, etc. These counting tasks appear in most of the areas where numbers are found. Counting should become intuitive and logical to provide a strong and natural base for building on more difficult math content afterward.

• Understand and apply properties of operations and the relationship between addition and subtraction:
Students should know the properties of operations: there are many ways to add numbers and find the same result (when counting money to add up to a certain amount); the importance of base 10 in additions and subtractions of roman digits (base 10 in the prices: 1$ is 100 pennies and 10 dimes). This is about learning how to reason with abstract properties and to apply properties and relations to mental objects. It is also very important for more advanced math skills.

• Work with equal groups of objects to gain foundations for multiplication:
Multiplication is a core concept in Math and is a basis for many math real-world applications (all types of data conversions in time, money, weights, etc.). How children internalize multiplication is important.

- Continuity between my goals and the learners' goals: intrinsic motivation

Students are likely to want gamified instruction and real-world related examples to catch their attention and keep motivated.

Students' goals & intrinsic motivation are also fostered by games with peers & partner work, socialization, or interactive activities.

They like hands-on, discovering something new and playing even for 5 min.

As a math research subject, I am interested in understanding whether the use of 3D shapes can be efficient to learn math basics and gain math sensitivity for children. These activities using 3D shapes can be beneficial for counting (like the base-10 objects adding up and subtracting to materialize the operations).

I therefore include in the course the use of 3D objects to model some properties of operations (base-10 with the ten frames used for addition and subtraction understanding), in gamified, contextualized, group tasks.

Students should have already approached the following knowledge from grade 1:

Operations and Algebraic Thinking
- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten
- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract

- Learner Characteristic as a Baseline Profile:

General guidelines:

Design for the lowest case, i.e. taking into consideration the lowest level of development possible. This could happen for the youngest children or children with late development or small mental disabilities.

Children of this age can get bored fast if the activity is too easy or too repetitive. The design of instructional activities must take it into account.

Use one main activity for every student so that they can help one another and be sure to provide a full range of difficulties in the same activity to target novices as well as advanced learners.

Always remember to foster children's interest in the tasks to have them focus on class.

Developmental Level: 6 -7 years old Cognitive changes, form fantasy to logic and reason.	Prior Experience: First math prior knowledge is variable among students, can be very different. Must integrate lessons from the	Individual Differences: Some will have already seen counting at home and have good idea of numbers and operations, others no.

Still need consistent direction from adults / structured tasks. Nevertheless invest more and more time in doing things in the expected manner. Latter phases of Piaget's preoperational period, when learn language. Start logical concepts in limited arenas and still controlled by perceptions in others. Conservation of numbers is understood at that age, but still difficult to conserve mass. At 7: ability to manipulate symbolic elements, begin to take another's person point of view. At 7 enter the operational period (until 12), facility with mental operations, logic (remain limited to actual physical objects). Ability to distinguish fantasy from reality, similarities btw objects, creative thinking to problem solving. Increased memory, attention span, greater impulse control. Children's metacognition starts at that age, know their needs to do better, learn or understand. They understand the essence of a problem, immutable nature of objects. Physical development: should be able to count on fingers	beginning, that is the interesting point. Already know how to count to 20/30, and how to add some numbers already. We want to go beyond to add/subtract/multiply, and confirm this number notions They can already recognize some errors in counting (own counting or from other people) They have already seen real world application of math This gives them prior idea on what math is and its utility (numbers/operations?) Social norms here can strongly impact learning! I assume that living in the USA they all would have seen watches / calendars Maybe could be interesting to understand how they count at home if from different cultures? Like calendars? Or higher numbers names? (is it 20 = two-ten like in Chinese or twenty (a new word))	Some will be interested in math and others no. Some will like counting, adding, subtracting, multiplying, others no. Some will think counting is intuitive. Some will be motivated by using 3D spatial reasoning. Others may not be motivated by the gamification part. Temperament may be a hard point, to have them concentrate on the task. Individual difference in the approach of math: visually or orally oriented? Do they tend to see 3D shapes and play with them or could they learn how to count through musical rhythm, adding sounds or sentences, using language, etc? Multiple activities, using body, visual etc, refining

According to: 0 BuildingBlocksTrajectoriesWithRef.pdf

- **Big Ideas Justification**

Throughout the educational design and particularly for assessment, instruction and evaluation research, the mentioned processes are justified by Big Ideas related to educational design (using the *BIJ* acronym when using Big Ideas for justification).

These Big Ideas are listed in another project conducted in group, for which here is a link: https://docs.google.com/document/d/1p1cZpKz1PKRc9Tgzz9yVl5pTX473mXq6GkFLryrmLmI/edit

Main Big Ideas referred in the document and their relations to this course:

The guiding principles
Alignment (goal, assessment, instruction, evaluation):

* Aligning the tasks to the development level and goals of students.

* Alignment between the goals' standards in math for 2nd graders and the assessment and instruction tasks.

How humans learn
- Prior knowledge:

* Assessing students' prior knowledge in the knowledge components identified.

* Correcting this prior knowledge if necessary.

* Activating it to build on new knowledge.

- Sense of belonging:

* Impose class norms and foster student integration in the class group to improve motivation.

- Motivation:

* Use contextualized tasks to foster students' motivation.

* Ground the tasks in real-world applications

* Include group tasks and interactivity.

- Metacognition:

* Provide students with feedback and opportunities to rethink content.

Ch3 How to implement educational design

Learners in context:

- Create natural critical learning environment

*Real world tasks

Design Goal

- We want our students to be: High level thinkers

* Clarify the goals and provide students with structured classes

- We want our education to achieve: Transfer of learning

* Students should work with analogies and review the content multiple times

Assessment

- Valid and reliable assessment

*Provide informal checks of understanding in contextualized tasks

- Self-evaluation

* Include informal self-assessment opportunities in group activities

Instruction

-Assessment as instructional technique

* Use the recipe design or other performance tasks as instruction

-Active learning

* Use concrete group tasks and foster manual interactions with the content

- **Goal Specification:**

To design the goals for Operations and Algebraic Thinking, I considered the 2nd grade standards in Pennsylvania, USA:

https://www.pakeys.org/wp-content/uploads/2017/11/2014-Pennsylvania-Learning-Standards-for-Early-Childhood-PreKindergarten.pdf

Here are the http://exdev.pdesas.org/Standard/StandardsDownloads

General goals for 2nd grade that I have selected for the scope of this individual project:

Operations and Algebraic Thinking:

I Represent and solve problems involving addition and subtraction.
II Understand and apply properties of operations and the relationship between addition and subtraction.
III Work with equal groups of objects to gain foundations for multiplication.

Insights from the interview with Holly, kindergarten teacher:

To explain addition with 2 / 3 digit-numbers, use calendars and ten frames to represent place-value etc. You can use ten frames to make cubes stacks

Emphasize the base 10: patterns in calendar (some kids say twenty-nine, then twenty-ten etc.)

Use visual hints and tools: Ten frames work well to recognize Ten and One because they are visual.

Use body language to make activities more interactive for children. Have students represent math concepts with their body in action: when comparing numbers, have them the crocodile mouth shape > or < to eat the bigger element.

According to Holly, data & conversion are mostly used as examples, good to know concepts, but not used as part of the core goals!

When it comes to making sense of units and conversion (like 60 s = 1 min, etc.). They should know the concept of conversion and some examples, but we usually don't use data or conversions to learn other things.

How to deal with diversity levels … There is always a baseline, and then it's nice to create partnered work with different levels: Good mix for peer interactions, motivation, help for all students.

How to transfer understanding to greater numbers? Use again ten frames

One of my goals was to study the impact of spatial reasoning for children on math abilities, they use shapes but according to Holly, they would be quite interested by a digital version, with interactive 3D and motivating activities.

Advice: Use Pinterest to find pictures of concepts and creative ideas and tools teachers use for instruction and assessment

Assessment Evidence: Aligning assessment to goals

Formative assessments:

- Ask questions during the activities. Include informal assessments such as exit tickets, question of the day with the answer at the beginning of one class.
- Reflect on what students learned, include an informal assessment routine.
- Prefer informal assessment to formal assessment for children of this age.

Summative assessments: Pre / mid / post / delayed?

- Include informal summative assessment in late classes: Help children retrieve what they learn and the structure of their understanding.
- Use formal assessments when conducting evaluation research.

Integrating feedback:

- The type of feedback (in group/individual, narrow/global, focus on errors/achievements/progress, etc.) depends on the group's ages and sensitivity: We will prefer group feedback, narrow and global depending on the goal, and focusing on progress and achievements.
- General feedback is aligned with group tasks and skills/disposition goals.
- Math concepts/knowledge are mostly specific. Thus, specific feedback is needed to foster deep understanding, this feedback will be given individually in appropriate times. Students should not feel much pressure, so this feedback should also focus on their progress.
- Balance negative and positive feedback with progress. Give links to other real-world applications to foster transfer of learning.

