AIFS Lecture 3: Solutions to Ax=b

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Overview

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Linear Combination

Definition

Linear Combination: Given a set of vectors, $B = \{\mathbf{v_1}, \ldots, \mathbf{v_n}\}$, its linear combination is a vector $\mathbf{v}'$, given by multiplying each vector by some scalar, a_i and summing the results.

$$\mathbf{v}' = \sum_{i=1}^{n} a_i \mathbf{v_i}$$

Linear (In)dependence

Definition

Linear (In)dependence: A set of vectors, $B = \{\mathbf{v_1}, \ldots, \mathbf{v_n}\}$, is said to be *linearly independent* if and only if no individual vector can be expressed as a linear combination of the others. Correspondingly, the set of vectors is said to be *linearly dependent* if there exists some set of scalars $\{a_1, \ldots, a_n\}$ such that some vector $\mathbf{v_k}$, $k \in \{1, \ldots, n\}$, can be expressed as,

$$\mathbf{v_k} = \sum_{i=1, i \neq k}^{n} a_i \mathbf{v_i}$$

Alternatively, the set of vectors, B is said to be linearly dependent if there exists some non-trivial set of scalars $\{a_1, \ldots, a_n\}$ such that,

$$\sum_{i=1}^{n} a_i \mathbf{v_i} = 0$$

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Span

Definition

Span: The span of a set of vectors, $B = \{\mathbf{v_1}, \ldots, \mathbf{v_n}\}$, is the space formed by all possible finite linear combinations of B.

$$span(B) = \left\{ \sum_{i=1}^{n} a_i \mathbf{v_i} \;\middle|\; n \in \mathbb{N} \wedge \mathbf{v_i} \in B \wedge a_i \in \mathbb{R} \right\}$$

Basis

Definition

Basis: The *Basis* of a vector space, V, is a set of vectors, B, such that it can uniquely span the space. The elements of a basis are called *basis vectors*.