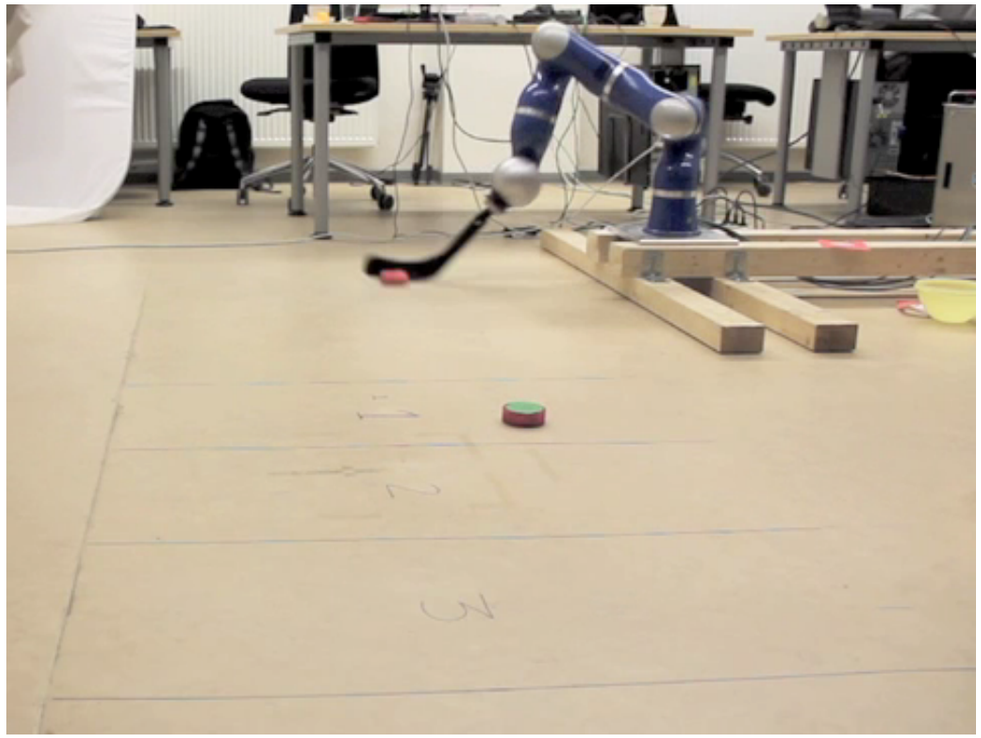
Robot Autonomy

Lecture 2:
MDPs and Reinforcement Learning

Oliver Kroemer

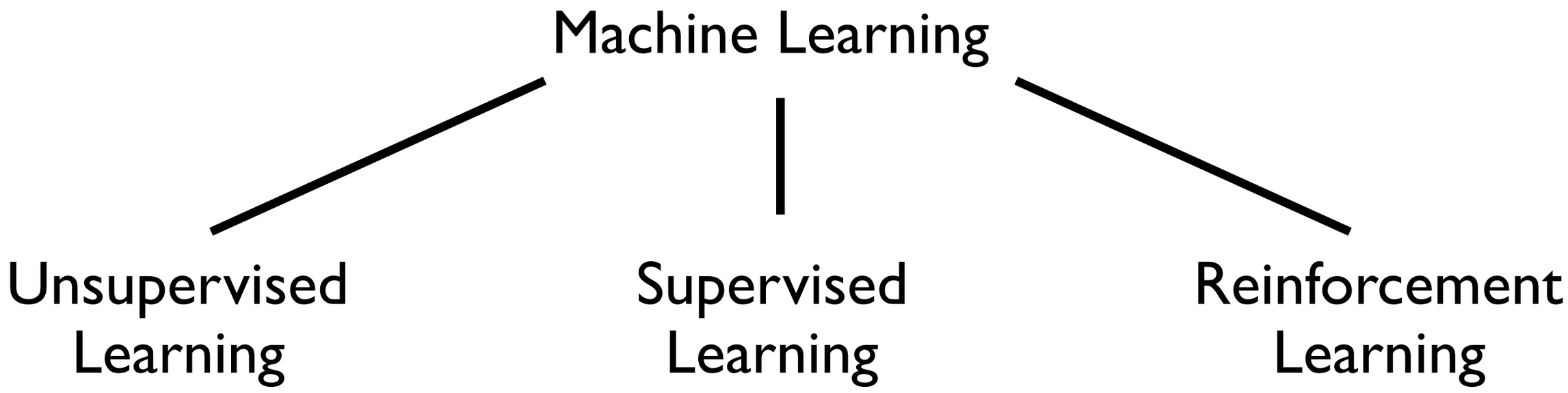
Motivation

- Want robots to perform complex adaptive behaviors

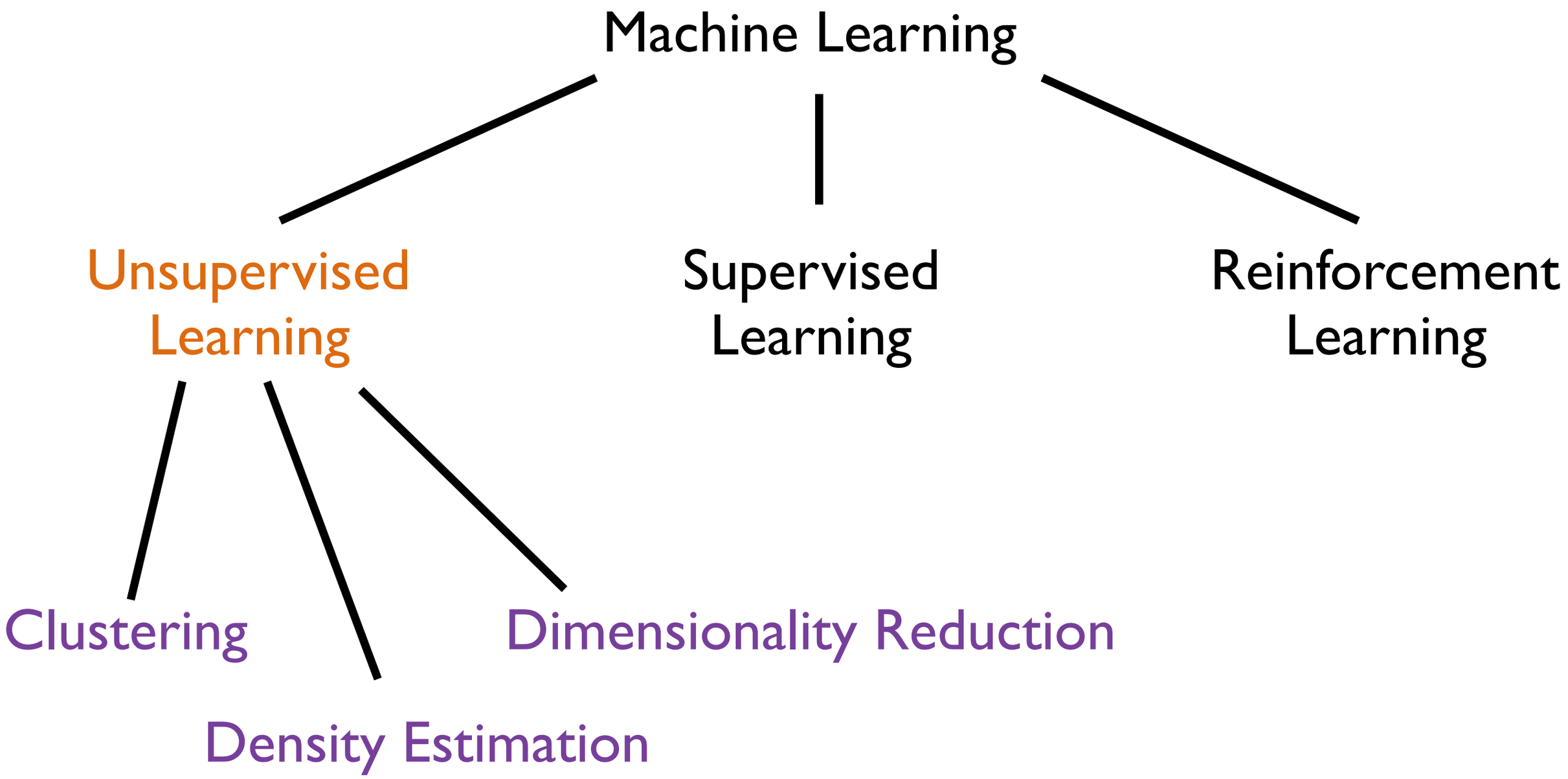


- Programming these behaviors by hand is not trivial
- Want to adapt to complex unstructured environments
- Often easier to have robot learn from experience