Important take-away: Metacognitive goals are aligned with 7 year old children, but we cannot be sure that all students will be developed enough to be able to think metacognitively

Conceptual Knowledge: 1	**Procedural Skills: 2**	**Dispositions: 3**
Students will:	Students will be able to:	Students will be:

1.1 Identify numbers from 1 to 1000 **1.1.1 Recognize numbers through / Use common forms of numerical representation (fingers, dots, 3D shapes, etc.)** - 5 fingers, dots on a dice, Cubes, pyramids, pennies, quarters, different weights (between 1g to 1kg), prices, distances on a ruler, etc. - 1 to 10 on fingers - 6 per dices but you can use more dices - Until hundreds with Calendars - Use ten frames to represent greater numbers **1.1.2 Know the vocabulary of numbers** - A whole number - Know the pattern of the words: - Digits - Ten multiples and teens - Use calendars to recognize the pattern tens/units - A two digits number is written as the tens digit followed by the unit digit. - Emphasizing base 10: patterns in calendar (some kids say twenty-nine, then twenty-ten etc.) **1.2 Identify comparison relations between numbers:** **1.2.1 Know the concept of place-value** - Know the names of places: ones, tens, hundreds, thousands - Use ten frames to recognize place value with visual tools. - I know that a bar of cubes are ten cubes. And a grid of bars are 100 cubes. - 142 has 1 hundreds, 4 tens and 2 ones - Definition of place-value: let's take a number: 142, you can make additions of numbers to get this number, for example 100 + 40 + 2. The first digits of these numbers are the same as the digits used for 142,	**2.1 read, write and skip count to 1000** - I read the numbers in letters (1 = one, etc.) I can write any numbers btw 1 and 1000 thanks to the rules I know and practice - write the numbers: in order to do that, learn how to write the digits as numbers, then write the digits corresponding to the 'ones'/'tens'/'hundreds' in the right order to form the new number. I can skip count 2 by 2 when practicing on a calendar: I detect the different colors of the numbers spaced 2 by 2 and I say them. I can skip count 2 by 2 without visual help. I can think about the same numbers spaced 2 by 2, adding 2 each time, and say them without visual help. - to skip count 2 by 2, imagine the number line: when you count regularly it is counting 1 by 1 because you add 1 to each number you just said. Now just add 2 to the number you previously said. - Use calendars with numbers ordered with place-value and keep track of the date, - Count the number of school days, keep track of that - rows of diff colors, visual hints -pocket charts	**3.1 I am concentrated when counting** **3.2 I am attentive to peers when they talk during the class, attentive to teachers** **3.3 I am interactive when a game for numerical play is happening** **3.4 I am respectful of others (teacher and peers) and their points of view when evaluating numerical expressions, playing numerical games, etc.**

because we write 142 = 1x100 + 4x10 + 2x1, so we use multiples of 1, 10, 100 to write the numbers. Here the 2 is the digit for "ones" (first digit), the 4 is the digit for "tens" (second digit), the 1 is the digit for "hundreds" (third digit). The place value is the name of when you place digits in an order like 142 to form a new number. **1.2.2 Know the concept of mental line, greater than, less than, or equality** - The mental number line is a line going toward the right, and you can place number on the line and the greater the number, the righter it is placed on the line. - The greater the numbers, the righters they fall on the line. - Signs <, >, = - Greater than: a number is greater than another number when there are more objects when we gather the corresponding number of objects. - A number is greater than another one if you count it or measure it, the number is higher - Less than: same explanation , shorter, lighter, fewer objects - Equality: same number of objects when we gather the corresponding number of objects or measure it. - -> less/greater, piles, equal sign, - Have students represent math concepts with their body in action: when comparing numbers, have them the crocodile mouth shape > or < to eat the bigger element. **1.2.3 Know the mathematical vocabulary of comparison** - greater than, less than, more, plus, less, addition, subtraction **1.3 Identify basic mathematical operation and their properties** **1.3.1 identify addition**	**2.2 apply addition and subtraction to numbers up to 1000.** - say you know what addition is, you want to add 32 and 59. You first add the ones digits: 2 and 9, it is 11. If the result is greater than 10, you keep the ones digit of your result to make the final digit: here 0. Then as the sum is greater than 10 you add 1 to the final 'tens' digit, and you repeat the step for the tens: 3 + 5 = 8 + 1 you just added, so 9 and 0 = 90. - same for subtraction but this time subtracting each time and if you cannot subtract one digit to another (8 to 3 for example), you create a 1 in front of the 8 so you subtract 3 to 18 which gives 15 so you write the 5 in the same place (same column) in the result, and you add the 1 to the digit on the left of the column you were working on, then you repeat the step with a shift to this column on the left. **2.2.1 Use mental strategies to add and subtract within 20.** - try to count in your head, saying: 5 + 3, it is 5 +3 or 6 +2 or 7 +1 or 8 + 0 so 8. - build upon experience /strategies I know how to add numbers in my head, because I especially know the one digits addition - same for subtraction in the opposite way. - depends kids, finger tricks, some need paper etc. **2.2.2 use place value understanding properties to add and subtract within 1000.** - same as above.	

- when you add objects to a group of objects to have a greater number of objects, you add objects, so it is an addition. If you have 5 matches, you want to have 8, so you add 3 objects to your group, this is an addition, you finally have 8 matches, so you note 5 + 3 = 8. - When I add two numbers, the result is the number I count or measure when I group the first two numbers of objects. - Adding 2 and 5 is the same than adding 5 and 2, it is symmetric. - The quantity in addition is not changing depending on how you add numbers: 10 two faces dices to add, the sum is always 10! - Addition is written with a "+" **1.3.2 identify subtraction** - same explanation as addition - subtracting 2 to 5 is possible but not subtracting 5 to 2 because 5 is greater than 2. You can remove 2 matches from a group of 5 matches but you can not remove 5 matches from a group of 2. - Written with a "-" **1.3.3 identify multiplication** - when you group matches per 3 and you make 4 groups, you can calculate the number of total matches by doing 3 + 3 + 3 + 3 or 4 times 3, it is a multiplication and it gives 12 **1.4 Know real-world examples of operations and data** **1.4.1 Know the relations between years, months, days, hours, minutes, seconds, etc.** - Use the data & conversions as examples, good to know, but not as the main basis for learning. - A year is 365 days, and 12 months, a month is 30 or 31 days (list of month days), a day is 24 hours, a	I use the columns to represent place value and hundreds, tens and units to perform an addition. 2.3 use real world objects to understand the values of numbers and operations - use groups of objects (matches, coins, pens, LEGO…) and count the numbers of objects in each group, then ask yourself which group has more objects, and how many objects you must add to the smaller to get as many objects as in the bigger. I can tell which group of objects has more elements I can add the exact number of missing elements to the group to get the adequate number of elements. **2.3.1 separate and join sets of objects to represent addition and subtraction** - same idea as just above. **2.3.2 Work with equal groups of objects to gain foundations for multiplication.** - make groups of objects with the same number of objects in each group, say 6 objects in each group. Then count how many groups of objects you have (say 3). You want to know the total number of objects. You make a multiplication of 6 by 3 so you add 3 times the number 6 so you do 6+6+… 3 times. Do the same for other numbers. - if **I** have groups constituted of the same number of objects, I can find the total number of objects performing one digit multiplication.	

week is 7 days, an hour is 60 minutes, a minute is 60 seconds. - Mostly minutes according to Holly: - A hour is 60 minutes, A min is 60s	**2.3.3 read time on digital clocks, compare amounts of time, apply operations to data in real-world stories about numbers (distances, weight, time, temperature, etc.)** - read the time on this digital clock. You must specify if it is the morning or afternoon. What is the longer amount of time? 3 minutes or 160 seconds? 2 hours or 110 minutes? 20 days or 3 weeks? I can read time on a digital clock. - create similar conversions raising interest and relating to daily tasks, like is the lunch break longer than one class? **2.3.4 convert basic units** - what are 12 quarters in dollar? What are 14 dimes or 562 pennies? I can use coins and dollar bills to reach a given amount. **2.4 communicate with think aloud while engaged in numerical thinking** - when counting 3 by 3 from 0 to 30, try to speak all the thoughts that comes into mind, if you imagine the numbers in between the numbers you count, say them to me. Describe what you think at the same time you think it. - if I ask you which number is the greatest among 42, 61, and 67, tell me what are your thoughts when you decide the answer: you could tell me: “ok so which one is greater, we must first compare the digit on the left, so here 4 < 6 so the biggest is 61 or 67, and then we must compare the digits going to the left, so here 7 > 1 so the biggest is 67”.	

Meta-Level Conceptual: 4	Meta-Level Procedural: 5	Meta-Level Dispositions: 6
4.1 I evaluate my own understanding of the numerical concepts (auto-assessment) (ex: I prefer additions because …) **4.2 I evaluate my lack of knowledge to fulfill a goal.** **4.3 I can say if I understood well the basic measurement units** **4.4 When I identify a lack of knowledge, I ask a question to a teacher or a peer** **4.5 When I have trouble mentally representing a concept (a number, a number comparison, etc.), I think about the same numbers of objects, how to build this number with ten frames, or all the physical and visual representations of the number comparison.** **4.6 I can say what I did good during a task to reach a goal.** **4.7 I know the upper limit of the numbers I can count until**	**5.1 I can evaluate if I performed a valid and logic operation** **5.2 I can design a plan to evaluate the skills I applied** **5.3 I evaluate the correctness and meaning of the applications' results.** **5.4 When I read, write and skip count to 1000, I can explain the procedure and evaluate my performing it.** **5.5 If I need a step or a hint to apply the skills, I realize it and I seek help.** **5.6 When I count mentally, I can think aloud if asked for it and explain my thinking.** **5.7 I can tell if I followed the method for addition, subtraction and multiplication.** **5.8 I can seek help and offer help if I recognize others' needs** **5.9 After listening to peers, I can tell what they said, the effects of their learning and peers reaction.**	**6.1 I evaluate my lack of attention to others.** **6.2 When counting, I can regulate my own concentration.** **6.3 When listening to peers, I can identify their level of attention, evaluate it and interact in response** **6.4 When I see other children struggling, I ask if they need help.** **6.5 When a numerical play is happening, I can evaluate my degree of interactivity with others and with the material, monitor it towards the disposition goal** **6.6 After a numerical play, I can recall the degree of interactivity and identifies the effects of my approach on the task.** **6.7 I can self-evaluate my progress on dispositions goals and how it affected my learning in general**

■ Assessment Design

- Overview of Assessment Evidence

General timeline, justifications for the assessment timeline:

- At the beginning of each lesson, informal assessment of the class's understanding of the previous lesson, say asking a question like: "skip count 3 by 3 to 20" and according to the feedback guidelines: Provide immediate feedback correcting students and integrate the students' level of understanding in class's planning. (formative assessment)

    - *BIJ: Assessment (one-minute essay) – Assessment as an instructional technique*

        - Holly: We use many informal assessments at the beginning of each class to understand if children understood the lesson of the day before, and if not, it influences the lesson of the day.

- After a group task, give a feedback focusing on the achievements and the progress needed. Do not target individual students but rather address the feedback to the group. (Formative assessment)

    - *BIJ: Self-evaluation – Sense of Belonging – High level Thinkers*
        - Holly: The feedback is better **given in groups** because it less targets one student in particular

- Summative assessment split in multiple assessments: 3 times throughout the course, in groups with feedback insisting on progress.

