Metamaterial-Based Passive Dynamic Walking: Design Optimization and Control

Abhishek Kishor, Anushtup Nandy, Barath Kumar J. K., Jairam Reddy
Department of Mechanical Engineering, Columbia University in the city of New York

***Abstract*—This paper presents a novel approach to dynamic locomotion by integrating metamaterial-based leg designs with optimized control strategies. The metamaterial leg replaces traditional spring-damper systems, offering intrinsic stiffness and damping for energy-efficient motion. Two computational models are employed: (1) an inverted pendulum model for gait initiation in Parkinson's Disease (PD) patients, simulating impaired anticipatory postural adjustments (APAs) with a robust PID controller; (2) a dynamic Raibert Hopper using a spring-loaded inverted pendulum (SLIP) framework, stabilized by proportional-derivative (PD) control incorporating neuromechanical insights to analyze reduced dorsiflexor activation in PD patients.**

The proposed system demonstrates performance comparable to that of traditional actuators while improving energy efficiency, robustness, and simplicity. Applications span rehabilitation robotics, bio-inspired systems, and dynamic locomotion research, advancing material-driven innovations for next-generation robotic systems.

***Index Terms*—control, rehabilitation, legged robots, metamaterial**

I. Introduction

Parkinson's disease (PD), a neurodegenerative disorder, severely impacts motor functions, particularly gait initiation, which is crucial for mobility and independence [5] [4]. In healthy individuals, anticipatory postural adjustments (APAs) activate the tibialis anterior (TA) muscle, enabling forward motion, shifting the center of mass (COM), and stabilizing the initial stance [6]. PD patients experience reduced or absent TA activation, impairing APAs, diminishing neuromechanical torque, reducing center of pressure (COP) excursion, and slowing gait initiation [2] [11]. These deficits increase fall risks and lower quality of life, necessitating innovative therapeutic solutions [1] [15] [18].

This study proposes a novel approach using metamaterial-based actuators instead of traditional spring-mass-damper systems, integrating external and internal noise to simulate realistic dynamic and static models. By employing an inverted pendulum model, it replicate healthy gait mechanics, account for impaired dorsiflexor activation in PD patients, and monitor lean angle. A robust PID controller compensates for disturbances, ensuring enhanced accuracy and stability [9]. This metamaterial-based system offers superior performance, improved cost-effectiveness, and greater adaptability, paving the way for a transformative method to address gait initiation challenges in PD patients.

A. Literature review

Recent advancements in rehabilitation technologies for PD patients have leveraged computational models [10] and bioinspired systems [17] to better understand and address gait disorders. The Neuro4PD model [12] integrates computational insights into humanoid robots to explore PD dynamics. Biomimetic neurostimulation frameworks [19] improve mobility by simulating mechanoreceptor functions. AI-driven approaches, such as deep learning models and bio-inspired feature selection, have enhanced early diagnosis and monitoring of PD [13]. Virtual reality (VR) offers an immersive, risk-free environment for patient-tailored rehabilitation. Simulation tools, such as static inverted pendulum models and dynamic Raibert Hopper frameworks, have provided valuable insights into the biomechanics of gait initiation [3]. Additionally, advancements in metamaterials—engineered structures with tailored mechanical properties—have shown promise in biomechanics, offering lightweight, cost-effective alternatives to traditional systems [7]. However, current solutions heavily depend on bulky and expensive electromechanical spring-mass actuators, necessitating simpler, more efficient designs.

B. Solution Approach

This study builds on recent advancements by integrating computational modeling, robust control, and metamaterial-based actuators to emulate and enhance gait initiation in PD patients. A metamaterial-based system replaces conventional spring-mass actuators, offering a mechanically simpler, lightweight, and cost-effective alternative. The approach incorporates both a static inverted pendulum model and a dynamic Raibert Hopper framework, allowing for comprehensive analysis of gait initiation mechanics. By leveraging a robust closed-loop PID controller, the system compensates for internal and external disturbances, ensuring accurate emulation of healthy gait dynamics. Additionally, the metamaterial-based actuator achieves performance comparable to traditional systems while significantly enhancing closed-loop time-domain specifications, including improved response speed, stability, and noise rejection capabilities. This innovative design addresses key neuromechanical deficits in PD patients, presenting an efficient and practical rehabilitation solution.

