banandan@andrew.cmu.edu
Barathwaj Subramaniam Anandan

1.1

Assuming, after normalising, the X_1 and X_2 (co-ordinate origin) is (0,0).

Converting them into homogenous co-ordinates

$$\tilde{x}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\tilde{x}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Relation with Fundamental Matrix is given by,

$$x_2^T F x_1 = 0$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{21} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{23} & f_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

Therefore,

$$[\ f_{31}\ f_{32}\ f_{33}] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

Finally,

$f_{33} = 0$.

1.2. Given that there is no rotation by only translation which is parallel to x axis.

Therefore,

P1 = K1[I|0]

P2 = K2[I|t]

$$F = [e']_\times P_2 P_{1+} = [e']_\times K\, K^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

The equation of the epipolar line is given by l = F$\tilde{x}$

Where $\tilde{x}$ = $[x\ y\ 1]^T$

$$l = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} [x\ y\ 1]^T = [0\ {-1}\ y]^T$$

but we know that the fundamental Matrix relation is, $x_2^T F x_1 = 0$

$$[x'\ \ y'\ \ 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} [x\ \ y\ 1]^T = 0$$

$[x'\ \ y'\ \ 1]\ [0\ {-1}\ \ y]^T = 0$

$-y' + y = 0$

$y = y'$

which is the equation of x axis and proves that two cameras are also parallel to x axis.

1.3 Since we have sensors to detect readings of R and t accurately, we can obtain the transformation matrix for the i^{th} frame from the 0^{th} frame,

$H_i = \begin{bmatrix} Ri & ti \\ 0 & 1 \end{bmatrix}$

Let's consider for example, for frame 4 and 5 relative transformation

${}_4H^5 = H_5 H_4^{-1}$

we get,

$$\begin{bmatrix} R_5 R_4^{-1} & -R_5 R_4^{-1} t_4 + t_5 \\ 0 & 1 \end{bmatrix}$$

Relating both, we can find R and t matrices,

$R = R_5 R_4^{-1}$ $\quad t = -R_5 R_4^{-1} t_4 + t_5$

Generally Essential matrix $E = t_x R$

Relating Fundamental matrix F and E , we have $E = K^{-1} F K$

$F = K E K^{-1} = \mathbf{K\, t_x\, R\, K^{-1}}$

1.4.

C1 - Camera centre

C2 - Mirror reflected camera centre.

X - image

X' -reflected image from the mirror

Projected point of X in C1 – x_1

Projected point of X in C2 – x_2

Projected point of X' in C1 – x_1'

Projected point of X' in C2 – x_2'

Relation of Fundamental matrix and the correspondence, we get

$x_1^T F x_2 = 0$ (1)

$x_2' F^T x_1 = 0$ (2)

since x_1' , x_1, x_2, x_2' are symmetric as it is given and equidistant from each other, we have

$x_2' F x_1 = 0$ (3)

adding (2) and (3), we get

$x_2' F x_1 + x_2' F^T x_1 = 0$

$F+F^T = 0$

$\mathbf{F = - F^T}$

Which proves that F is a skew-symmetric matrix.

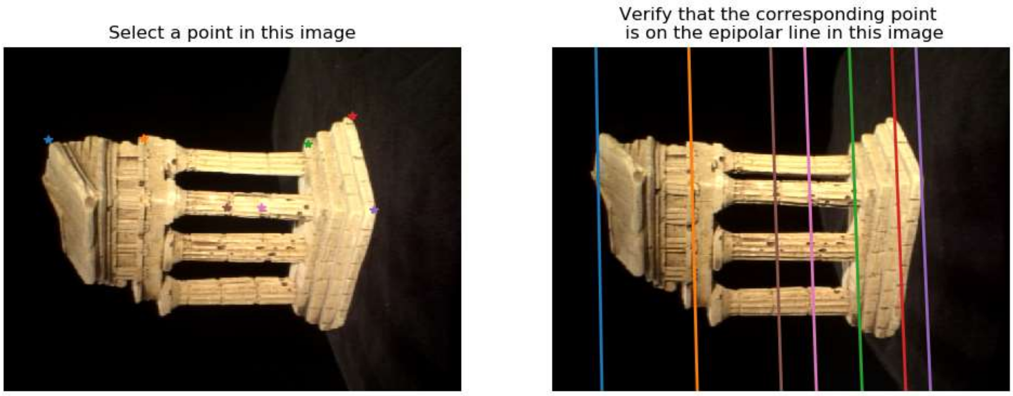
2.1 the 8 point algorithm

F matrix:

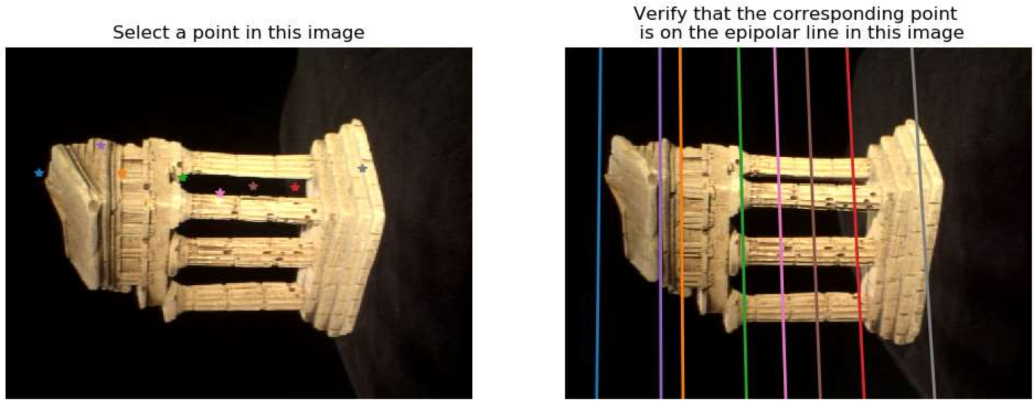
[[-5.39510268e-04 -5.34867386e-02 7.19762206e-01]

 [-2.43579764e-02 1.45738860e-03 -1.05587609e-02]

 [-6.91351246e-01 1.93936474e-02 -4.10639901e-03]]



2.2 7 point Algorithm:



F(seven point) = [[-6.09080639e-09 1.73057621e-08 9.40992608e-04]

[-1.62234195e-07 -8.63467505e-10 1.78627741e-05]

[-9.05544907e-04 -6.71986149e-06 -3.35266556e-03]]

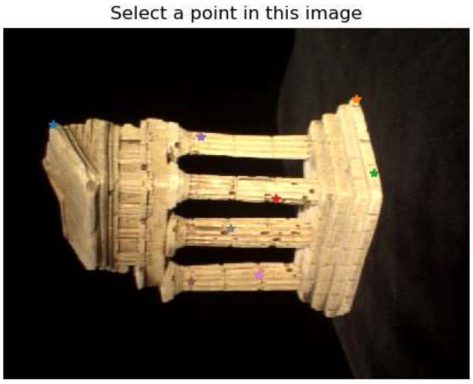
3.1 Essential Matrix Estimation from F and Intrinsics K1 and K2:

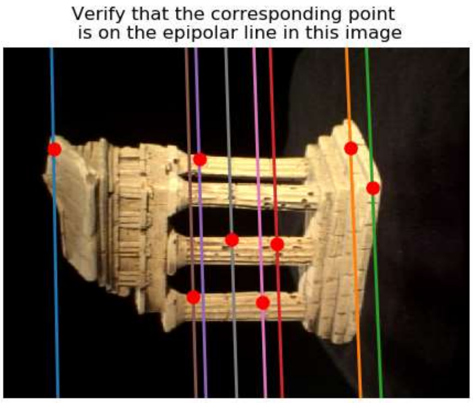
[[-3.04477699e-03, -3.02949405e-01, 1.66026654e+00],
[-1.37963814e-01, 8.28452408e-03, -5.12671289e-02],
[-1.66531742e+00, -1.26601678e-02, -1.36557606e-03]]

3.2 Expression for A:

$$A = \begin{bmatrix} yp_3^T - p_2^T \\ p_1^T - xp_3^T \\ y'p'^T_3 - p'^T_2 \\ p'^T_1 - x'p'^T_3 \end{bmatrix}$$