    Holly: Children do not like negative feedback, so the feedback should focus on **positive outcomes**. It should target a **need of progress.**

    - *BIJ: Metacognition – Valid and reliable assessment*

Specific assessment tasks, formative/summative:

Performance Tasks :	Other Evidence
1.Performance Tasks: Formative assessment to understand what they really know and how they make connections between tasks One task that integrates across goals to accomplish a real-world task as a formative assessment: Individual (basic task) 1.1 Pick 5 real-life objects you really know well. Then describe them using numerical vocabulary, with as many details as you can. Cognitive level goals: 1.1-Identify numbers from 1 to 1000 1.1.2-know the vocabulary of numbers 2.1 – 2.2 – 2.3 3.1 – 3.4 Meta-cognitive level goals: 4.1 – 4.2 – 4.4 – 4.5 – 4.6 5.1 – 5.2 – 5.3 - 5.4 – 5.5 – 5.6 6.2 - 6.7 Teacher's action: - Monitor the choice of objects, and the vocabulary used to describe the objects. Ask questions about the reasons behind those numbers (why a dice has 6 sides for example…), reality of numbers, comparing the numbers in objects, how children describe the objects using numbers. Personalize the feedback and assessment, going further if the kid shows	**3. Other Evidence: Formative assessment** Preferably emphasize this in the beginning of the year to know what their prior level is and note their performance to establish progress! 3.1 Questions during class and during activities, about the Knowledge, and ask for the children to perform the activities and show you their procedural skills: 1.Conceptual Knowledge 2.Procedural Skills Questions like: "Do you know what are the 3 columns names for place-value?" "Can you skip count 2 by 2 from 10 up to 30?" 3.+4. C / MC dispositions Ex: How do you react when your friend needs help? If he asks you for help but you already helped him 10 times? 3.2 Observations during in class activities to test dispositions in context specifically: MC Dispositions 4. Ex: observe the children monitoring their seeking help or offering help in reaction to peers' reactions; Dispositions 3. Ex: observe the behavior of children when listening to others to attest concentration and attention

good understanding of numbers (like, could you sum up those numbers? Etc.) - and record children's actions Feedback given: Focus on progress and sense-making, explanations and real-world examples	3.3 Question of the day each morning to the class. This is used to reflect on what they learnt from the previous class content. It is a morning routine: 1 Conceptual K 2. Procedural Skills
Individual (more advanced task with data and measurement): 1.2 Create your own cooking recipe for one person, listing the ingredients and the weights, and steps. Add a price to each item to purchase as in a real shopping list, and sum the prices to get the final price. (And why not cook them at the end haha) Cognitive level goals: 1.1-Identify numbers from 1 to 1000 1.1.2-know the vocabulary of numbers 1.3 - 1.4 2.1 – 2.2 – 2.3 3.1 - 3.3 – 3.4 Meta-cognitive level goals: 4.1 – 4.2 – 4.3 - 4.4 – 4.6 5.1 – 5.2 – 5.3 - 5.4 – 5.5 – 5.7 – 5.8 – 5.9 6.2 - 6.3 – 6.4 – 6.6 - 6.7 Teacher's action: - Monitor the choice of recipe: length, real-world "ingredients" … ask questions about the weights of the ingredients, if it is realistic, etc. Ensure the steps and prices are coherent. Give feedback on the prices' addition when doing the task.	**4. Other Evidence: Summative informal assessments** 4.1 Think-aloud when practicing tasks that need strategy and planning: Conceptual K Procedural Skills MC Conceptual MC Procedural Can you think aloud while adding 45 and 86 through paper& pencil AND through mental additions? 4.2 In group summative assessments when doing activities: trying to get what they learnt from the class: Conceptual K Procedural Skills MC Conceptual MC Procedural example: asking to children to skip count 3 by 3 to 20 in a group while doing counting activities. Or: Now you know how to skip count 3 by 3, what do you need to skip count 5 by 5?

<table>
<tr>
<td>- Students should fill a predefined grid used as a scaffold, with different sections (see Instruction, below)

-Record children's actions to monitor progress toward goals, about dispositional goals. (the skills and knowledge are recorded on the scoring grid)

Feedback given: Focused on progress toward counting, making sense of units and explanations for ingredients and weights.

2.Performance Tasks: Summative informal assessments:

In group, global task that integrates data, measurement, and additions, and multiplications. To be used by the end of the course

2.1 Draw multiple situations of you daily life representing all the numbers that appear in the situation (the time with a clock, the number of objects, the age of the characters, etc.) and draw them in the environment. Explain the examples of numbers you give and use additions to sum up all the numbers you found. Explain your drawing to your peers, ask questions about your peer's work.

Cognitive level goals:

1.1-Identify numbers from 1 to 1000

1.2-know the vocabulary of numbers

1.4-know real-world examples of operations and data

2.1 – 2.2 – 2.4

3.3 – 3.4

Meta-cognitive level goals:

4.1 – 4.2 – 4.4 – 4.6</td>
<td>4.3-Class summative assessment, 3 times throughout the course. Assess individual progress with summative tasks like 4.1 or 4.2:

Conceptual K

Procedural Skills

MC Conceptual

MC Procedural

5.Student Self-Assessment / Reflection

(after googling metacognitive prompts for 2nd grader in maths) Self-assessment questions stored in a journal to make them reflect on measurement units or their own ability to solve math problems using their knowledge of words, numbers, pictures.

Self-assessment reports, maybe through peer comparisons or other “imaginary” students doing something, comparing to what they actually do

Keeping track of their progress helps them self-assess their learning and understand their abilities.

Holly emphasized the needs of body/visual activities for the kids</td>
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5.1 – 5.4 – 5.5 – 5.6 – 5.7 6.1 – 6.3 – 6.4 – 6.7 Teacher's action: -Monitor the drawings in groups, ask questions about the reality of numbers, how children describe the scene using numbers -Record children's actions Feedback given: Focused on progress and good drawings.	

- Specific Assessments

	Cognition (Key Goal Focus)	Observation (Tasks Yield Evidence)	Interpretation (Rubric Criteria & Levels)
Conceptual	1.2.2 Know the concept of mental line, greater than, less than, or equality	Students will spot an error in a number line (numbers not at the right place). Students will place numbers on a number line until 1000. Students will coach a peer to place numbers on a number line (say order the numbers 2; 54; 63; 12; 136; 789; 100; 199; 700) ➔ Context is structured ➔ Directions: only the task assignment ➔ Progressive scaffolds about their reflection on number comparison, number line, etc ➔ Objective: asking about knowledge.	When getting it right: level 0: number line to 10 level 1: to 50 level 2: to 100 level 3: to 1000 *ask help to peers or teachers *while coaching a peer, adapting explanation level to the peer's understanding Level 0: does not know about the error and does not try to catch it Lvl 1: tries to catch error but can't find it Lvl 2: *spot the error themselves with guidance Lvl 3: *self-correct

Procedural	2.2 apply addition and subtraction to numbers up to 1000	Add 4 and 7 Add 19 and 86 (over 100) Add the numbers 42 and 521 (3 digit numbers to add) Subtract 3 from 8 Subtract 29 from 37 (carrying, simple version) Subtract 357 from 615 (carrying, hard) Coach a peer to add and subtract at different levels. Other features: using tools, using scaffolds, etc. Think aloud when working. ➔ Can be mentally, or using a paper, or using ten frames to help. ➔ Directions: only the task assignment, and asking to use a type of support objects. ➔ Progressive scaffolds depending on their progress, about operation properties and methods, etc. Objective: asking about skills and knowledge	Level 0: add 4 and 3 Level 1: add 23 and 76. Level 2: add 215 and 745 Levels for subtraction depending on whether or not carrying and number of digits. Same levels from easier to difficult: Meta level0: using ten frames Meta level 1: using paper ML2: mentally *think-aloud: - cannot explain the process and fails - can explain the process but fails to do it - can explain the process and apply it If they get it wrong: *retry *ask help to peers or teachers *while coaching a peer, adapting explanation level to the peer's understanding
Dispositional	3.2.1 attentive to peers when they talk during the class, attentive to teachers	Observe children's attention when listening to others Ask to sum up the other's speech or question about what they said Ask them if they think listening to other is important ➔ Context is natural ➔ No directions ➔ No scaffold ➔ Objective: dispositions	Criteria: *does not talk during others' speech *seems concentrated on others' speech *ask questions about their speech *react to others' speech flow (jokes or help seeking etc) *can rephrase the other's speech

- **Instructional design – General description**

- Instruction

General Classroom Climate:

Thanks to insights from Holly and the developmental levels identified for students in grade 2:

- In-classroom instruction (Holly emphasized the fact that it was hard to run kindergarten classes remotely! It is always hard to keep children's attention, there is a need to change the activities every 5 min!)
- Use of many group activities, and if possible, creation of multi-level groups to foster peer help and learning by explaining, it does work well with children according to Holly.
- *BIJ: Active learning*

- The children can move in the class, but we want them to sit all at the same time to listen to the teacher and practice self-activities in calm, so we need at least as many chairs as students.
- The class have playing areas with objects for learning, hand drawing / building objects (paper, artistic material etc.), books for reading in the different disciplines they are taught and other books to foster curiosity and open mindedness (I imagine the entire class, not only for math learning. It is important to have the entire picture to draw coherent conclusions).
- To facilitate belonging, we will use group activities to mix children and let them know each other, we will include free time where they can take possession of the class's material and go over it to discover the place, and make it familiar to them.
- *BIJ: Sense of belonging*

Established Daily / Weekly Routine:

- **What overall routines will you establish to guide the flow of each session, each week, etc.?**
  - The classes will last 45 min each week for as many weeks as needed to go over the 10h course (so ~ 14 classes)
  - At the beginning of each class, I will ask about the last class's content as a formative assessment, informal to the entire class to know if I have to go over the same material during the next class → 5 min.

Then, I will have two possible schedules for the class:

If nothing is new in the lesson, i.e. the content targeted during this class has already been started in a previous lesson and the learning tools I plan to use are similar to the previous tools already used:

  - Then I plan to have a work session with activities and informal assessment, with individual and group activities for the rest of the class ~ 40 min.
  - My job during these 40 min:
  - Explain the activity that is targeted during this particular class session, by orally giving explanations to children, or showing them how to do the activity using the objects as they are meant to do it. Explaining the activity helps children understand what they have to do.
- *BIJ: Alignment – Transfer of learning*

  - Make the groups or at least make sure the groups are mixed in levels: based on the notes taken before and informal/formal assessments, ensure no groups have only poor or highly achieving students.
- *BIJ: High Level Thinkers*

  - Help struggling groups buy showing them how to do the task when they cannot do it on their own.
- *BIJ: Metacognition*

  - After the group task, give a feedback focusing on the achievements and the progress needed. Do not target individual students but rather address the feedback to the group: going over each group, maybe ~10 min during the activities time.

