AIFS ML Lecture 4: Machine Learning Basics

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Overview

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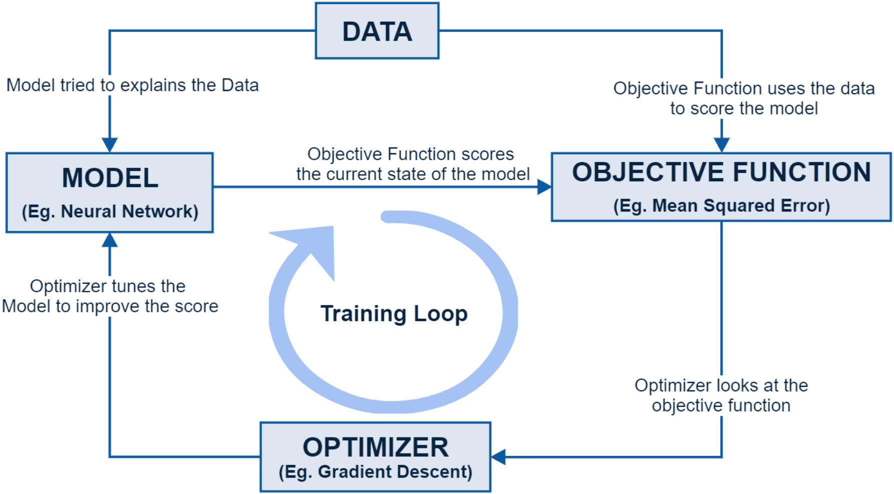
Recap - Training Loop



Recap - Learning Styles

- Based on Supervision
  - **Supervised:** The dataset provides a supervisory signal in the form of a label.
  - **Unsupervised:** No explicit prediction or inference, the goal of the algorithm is to extract patterns about the underlying data generating process.
  - **Reinforcement:** The science of sequential decision making for artificial agents, such that the decisions taken by the agent in some environment maximizes a notion of cumulative reward.
- Based on Mode of Consuming Data
  - **Online:** During each iteration of the training loop, the learning algorithm processes a small subset of the dataset called a mini-batch. This allows the algorithm to learn from new data on-the-fly.
  - **Batch:** During each iteration of the training loop, the learning algorithm processes the entire dataset. These algorithms cannot learn incrementally from new data.

Recap - Learning Styles

- Based on the Type of Model
  - **Non-parametric Model:** Memory-based models whose structure is determined from the data rather than specified apriori. They cannot be parameterized by a finite number of parameters.
  - **Parametric Model:** An explicit mathematical model with finite adaptable parameters for modelling the data generating process. The model is trained to fit the data by adjusting the parameter values.

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  - **Optimizer:** Closed Form Solution

$$\mathbf{w}^* = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\mathbf{y}$$

Data

- Each record (data-point) in the dataset is represented as a row in a matrix $\boldsymbol{X}$
- If there is N records each having D features, the matrix $\boldsymbol{X}$ has shape $N \times D$

$$\boldsymbol{X} = \begin{bmatrix} \leftarrow & \mathbf{x}_1 & \rightarrow \\ \leftarrow & \mathbf{x}_2 & \rightarrow \\ & \vdots & \\ \leftarrow & \mathbf{x}_N & \rightarrow \end{bmatrix}_{N \times D}$$

- Each record $\mathbf{x}_i$ has D elements, which corresponds to the D features of a data-point.

Model

- As the name suggests, *linear* regression uses a model that is both linear in the parameters and inputs.

$$\hat{y} = w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots w_D x_D + b$$

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- The term b is the y-intercept, or often called as the bias.
- If b is written as $w_0 x_0$ where x_0 is set to 1, then:

$$\hat{y} = w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots w_D x_D + w_0 x_0$$

Or more succinctly,

$$\hat{y} = \mathbf{w}^T \mathbf{x}$$

Where,

$\mathbf{x} = (x_1, \dots, x_D, x_0)$

$\mathbf{w} = (w_1, \dots, w_D, w_0)$

Loss Function

- When we use **mean squared-error** (MSE) to measure the loss function, linear regression is called ordinary least squares (OLS).
- The "error" in Mean squared-error is the Euclidean distance between the true label and the prediction of the model. For data-point $\mathbf{x}_i$ the prediction is $\hat{y}_i = \mathbf{w}^T\mathbf{x}_i$, so the error is:

$$\hat{y}_i - y_i$$

  Where, y_i is the true label (supervisor signal) from the dataset.
- So the mean of all the squared-errors is given by,

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)$$

$$\text{MSE} = \frac{1}{N}||\hat{\mathbf{y}} - \mathbf{y}||_2^2$$

  Where,

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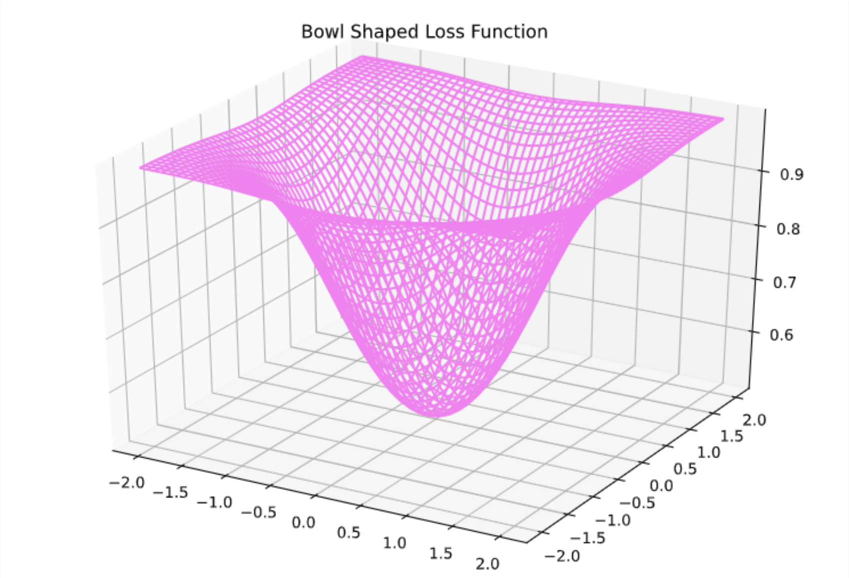
Loss function Illustration



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- Putting $\mathbf{w}^\star$ in the model gives you the linear regression solution.

$$\hat{y} = {\mathbf{w}^\star}^T \mathbf{x}$$