II. Methodology

The proposed methodology integrates material-driven innovations with computational modeling and control strategies to address dynamic locomotion challenges. By combining insights from biomechanics, robotics, and control theory, this study develop a comprehensive framework that incorporates two key models: Subsections II-B a Parkinson's Disease (PD)

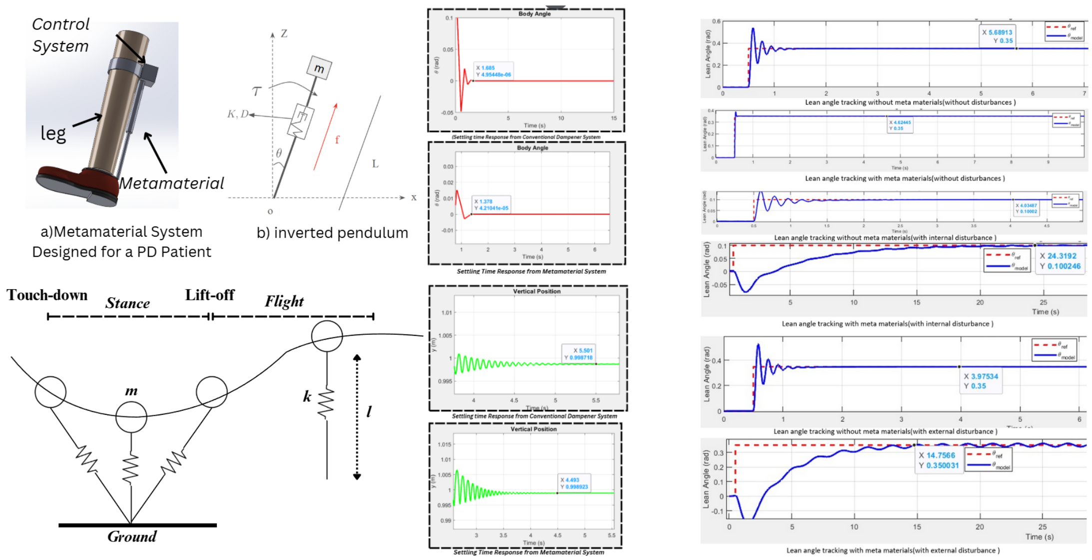
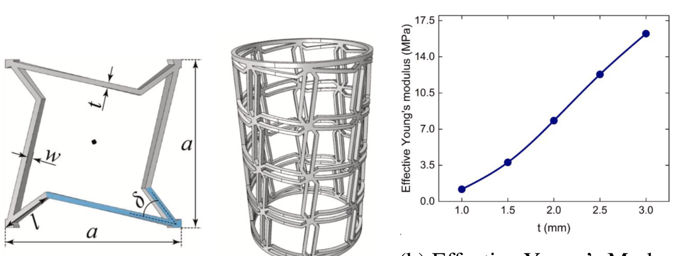


Fig. 1: SLIP and Inverted pendulum represenation and plots for comparison with metamaterials

gait initiation model using an inverted pendulum, and II-C a dynamic Raibert Hopper based on the spring-loaded inverted pendulum (SLIP). These models are unified by their use of metamaterial-based legs (II-A), which replace conventional spring-damper systems, offering intrinsic stiffness & damping properties for energy-efficient motion. Each subsection details the mathematical formulation and control strategies employed.

A. Design and Modeling of Metamaterial

This section introduces innovative tubular compression-torsion mechanical metamaterials, Fig 2, with potential applications as actuators. These metamaterials are parametrically designed using the finite element method (FEM) with ABS material, building on the methodology established by Montazeri et. al. [7]. The research explores the influence of section geometry on the metamaterial's twisting capability and mechanical properties.



(a) Unit Cell and Full Array Metamaterial

(b) Effective Young's Modulus Structures with different t(thickness)

Fig. 2: Metamaterial Geometry Construction and FEA Results

The effective Young's modulus of the metamaterial, E_e, can be customized by modifying design parameters. This modulus is crucial for determining the stiffness of the structure. The stiffness, in turn, is used to calculate the damping characteristics of the system.