If there is something new in the lesson, i.e. the content targeted during this class has never been taught previously or the learning tools I plan to use are new:

  - In this case I will target the instruction of the new content (always in relation with what was taught before) in relation with assessment and understanding children's prior knowledge
  - I will start by asking them questions about the new content in an introductory session to get their prior knowledge of the new material, maybe for 15 min.
- *BIJ: Prior Knowledge*

  - Then I will plan the activities (individually or in groups) for instruction and informal, formative assessment (used for both instruction and assessment), for the remaining 25 min:
- *BIJ: Assessment as instructional technique*

  - My job during these 25 min:
  - Explain the activity that is targeted during this particular class session, by orally giving explanations to children, or showing them how to do the activity using the objects as they are meant to do it. Explaining the activity helps children understand what they have to do.
- *BIJ: Alignment – Transfer of learning*

    - Make the groups or at least make sure the groups are mixed in levels: based on the notes taken before and informal/formal assessments, ensure no groups have only poor or highly achieving students.
- *BIJ: High Level Thinkers*
    - Help struggling groups buy showing them how to do the task when they cannot do it on their own.
- *BIJ: Metacognition*
    - After the group task, give a feedback focusing on the achievements and the progress needed. Do not target individual students but rather address the feedback to the group: going over each group, maybe ~10 min during the activities time.

- **What class norms would you establish? How?**

- *BIJ: Sense of Belonging*

I will set multiple norms during the class to create a sense of belonging and to provide a behavioral framework for students.

I will mainly establish these norms at the beginning of the course, by clearly stating them and showing them in class. I can write them down for example on a paper and hang it on the wall.

I plan to add the rationale behind the norm on the rule statement, for students to understand why they are forced to raise their hand or leave class after the dismissal bell.

- Raise your hand to ask a question in class
- Be polite when talking to the teacher.
- Listen to your peer when they are talking
- Ask for help if you need to
- Check the date everyday as a class (have one student write it on the board for example)
- Do not leave class before the dismissal bell
- Always bring your material to write and draw in class

*I use the WHERETO elements to specifying the key concepts of my teaching approaches (*UbD p.197)

• Where & Why? Ensure that students understand WHERE the unit is headed and WHY.

- Recall the objective of the activities when we run them

- *BIJ: High level Thinkers*

- The students are 8 years old so I will not spend too much time focusing explicitly on the overall course structure, however I will:
- Include performance tasks in the activities, relating the activity to real-world needs and situations.

- *BIJ: Create natural critical learning environment*

- At the beginning of each class, I will go over the previous class's take away, check if the students understood it.

- *BIJ: Self-evaluation*

- When helping children with the activities in class, I will emphasize how they can use what they already learned in previous classes, to show them their own progress and how the course is built.

- *BIJ: Transfer of learning*

• Hook. HOOK students in the beginning and HOLD their attention throughout.

- Holding students' attention at this age (according to Holly) is mainly supported by providing activities that catch students' attention
- I will provide activities that are in students' proximal zone of development, making groups of levels (according to the prior knowledge test I ran when the content/tool was first mentioned in class), with low to high achieving students (say groups of 5), in order to

- *BIJ: Motivation*

- Give the opportunity to the low achieving students to learn by seeing how their peers do and try to copy them
- Enable the high-achieving students to teach their skills to their peers, to reinforce their learning and create a better sense of belonging for all

- *BIJ: Alignment – Sense of belonging*

- Face different ways of achieving the tasks within a group, to give something to discuss when assessing informally in the group.
- I will HOOK / HOLD by providing real-world related tasks and activities with mentioning / using material the students already know (technologies, or generational culture)

- *BIJ: Education Technology – Create natural critical learning environment*

• Equip. EQUIP students with necessary experiences, tools, knowledge, and know-how to meet performance goals.

- During the planned instructional sessions, I will provide the necessary tools and feedback on the tools' use to meet performance goals (skills and knowledge, C + MC):
  for example, when it comes to addition and subtraction, I will provide them with examples and ten frames & calendars for them to understand the content and apply the expected skills.

- *BIJ: Active Learning*

- During the planned instructional sessions, I will provide the necessary interactions and feedback to meet performance goals (dispositions Cognitive + MC):
  e.g. when showing concentration while counting, I will provide tasks where they will be able to count in groups (say: they have to represent numbers with ten frames and then add them all) and give feedback, asking them to count in front of me and observing their concentration, even when peers are around.

- *BIJ: Active Learning*

- During the "prior knowledge assessment" time, I will activate students' prior knowledge and correct initial misconceptions, providing a first small theoretical framework for the know-how goals (skills and knowledge, C + MC):
  I will ask questions about what they know about the subject (say do they know how to subtract) and correct their misconceptions by orally explaining them the valid concepts and showing them how it is working (say, this is how to subtract).

- *BIJ: Prior Knowledge – Assessment as an instructional technique*

• Rethink / Reflect / Revise. Provide students with numerous opportunities to RETHINK big ideas, REFLECT on progress, and REVISE their work.

- When going over the past lessons' understanding during the beginning of the class's short formative assessments, I will provide students an opportunity to rethink big ideas:

  I will ask them to explain me the content of the previous class, after that I will correct their errors or reexplain the content briefly. They will have the opportunity to think about what they really know about this content,

and then I will show them how this content can be useful for the class's big ideas by taking a higher perspective on the course.

- *BIJ: High Level Thinkers – Metacognition*

- When emphasizing the use of previous content during the activities run with students, I will provide an opportunity to Revise their previous work and learning and Adapt it to the newly learned material.

  I will ask them to evaluate what they are able to do in the activity. I will show them that they integrated well the new content in the activity, so they will understand their progress. I will ask them what they learned during the previous class.

- *BIJ: Self-evaluation*

• Evaluate. Build in opportunities for students to EVALUATE progress and self-assess.

- I will provide feedback during the activities to ask for self-reflection about their integration of the content, helping them evaluate their progress and self-assess:

  When doing an activity to learn how to subtract numbers, I will ask them what they are able to do, and point out that they use the new method (carrying) and that they are able to subtract numbers with more digits than before. For self-assessment, I will ask them what they think they are able to do.

- *BIJ: Assessment as instructional technique – Self-evaluation*

• Tailor to Context & Learner Characteristics. Be TAILORED to reflect individual talents, interests, styles, and needs.

- When assessing prior knowledge before presentation of the content, I will provide students with an opportunity to explain their ideas and their individual differences in relation with the content taught:

- For read, write, and skip count: I will ask them how they would skip count before showing them techniques to do it. Some students will give valid techniques, others will give wrong techniques. They will emphasize different types of techniques, and I will give feedback to correct the ideas and show how each intervention helps refine the techniques for skip-count. Say, if students say: "I would skip-count by drawing the numbers and hiding the one in the middle", then I would say that it is one way to do it, and ask this student to show the class how to do this, to help understand the advantages and drawbacks of this method.

- *BIJ: Prior Knowledge*

- When going over the groups during the activities, I will ask students to explain how they do the tasks and why they gain to express their individuality in the task. I will emphasize the acceptation of their styles still grounded in reaching performance.

- Same as above, if a student skips count by drawing numbers then I would agree and push him to continue, if it is helping. I would show him other ways to do it also.
- *BIJ: Metacognition – Sense of Belonging*

- When helping struggling groups, I will understand why the group is struggling, and what are the individual needs in the groups, and redirect the activity towards these cross-group / individual needs.

  Let us take the example of skip counting.
- I will understand the group & individual struggles by seeing where the group is stuck in the skip counting task, and what are the individual achievements made. I will ask each group member to explain what he would do to solve the problem of skip-counting. I will select the step where they are struggling when applying the methods and explain them again and showing them how to do it. Then ask them to reproduce it.
- *BIJ: Create natural learning environment*

• Organize to Optimize. Be ORGANIZED to optimize deep understanding as opposed to superficial coverage.

- Through the planning of the class, with 45 min classes each week throughout a longer period, I will optimize deep understanding and integration of new content in learning.
- *BIJ: High Level Thinkers*

- Through the organization of each class, I will give a consistent framework to make students feel comfortable, safe, and focused on learning.
- *BIJ: Sense of belonging – Motivation*

- By going over the past lessons at the beginning of each lesson, I will use spacing and repetition effects for memorization and metacognition development.
- *BIJ: Metacognition*

- By giving precise feedback during the activities and insights about how the content relates to the previously learned content, I will foster deep understanding.
- *BIJ: Active Learning – Transfer of Learning*

- Designed Flow of instructional elements:

Keep track of the date everyday

	Flow of instructional elements / assessments	Direct instruction vs. facilitation vs. coaching	Alignment with goals	Justifications using big ideas
Week 1	Introduction of the lesson: **Hook** students, show the final tasks they will be able to do and what they will use: **Where & Why** *adding 2 numbers with objects (ten frames, calendars, etc.) *writing a full recipe *drawing where numbers are in real-life Etc. Class Norms – Set the Goals: refer to the Norms section. Prior Knowledge assessment about the class general content: *level in 1.1 / 1.2 / 1.3 – 2.1 / 2.2 with class scaled assessment about these goals. Instructional activities + Informal formative assessment + feedback - Assessment: 3.2 *Recognize numbers from 1 to 100 *Use fingers, dices, calendars *During group activities, use informal formative assessments watching students do the tasks to get individual levels.	Direct Instruction Facilitation through Instructional activities Coaching through feedback	1.1 Identify numbers from 1 to 1000 3.1 Concentration when counting 3.2 Attentive to peers 4.1 Self-evaluation of understanding 4.4 Ask a question 4.7 know my count limit 5.6 think-aloud when counting 6.1 evaluate lack of attention 6.6 recall interactivity level	Introduction of the lesson: Alignment - Motivation Norm setting: Sense of belonging Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning
Week 2	Formative Assessment about previous class: 3.3 *Ask if the class can recognize numbers from 1 to 100 **Rethink/Reflect/Revise** Prior Knowledge assessment about new content	Direct Instruction Facilitation through Instructional activities	1.1.2.Know the vocabulary of numbers 4.2 I evaluate my lack of knowledge to fulfill a goal	Summative assessment about previous class: Self-evaluation - Metacognition Prior Knowledge assessment: Prior Knowledge

