Advanced methods of differentiation

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The derivative of a function of a real variable measures the sensitivity to change of the function value (output value)with respect to a change in its argument (input value). The process of finding a derivative is called differentiation. The reverse process is called antidifferentiation. The fundamental theorem of calculus relates antidifferentiation with integration.Differentiation and integration constitute the two fundamental operations in single-variable calculus. So it it is very important totake a derivative fast and correct

Let's take the derivative of the following function:

$$y = x^5 - x^4 + \cos(\ln(x^2 - \frac{1}{\tan(x)})) + \sin(x^{x^x}) \cdot e^{x^{10} - \cos(x)} \tag{1}$$

Taking derivative of:

$$y = x \tag{2}$$

After long calculations we get:

$$\frac{dy}{dx} = 1 \tag{3}$$

I guess the differential of:

$$y = \cos(x) \tag{4}$$

Results in:

$$\frac{dy}{dx} = \sin(x) \cdot (-1) \cdot 1 \tag{5}$$

Taking derivative of:

$$y = x \tag{6}$$

After long calculations we get:

$$\frac{dy}{dx} = 1 \tag{7}$$

Taking derivative of:

$$y = x^{10} \tag{8}$$

Have value of:

$$\frac{dy}{dx} = 10 \cdot x^{10-1} \cdot 1 \tag{9}$$

Taking derivative of:

$$y = x^{10} - \cos(x) \tag{10}$$

Have value of:

$$\frac{dy}{dx} = 10 \cdot x^{10-1} \cdot 1 - \sin(x) \cdot (-1) \cdot 1 \tag{11}$$

The Differential of the function:

$$y = e^{x^{10} - \cos(x)} \tag{12}$$

Have value of:

$$\frac{dy}{dx} = e^{x^{10} - \cos(x)} \cdot (10 \cdot x^{10-1} \cdot 1 - \sin(x) \cdot (-1) \cdot 1) \tag{13}$$

Taking derivative of:

$$y = x \tag{14}$$

Results in:

$$\frac{dy}{dx} = 1 \tag{15}$$

The Differential of the function:

$$y = \ln(x) \tag{16}$$

Results in:

$$\frac{dy}{dx} = \frac{1}{x} \tag{17}$$

The Differential of the function:

$$y = x \tag{18}$$

Have value of:

$$\frac{dy}{dx} = 1 \tag{19}$$

Taking derivative of:

$$y = \ln(x) \tag{20}$$

Results in:

$$\frac{dy}{dx} = \frac{1}{x} \tag{21}$$

The Differential of the function:

$$y = x \tag{22}$$

Have value of:

$$\frac{dy}{dx} = 1 \tag{23}$$

The Differential of the function:

$$y = x \cdot \ln(x) \tag{24}$$

After long calculations we get:

$$\frac{dy}{dx} = 1 \cdot \ln(x) + x \cdot \frac{1}{x} \tag{25}$$

Obviously that this mathematical function after differentiation

$$y = e^{x \cdot \ln(x)} \tag{26}$$

After long calculations we get:

$$\frac{dy}{dx} = e^{x \cdot \ln(x)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x}) \tag{27}$$

Obviously that this mathematical function after differentiation

$$y = x^x \tag{28}$$

Have value of:

$$\frac{dy}{dx} = e^{x \cdot \ln(x)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x}) \tag{29}$$

Taking derivative of:

$$y = x^x \cdot \ln(x) \tag{30}$$

Have value of:

$$\frac{dy}{dx} = e^{x \cdot \ln(x)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x}) \cdot \ln(x) + x^x \cdot \frac{1}{x} \tag{31}$$

I guess the differential of:

$$y = e^{x^x \cdot \ln(x)} \tag{32}$$

Results in:

$$\frac{dy}{dx} = e^{x^x \cdot \ln(x)} \cdot (e^{x \cdot \ln(x)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x}) \cdot \ln(x) + x^x \cdot \frac{1}{x}) \tag{33}$$

The Differential of the function:

$$y = x^{x^x} \tag{34}$$

After long calculations we get:

$$\frac{dy}{dx} = e^{x^x \cdot \ln(x)} \cdot (e^{x \cdot \ln(x)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x}) \cdot \ln(x) + x^x \cdot \frac{1}{x}) \tag{35}$$

Taking derivative of:

$$y = \sin(x^{x^x}) \tag{36}$$

After long calculations we get:

$$\frac{dy}{dx} = \cos(x^{x^x}) \cdot e^{x^x \cdot \ln(x)} \cdot (e^{x \cdot \ln(x)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x}) \cdot \ln(x) + x^x \cdot \frac{1}{x}) \tag{37}$$