Machine Learning

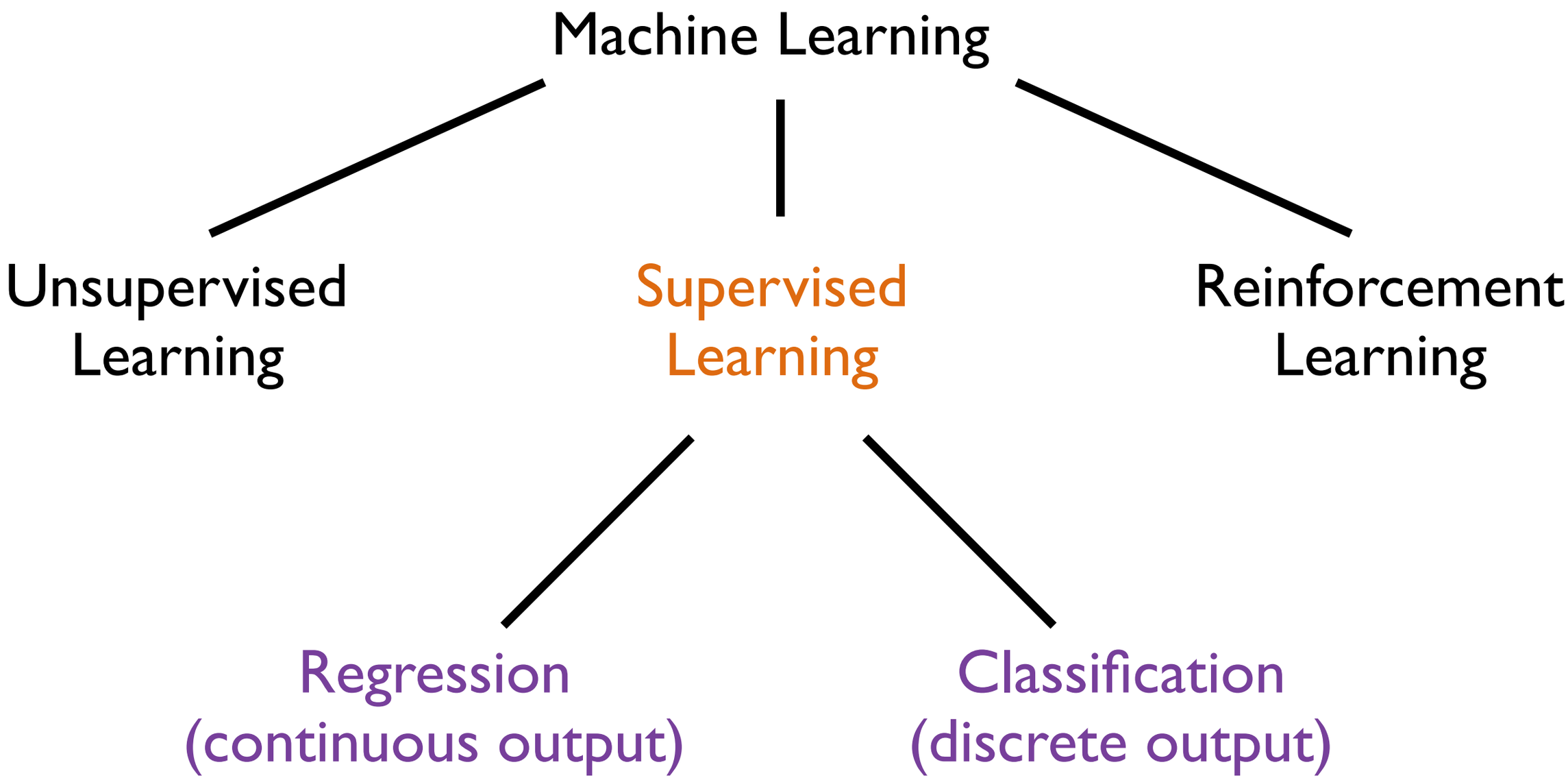


Machine Learning



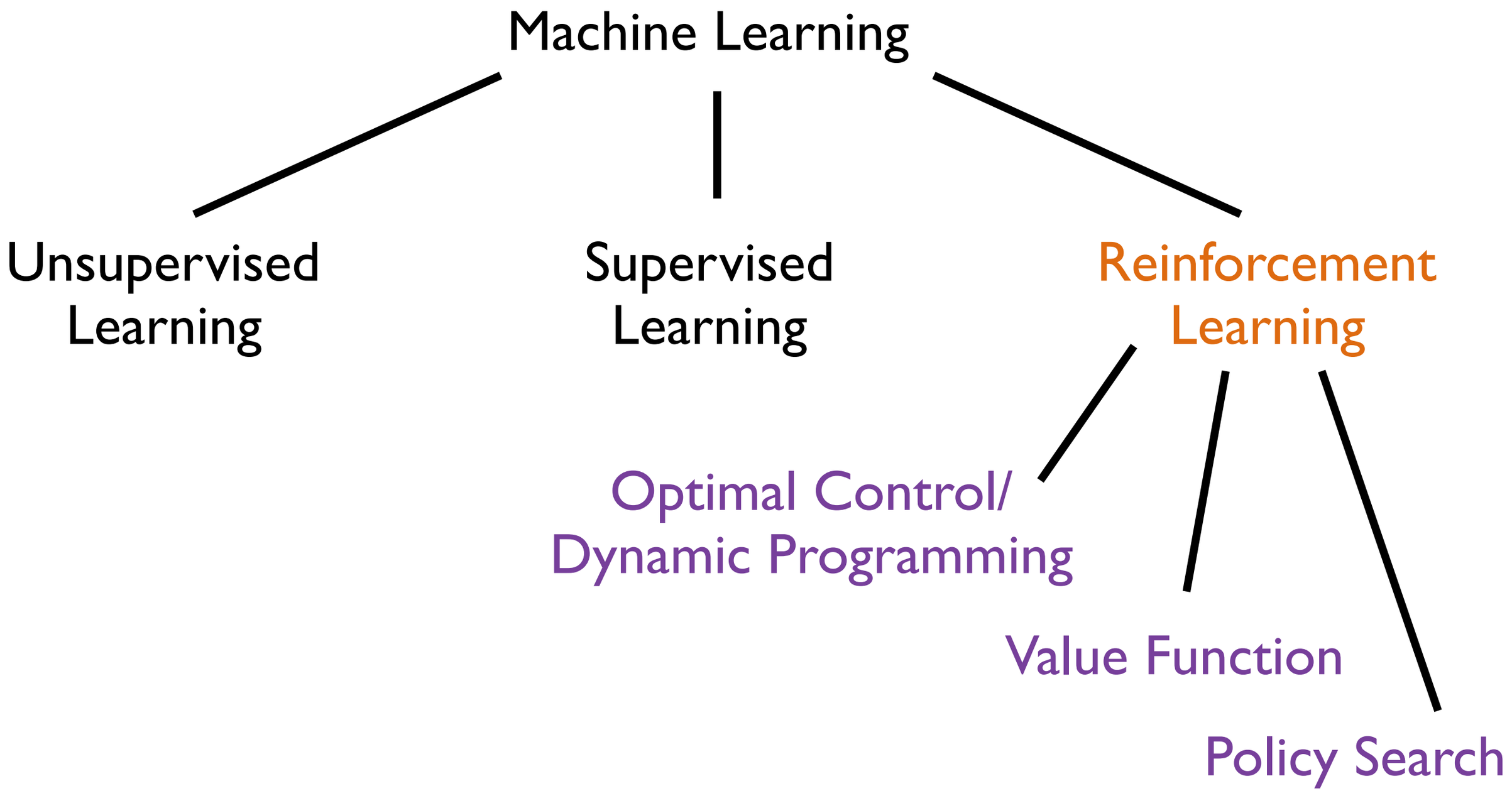
learn structure from input information only