	*level at 1.1.2 Instructional activities + informal formative assessment + feedback – Assessment: 3.1 *vocabulary of numbers: describe numbers with the good vocabulary: whole numbers, digits, tens etc. *use calendars in groups to recognize patterns tens/units **Hook & Hold** *emphasizing base 10 with group activity *ask children to listen to peers to identify their level of attention, for informal formative assessment of MC goals. **Evaluate**	Coaching through feedback	5.4 When I read, write and skip count to 1000, I can explain the procedure and evaluate my performing it. 5.5 If I need a step or a hint to apply the skills, I realize it and I seek help. 5.6 think-aloud when counting 6.2 When counting, I can regulate my own concentration. 6.3 When listening to peers, I can identify their level of attention, evaluate it and interact in response 6.4 When I see other children struggling, I ask if they need help.	Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning
Week 3	Formative Assessment about previous class 3.3 *Ask if the class knows the vocabulary of numbers Prior Knowledge assessment about new content *level at 1.2 Instructional activities + informal formative assessment + feedback- Assessment: 3.1 *concept of place value explained in group tasks with visual tools: ten frames *mental line, number comparisons: ask students to draw a mental line, use their arms to represent the concepts of > and < **Tailor**	Direct Instruction Facilitation through Instructional activities Coaching through feedback	1.2-Identify comparisons relations between numbers 3.1 Concentration when counting 3.2 Attentive to peers 4.6 I can say what I did good during a task to reach a goal. 5.2 I can design a plan to evaluate the skills I applied 5.3 I evaluate the correctness and meaning of the applications' results.	Summative assessment about previous class: Self-evaluation - Metacognition Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning

	*use groups of objects to make them experiment equality		6.1 I evaluate my lack of attention to others. 6.2 When counting, I can regulate my own concentration.	
Week 4	Formative Assessment about previous class 3.3 *Ask if the class can explain the concept of place-value **Rethink/Reflect/Revise** *Ask two students to show the class how to use ten frames to represent a 3-digit number Formative assessment 1.1 Formative assessment, individual performance task: describe real-life objects with numerical vocabulary (see assessment section) **Hook & Hold** **Equip**	Direct Instruction Facilitation through Instructional activities Coaching through feedback	1.2 .Identify comparisons relations between numbers 3.1 Concentration when counting 3.2 Attentive to peers 4.2 I evaluate my lack of knowledge to fulfill a goal. 4.4 When I identify a lack of knowledge, I ask a question to a teacher or a peer 5.4 When I read, write and skip count to 1000, I can explain the procedure and evaluate my performing it. 6.7 I can self-evaluate my progress on dispositions goals and how it affected my learning in general	Summative assessment about previous class: Self-evaluation - Metacognition Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning
Week 5	Prior Knowledge assessment about new content 3.3 *level at 1.3 + 2.3.1 Instructional activities + feedback *identify addition with groups of objects *identify subtraction with groups of objects *identify multiplication grouping objects While doing the activity:	Direct Instruction Facilitation through Instructional activities Coaching through feedback	2.1 Read, write and skip count to 1000 1.3. Identify basic mathematical operation and their properties 3.3 I am interactive when a game for numerical play is happening 4.2 I evaluate my lack of knowledge to fulfill a goal. 4.4 When I identify a lack of knowledge, I	Course evaluation: Valid and reliable assessment – Create natural critical learning environment Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning

	First Third-term course evaluation - Metacognitive abilities assessment Summative assessment Think-Aloud like assessment 4.1. Example tasks for each student about 1.1 – 1.2. **Rethink/Reflect/Revise** **Equip**		ask a question to a teacher or a peer 4.7 I know the upper limit of the numbers I can count until 5.5 If I need a step or a hint to apply the skills, I realize it and I seek help. 5.8 I can seek help and offer help if I recognize others' needs 5.9 After listening to peers, I can tell what they said, the effects of their learning and peers reaction. 6.4 When I see other children struggling, I ask if they need help.	
Week 6	Formative Assessment about previous class 3.3 *can the class explain what is addition, subtraction and multiplication? **Rethink/Reflect/Revise** *can the class represent addition, subtraction and multiplication with groups of objects? Prior Knowledge assessment about new content *level at 2.1 Instructional activities + informal formative assessment + feedback - Assessment: 3.2 *read, identify and write the numbers in letters **Evaluate** *practice skip-count on calendars, starting two-by-two	Direct Instruction Facilitation through Instructional activities Coaching through feedback	Identify comparisons relations between numbers 2.1 Read, write and skip count to 1000 3.4 I am respectful of others (teacher and peers) and their points of view when evaluating numerical expressions, playing numerical games, etc. 4.5 When I have trouble mentally representing a concept (a number, a number comparison, etc.), I think about the same numbers of objects, how to build this number with ten frames, or all the physical and visual	Summative assessment about previous class: Self-evaluation - Metacognition Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning

			representations of the number comparison. 5.1 I can evaluate if I performed a valid and logic operation 6.5 When a numerical play is happening, I can evaluate my degree of interactivity with others and with the material, monitor it towards the disposition goal	
Week 7	Formative Assessment about previous class 3.3 *Can the class write numbers in letters? *Can children skip-count 2 by 2 Instructional activities + informal formative assessment + feedback-Assessment: 3.1 *read, identify and write the numbers in letters *practice skip-count on calendars, starting two-by-two **Tailor** *show real-world examples where skip-counting and writing numbers in letters are useful: streets' numbers, checks, etc. **Hook & Hold**	Direct Instruction Facilitation through Instructional activities Coaching through feedback	1.3 Identify comparisons relations between numbers 2.1 Read, write and skip count to 1000 3.3 Playing interactively 3.4 Respectful of others 4.2 I evaluate my lack of knowledge to fulfill a goal. 4.4 When I identify a lack of knowledge, I ask a question to a teacher or a peer 5.4 When I read, write and skip count to 1000, I can explain the procedure and evaluate my performing it. 6.7 I can self-evaluate my progress on dispositions goals and how it affected my learning in general	Summative assessment about previous class: Self-evaluation - Metacognition Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning
Week 8	Formative Assessment about previous class 3.3 *knowledge of time conversions	Direct Instruction Facilitation through	1.4 Identify basic mathematical operation and their properties	Summative assessment about previous class: Self-evaluation - Metacognition

	*add weights or distances **Rethink/Reflect/Revise** Prior Knowledge assessment about new content *level at 2.2.2 Instructional activities + informal formative assessment + feedback - Assessment: 3.2 *how to conduct an addition with paper & pencil using place-value **Equip** *how to conduct a subtraction with paper & pencil using place-value **Equip**	Instructional activities Coaching through feedback	2.2.2 use place value understanding properties to add and subtract within 1000. 3.3 I am interactive when a game for numerical play is happening 3.4 I am respectful of others (teacher and peers) and their points of view when evaluating numerical expressions, playing numerical games, etc. 4.2 I evaluate my lack of knowledge to fulfill a goal. 4.4 When I identify a lack of knowledge, I ask a question to a teacher or a peer 5.5 If I need a step or a hint to apply the skills, I realize it and I seek help. 5.6 When I count mentally, I can think aloud if asked for it and explain my thinking.	Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning
Week 9	Formative Assessment about previous class 3.3 *knowledge about addition & subtraction methods **Rethink/Reflect/Revise** *Ask if the class can conduct addition & subtraction Prior Knowledge assessment about new content *level at 2.2.1 Instructional activities + informal formative assessment. - Assessment: 3.1 + feedback Goals: 2.2.1 – 2.2.2	Direct Instruction Facilitation through Instructional activities Coaching through feedback	2.2 apply addition and subtraction to numbers up to 1000. 3.1 I am concentrated when counting 3.2 I am attentive to peers when they talk during the class, attentive to teachers 4.5 When I have trouble mentally representing a concept (a number, a number comparison, etc.), I think about	Summative assessment about previous class: Self-evaluation - Metacognition Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning

	*mental strategies to add numbers, starting with one-digits **Equip** Evaluation Research Task: Experimental group: *how to actively practice place-value for addition using ten frames to represent hundreds/tens/units **Equip** *how to actively practice place-value for subtraction using ten frames to represent hundreds/tens/units **Equip** Control group: *how to actively practice place-value for addition using pictures of numbers to represent hundreds/tens/units **Equip** *how to actively practice place-value for subtraction using pictures of numbers to represent hundreds/tens/units **Equip**		the same numbers of objects, how to build this number with ten frames, or all the physical and visual representations of the number comparison. 4.6 I can say what I did good during a task to reach a goal. 5.7 I can tell if I followed the method for addition, subtraction and multiplication. 5.8 I can seek help and offer help if I recognize others' needs 6.5 When a numerical play is happening, I can evaluate my degree of interactivity with others and with the material, monitor it towards the disposition goal 6.6 After a numerical play, I can recall the degree of interactivity and identifies the effects of my approach on the task.	
Week 10	Prior Knowledge assessment about new content 3.3 *level at 2.3 Instructional activities + feedback – 2.2.1 – 2.2.2 – 2.3 *conduct addition, subtraction	Direct Instruction Facilitation through Instructional activities Coaching through feedback	2.2 Apply addition and subtraction to numbers up to 1000 2.3 Use real world objects to understand the values of numbers and operations	Course evaluation: Valid and reliable assessment – Create natural critical learning environment Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback:

