Gain in Multi-mode Simulations

Zachary Ziegler

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Motivation

- Gain competition is a crucial intermodal interaction process in fully multimode cavities
  - Especially for the cases we've seen experimentally and numerically where HOM mode-locking occurs.

- If gain competition is not modeled in a way that takes into account the spatial dimensions, we don't expect the simulation to be even qualitatively accurate.

Saturating gain

$$\frac{\partial}{\partial z} A(x,y,z,\omega) = \frac{\frac{g}{2} f(\omega)}{1+\frac{\int dt \left| A(x,y,z,t) \right|^2}{I_{sat}}} A(x,y,z,\omega)$$

Spatially dependent saturation

Modal decomposition: $$A(x,y,z,\omega) = \sum_n F_n(x,y) A_n(\omega,z)$$

Inner product: $$\int F_n(x,y) F_p(x,y) \, dx \, dy = \delta_{np}$$

$$\sum_n F_n(x,y) \frac{\partial}{\partial z} A_n(\omega,z) = \frac{\frac{g}{2} f(\omega)}{1+\sum_{l,m} F_l(x,y) F_m(x,y) \frac{\int dt \, A_l^*(t,z) A_m(t,z)}{I_{sat}}} \sum_n F_n(x,y) A_n(\omega,z)$$

Taylor expansion

$$\sum_n F_n \partial_z A_n \approx \frac{g}{2} f \sum_n F_n A_n - \frac{g}{2} f \sum_{l,m,n} F_l F_m F_n A_n B_{lm} \qquad B_{lm} \equiv \frac{\int dt\, A_l^* A_m}{I_{sat}}$$

Apply inner product: $\int dx\, dy\, F_p(x,y)$

$$\partial_z A_p \approx \frac{g}{2} f A_p - \frac{g}{2} f \sum_{l,m,n} S_{plmn}^R B_{lm} A_n \qquad S_{plmn}^R \equiv \int dx\, dy\, F_p F_l F_m F_n$$

In matrix form:

$$\begin{bmatrix} \partial_z A_1 \\ \partial_z A_2 \\ \partial_z A_3 \end{bmatrix} \approx \begin{bmatrix} 1 - \sum_{l,m} S_{1lm1}^R B_{lm} & \sum_{l,m} S_{1lm2}^R B_{lm} & \sum_{l,m} S_{1lm3}^R B_{lm} \\ \sum_{l,m} S_{2lm1}^R B_{lm} & 1 - \sum_{l,m} S_{2lm2}^R B_{lm} & \sum_{l,m} S_{2lm3}^R B_{lm} \\ \sum_{l,m} S_{3lm1}^R B_{lm} & \sum_{l,m} S_{3lm2}^R B_{lm} & 1 - \sum_{l,m} S_{3lm3}^R B_{lm} \end{bmatrix} \begin{bmatrix} \frac{g}{2} f A_1 \\ \frac{g}{2} f A_2 \\ \frac{g}{2} f A_3 \end{bmatrix}$$

Taylor expansion

$$\begin{bmatrix} \partial_z A_1 \\ \partial_z A_2 \\ \partial_z A_3 \end{bmatrix} \approx \begin{bmatrix} 1-\sum_{l,m} S^R_{1lm1} B_{lm} & \sum_{l,m} S^R_{1lm2} B_{lm} & \sum_{l,m} S^R_{1lm3} B_{lm} \\ \sum_{l,m} S^R_{2lm1} B_{lm} & 1-\sum_{l,m} S^R_{2lm2} B_{lm} & \sum_{l,m} S^R_{2lm3} B_{lm} \\ \sum_{l,m} S^R_{3lm1} B_{lm} & \sum_{l,m} S^R_{3lm2} B_{lm} & 1-\sum_{l,m} S^R_{3lm3} B_{lm} \end{bmatrix} \begin{bmatrix} \frac{g}{2} f A_1 \\ \frac{g}{2} f A_2 \\ \frac{g}{2} f A_3 \end{bmatrix}$$

The S tensor is calculated ahead of time, the B tensor is calculated at each step, the matrix multiplication is applied to get the coupled differential equations, and the equations are solved numerically, e.g.

$$\frac{\partial \vec{A}}{\partial z} = T \frac{g}{2} f \vec{A} \quad \rightarrow \quad \vec{A}(z+\Delta z) = \vec{A}(z) + \Delta z\, T \frac{g}{2} f \vec{A}(z)$$

The expansion's accuracy relies on the off diagonal elements being small. In ALL our lasers that is necessarily violated because the large signal gain is limited by the gain saturation.