Machine Learning



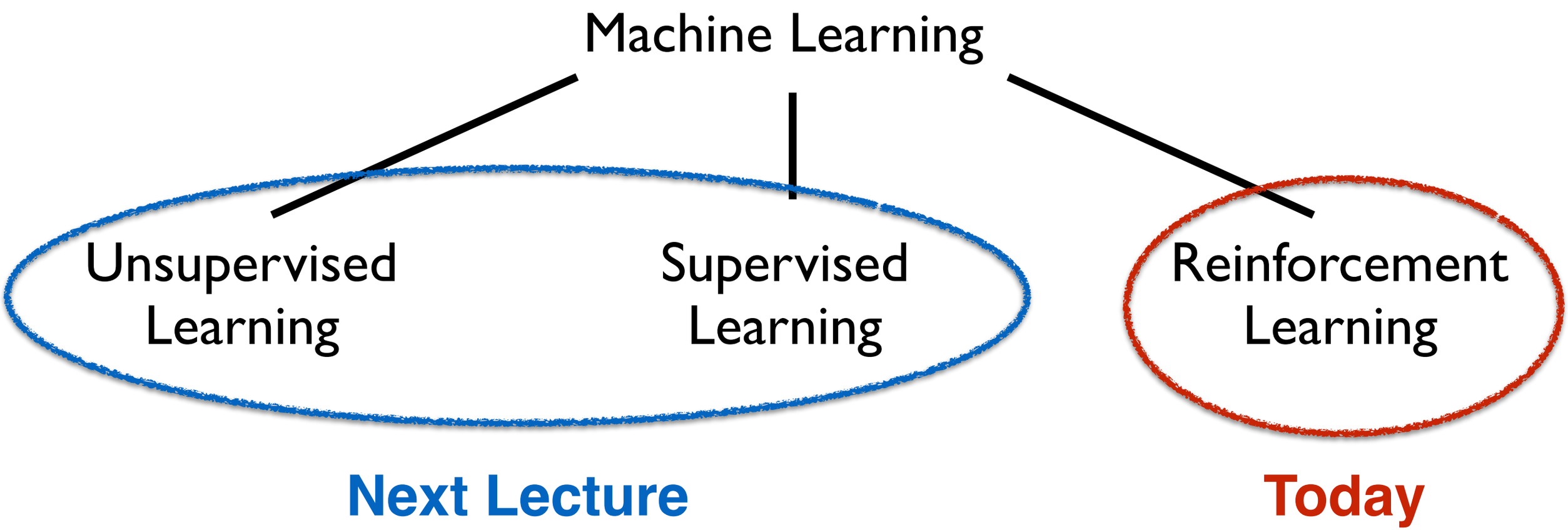
learn mapping from input-output pairs

Machine Learning



learn policy from **reward** for a given state action pair

Machine Learning

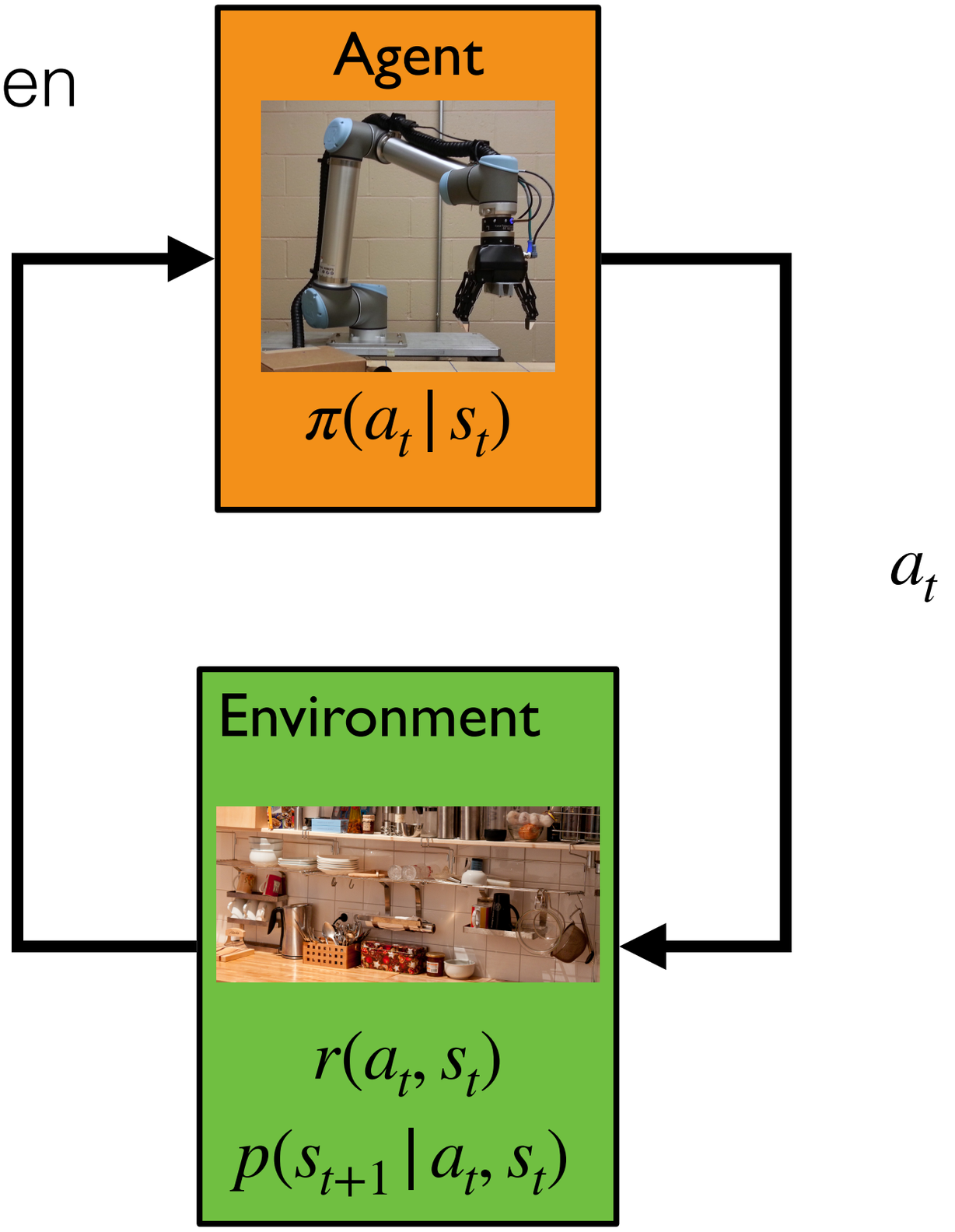


- Robots can acquire information from many sources
- Will be focusing on learning from own experiences

Markov Decision Processes

Markov Decision Processes

Task:
Cleaning up a kitchen



Markov Decision Processes

- State space

$$s_t \in \mathbb{S}$$

- Action space

$$a_t \in \mathbb{A}$$

- Transition probability

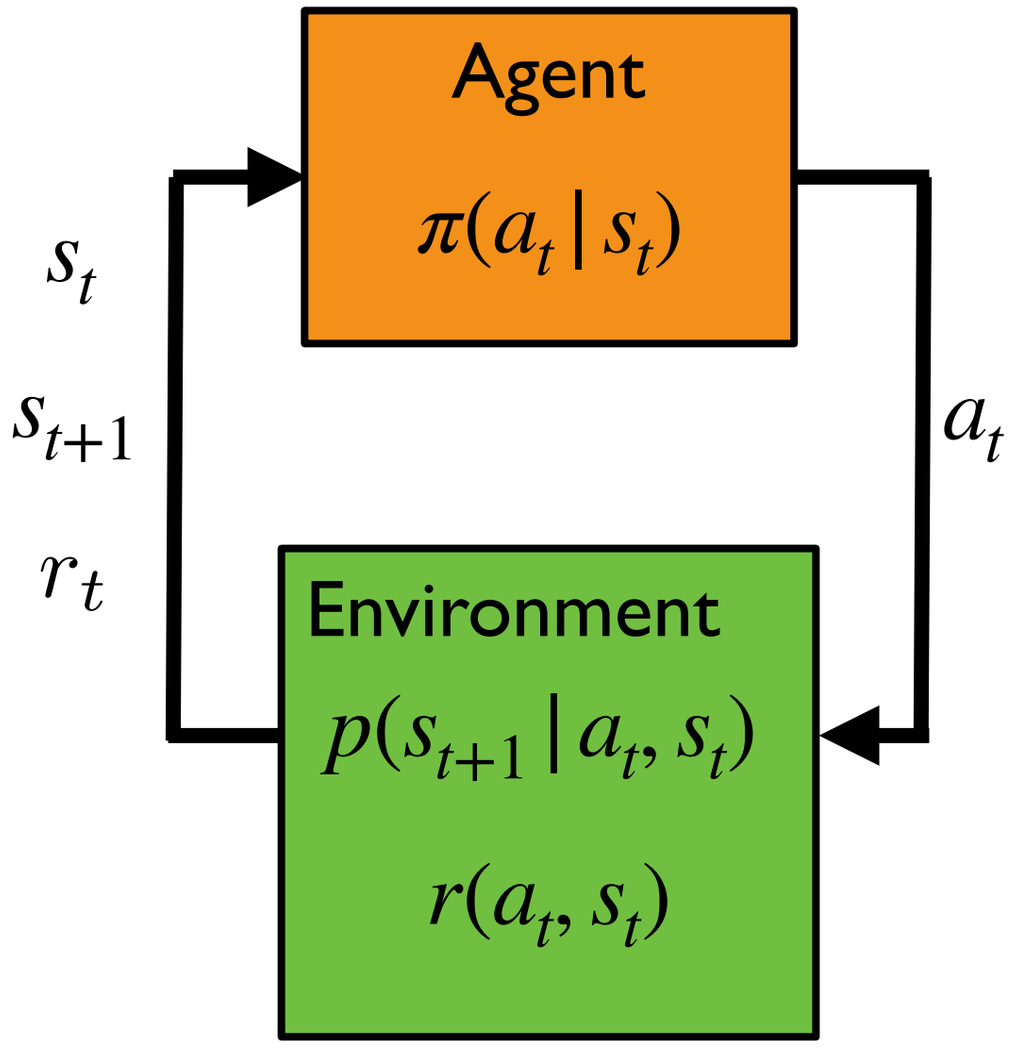
$$p(s_{t+1} \mid a_t, s_t)$$

- Reward function

$$r(a_t, s_t)$$

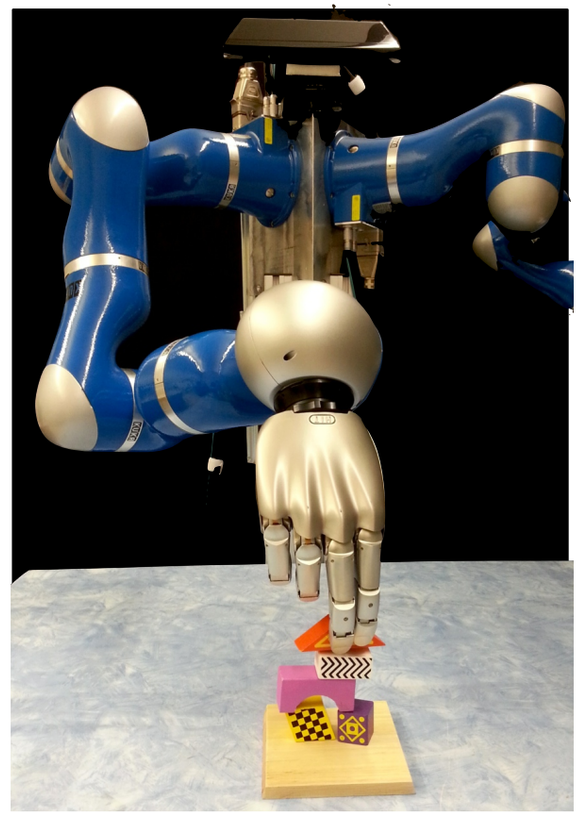
- Discount factor

$$0 \leq \gamma \leq 1$$



State and Markov Property

- State
  - positions of the hand and blocks
  - finger angles and contact forces
  - …