	*mental strategies to add numbers, starting with one-digits *use groups of objects to practice multiplication While doing the activity: Second Third-term course evaluation in group – Metacognitive abilities assessment Summative assessment like assessment 4.2. **Hook & Hold** Example tasks for each student about 1.3, 2.1, 2.2, reflect on progress. **Rethink/Reflect/Revise** **Evaluate**		3.3 I am interactive when a game for numerical play is happening 3.4 I am respectful of others (teacher and peers) and their points of view when evaluating numerical expressions, playing numerical games, etc. 4.2 I evaluate my lack of knowledge to fulfill a goal. 4.6 I can say what I did good during a task to reach a goal. 5.2 I can design a plan to evaluate the skills I applied 5.3 I evaluate the correctness and meaning of the applications' results. 5.7 I can tell if I followed the method for addition, subtraction and multiplication 6.1 I evaluate my lack of attention to others. 6.2 When counting, I can regulate my own concentration. 6.6 After a numerical play, I can recall the degree of interactivity and identifies the effects of my approach on the task.	Alignment – Assessment as instructional technique – Active learning

Week 11	Formative Assessment about previous class 3.3 *Can children conduct written & mental addition? **Rethink/Reflect/Revise** *Can they conduct written & mental subtraction? Instructional activities + feedback - Assessment: 1.2 Formative assessment, individual performance task: cooking recipe design (see assessment section) **Hook & Hold**	Direct Instruction Facilitation through Instructional activities Coaching through feedback	1.3. Identify basic mathematical operation and their properties 1.4. Know real-world examples of operations and data 2.2 Apply addition and subtraction to numbers up to 1000 2.3 Use real world objects to understand the values of numbers and operations 3.3 Playing interactively 3.4 Respectful of others 4.3 I can say if I understood well the basic measurement units 5.2 I can design a plan to evaluate the skills I applied 5.4 When I read, write and skip count to 1000, I can explain the procedure and evaluate my performing it. 5.5 If I need a step or a hint to apply the skills, I realize it and I seek help. 6.2 When counting, I can regulate my own concentration. 6.5 When a numerical play is happening, I can evaluate my degree of interactivity with others and with the	Summative assessment about previous class: Self-evaluation - Metacognition Prior Knowledge assessment: Prior Knowledge Formative assessment activity: Valid and reliable assessment – Create natural critical learning environment

			material, monitor it towards the disposition goal 6.7 I can self-evaluate my progress on dispositions goals and how it affected my learning in general	
Week 12	Formative Assessment about previous class 3.3 *Can children conduct written & mental addition? *Can they conduct written & mental subtraction? **Rethink/Reflect/Revise** Prior Knowledge assessment for new content: *level at 1.4 & 2.3.3 / 2.3.4 Instructional activities + informal formative assessment + feedback - Assessment: 3.2 Goals: 2.2 – 2.3 – 1.4 *practice addition & subtraction with real-world data **Equip** *practice conversions with time, distances, weights **Evaluate** *read time	Direct Instruction Facilitation through Instructional activities Coaching through feedback	1.4. Know real-world examples of operations and data 2.3 Use real world objects to understand the values of numbers and operations 2.4 Communicate with think aloud while engaged in numerical thinking 3.3 I am interactive when a game for numerical play is happening 3.4 I am respectful of others (teacher and peers) and their points of view when evaluating numerical expressions, playing numerical games, etc. 4.3 I can say if I understood well the basic measurement units 5.1 I can evaluate if I performed a valid and logic operation 5.7 I can tell if I followed the method for addition, subtraction and multiplication 6.7 I can self-evaluate my progress on dispositions goals and how it affected	Summative assessment about previous class: Self-evaluation - Metacognition Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning

			my learning in general	
Week 13	Formative Assessment about previous class: 3.3 *can children read time and practice operations with data? Prior Knowledge assessment about new content *level at 2.4 Instructional activities + informal formative assessment + feedback - Assessment: 3.1 *practice addition & subtraction with real-world data *use think aloud in groups when counting and conducting operations **Rethink/Reflect/Revise**	Direct Instruction Facilitation through Instructional activities Coaching through feedback	2.4 Communicate with think aloud while engaged in numerical thinking 4.4 When I identify a lack of knowledge, I ask a question to a teacher or a peer 5.8 I can seek help and offer help if I recognize others' needs 5.9 After listening to peers, I can tell what they said, the effects of their learning and peers reaction. 6.3 When listening to peers, I can identify their level of attention, evaluate it and interact in response	Summative assessment about previous class: Self-evaluation - Metacognition Prior Knowledge assessment: Prior Knowledge Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning
Week 14	Closing course – final evaluation course with children informal surveys + teacher's feedback **Rethink/Reflect/Revise** Instructional activities + feedback - Assessment: 2.1 Summative assessment, Performance task in groups: drawings of the daily life situations with numbers (see assessment section) **Evaluate**	Facilitation through Instructional activities Coaching through feedback	2.4 Communicate with think aloud while engaged in numerical thinking 4.1 I evaluate my own understanding of the numerical concepts (auto-assessment) 5.7 I can tell if I followed the method for addition, subtraction and multiplication.	Evaluation course: Valid and reliable assessment – Self-evaluation - Metacognition Instructional activities / informal formative assessment / feedback: Alignment – Assessment as instructional technique – Active learning

Examples of Specific Activities & Experiences:

Learn the vocabulary of numbers and how to recognize them: Week 2

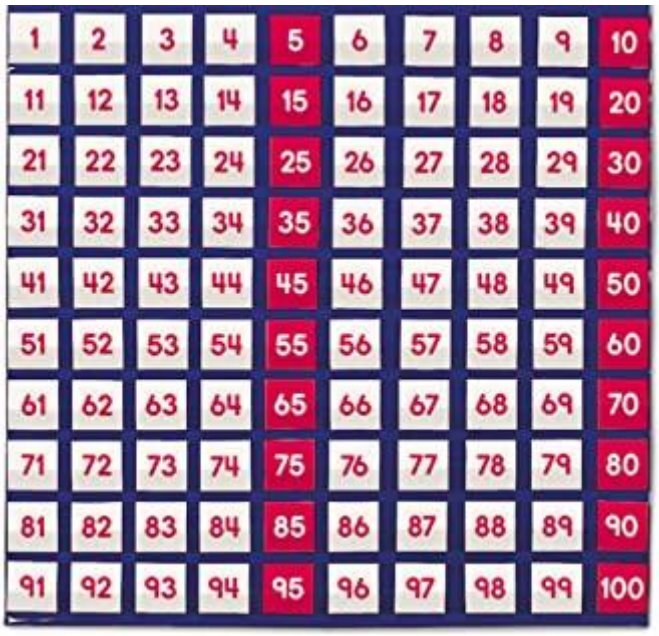
Approach: guided discovery & Feedback

Context: First lessons of the course, need to assess global level and individual

Directions:

In groups, match numbers' names in letters with numbers in digits by manually placing papers with numbers on the calendar with numbers. You can see both the numbers with digits and the numbers written with letters.

Representations: calendars with numbers from 1 to 100, ordered in columns. All numbers should be of the same color, except the 5's and 10's like this, to focus on numbers' names and distinctions between every 10's.



Resources: Use cut pieces of paper with the numbers from 1 to 100 in letters. "one" is one paper, "two" is another one, "three", … "forty-six", "one hundred". (students can make them beforehand or just print them)

Student actions & interactions: place the paper numbers on the corresponding number, interacting with peers to perform the task.

Guidance: No guidance from the teacher, except if the group struggles.

Different learners' accommodations:

Help can be provided by the teacher; they can explain once again the different names and how they relate to the structure of numbers: "thirty-four" is thirty which corresponds to the 30's and four is added to the 30's, so "thirty-four" is 34".

The task can be done individually also, if the students have trouble interacting with others: Then just let the student do the task on their own and give feedback.

Learners can perform the task on a digital platform if they have trouble peer-interacting or if they are given online classes (due to covid19 for example), then use google documents and zoom calls to let students interact with online calendars' pictures and numbers' names to move around.

Products: An ordered calendar with numbers in digits and letters.

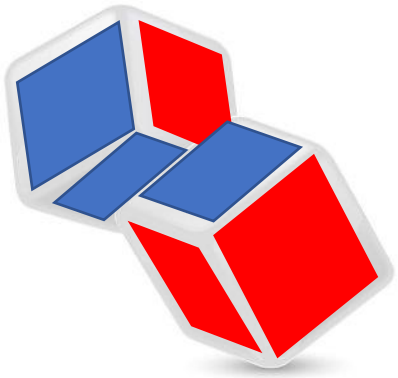
Learn addition & subtraction with real-world data, week 5

Approach: guided discovery

Context: Students learn the properties of addition with a bicolor dice: there are multiple ways to add up numbers to reach a certain number.

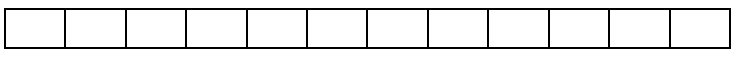
Directions: In groups of 4, roll a bicolor dice 12 times (vary the number of rolls) and note how many times each color comes up, and add the number for both colors; observe that it always add up to 12, this is a property of addition.

Dices with two colors: blue and red.

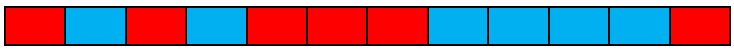


Scaffolds used when performing the task:

Students take a paper and draw 12 squares like this:



Then they draw a point in the next empty slot in blue or red depending on the color obtained with the dice, one for each roll, it could look like this at the end:



Representations: bicolor dice gives a pair of random numbers adding up to the total number of rolls.

Resources: Bicolor dice, pen to note results, blue and red (just the same colors as the dice)

Student actions & interactions:

Roll a dice, add "one" to the corresponding color until you rolled the total number of times

Guidance: Teacher walks in the classroom across groups to help and give feedback on the theory behind to explain addition properties.

Different learners' accommodations: It is possible to do this with a regular dice for color blind for example.

Products: No products, the results are the understanding of addition properties.

Learn to skip-count, week 6 & 7

Approach: guided discovery & Feedback

Context: learners learn how to skip count by trying and failing, finding the best methods to do it

Directions: In groups of 5, complete a paper sheet with skip counted numbers from 0 to 10*number to skip count with (like 0->20 / 0 -> 30 etc.) such as this one:

2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90

Students just have the first number on the far left column at the beginning of the task.

Representations & Resources: paper with lines for each number from 2 to 9 and cases for skip counting.

Student actions & interactions: Try to skip count as they can, intuitively. In groups, they can share their methods. They must understand others' methods and evaluate them.

Guidance: The teacher comes across all groups and ask a think aloud of one student for this task. This gives a method, and then ask others to try the same method and evaluate it.

