Advanced methods of differentiation

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The derivative of a function of a real variable measures the sensitivity to change of the function value (output value)with respect to a change in its argument (input value). The process of finding a derivative is called differentiation. The reverse process is called antidifferentiation. The fundamental theorem of calculus relates antidifferentiation with integration.Differentiation and integration constitute the two fundamental operations in single-variable calculus. So it it is very important totake a derivative fast and correct

Let's take the derivative of the following function:

$$y = x^5 - x^4 + \sin(\ln(x^2 + \frac{1}{\tan(x)})) - (e^x + 3^x) \cdot \cos(x^{10}) \tag{1}$$

Taking derivative of:

$$y = x \tag{2}$$

Results in:

$$\frac{dy}{dx} = 1 \tag{3}$$

I guess the differential of:

$$y = x^{10} \tag{4}$$

Have value of:

$$\frac{dy}{dx} = 10 \cdot x^{10-1} \cdot 1 \tag{5}$$

Taking derivative of:

$$y = \cos(x^{10}) \tag{6}$$

Results in:

$$\frac{dy}{dx} = \sin(x^{10}) \cdot (-1) \cdot 10 \cdot x^{10-1} \cdot 1 \tag{7}$$

I guess the differential of:

$$y = x \tag{8}$$

After long calculations we get:

$$\frac{dy}{dx} = 1 \tag{9}$$

The Differential of the function:

$$y = 3^x \tag{10}$$

Have value of:

$$\frac{dy}{dx} = 3^x \cdot \ln(3) \cdot 1 \tag{11}$$

The Differential of the function:

$$y = x \tag{12}$$

Results in:

$$\frac{dy}{dx} = 1 \tag{13}$$

The Differential of the function:

$$y = e^x \tag{14}$$

Have value of:

$$\frac{dy}{dx} = e^x \cdot 1 \tag{15}$$

Obviously that this mathematical function after differentiation

$$y = e^x + 3^x \tag{16}$$

Results in:

$$\frac{dy}{dx} = e^x \cdot 1 + 3^x \cdot \ln(3) \cdot 1 \tag{17}$$

I guess the differential of:

$$y = (e^x + 3^x) \cdot \cos(x^{10}) \tag{18}$$

Have value of:

$$\frac{dy}{dx} = (e^x \cdot 1 + 3^x \cdot \ln(3) \cdot 1) \cdot \cos(x^{10}) + (e^x + 3^x) \cdot \sin(x^{10}) \cdot (-1) \cdot 10 \cdot x^{10-1} \cdot 1 \tag{19}$$

The Differential of the function:

$$y = x \tag{20}$$

Results in:

$$\frac{dy}{dx} = 1 \tag{21}$$

Obviously that this mathematical function after differentiation

$$y = \tan(x) \tag{22}$$

After long calculations we get:

$$\frac{dy}{dx} = \frac{1}{(\cos(x))^2} \tag{23}$$

Taking derivative of:

$$y = 1 \tag{24}$$

After long calculations we get:

$$\frac{dy}{dx} = 0 \tag{25}$$

The Differential of the function:

$$y = \frac{1}{\tan(x)} \tag{26}$$

After long calculations we get:

$$\frac{dy}{dx} = \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)} \tag{27}$$

Taking derivative of:

$$y = x \tag{28}$$

Have value of:

$$\frac{dy}{dx} = 1 \tag{29}$$

I guess the differential of:

$$y = x^2 \tag{30}$$

After long calculations we get:

$$\frac{dy}{dx} = 2 \cdot x^{2-1} \cdot 1 \tag{31}$$

The Differential of the function:

$$y = x^2 + \frac{1}{\tan(x)} \tag{32}$$

After long calculations we get:

$$\frac{dy}{dx} = 2 \cdot x^{2-1} \cdot 1 + \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)} \tag{33}$$

Obviously that this mathematical function after differentiation

$$y = \ln(x^2 + \frac{1}{\tan(x)}) \tag{34}$$

Have value of:

$$\frac{dy}{dx} = \frac{2 \cdot x^{2-1} \cdot 1 + \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 + \frac{1}{\tan(x)}} \tag{35}$$

The Differential of the function:

$$y = \sin(\ln(x^2 + \frac{1}{\tan(x)})) \tag{36}$$

Have value of:

$$\frac{dy}{dx} = \cos(\ln(x^2 + \frac{1}{\tan(x)})) \cdot \frac{2 \cdot x^{2-1} \cdot 1 + \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 + \frac{1}{\tan(x)}} \tag{37}$$

The Differential of the function:

$$y = x \tag{38}$$

After long calculations we get:

$$\frac{dy}{dx} = 1 \tag{39}$$

I guess the differential of:

$$y = x^4 \tag{40}$$

After long calculations we get:

$$\frac{dy}{dx} = 4 \cdot x^{4-1} \cdot 1 \tag{41}$$

I guess the differential of:

$$y = x \tag{42}$$

After long calculations we get:

$$\frac{dy}{dx} = 1 \tag{43}$$

Taking derivative of:

$$y = x^5 \tag{44}$$

Have value of:

$$\frac{dy}{dx} = 5 \cdot x^{5-1} \cdot 1 \tag{45}$$

Taking derivative of:

$$y = x^5 - x^4 \tag{46}$$

Results in:

$$\frac{dy}{dx} = 5 \cdot x^{5-1} \cdot 1 - 4 \cdot x^{4-1} \cdot 1 \tag{47}$$

I guess the differential of:

$$y = x^5 - x^4 + \sin(\ln(x^2 + \frac{1}{\tan(x)})) \tag{48}$$

After long calculations we get:

$$\frac{dy}{dx} = 5 \cdot x^{5-1} \cdot 1 - 4 \cdot x^{4-1} \cdot 1 + \cos(\ln(x^2 + \frac{1}{\tan(x)})) \cdot \frac{2 \cdot x^{2-1} \cdot 1 + \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 + \frac{1}{\tan(x)}} \tag{49}$$