- Markov property
  - all future states are independent of the past states given the current state (the MDP is memoryless).

$$p(s_{t+1} \mid a_t, s_t, s_{t-1}, \ldots, s_1) = p(s_{t+1} \mid a_t, s_t)$$

  - transition distribution and policy depend only on current state

Markov Decision Process

- Goal:
  learn a policy π^* that maximises the expected return
- Policy

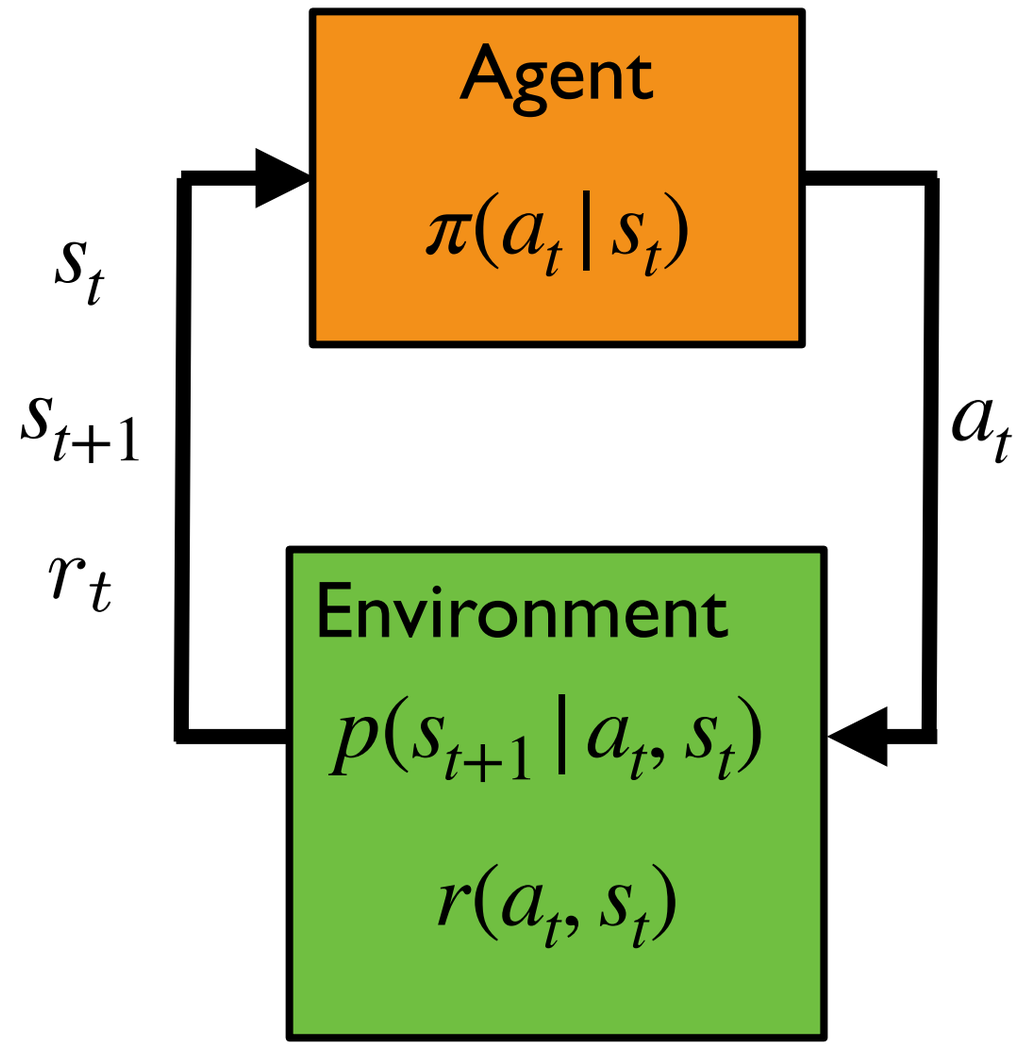
$$\pi(a_t \mid s_t)$$

- Expected Return

$$J(\pi) = E\left[\sum_{t=0}^{N} r(a_t, s_t)\right]$$

$$J(\pi) = E\left[\sum_{t=0}^{\infty} \gamma^t r(a_t, s_t)\right]$$

$$0 \leq \gamma < 1$$



Value Functions

Value Function

- Want the policy to generally choose actions that
  1. Give high immediate rewards
  2. Transition to good states for acquiring more rewards in future
- Need to determine how "good" a state is to be in

Value Function

- Want the policy to generally choose actions that
  1. Give high immediate rewards
  2. Transition to good states for acquiring more rewards in future
- Need to determine how "good" a state is to be in

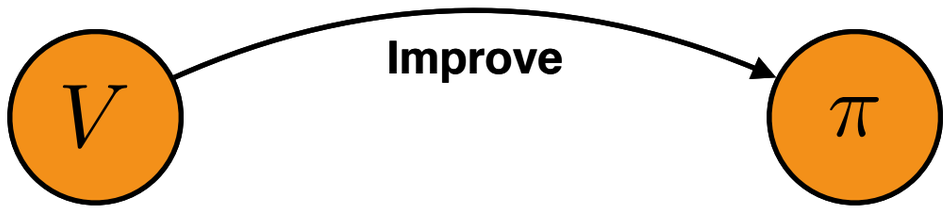
- Value function computes expected future discounted reward

$$V^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t r(a_t, s_t) \,|\, s_0 = s\right]$$

  - given the initial state and current policy
- Value represents the reward-to-go given the policy

Generalized Policy Iteration

- Value function allows the robot to find better policies



- Policy improvement:
  - Given a value function, compute an improved policy

New Policy

$$\pi'(s_t) = \arg\max_a (r(a, s_t) + \gamma E[V^{\pi}(s_{t+1})])$$

Old Policy

Immediate Reward

"Good" Next State

Generalized Policy Iteration

- Lecture example:
  - V(sitting in front 3 rows)=100
  - V(sitting in back rows)= -50
- Who would stay in their seat? Who would move?

Generalized Policy Iteration

- Value function allows the robot to find better policies

- Policy evaluation:
  - Given a policy, compute the corresponding value function
  - Values will increase with more iterations for even better policies
- An optimal policy results in an optimal value function

$$\pi^* \rightarrow V^*(s) \qquad V^{\pi}(s) \leq V^*(s) \forall s$$

How to compute the value function?
Need estimates of V(s)

Monte Carlo Estimates

- The value function can be written as the expectation

$$V^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t r(a_t, s_t) \,|\, s_0 = s\right]$$