Exact solution

$$\frac{\partial A(x,y,z,\omega)}{\partial z}\left(1+\frac{\int dt\left|A(x,y,z,t)\right|^2}{I_{sat}}\right)=\frac{g}{2}f(\omega)A(x,y,z,\omega)$$

Decompose into modes: $A(x,y,z,\omega)=\sum_n F_n(x,y)A_n(\omega,z)$

$$\sum_n F_n\frac{\partial A_n}{\partial z}+\sum_{l,m,n}F_l F_m F_n\frac{\partial A_n}{\partial z}\frac{\int dt\,A_l^* A_m}{I_{sat}}=\frac{g}{2}f\sum_n F_n A_n$$

Apply inner product: $\int dx\,dy\,F_p(x,y)$

$$\frac{\partial A_p}{\partial z}+\sum_{l,m,n}S^R_{plmn}\frac{\partial A_n}{\partial z}B_{lm}=\frac{g}{2}f\,A_p$$

Exact solution

$$\frac{\partial A_p}{\partial z} + \sum_{l,m,n} S^R_{plmn} \frac{\partial A_n}{\partial z} B_{lm} = \frac{g}{2} f A_p$$

In matrix form:

$$\begin{bmatrix} 1+\sum_{l,m} S^R_{1lm1} B_{lm} & \sum_{l,m} S^R_{1lm2} B_{lm} & \sum_{l,m} S^R_{1lm3} B_{lm} \\ \sum_{l,m} S^R_{2lm1} B_{lm} & 1+\sum_{l,m} S^R_{2lm2} B_{lm} & \sum_{l,m} S^R_{2lm3} B_{lm} \\ \sum_{l,m} S^R_{3lm1} B_{lm} & \sum_{l,m} S^R_{3lm2} B_{lm} & 1+\sum_{l,m} S^R_{3lm3} B_{lm} \end{bmatrix} \begin{bmatrix} \partial_z A_1 \\ \partial_z A_2 \\ \partial_z A_3 \end{bmatrix} = \begin{bmatrix} \frac{g}{2} f A_1 \\ \frac{g}{2} f A_2 \\ \frac{g}{2} f A_3 \end{bmatrix}$$

$$T \frac{\partial \vec{A}}{\partial z} = \frac{g}{2} f \vec{A} \quad \rightarrow \quad \vec{A}(z+\Delta z) = \vec{A}(z) + \Delta z \, T^{-1} \frac{g}{2} f \vec{A}(z)$$

Calculation of T matrix - O(modes^4)

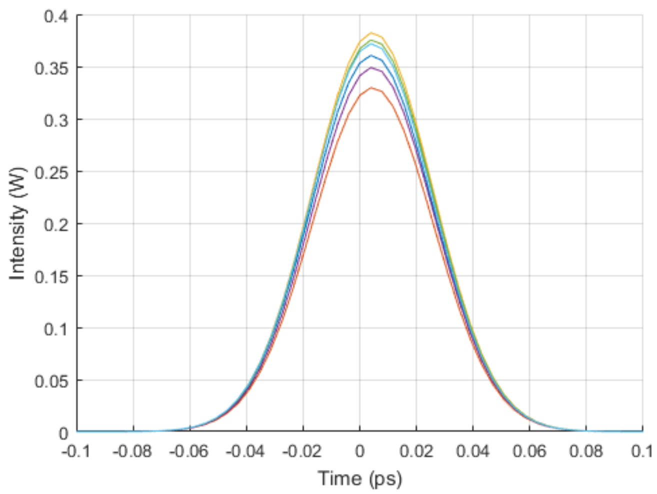
Inversion - O(modes^3)

Matrix multiplication - O(modes^2)