Different learners' accommodations:

- The task can be done individually, as above
- Digitally also with a computer interface
- For students with an attention issue, the task can be shifted toward more manual activities such as adding two objects to a group of objects at a time and then counting the objects each time another one is added and write the results on a similar sheet. Integrating an extrinsic motivation like the first students to finish without any error, have a bonus. They can start free activities as soon as they finished for example.
- For students with a motor issue, they can share the task with other students and assign individual roles as finding the right number, and another student writes it.
- For students with sight issues, you can revise the class design with pairing these students with students that are at ease with the notions, and have them work together, focusing on naming the numbers and mentally counting.

Products: A grid completed with skip count numbers per group

Learn place-value concept to represent numbers, week 8

Approach: direct instruction & guided discovery

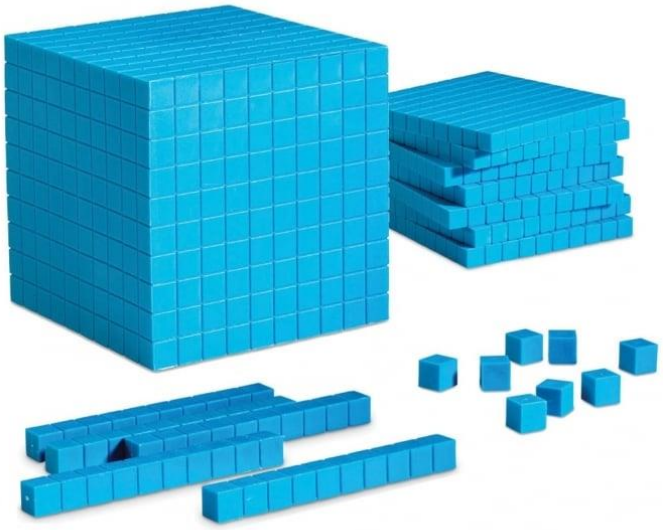
Context: learn place value with ten frames

Directions: In groups, combine hundreds/tens/units to add up to a number given by the teacher.

Representations & Resources: Ten frames represent place value in numbers, being in base 10, like this:

For this type of ten frames, the yellow squares are units, the green are tens, and blue are hundreds.

Other type of ten frames, with more 3D shapes (not flat pieces):



Student actions & interactions: Interact with peers to collectively organize units / tens / hundreds to represent a given number. Show the teacher where are the hundreds/tens/units.

Guidance: The teacher first explains in class what are the hundreds/tens/units and how to represent them using ten frames. They give the numbers for students to add up to. Then they help students in group activities, give feedback. Feedback is about the use of ten frames to represent place-value and how to spot place value in numbers.

To provide more challenge for more advanced students, propose to represent the same numbers as before but with a different number of frames (like, replacing some hundreds by 10s of tens)

Different learners' accommodations: can be done individually, be helped by peers in groups or provide help to peers. For peers with sight, attention disorders, etc: refer to the above section.

Products: Frames representing a number. Teachers validate them, give feedback and note down what the students did well and what are the muddy points.

Performance task: design a cooking recipe and course list with prices, week 11

Approach: guided discovery & Feedback

Context: Create your own recipe for one person, work with real data. Then give a price for each item on the shopping list and add up all the prices to find the final price of your purchases. Have students fill in the following tables and answer the following questions:

Ingredients	Weights / qtity for 1 person	Price for 1 person
Provide 6 lines as a maximum		
FINAL TOTAL	No need to add the weights	(sum here all the prices to get the total price for 1):

Write the steps for your recipe: (6 lines, one line each step)

Step 1	
Step 2	
Step 3	
Step 4	
Step 5	
Step 6	

What would be the price of your recipe for 1 person if you decide to remove one item from the recipe? Removed item:

Final price after removing the item:

Ingredients	W/qtity for 2 people	Price for 2 people
Provide 6 lines as a maximum		
FINAL TOTAL	Total price:	(sum here all the prices to get the total price for 2):

Directions: Create your own cooking recipe for one person, listing the ingredients, the weights, and the steps: Fill in the assessment sheet with the names of ingredients, and all the information. Then duplicate the recipe for 2 people.

Representations & Resources: use old cooking books (given by parents?) to cut pictures of ingredients and final products,

Student actions & interactions: come up with a sheet with a list of ingredients, their quantity, and the directions for cooking. Imagine a cooking recipe and note down the steps. Reflect about the reality/coherence of weights and quantities.

Guidance: The teacher explains the task, helps students precise / refine their ideas, ask questions about coherence and reality of weights and quantities. We could say that students will vote for the best recipe and cook it in class

Different learners' accommodations: can be done in groups if needed

Products: A recipe for one/three people.

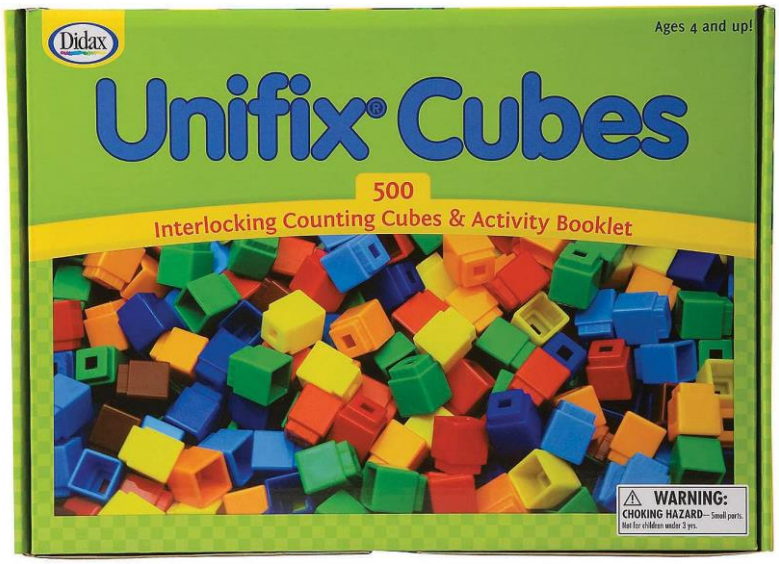
■ Evaluation Research

- Evaluate educational impact

This research investigates the impact of practicing addition and subtraction with ten frames on place-value understanding and paper & pencil addition and subtraction. This research is conducted during weeks 8 and 9.

During week 8, students from both groups learn how to conduct addition and subtraction on paper with paper & pencil. The following week, the experimental group uses ten frames in group to add and subtract, when the control group uses colored objects to practice addition and subtraction in group.

The colored objects can be unifix cubes like the below ones, in sufficient amounts (like 60, with 20 of each color, representing units, tens, hundreds).



BIJ: Active learning – Transfer of Learning

Research Question(s): Do ten frames improve students' understanding of addition and subtraction?

Hypotheses & Related Predictions: Students learn addition and subtraction better when they practice using ten frames and blocks of units representing the base-10 tens and hundreds, than simple unifix colored cubes where one only cube represents hundreds/tens/units depending on its color.

Experimental Design (experimental and control groups, independent and dependent variables, covariates related to individual differences, etc.):

Independent variable: ten frames and unifix cubes

Dependent variable: students' improvement between pre-test and post-test, measuring students' performance at the following task: individually add and subtract numbers with paper & pencil. Conduct the pre-test at the end of the week 8 class after the explanations and training about how to conduct addition & subtraction with paper & pencil. Students take the post-test on the week after the experiment (week 9). Make the assessment short, with

6 additions (1 with units / 3 with tens (1 without carry, 2 with carry) / 3 with hundreds (1 without carry, 2 with carry))

6 subtractions (1 with units / 3 with tens (1 without carry, 2 with carry) / 3 with hundreds (1 without carry, 2 with carry))

Give enough time for students, 15 to 20 minutes.

Covariates: individual differences controlled with random assignment, even though the sample is small.

Method (subjects, procedure, materials, timeline): Experimental and control group procedure. Half class in each group, with random assignment. Teachers will take notes on how groups score with the manipulatives and check whether their scores are correlated to the improvement score (dependent measure). This helps ensure that the effect is actually the expected one.

During week 8, the two groups are not formed yet. All students practice pencil & paper addition & subtraction, as a class, during 45 min. The teacher introduces the notion of carry in subtraction and explains how to conduct the subtraction using carry.

During week 9, students are split in both groups. The experimental group works with ten frames to understand base 10 and to represent the groups of hundreds/tens/units to add and subtract and insists on the representation of carry during subtraction. The control group works with unifix cubes to represent a hundred / a ten or a unit. They make groups of objects to represent numbers and carry when subtracting.

Using unifix cubes is expected to be more time consuming than ten frames. To take this variable into account, the teacher will provide both groups with 30 minutes for practicing with their respective tool. The teacher will note the number of problems done by each group within the 30 minutes to correlate this information to the final improvement. We want to ensure that it is not the number of problems in the training that influences the dependent variable, if not the type of tool used.

Week 10: Conduct the assessment

Materials: Unifix cubes and ten frames, in the same quantities.

Data Collection & Scoring: The data collected is the assignment of children in both groups, with a list of names. The teacher collects the post-test results during week 10 on paper sheets. The test scoring has multiple categories:

1. Numerical application is correct (no errors when adding digits, which is the basis of operations)

2. The method is correct when adding and subtracting (knows the method, starting to the right, adding column by column, knows about the carry)

3. The use of carry is well conducted.

For each question, the teacher will note the following points:

- Is simple calculation conducted well?
    - No calculation error: 184

      - 56

      = 138

There is no calculation error, but the student just forgot the carry (we assume that if there is no evidence of the carry and an error exactly matching the case without carry, the student actually forgot the carry and did not make a calculation error).

- Calculation error: 184
- - 56

= 125

There is a calculation error, because the carry was taken into account but the result is false for the units.

- Is there evidence that the student knows the method for adding and subtracting?
  - Evidence of the alignment in columns
  - Starting to the right
  - Adding column by column
  - Evidence of the use of carrying

- Is carrying employed well?
  - Check for a correct answer whenever carrying is necessary, for the digit in the corresponding column

Assessment of Design Quality (sampling, validity, reliability, triangulation, possible confounds …):

Random sampling increases the likelihood of controlled individual variables.