The effective Young's modulus of the metamaterial, E_e, can be customized by modifying design parameters. This modulus is crucial for determining the stiffness of the structure, which in turn is used to calculate the damping characteristics of the system. The effective Young's modulus is defined as, $E_e(z) = \frac{\Delta z/H}{F(z)/A}$, where Δz is the displacement along the axial direction, H is the original length of the structure, $F(z)$ is the applied force in the axial direction, and A is the cross-sectional area.

The stiffness of the structure, k, is expressed as, $k = \frac{L}{EA}$, where L is the length of the structure, E is the material's Young's modulus, and A is the cross-sectional area.

The results demonstrate the tunability of the metamaterial by adjusting unit cell design parameters, showcasing properties such as effective Young's modulus customization.

B. Parkinson's Disease (PD) Gait Initiation Model

The dynamics of gait initiation in Parkinson's Disease (PD) patients are modeled as a single-link inverted pendulum with a rotational joint at the ankle, as seen in Fig 1 (b). This model represents the human body as a single rigid link pivoting around the ankle. The governing equation is:

$$\ddot{\theta} = \frac{T_{\text{Ank}} + T_{\text{instability}} - b\dot{\theta} - (k_{\text{eff}} + m_B g L_{\text{COM}})\,\theta}{J_B} \tag{1}$$

where $T_{\text{Ank}} = T_F + T_{\text{PID}} + T_{\text{tremor}}$. Here: - J_B: Body moment of inertia about the ankle. - b: Damping coefficient. - k_{eff}: Effective stiffness from mechanical and gravitational forces. - m_B, g, L_{COM}: Body mass, gravity, and center of mass (COM) distance from the ankle.

Forward kinematics are derived using Denavit-Hartenberg parameters with the transformation matrix:

$$T = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & L_{\mathrm{COM}}\cos(\theta) \\ \sin(\theta) & \cos(\theta) & 0 & L_{\mathrm{COM}}\sin(\theta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The COM position is given by:

$$x_B = L_{\mathrm{COM}}\cos(\theta), \tag{3}$$

$$y_B = L_{\mathrm{COM}}\sin(\theta). \tag{4}$$

A robust PID controller stabilizes the system by minimizing the error between the desired $\theta_{\text{ref}}(t)$ and actual lean angles $\theta(t)$. The error & controller torque are $e(t) = \theta_{\text{ref}}(t) - \theta(t)$:

$$T_{\mathrm{PID}} = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt}, \tag{5}$$

where K_p, K_i, and K_d are proportional, integral, and derivative gains, respectively. Jacobian analysis provides linear velocity components for COM motion:

$$J_v = \begin{bmatrix} -L_{\mathrm{COM}}\sin(\theta) \\ L_{\mathrm{COM}}\cos(\theta) \end{bmatrix}. \tag{6}$$

This model simulates impaired anticipatory postural adjustments (APAs) in PD patients [1], [5], [6] by reducing dorsiflexor torque [18]. Key outputs include lean angle (θ) and center of pressure (COP), which are analyzed under both normal and impaired conditions [12]. Furthermore, it extends prior work by incorporating neuromechanical insights into reduced tibialis anterior activation in PD patients [11] leveraging metamaterial-based designs for enhanced performance [19].

C. Dynamic Raibert Hopper (SLIP Model)

The Raibert Hopper is modeled as a spring-loaded inverted pendulum (SLIP), alternating between flight and stance phases, seen in Fig 1 (c). During flight, free-fall dynamics are $\ddot{x} = 0, \quad \ddot{y} = -g, \quad \ddot{\theta} = 0$. where x, y are horizontal and vertical positions, and θ is body angle. In stance, the leg acts as a damped spring with force components:

$$F_{\text{spring}} = k_s(L_0 - L) - b_s\dot{L}, \tag{7}$$

$$F_x = F_{\text{spring}}\frac{x - x_{\text{foot}}}{L}, F_y = F_{\text{spring}}\frac{y}{L} - mg \tag{8}$$

where k_s, b_s are stiffness and damping coefficients, L_0 is natural leg length, and L is current leg length.