- We can approximate this expectation based on samples

$$E[y] \approx n^{-1} \sum_{i=1}^{n} y_i$$

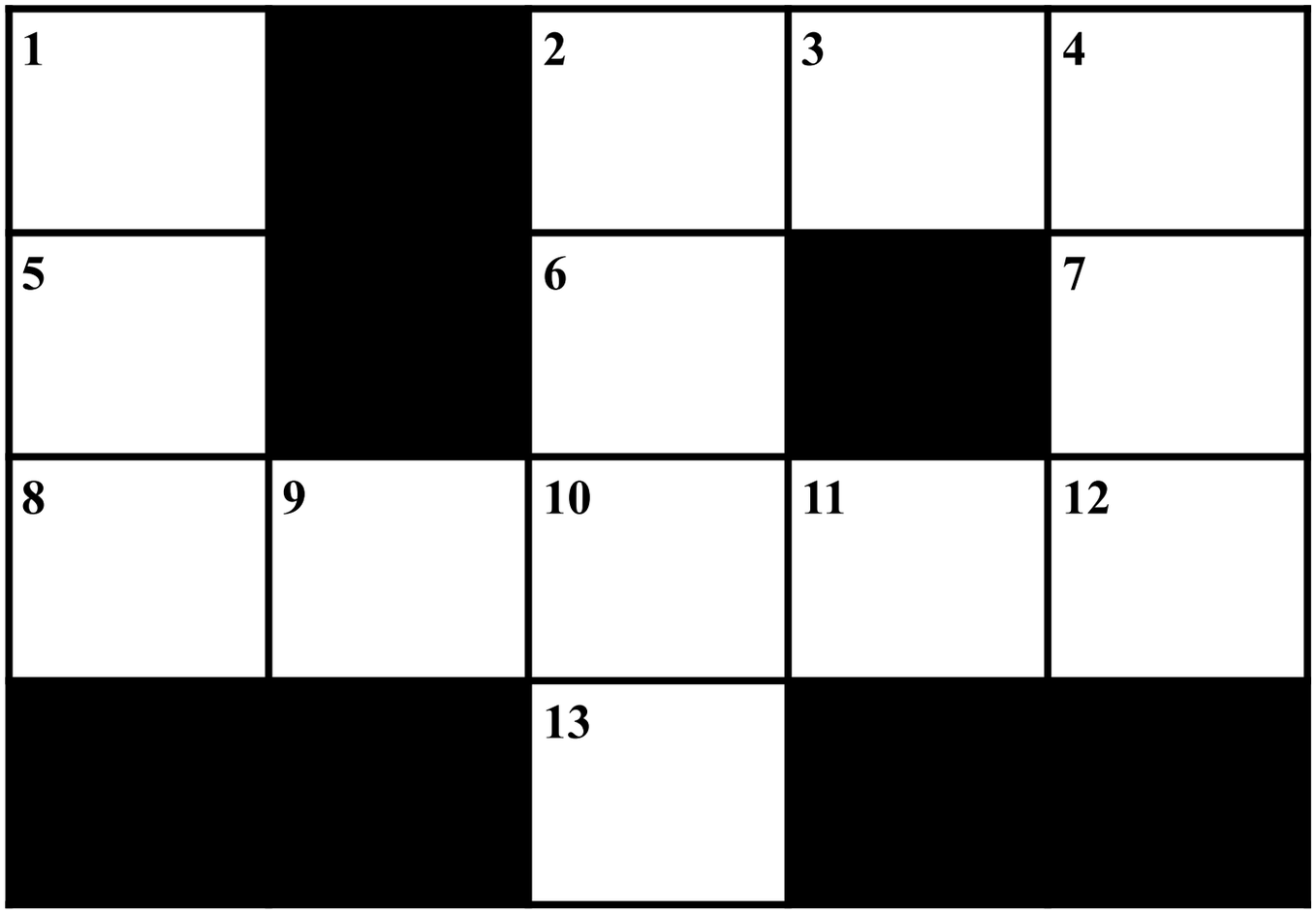
$$V^{\pi}(s) \approx n^{-1} \sum_{i=1}^{n} \sum_{t=0}^{\infty} \gamma^t r(a_{it}, s_{it})$$

  - where $s_{i0} = s$ and n is the number of times agent visited s

Gridworld Example

- Consider the following example problem

State space:

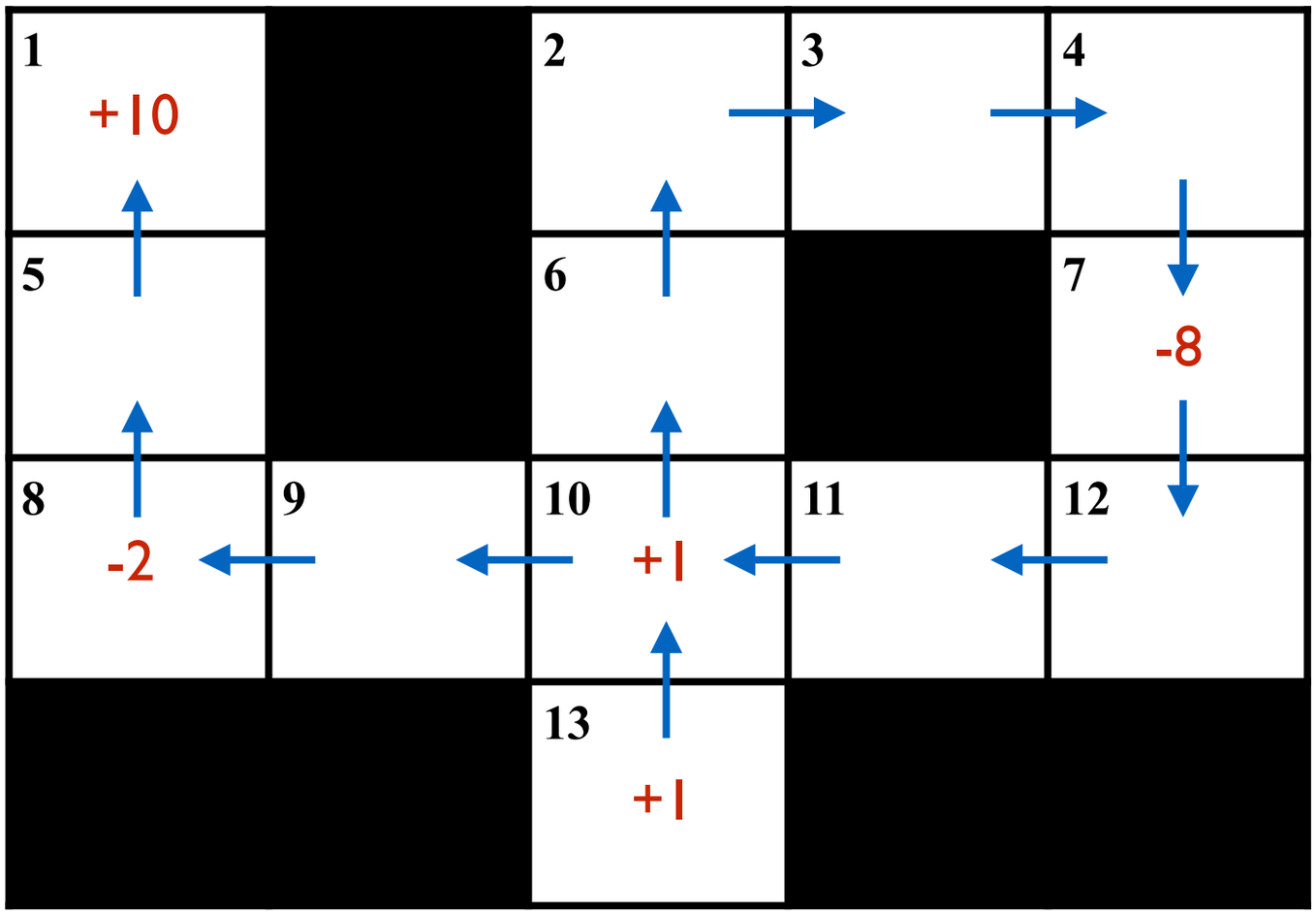


Action space:

left, right, up, down, and wait

Gridworld Example

- Consider the following rewards and current policy

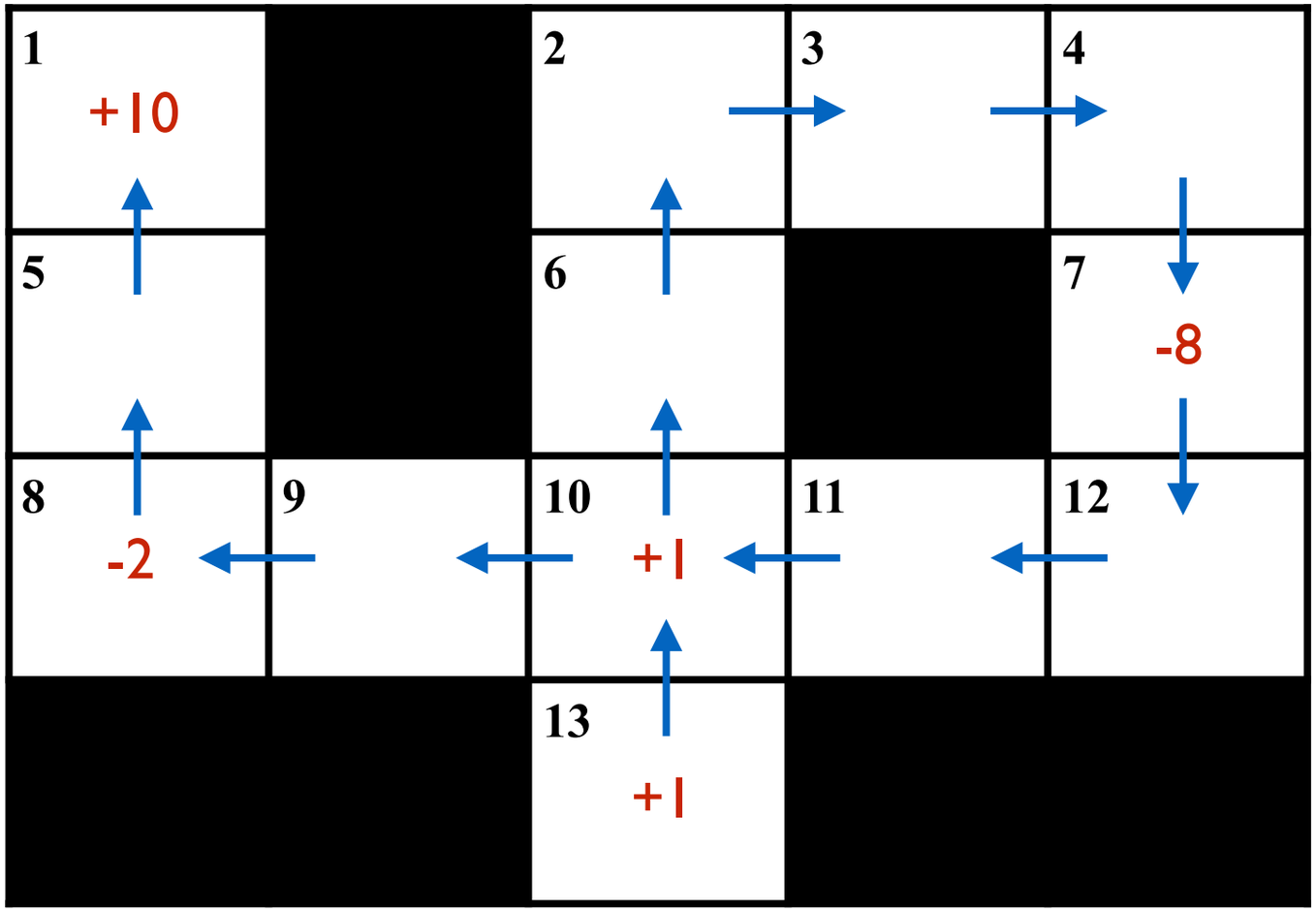


$$\gamma = 0.95$$

- What is the value of each state under this policy?

Monte Carlo Estimates

- Assume we have acquired the samples along the path



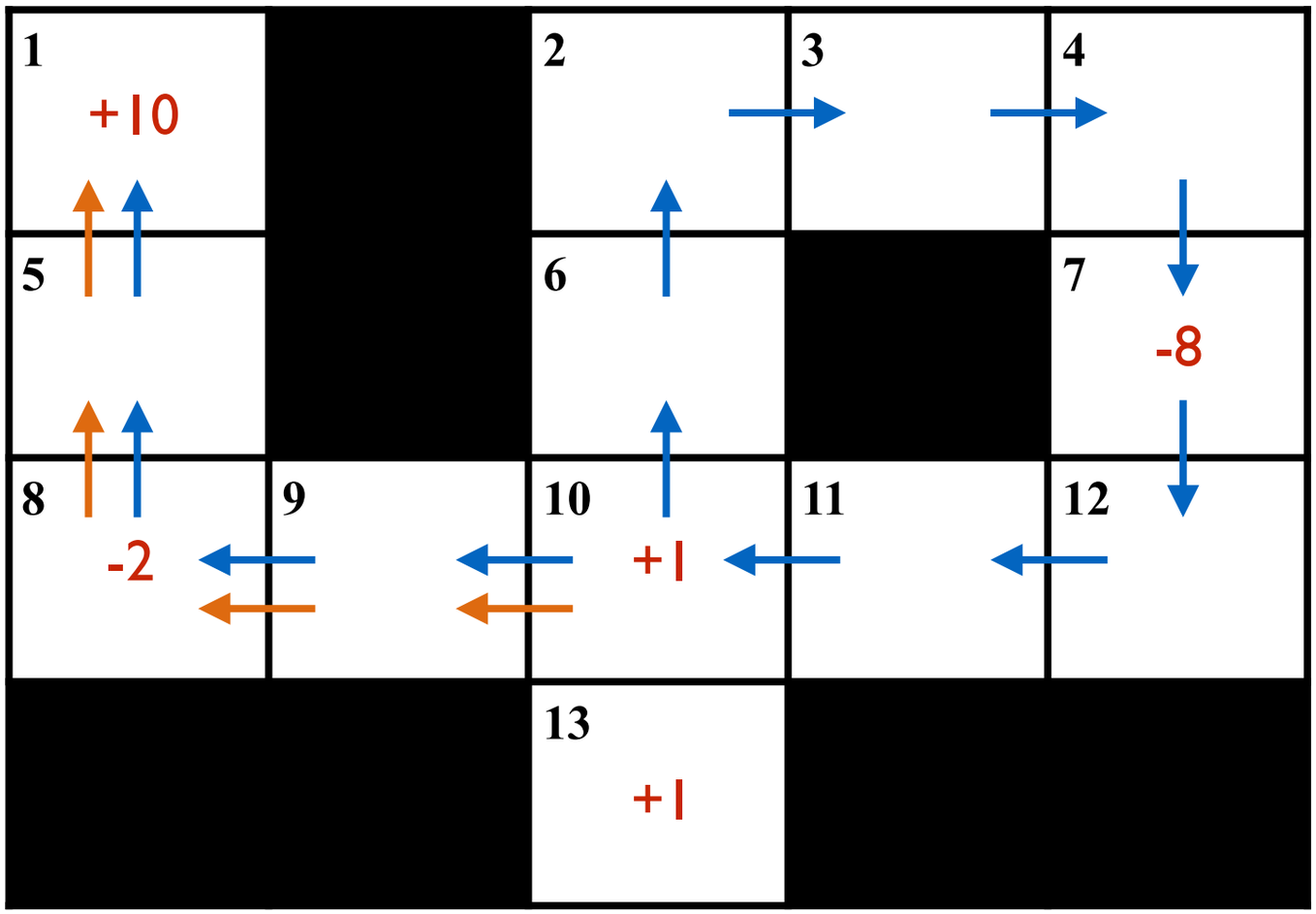
$$\gamma = 0.95$$

$$V^{\pi}(s = 13) = 1 + 0.95^{1}1 + 0.95^{6}(-8) + 0.95^{9}1 + 0.95^{11}(-2) + 0.95^{13}10$$

$$V^{\pi}(s = 13) = 0.695$$

Monte Carlo Estimates

- Assume we have acquired the samples along the path



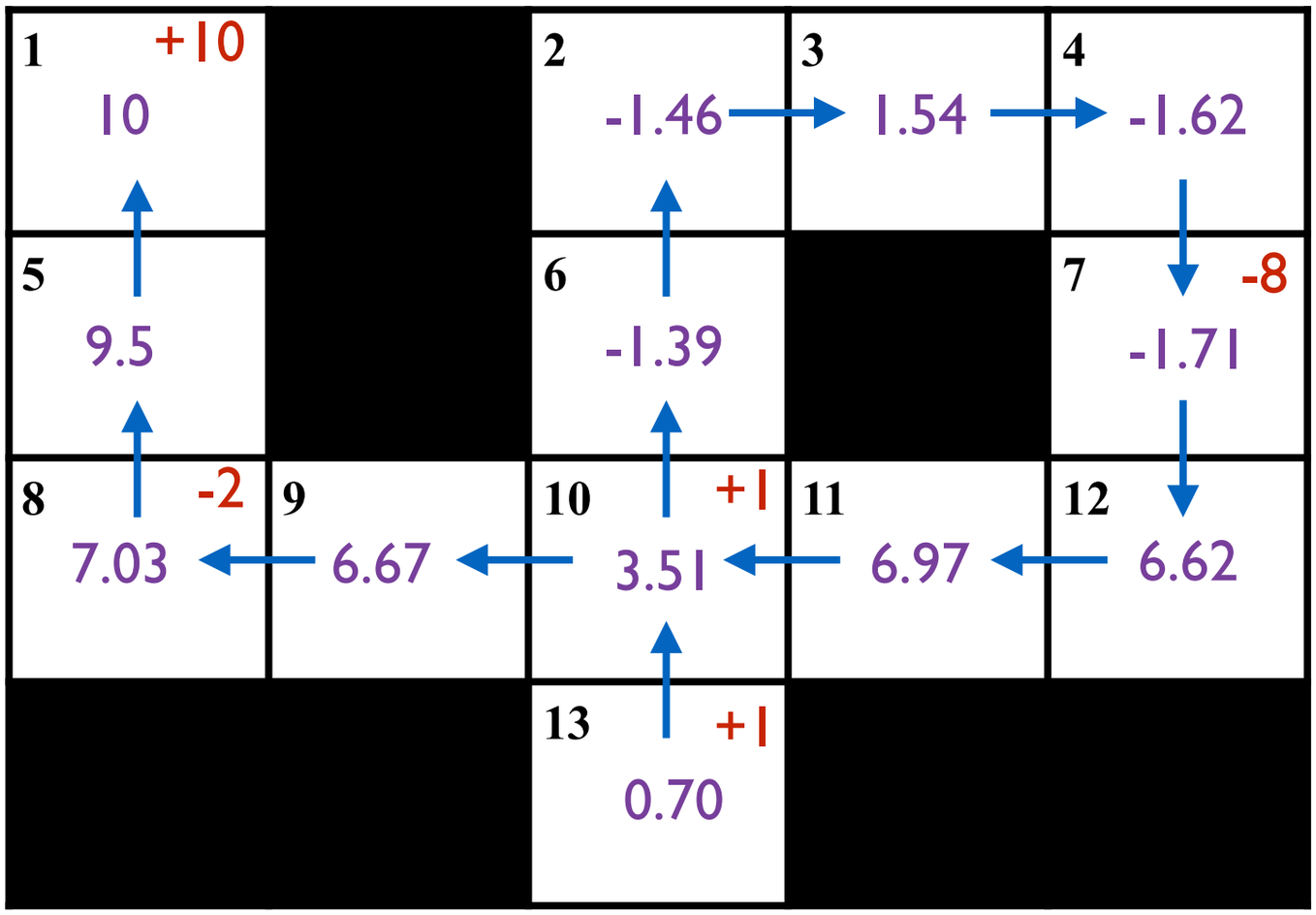
$$\gamma = 0.95$$

$$V^{\pi}(s=10) = \frac{1}{2}(1 + 0.95^5(-8) + 0.95^8 1 + 0.95^{10}(-2) + 0.95^{12}10)$$

$$+\frac{1}{2}(1 + 0.95^2(-2) + 0.95^4 10) = -0.16 + 3.67 = 3.51$$

Monte Carlo Estimates

- Resulting values:



$$\gamma = 0.95$$

- Could use the new estimates to update policy for state 10
- Monte Carlo has large variances for less constrained problems
- Low value for state 13 given transition to state 10's value