Providing similar instruction about operations during the first week to both groups ensures they similarly build on prior knowledge. The presentation of different instructional material for the experiment less affects how they build on prior knowledge.

Internal validity:

The addition of little context in the tasks: Lots of context could be distracting or word problems that might make it difficult for students working in a second language.

Use a scoring grid for the result task, with 3 sections to assess students' performance at three levels:

Students can apply simple operations on digits (the student validates this item if more than 4/5 of their one-digit addition or subtraction is correct)

Students know when to apply the methods for adding and subtraction: level 0 in all one digit operations / level 1 in two digits operations / level 2 in three digits operations

Students know how to conduct subtraction with carry: if students found the right answer using carry and show notes evidencing their use of carry

We seek to identify the effect of instructional activities and in particular ten frames on the second and third objectives only.

Both instructional activities are active learning, and group learning. This controls for other covariables like active learning / group interactions / peer feedback.

Reliable:

The only difference that exists between the two groups are the instructional tasks. They receive similar guidelines and objectives when practicing the activities: They must add or subtract the numbers given using their corresponding tools.

Children are not aware that they are being part of an evaluation research.

- Evaluate educational implementation

Fidelity check of teacher following the proposed instruction and assessment design

Data collection:

The teacher has given the same guidelines for both groups and has not informed students about the real purpose of both groups.

The teacher emphasized that the instructional experimental activity is just an activity and not a test, when splitting the class into two groups, if students ask question (they probably will, because they are being assigned to activities)

The teacher did not inform students about the test the following week.

The teacher walks around the classroom and gives as little feedback as possible, just enough for students to understand how to represent numbers and carrying. They give additional tasks when students finish their ones, during the activity.

A better situation would be to have another teacher from another class that doesn't know the study that comes in the class for this particular day and give the feedback. This way, we would avoid implicit / unconscious bias from teacher's feedback, knowing the purpose of the study. And if both groups have the same new teacher, this covariable doesn't influence the study results. If no teacher is available, the teacher must give minimal feedback and just enough for students to progress in their activity.

Forms used:

Form for week 8:

- The teacher didn't mention the study, nor did he mention the groups, or tests.
- The teacher didn't separate groups beforehand (to account for unconscious bias in initial instruction)

Form for conducting the activity (week 9) with guidelines on the appropriate behavior and tasks given to students:

- The teacher has given the same guidelines for both groups:
    - You will have 30 minutes to solve how many problems as possible.
    - You will have to use the tools you are assigned to, in order to solve the problems.
    - When you face a problem, you can call me, and I will help you understand how to use the tool to solve the problem.
- The teacher creates random groups of 4 within each experimental/control group.
- The teacher emphasized that the instructional experimental activity is just an activity and not a test, when splitting the class into two groups, if students ask question (they probably will, because they are being assigned to activities)
- The teacher did not inform students about the test the following week.
- The teacher gave as little feedback as possible: they gave progressive hints to ensure that students try to understand how their tool relates to the task
    - For example, if the students use ten frames but cannot solve the subtraction 153 – 86:
        - The teacher asks students to reproduce the paper & pencil subtraction method by using the ten frames to represent the numbers.
        - If there are still issues, the teacher shows how to do it, and briefly explains the steps:
        - Represent each number with hundreds/tens/units. Remove the right numbers of digits in each base 10, and if impossible, replaces one base 10 by 10 objects from the smaller base 10 (replace 100 by 10*10 or 10 by 10*1) then relates to the use of carry.

    - They give just enough feedback for students to understand how to represent numbers and carrying. They give additional tasks when students finish their ones, during the activity.

Form for assessing students, giving the test week 10:

- The teacher did not separate students in groups
- They gave the post-test individually
- They answered the questions in the class if they concern something else than the experimental purpose
- They keep track of time and give enough time for students, around 15/20 min.

When assessing students:

- Students know when to apply the methods for addition and subtraction when there is evidence that they used column operations & mentioned/wrote a carry & went from right to left & performed the appropriate operation in the context (addition or subtraction), in at least 2/3 of the questions they started within the given time.

- Students know how to conduct addition & subtraction with carry: if students found the right answer using carry and show notes evidencing their use of carry, at least in 2/3 of the carry tasks.

The teacher then grades each component among these two by counting the number of operations where the student completed each one. There are two sections, one for methods and one for subtraction with carry.

- The score in both sections is a number between 0 and 12 (there are 12 questions)
- The teacher then calculates the mean of each group's results for both goals.
- There is an effect for each section if the difference in achievements is statistically significant at a 95% conducting a t-test on mean differences.
- The hypothesis is validated if at least one of both sections show a significant effect.

There should be at least 20 children in each group to expect significant results. One way to deal with this (the current class has ~ 20 students in total, so 10 in each group) is to merge two classes to conduct the study.

Planned scoring:

Scoring criteria:

The evaluation researcher must follow these steps detailed in a form for evaluation.

The teacher is doing the right thing for both groups and not biasing the study:

- If the teacher or an external assessment (evaluation researcher can answer the above form) answers negatively to all the above questions

Fidelity check of students participating in instruction and assessment as anticipated

Data collection:

Instruction week 8: students participate in the instructional activity to learn about addition & subtraction methods with paper & pencil.

Instruction week 9: students are interacting with their peers during instructional activities in their group. They are responsive to feedback given during the activity (like, you should consider that one ten is equal to ten ones and the same for hundreds and tens, so that you can

replace one hundred or one ten by their equivalent in tens and units when subtracting). They use the assigned material to perform operations and practice.

Assessment week 10: Students engage in the test as a practice activity, as they do not know the purpose of the test. They can be motivated by the teacher saying that they will have a break when they finish this activity, or they will have another reward. They should not talk to others during the test.

Forms used:

Week 8:

did students write at least 10 operations during class?

did they listen to the instructions given by the teacher?

did they practice addition & subtraction with carry at least once for each?

Week 9:

did students interact with peers during instructional activities?

did all group achieve at least 5 additions and 5 subtractions, with at least 2 with hundreds and 2 with carry?

are students not aware of the purpose of the activities? (they should not know about the comparison between both groups).

Week 10:

Are students talking to others during the test?

Are they actively engaged in the test? (did they write notes on the paper sheets showing that they tried to solve the operations?)

Planned scoring:

Week 8:

8.1 How many operations did students write in class?

- < 5: low
- 5 – 10: medium
- > 10: high

8.2 did they listen to the instructions given by the teacher?

- They practice with operations Yes / No

- They do it individually, no interactions with others Yes / No

8.3 did they practice addition & subtraction with carry at least once for each?

- No
- 1 – 2 carrying operations: low
- 3 - 5 carrying operations: medium
- 6> carrying operations: high

Week 9:

9.1 did students interact with peers during instructional activities?

- Started at least one discussion Yes / No
- Answered to at least one question Yes / No
- Showed at least one operation resolution Yes / No

9.2 did all group achieve at least 5 additions and 5 subtractions, with at least 2 with hundreds and 2 with carry?

- Yes/No

9.3 are students not aware of the purpose of the activities? (they should not know about the comparison between both groups).

- Yes / No

9.4 Are students engaged in the task?

- Students are interacting with others (see above)
- Students proposed resolution paths for operations:
    - <= 1: low
    - 2 – 4: medium
    - > 4: high

Week 10:

10.1 Are students talking to others during the test?

- Yes / No

10.2 Are they actively engaged in the test? Yes/No

10.3 They wrote notes on the paper sheets showing that they tried to solve the operations Yes/No

With the nuanced scores in listening and interacting, it will be possible to analyze whether great listeners / interactive / engaged students scored better than students behaving poorly in these categories, with the middle group separate.

The other answers will help categorize the variables concerning students' behaviors, so that we can model the correlations between students' characteristics and their impact on students' final learning.

If the answers from the following questions: 8.3: No / 9.4 <= 1 ; then the student is discarded from the experiment.

- **Initial Resources Available**

- Personal experience as a student / teacher

I already tried to think about the best approach for children to like Math and feel comfortable with it. I personally feel comfortable with it and would like to also understand why.

I gave Math classes to help students (homework, etc.) and I participated in a Math online classes project (on adaptive feedback).

- Articles/References: standards

Article on Math 2nd graders:
https://www.education.com/magazine/article/Math_for_Second_Graders/

Common Core State Standards for Math: CCSSI_MathStandards.pdf

Doug Clement Research-based instructions for young children / And related papers:

Why a Colorado Researcher Believes Preschoolers Should Learn--and Play with--Math, July 14, 2017 *Chalkbeat*

Five Compelling Reasons to Teach Spatial Reasoning to Young Children, January 17, 2017, *KQED MindShift*

Commentary: Good Math Skills Begin at Home, January 2, 2017, *The Philadelphia Inquirer*

Turns Out, Counting on Your Fingers Makes You Smarter, October 23, 2016, *Wall Street Journal*

STEM starts early: Grounding science, technology, engineering, and math education in early childhood, Elisabeth R. McClure, Lisa Guernsey, Douglas H. Clements, Susan Nall Bales, Jennifer Nichols, Nat Kendall-Taylor, Michael H. Levine, 2017

Children developmental levels at 6-8 years old:

https://www.scholastic.com/parents/family-life/creativity-and-critical-thinking/development-milestones/cognitive-development-6-7-year-olds.html#:~:text=At%20about%20the%20age%20of,are%20about%2012%20years%20old.&text=According%20to%20his%20work%2C%20children's,the%20same%20timeframe%20without%20it.

- Educational content for this domain.

Multiple educational material for Math education in 2nd grade:

- worksheets: https://www.education.com/worksheets/second-grade/math/, varied and neutral, with the raw material to teach, useful.
- A lot of game ideas, in online games with pictured characters, animals, counting etc. Games use clocks, coins, counters, 3D shapes, etc. https://www.weareteachers.com/second-grade-math-games/

- Real Math: Building Blocks Edition 0 BuildingBlocksTrajectoriesWithRef.pdf

There are interesting ideas about real-world activities. It is important to remind that we must focus on aligned assessment and instruction and not rely on teacher focused activities.

- Teacher help

Thanks to my instructors, I had the chance to discuss with Holly Blinzman, a kindergarten teacher.

She gave me interesting ideas on assessment and instruction, as well as on class conduction and context for young children.