A proportional-derivative (PD) controller regulates body angle and foot placement control adjusts horizontal foot position during flight to ensure stability:

$$\tau = K_p(\theta_{\text{desired}} - \theta) - K_d\dot{\theta}, \;\; x_{\text{foot}} = x + \alpha\dot{x} \tag{9}$$

where α is a gain for foot placement adjustment. This approach builds on foundational work by Raibert [14], demonstrating how dynamic stability can be achieved through simple control principles. The metamaterial-based leg design replaces traditional spring-damper systems, reducing complexity while enhancing energy efficiency [7], [16].

III. Results

To model a normal human using conventional prosthetics, the system parameters were set as follows: body weight of 72 kg, spring stiffness of 7500 N/m, and damping coefficient of 5 kg/s. When designing and developing prosthetics incorporating metamaterials using ABS, the body weight was reduced to 70 kg while maintaining the spring stiffness at 7400 N/m and increasing the damping coefficient to 5.5 kg/s.

The performance of the metamaterial-integrated system was evaluated using mass-spring-damper (MSD) parameters in the context of dynamic locomotion systems with PID control. A schematic representation of the setup is shown in Fig. 1 (d) and (e).

Performance tests were conducted to compare the behavior of systems with and without metamaterials under three scenarios: no disturbances, internal disturbances simulating gait instabilities, and external disturbances mimicking environmental impacts.

In all scenarios, the metamaterial-enhanced system exhibited superior performance, characterized by faster stabilization, reduced overshoot, and improved overall stability. Under undisturbed conditions, the metamaterial system achieved quicker damping and stabilization. During internal disturbance tests, it effectively suppressed oscillations, reducing the workload on the PID controller. For external perturbations, the system efficiently dissipated shocks and rapidly restored equilibrium.

Simulations using the Raibert Hopper further validated these results, demonstrating reduced settling times for vertical position and body alignment. These findings confirm that integrating metamaterials significantly enhances the mechanical response of dynamic systems. This improvement complements control strategies, offering superior resilience and adaptability in various operating conditions.

IV. Conclusion and Future Work

This study demonstrated the potential of metamaterials to revolutionize prosthetic ankle exoskeleton design by simplifying numerous discrete components into a single element capable of delivering functionality comparable to traditional spring-mass-damper systems. The designed controller and metamaterial actuators provided insights into both normal and impaired gait mechanics, addressing challenges such as impaired anterior tibial activation in Parkinson's patients. The findings highlight that metamaterials offer a lighter, cost-effective, mechanically streamlined, and reliable alternative, paving the way for advancements in prosthetic technology and gait rehabilitation.

Future work will focus on experimental validation to confirm the practical applicability of these concepts. This includes adapting the approach to various body parts and impairments to enhance versatility [13]. Additionally, exploring complex dynamics, leveraging optimization techniques for greater accuracy, and developing advanced multi-material and multi-directional metamaterial designs will expand the scope and effectiveness of this innovative technology [8].

References

[1] Anne Burleigh-Jacobs, Fay B. Horak, John G. Nutt, and Jose A. Obeso. Step initiation in parkinson's disease: Influence of levodopa and external sensory triggers. *Movement Disorders*, 12(2):206–215, March 1997.

[2] H A Chang, T Y Chuang, S J Lee, S F Liao, H C Lee, Y H Shih, and H Cheng. Temporal differences in relative phasing of gait initiation and first step length in patients with cervical and lumbosacral spinal cord injuries. *Spinal Cord*, 42(5):281–289, February 2004.

[3] Shriniket Dixit, Khitij Bohre, Yashbir Singh, Yassine Himeur, Wathiq Mansoor, Shadi Atalla, and Kathiravan Srinivasan. A comprehensive review on ai-enabled models for parkinson's disease diagnosis. *Electronics*, 12(4):783, February 2023.

[4] Rodger J. Elble, Charles Moody, Keith Leffler, and Raj Sinha. The initiation of normal walking. *Movement Disorders*, 9(2):139–146, January 1994.

[5] Suzanne E Halliday, David A Winter, James S Frank, Aftab E Patla, and François Prince. The initiation of gait in young, elderly, and parkinson's disease subjects. *Gait amp; Posture*, 8(1):8–14, August 1998.