Obviously that this mathematical function after differentiation

$$y = \sin(x^{x^x}) \cdot e^{x^{10} - \cos(x)} \tag{38}$$

Results in:

$$\begin{aligned} \frac{dy}{dx} &= \cos(x^{x^x}) \cdot e^{x^x \cdot \ln(x)} \cdot (e^{x \cdot \ln(x)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x}) \cdot \ln(x) + x^x \cdot \frac{1}{x}) \\ &\cdot e^{x^{10} - \cos(x)} + \sin(x^{x^x}) \cdot e^{x^{10} - \cos(x)} \cdot (10 \cdot x^{10-1} \cdot 1 - \sin(x) \cdot (-1) \cdot 1) \end{aligned} \tag{39}$$

Obviously that this mathematical function after differentiation

$$y = x \tag{40}$$

After long calculations we get:

$$\frac{dy}{dx} = 1 \tag{41}$$

Obviously that this mathematical function after differentiation

$$y = \tan(x) \tag{42}$$

Results in:

$$\frac{dy}{dx} = \frac{1}{(\cos(x))^2} \tag{43}$$

The Differential of the function:

$$y = 1 \tag{44}$$

After long calculations we get:

$$\frac{dy}{dx} = 0 \tag{45}$$

I guess the differential of:

$$y = \frac{1}{\tan(x)} \tag{46}$$

Results in:

$$\frac{dy}{dx} = \frac{0 \cdot \tan(x) - 1 \cdot \dfrac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)} \tag{47}$$

The Differential of the function:

$$y = x \tag{48}$$

Have value of:

$$\frac{dy}{dx} = 1 \tag{49}$$

Obviously that this mathematical function after differentiation

$$y = x^2 \tag{50}$$

After long calculations we get:

$$\frac{dy}{dx} = 2 \cdot x^{2-1} \cdot 1 \tag{51}$$

The Differential of the function:

$$y = x^2 - \frac{1}{\tan(x)} \tag{52}$$

After long calculations we get:

$$\frac{dy}{dx} = 2 \cdot x^{2-1} \cdot 1 - \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)} \tag{53}$$

Taking derivative of:

$$y = \ln(x^2 - \frac{1}{\tan(x)}) \tag{54}$$

Results in:

$$\frac{dy}{dx} = \frac{2 \cdot x^{2-1} \cdot 1 - \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 - \frac{1}{\tan(x)}} \tag{55}$$

Taking derivative of:

$$y = \cos(\ln(x^2 - \frac{1}{\tan(x)})) \tag{56}$$

Have value of:

$$\frac{dy}{dx} = \sin(\ln(x^2 - \frac{1}{\tan(x)})) \cdot (-1) \cdot \frac{2 \cdot x^{2-1} \cdot 1 - \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 - \frac{1}{\tan(x)}} \tag{57}$$

Obviously that this mathematical function after differentiation

$$y = x \tag{58}$$

Have value of:

$$\frac{dy}{dx} = 1 \tag{59}$$

I guess the differential of:

$$y = x^4 \tag{60}$$

After long calculations we get:

$$\frac{dy}{dx} = 4 \cdot x^{4-1} \cdot 1 \tag{61}$$

Taking derivative of:

$$y = x \tag{62}$$

Have value of:

$$\frac{dy}{dx} = 1 \tag{63}$$

I guess the differential of:

$$y = x^5 \tag{64}$$

After long calculations we get:

$$\frac{dy}{dx} = 5 \cdot x^{5-1} \cdot 1 \tag{65}$$

Obviously that this mathematical function after differentiation

$$y = x^5 - x^4 \tag{66}$$

Results in:

$$\frac{dy}{dx} = 5 \cdot x^{5-1} \cdot 1 - 4 \cdot x^{4-1} \cdot 1 \tag{67}$$

I guess the differential of:

$$y = x^5 - x^4 + \cos(\ln(x^2 - \frac{1}{\tan(x)})) \tag{68}$$

Results in:

$$\frac{dy}{dx} = 5 \cdot x^{5-1} \cdot 1 - 4 \cdot x^{4-1} \cdot 1 + \sin(\ln(x^2 - \frac{1}{\tan(x)})) \cdot (-1) \cdot \frac{2 \cdot x^{2-1} \cdot 1 - \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 - \frac{1}{\tan(x)}} \tag{69}$$

Obviously that this mathematical function after differentiation

$$y = x^5 - x^4 + \cos(\ln(x^2 - \frac{1}{\tan(x)})) + \sin(x^{x^x}) \cdot e^{x^{10} - \cos(x)} \tag{70}$$

