Pattern recognition system
Exercise Project

Report Assignment n°1

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I

Let us calculate $P(S_t = j)\ \forall j \in \{1,2\}$ and for t = 1,2,3,...

We will do it $\forall t \in \{1..3\}$ and notice that this is constant $\forall t$.

— For t = 1, the probabilities are given by the matrix q_j. Thus, $P(S_1 = 1) = 0.75$ and $P(S_1 = 2) = 0.25$.

$\forall t \geq 2$, we will use the following formula :

— $P(S_t = j) = \sum_{i=1}^{2} P(S_t = j, S_{t-1} = i) = \sum_{i=1}^{2} P(S_t = j | S_{t-1} = i) P(S_{t-1} = i)$

For the case t = 2, we have :

— $P(S_t = j) = \sum_{i=1}^{2} a_{ij} q_i$

We thus obtain $P(S_2 = 1) = 0.75$ and $P(S_2 = 2) = 0.25$. We immediately notice that

$$\forall i\ P(S_2 = i) = P(S_1 = i)$$

. By recurrency, we have $\forall t\ \Pr(S_t = j)$ constant.

II

After we generated 10 000 state integers, we found the following probabilities :

$$\Pr(S_t = 1) = 0.7525\ and\ \Pr(S_t = 2) = 0.2475$$

III @HMM/rand

III.1 Theorical calculation

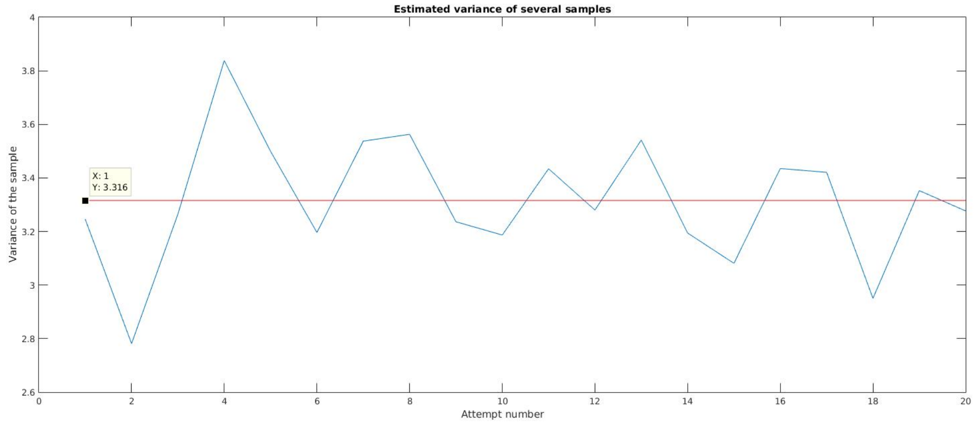
Let us now calculate $E[X_t]$ and $Var[X_t]$.

— $E[X_t]$: the book gives the formula $E[X] = E_S[E_X[X|S]]$, and according to the two different possible values of S, X density probability function is either b_1 or b_2.
Then, for j = 1, $E[X|S] = \mu_1$; and for j = 2, $E[X|S] = \mu_2$.
$E[X] = P(S = 1) * \mu_1 + P(S = 2) * \mu_2 = 0 + 3 * 0.25 = 0.75$.