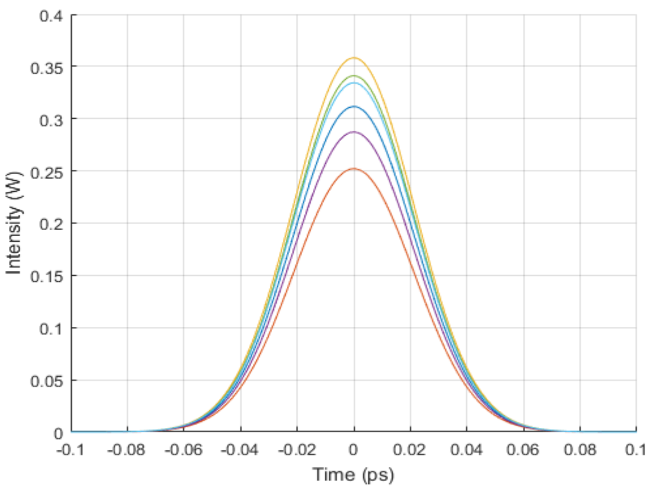
The overall computational complexity is the same as for the nonlinear term, and in fact it's doing almost exactly the same calculation so the slowdown of including this can be negligible

Results

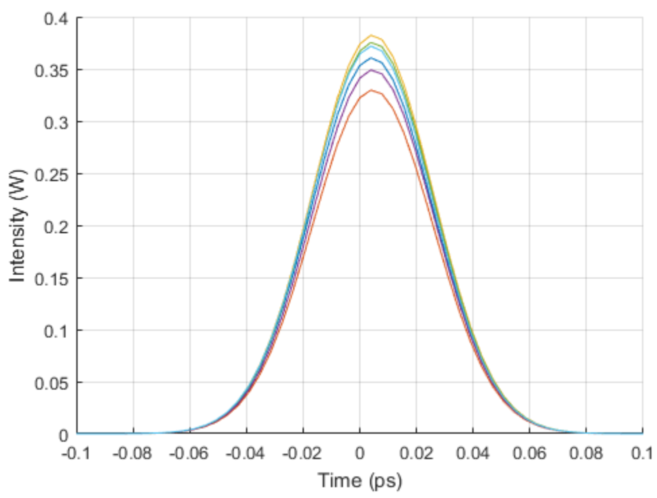
Single field “true” gain



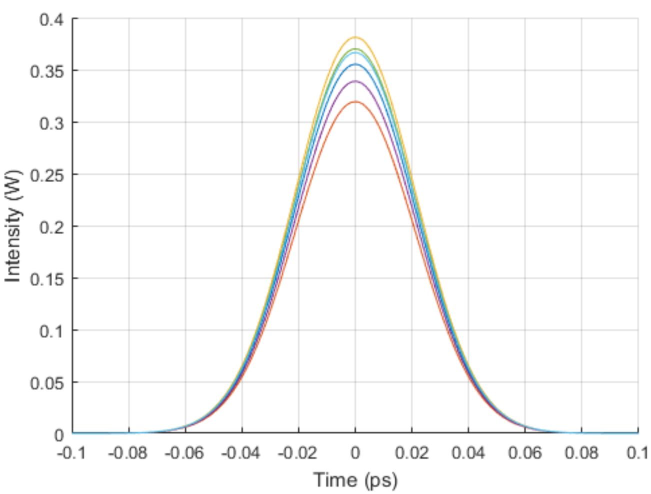
Taylor expansion mode resolved



Single field “true” gain



This solution mode resolved



Generalization to GMMNLSE

$$\frac{\partial}{\partial z} A(x,y,z,\omega) = \frac{\frac{g}{2} f(\omega)}{1 + \frac{\int dt \left| A(x,y,z,t) \right|^2}{I_{sat}}} A(x,y,z,\omega) + G(x,y,z,\omega,A)$$

$$\frac{\partial A(x,y,z,\omega)}{\partial z} \left(1 + \frac{\int dt \left| A(x,y,z,t) \right|^2}{I_{sat}} \right) = \frac{g}{2} f(\omega) A(x,y,z,\omega) + \left(1 + \frac{\int dt \left| A(x,y,z,t) \right|^2}{I_{sat}} \right) G(x,y,z,\omega,A)$$

There will be other terms here that are not exactly accounted for, but they will be small if the step size is smaller than all nonlinear lengths and a split-step approach is used:

1. $\vec{A}(z+\Delta z) = \vec{A}(z) + \Delta z\, T^{-1} \frac{g}{2} f\, \vec{A}(z)$
2. $\frac{\partial A_p}{\partial z} = D A_p + N A_p$