Results in:

$$\begin{aligned}\frac{dy}{dx} = {} & 5 \cdot x^{5-1} \cdot 1 - 4 \cdot x^{4-1} \cdot 1 + \sin(\ln(x^2 - \frac{1}{\tan(x)})) \cdot (-1) \cdot \frac{2 \cdot x^{2-1} \cdot 1 - \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 - \frac{1}{\tan(x)}} \\ & + \cos(x^{x^x}) \cdot e^{x^x \cdot \ln(x)} \cdot (e^{x \cdot \ln(x)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x}) \cdot \ln(x) + x^x \cdot \frac{1}{x}) \\ & \cdot e^{x^{10} - \cos(x)} + \sin(x^{x^x}) \cdot e^{x^{10} - \cos(x)} \cdot (10 \cdot x^{10-1} \cdot 1 - \sin(x) \cdot (-1) \cdot 1)\end{aligned} \tag{71}$$

Now simplify the following:

$$y' = 5 \cdot x^{5-1} \cdot 1 - 4 \cdot x^{4-1} \cdot 1 + \sin(\ln(x^2 - \frac{1}{\tan(x)})) \cdot (-1) \cdot \frac{2 \cdot x^{2-1} \cdot 1 - \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 - \frac{1}{\tan(x)}}$$
$$+ \cos(x^{x^x}) \cdot e^{x^x \cdot \ln(x)} \cdot (e^{x \cdot \ln(x)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x}) \cdot \ln(x) + x^x \cdot \frac{1}{x})$$
$$\cdot e^{x^{10} - \cos(x)} + \sin(x^{x^x}) \cdot e^{x^{10} - \cos(x)} \cdot (10 \cdot x^{10-1} \cdot 1 - \sin(x) \cdot (-1) \cdot 1) \quad (72)$$

No need to tell you that:

$$y = 5 - 1 \quad (73)$$

Equals to:

$$y = 4 \quad (74)$$

No need to tell you that:

$$y = 5 \cdot x^4 \cdot 1 \quad (75)$$

Have the same value as:

$$y = 5 \cdot x^4 \quad (76)$$

No need to tell you that:

$$y = 4 - 1 \quad (77)$$

Have the same value as:

$$y = 3 \quad (78)$$

Easy to notice that:

$$y = 4 \cdot x^3 \cdot 1 \quad (79)$$

Equals to:

$$y = 4 \cdot x^3 \quad (80)$$

Easy to notice that:

$$y = 2 - 1 \quad (81)$$

Like:

$$y = 1 \quad (82)$$

It's obvious to a 3rd grade that:

$$y = x^1 \quad (83)$$

Like:

$$y = x \tag{84}$$

No need to tell you that:

$$y = 2 \cdot x \cdot 1 \tag{85}$$

Is 100% the same to the:

$$y = 2 \cdot x \tag{86}$$

Easy to notice that:

$$y = 0 \cdot \tan(x) \tag{87}$$

Is 100% the same to the:

$$y = 0 \tag{88}$$

Nothing is easier that understanding that:

$$y = 1 \cdot \frac{1}{(\cos(x))^2} \tag{89}$$

Have the same value as:

$$y = \frac{1}{(\cos(x))^2} \tag{90}$$

Nothing is easier that understanding that:

$$y = 0 - \frac{1}{(\cos(x))^2} \tag{91}$$

Have the same value as:

$$y = \frac{1}{(\cos(x))^2} \tag{92}$$

If your iq is more than 40 you'll get that:

$$y = 1 \cdot \ln(x) \tag{93}$$

Equals to:

$$y = \ln(x) \tag{94}$$

It's obvious to a 3rd grade that:

$$y = 10 - 1 \tag{95}$$

Have the same value as:

$$y = 9 \tag{96}$$

It's obvious to a 3rd grade that:

$$y = 10 \cdot x^9 \cdot 1 \tag{97}$$

Like:

$$y = 10 \cdot x^9 \tag{98}$$

It's obvious to a 3rd grade that:

$$y = \sin(x) \cdot (-1) \cdot 1 \tag{99}$$

Is 100% the same to the:

$$y = \sin(x) \cdot (-1) \tag{100}$$

The final derivative of the given function is:

$$y' = 5 \cdot x^4 - 4 \cdot x^3 + \sin(\ln(x^2 - \frac{1}{\tan(x)})) \cdot (-1) \cdot \frac{2 \cdot x - \frac{\frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 - \frac{1}{\tan(x)}} + \cos(x^{x^x}) \cdot e^{x^x \cdot \ln(x)} \cdot (e^{x \cdot \ln(x)} \cdot (\ln(x) + x \cdot \frac{1}{x}) \cdot \ln(x) + x^x \cdot \frac{1}{x}) \cdot e^{x^{10} - \cos(x)} + \sin(x^{x^x}) \cdot e^{x^{10} - \cos(x)} \cdot (10 \cdot x^9 - \sin(x) \cdot (-1)) \tag{101}$$

Materials used in my report:

1. Redkozubov's conspects and lections
2. "Collection of problems in mathematical analysis" by Kudriavcev L.D.

My github repository

Special thanks to Vasenin Egor for helping me create this report in LaTex