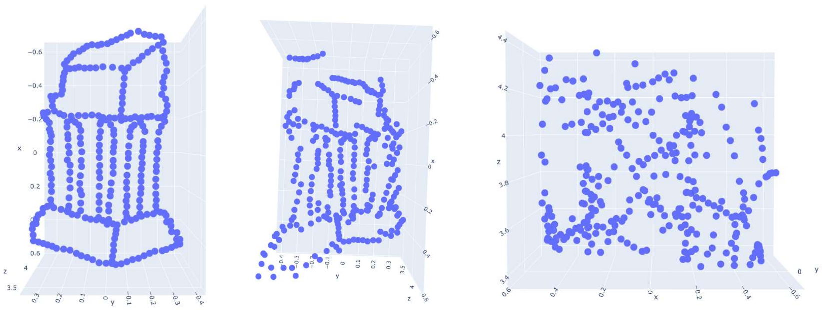
4.1 Epipolar Correspondences;





The F, points1 and points2 which were used to generate F is saved as ‘q4_1.npz’

4.2 3D Visualization- Temple Coordinates:

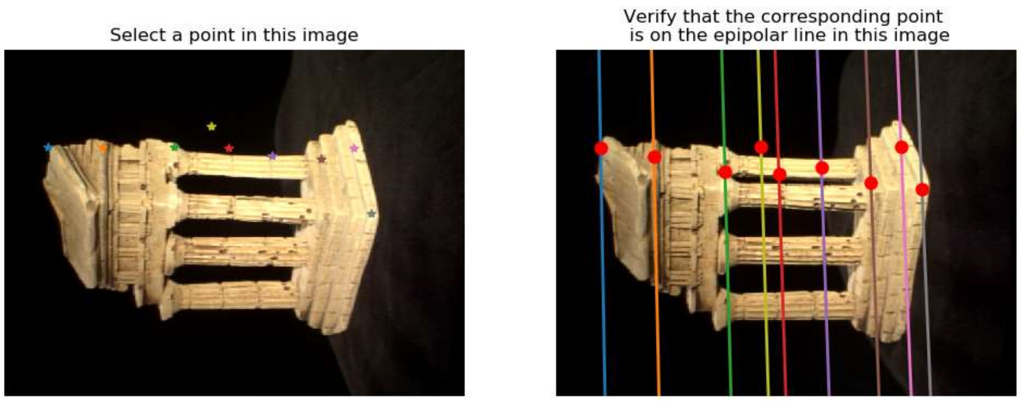


5.1: RANSAC:

Eightpoint with noisy Correspondences:

It can be seen that the eight point works well for noiseless correspondences. But if there is noise like the image above, it seems to give random inaccurate results.

RANSAC 7 point with noisy Correspondences:



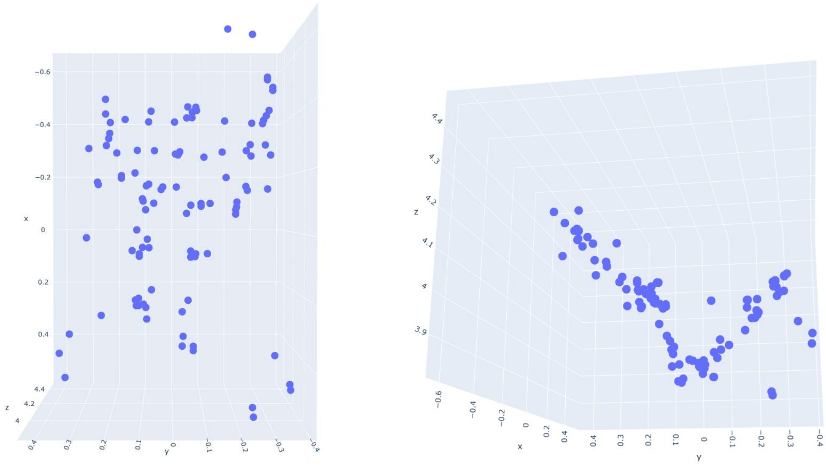
On the other hand, RANSAC when added with 7 point algorithm in order to remove the outliers, gives decent results as it removes noise(outliers) and the f is chosen based on the amount of inliers present.

The error metric used:

Given the noisy correspondences p1 and p2 , generally for the points to be inliers it has to lie on the epipolar line. So that $p_2^T F \, p_1 \ = 0$, where f is calculated randomly using 7 point algorithm. Since RANSAC helps us to find the points that lie closer to the line, we use a threshold value (e.g.: 0.001) to say that a point is an inlier if it lies within the threshold limit for a given calculated F, fundamental matrix. So the f with the most inliers is selected and that gives the best estimate of the correspondences.

5.3.

Error before Bundle Adjustment: 27985.398180529548



Error after Bundle adjustment: 12.534524154494393