[6] R. Lepers and Y. Brenire. The role of anticipatory postural adjustments and gravity in gait initiation. *Experimental Brain Research*, 107(1), November 1995.

[7] Amin Montazeri, Mahdi Naderinejad, Maryam Mahnama, and Amirhosein Hasani. 3d-printed twisting tubular metamaterials with tunable mechanical and torsional characteristics. *International Journal of Mechanical Sciences*, 262:108719, January 2024.

[8] Nature Scientific Reports. A novel metamaterial design for energy-efficient robotics applications. *Scientific Reports*, 13, 2023. https://www.nature.com/articles/s41598-023-32106-5.

[9] S Nogueira, E Ferreira, D Geisinger, C San Roman, and H Suarez. Model of postural control system applied in parkinson's disease patients. In *2010 Annual International Conference of the IEEE Engineering in Medicine and Biology*, page 5452–5455. IEEE, August 2010.

[10] Ümit Önen, Fatih M. Botsalı, Mete Kalyoncu, Yusuf Şahin, and Mustafa Tınkır. *Design and Motion Control of a Lower Limb Robotic Exoskeleton*. InTech, May 2017.

[11] Kerstin D. Pfann, Aron S. Buchman, Cynthia L. Comella, and Daniel M. Corcos. Control of movement distance in parkinson's disease. *Movement Disorders*, 16(6):1048–1065, November 2001.

[12] Jhielson M. Pimentel, Renan C. Moioli, Mariana F. P. de Araujo, Caetano M. Ranieri, Roseli A. F. Romero, Frank Broz, and Patricia A. Vargas. Neuro4pd: An initial neurorobotics model of parkinson's disease. *Frontiers in Neurorobotics*, 15, 2021.

[13] S. Priyadharshini, K. Ramkumar, Subramaniyaswamy Vairavasundaram, K. Narasimhan, S. Venkatesh, P. Madhavasarma, and Ketan Kotecha. Bio-inspired feature selection for early diagnosis of parkinson's disease through optimization of deep 3d nested learning. *Scientific Reports*, 14(1), October 2024.

[14] M. H. Raibert, H. B. Brown Jr., M. Chepponis, E. Hastings, J. Koechling, K. N. Murphy, S. S. Murthy, and A. J. Stentz. Progress report: October 1982 - october 1983. Technical report, The Robotics Institute and Department of Computer Science, Carnegie-Mellon University, Pittsburgh, PA, December 1983.

[15] Mark W. Rogers, Robert Kennedy, Sonia Palmer, Monika Pawar, Maggie Reising, Katherine M. Martinez, Tanya Simuni, Yunhui Zhang, and Colum D. MacKinnon. Postural preparation prior to stepping in patients with parkinson's disease. *Journal of Neurophysiology*, 106(2):915–924, August 2011.

[16] Science Daily. Soft robotic grippers with advanced metamaterials enable new possibilities in automation, February 2022. https://www.sciencedaily.com/releases/2022/02/220228150649.htm.

[17] Shakila Shafiq, Sabbir Ahmed, M. Shamim Kaiser, Mufti Mahmud, Md. Shahadat Hossain, and Karl Andersson. Comprehensive analysis of nature-inspired algorithms for parkinson's disease diagnosis. *IEEE Access*, 11:1629–1653, 2023.

[18] David E. Vaillancourt, Janey Prodoehl, Molly M. Sturman, Roy A.E. Bakay, Leo Verhagen Metman, and Daniel M. Corcos. Effects of deep brain stimulation and medication on strength, bradykinesia, and electromyographic patterns of the ankle joint in parkinson's disease. *Movement Disorders*, 21(1):50–58, August 2005.

[19] Giacomo Valle, Natalija Katic Secerovic, Dominic Eggemann, Oleg Gorskii, Natalia Pavlova, Francesco M. Petrini, Paul Cvancara, Thomas Stieglitz, Pavel Musienko, Marko Bumbasirevic, and Stanisa Raspopovic. Biomimetic computer-to-brain communication enhancing naturalistic touch sensations via peripheral nerve stimulation. *Nature Communications*, 15(1), February 2024.