Temporal Difference Learning

- The value function can be written in recursive form

$$V^{\pi}(s) = E[r(a_0, s_0) + \gamma V^{\pi}(s') \,|\, s_0 = s]$$

next state

- Estimate of value $V^{\pi}(s_t)$ is thus given by single step

$$r_t + \gamma V^{\pi}(s_{t+1})$$

- Bootstrap estimate using the current value estimates i.e., value of next state is just an estimate

Temporal Difference Learning

- Approximate value using a weighted average

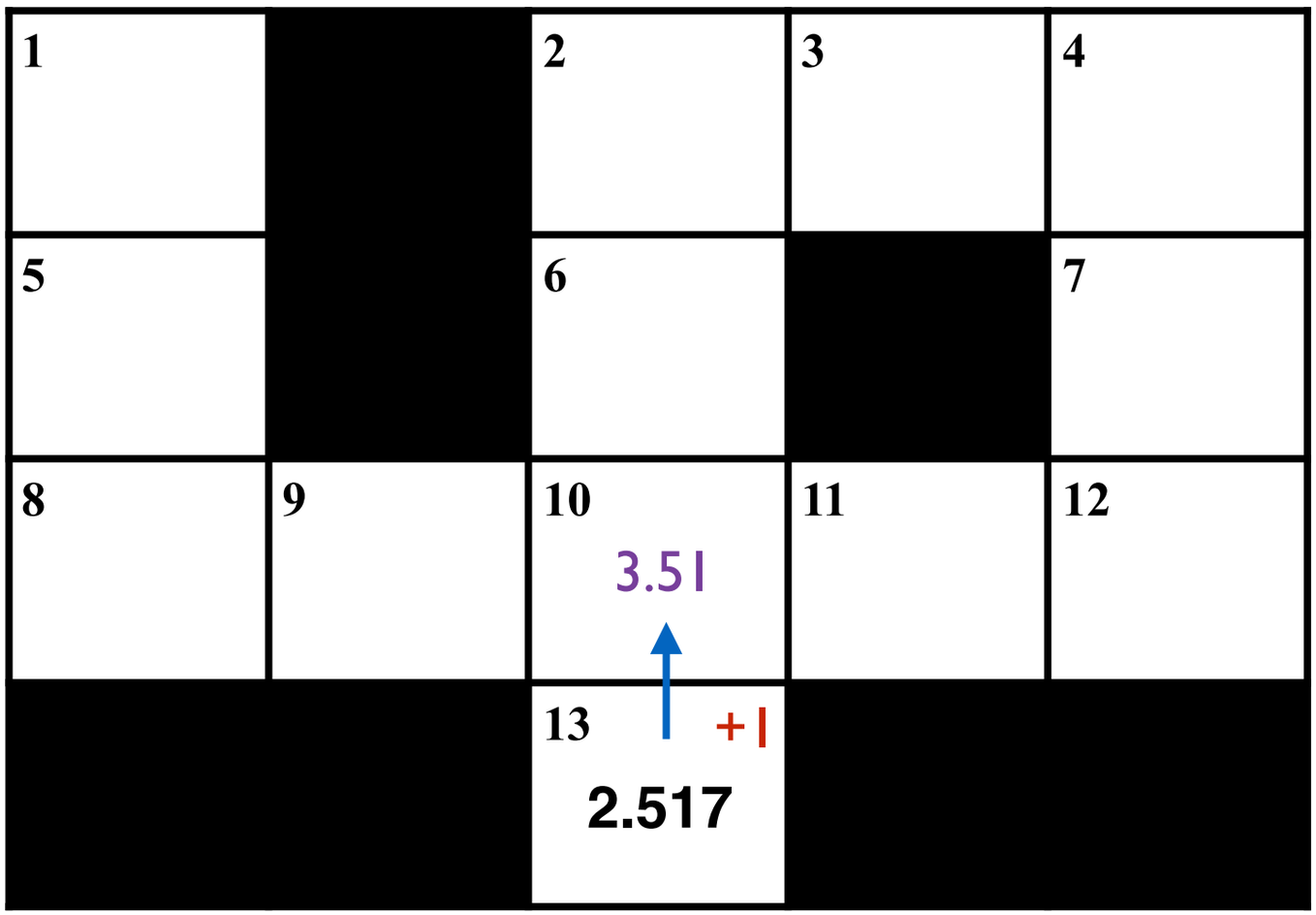
$$V^{\pi}_{\text{new}}(s_t) = (1-\alpha)V^{\pi}(s_t) + \alpha(r_t + \gamma V^{\pi}(s_{t+1}))$$

$$V^{\pi}_{\text{new}}(s_t) = V^{\pi}(s_t) + \underbrace{\alpha(r_t + \gamma V^{\pi}(s_{t+1}) - V^{\pi}(s_t))}_{\text{TD Error}}$$

- TD error between expected and estimated values
- Expected TD error will tend towards zero

Temporal Difference Example

- Lets consider the update for this one step



$$\gamma = 0.95$$

$$\alpha = 0.5$$

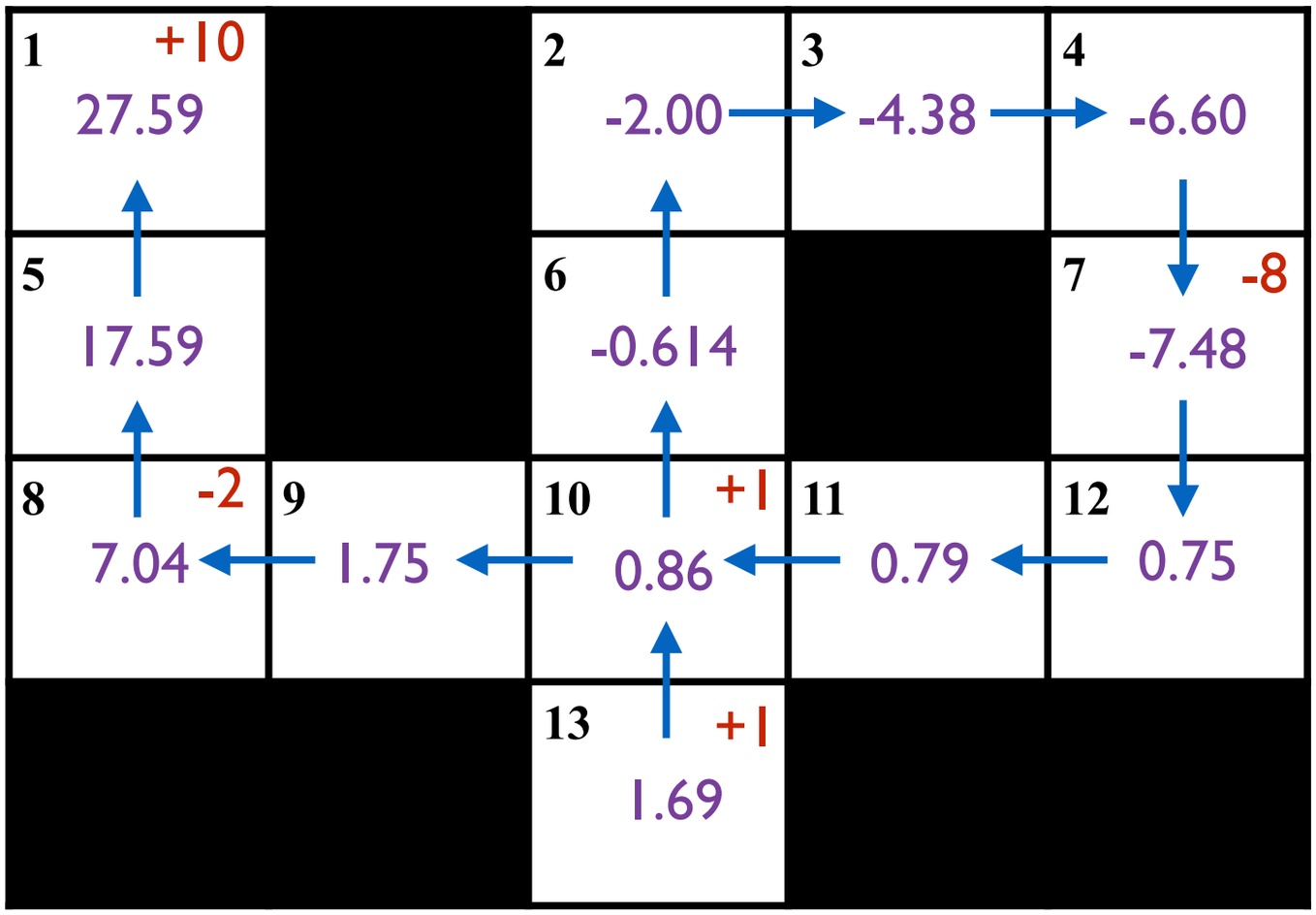
$$V^{\pi}_{\text{new}}(s_t) = V^{\pi}(s_t) + \alpha(r_t + \gamma V^{\pi}(s_{t+1}) - V^{\pi}(s_t))$$