I guess the differential of:

$$y = x^5 - x^4 + \sin(\ln(x^2 + \frac{1}{\tan(x)})) - (e^x + 3^x) \cdot \cos(x^{10}) \tag{50}$$

Results in:

$$\begin{split}\frac{dy}{dx} = 5 \cdot x^{5-1} \cdot 1 - 4 \cdot x^{4-1} \cdot 1 + \cos(\ln(x^2 + \frac{1}{\tan(x)})) \cdot \frac{2 \cdot x^{2-1} \cdot 1 + \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 + \frac{1}{\tan(x)}} \\ - (e^x \cdot 1 + 3^x \cdot \ln(3) \cdot 1) \cdot \cos(x^{10}) + (e^x + 3^x) \cdot \sin(x^{10}) \cdot (-1) \cdot 10 \cdot x^{10-1} \cdot 1\end{split} \tag{51}$$

Now simplify the following:

$$\begin{split}y' = 5 \cdot x^{5-1} \cdot 1 - 4 \cdot x^{4-1} \cdot 1 + \cos(\ln(x^2 + \frac{1}{\tan(x)})) \cdot \frac{2 \cdot x^{2-1} \cdot 1 + \frac{0 \cdot \tan(x) - 1 \cdot \frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 + \frac{1}{\tan(x)}} \\ - (e^x \cdot 1 + 3^x \cdot \ln(3) \cdot 1) \cdot \cos(x^{10}) + (e^x + 3^x) \cdot \sin(x^{10}) \cdot (-1) \cdot 10 \cdot x^{10-1} \cdot 1\end{split} \tag{52}$$

It's obvious to a 3rd grade that:

$$y = 5 - 1 \tag{53}$$

Is 100% the same to the:

$$y = 4 \tag{54}$$

No need to tell you that:

$$y = 5 \cdot x^4 \cdot 1 \tag{55}$$

Like:

$$y = 5 \cdot x^4 \tag{56}$$

It's obvious to a 3rd grade that:

$$y = 4 - 1 \tag{57}$$

Is 100% the same to the:

$$y = 3 \tag{58}$$

Nothing is easier that understanding that:

$$y = 4 \cdot x^3 \cdot 1 \tag{59}$$

Equals to:

$$y = 4 \cdot x^3 \tag{60}$$

It's obvious to a 3rd grade that:

$$y = 2 - 1 \tag{61}$$

Like:

$$y = 1 \tag{62}$$

Nothing is easier that understanding that:

$$y = x^1 \tag{63}$$

Like:

$$y = x \tag{64}$$

Easy to notice that:

$$y = 2 \cdot x \cdot 1 \tag{65}$$

Equals to:

$$y = 2 \cdot x \tag{66}$$

Even blind person sees that:

$$y = 0 \cdot \tan(x) \tag{67}$$

Like:

$$y = 0 \tag{68}$$

If your iq is more than 40 you'll get that:

$$y = 1 \cdot \frac{1}{(\cos(x))^2} \tag{69}$$

Is 100% the same to the:

$$y = \frac{1}{(\cos(x))^2} \tag{70}$$

Even blind person sees that:

$$y = 0 - \frac{1}{(\cos(x))^2} \tag{71}$$

Equals to:

$$y = \frac{1}{(\cos(x))^2} \tag{72}$$

Even blind person sees that:

$$y = e^x \cdot 1 \tag{73}$$

Have the same value as:

$$y = e^x \tag{74}$$

Nothing is easier that understanding that:

$$y = 3^x \cdot \ln(3) \cdot 1 \tag{75}$$

Like:

$$y = 3^x \cdot \ln(3) \tag{76}$$

Even blind person sees that:

$$y = 10 - 1 \tag{77}$$

Like:

$$y = 9 \tag{78}$$

Even blind person sees that:

$$y = 10 \cdot x^9 \cdot 1 \tag{79}$$

Like:

$$y = 10 \cdot x^9 \tag{80}$$

The final derivative of the given function is:

$$y' = 5 \cdot x^4 - 4 \cdot x^3 + \cos(\ln(x^2 + \frac{1}{\tan(x)})) \cdot \frac{2 \cdot x + \frac{\frac{1}{(\cos(x))^2}}{\tan(x) \cdot \tan(x)}}{x^2 + \frac{1}{\tan(x)}} \tag{81}$$
$$- (e^x + 3^x \cdot \ln(3)) \cdot \cos(x^{10}) + (e^x + 3^x) \cdot \sin(x^{10}) \cdot (-1) \cdot 10 \cdot x^9$$

Materials used in my report:

1. Redkozubov's conspects and lections
2. "Collection of problems in mathematical analysis" by Kudriavcev L.D.

My github repositorySpecial thanks to Vasenin Egor for helping me create this report in LaTex