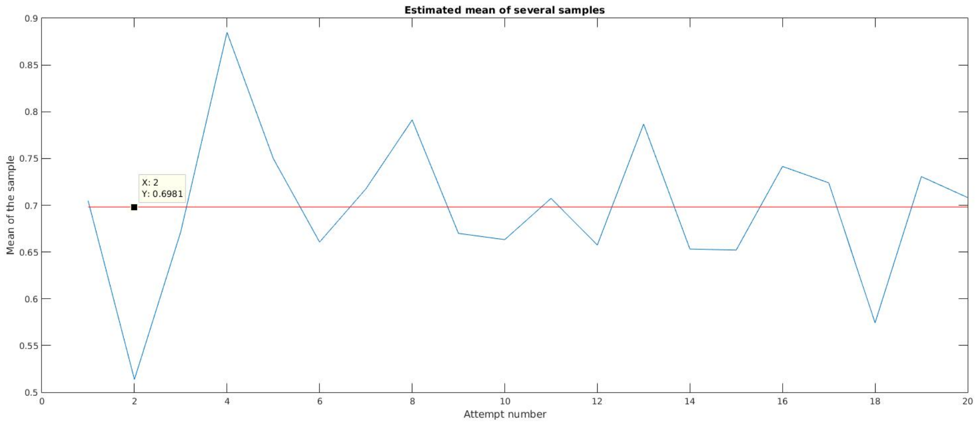
— $Var[X_t]$: according to the book, $Var[X_t] = E_S[Var_X[X|S]] + Var_S[E_X[X|S]]$.
Thanks to the same observation as before, the expression becomes :
$Var[X_t] = [0.75 * \sigma_1 + 0.25 * \sigma_2] + [0.75 * \mu_1^2 + 0.25 * \mu_2^2 - 0.75^2] = 1 + 0.25 * 9 - 0.75^2 = 2.6875$.

III.2 Measures

For these measures, we use the section "Test @HMM/rand" in the file main.m As we can see on the figures 1a and 1b the experimental values are close from the theoretical ones.



(a) $Var(X) \simeq 3.316$



(b) $E(X) \simeq 0.6981$

FIGURE 1 – Validation of @HMM/rand
Blue : Plot of the 20 attempts
Red : Mean over the 20 attempts

IV HMM behavior

For this part, we changed the behavior of the @HMM/rand function, in order to get a vector from b_1 (not a scalar like the previous question). So here, each sample is a vector $x_t \sim N(\mu_j, \sigma_j^2)$ (where j=1 or 2).

The following figure shows the vector ploted for different values of t.

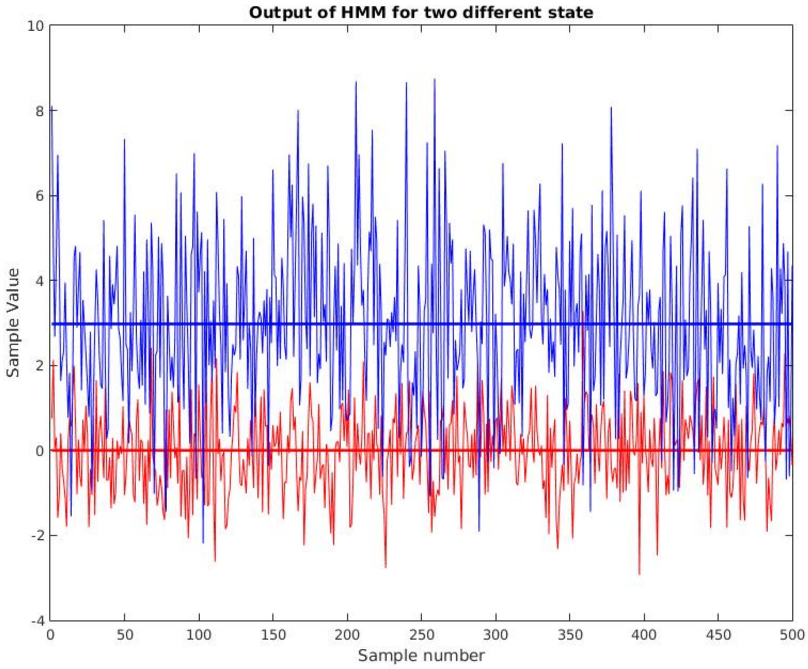


FIGURE 2 – Blue : Sample at a time t_1. $\mu = -0.0065$ and $\sigma^2 = 0.985$
Red : Sample at a time t_2. $\mu = 2.9759$ and $\sigma^2 = 3.8953$
The width lines represent the mean of the samples

Note

It is easy to find out in which state the system was when the sample was produced. On the figure 2 the samples have very different means (0 et 3) et very different variance.