
Pattern recognition system

Exercise Project

REPORT ASSIGNMENT N°1

Etudiant

Antoine HONORÉ
honore@kth.se

Etudiant

Audrey BROUARD
brouard@kth.se



I

Let us calculate $P(S_t = j) \forall j \in \{1, 2\}$ and for $t = 1, 2, 3, \dots$

We will do it $\forall t \in \{1..3\}$ and notice that this is constant $\forall t$.

— For $t = 1$, the probabilities are given by the matrix q_j . Thus, $P(S_1 = 1) = 0.75$ and $P(S_1 = 2) = 0.25$.

$\forall t \geq 2$, we will use the following formula :

$$— P(S_t = j) = \sum_{i=1}^2 P(S_t = j, S_{t-1} = i) = \sum_{i=1}^2 P(S_t = j | S_{t-1} = i) P(S_{t-1} = i)$$

For the case $t = 2$, we have :

$$— P(S_t = j) = \sum_{i=1}^2 a_{ij} q_i$$

We thus obtain $P(S_2 = 1) = 0.75$ and $P(S_2 = 2) = 0.25$. We immediately notice that

$$\forall i \ P(S_2 = i) = P(S_1 = i)$$

. By recurrency, we have $\forall t \ \Pr(S_t = j)$ constant.

II

After we generated 10 000 state integers, we found the following probabilities :

$$\Pr(S_t = 1) = 0.7525 \text{ and } \Pr(S_t = 2) = 0.2475$$

III @HMM/rand

III.1 Theoretical calculation

Let us now calculate $E[X_t]$ and $Var[X_t]$.

— $E[X_t]$: the book gives the formula $E[X] = E_S[E_X[X|S]]$, and according to the two different possible values of S , X density probability function is either b_1 or b_2 .

Then, for $j = 1$, $E[X|S] = \mu_1$; and for $j = 2$, $E[X|S] = \mu_2$.

$$E[X] = P(S = 1) * \mu_1 + P(S = 2) * \mu_2 = 0 + 3 * 0.25 = 0.75.$$

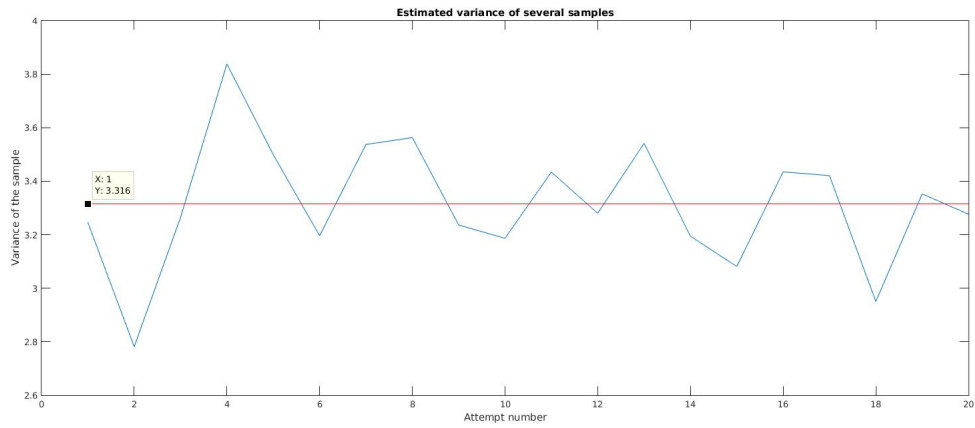
— $Var[X_t]$: according to the book, $Var[X_t] = E_S[Var_X[X|S]] + Var_S[E_X[X|S]]$.

Thanks to the same observation as before, the expression becomes :

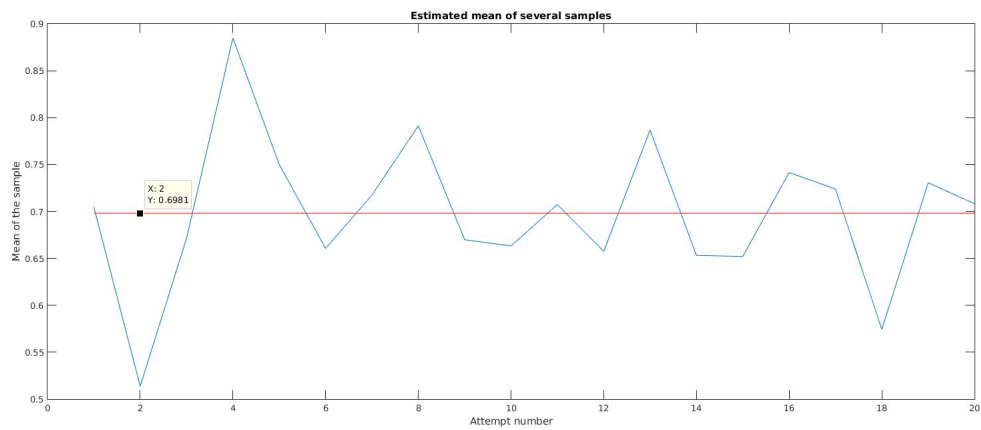
$$Var[X_t] = [0.75 * \sigma_1 + 0.25 * \sigma_2] + [0.75 * \mu_1^2 + 0.25 * \mu_2^2 - 0.75^2] = 1 + 0.25 * 9 - 0.75^2 = 2.6875.$$

III.2 Measures

For these measures, we use the section “Test @HMM/rand” in the file main.m As we can see on the figures 1a and 1b the experimental values are close from the theoretical ones.



(a) $Var(X) \simeq 3.316$



(b) $E(X) \simeq 0.6981$

FIGURE 1 – Validation of @HMM/rand

Blue : Plot of the 20 attempts
 Red : Mean over the 20 attempts