$$V^{\pi}_{\text{new}}(s = 13) = 0.7 + 0.5(1 + 0.95(3.51) - 0.7)$$

$$V^{\pi}_{\text{new}}(s = 13) = 0.7 + 0.5(3.6345) = 2.517$$

Temporal Difference Example

- After looping through the samples ten times



$$\gamma = 0.95$$

- Can update policy along the way
- Estimates are more consistent with lower variance
- Takes a while to propagate distant rewards

State-Action Values

- Policy improvements currently rely on transition model

$p(x_{t+1}|u_t, x_t)$

$$\pi'(x_t) = \arg\max_u \left(r(u, x_t) + \gamma E\left[V^\pi(x_{t+1})\right]\right)$$

- Learn state-action values directly

$$Q^\pi(x, u) = E\left[\sum_{t=0}^{\infty} \gamma^t r(x_t, u_t) | x_0 = x, u_0 = u\right]$$

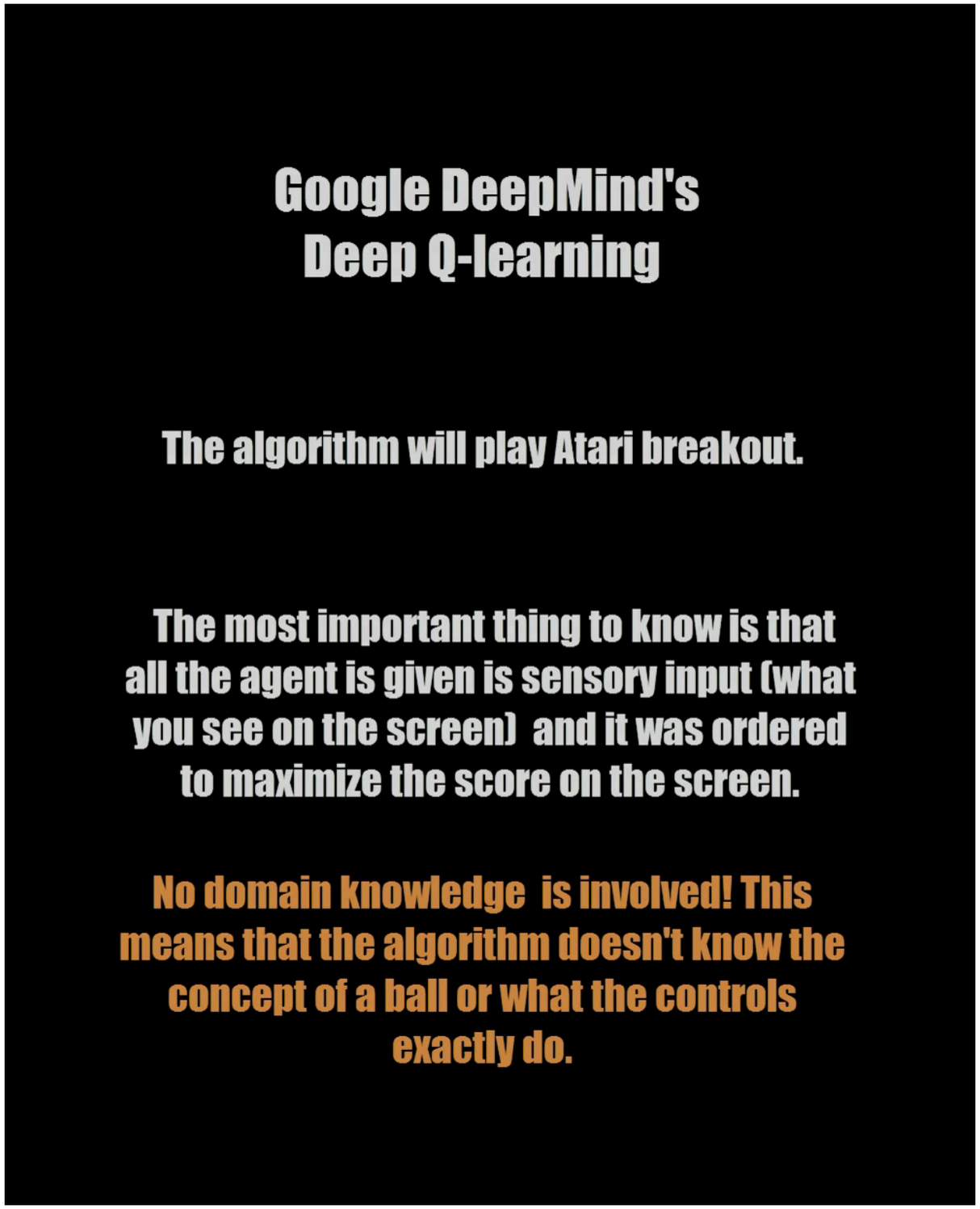
$$Q^\pi(x, u) = r(x, u) + \gamma E\left[V(x')\right]$$

  - Reward and transition to next state are implicit

- Policy improvement is then given by

$$\pi'(x) = \arg\max_u Q^\pi(x, u)$$

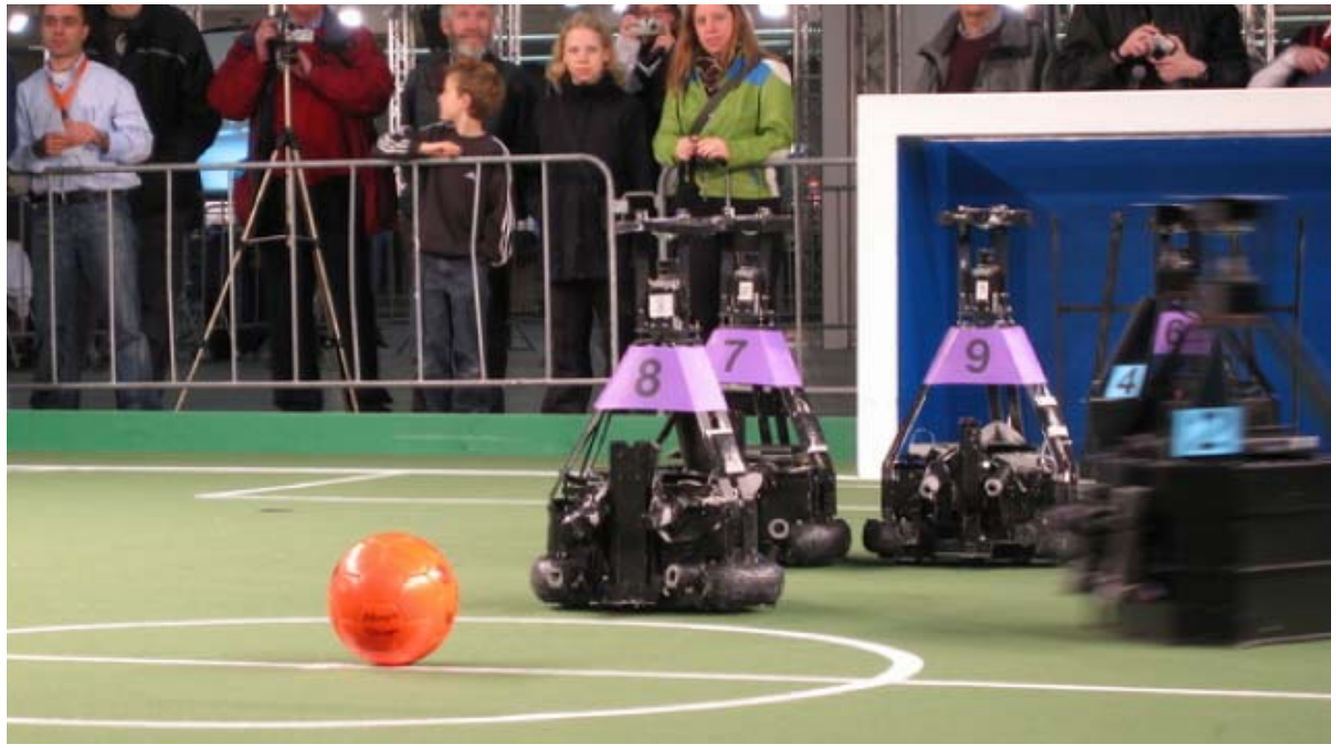
Atari Example

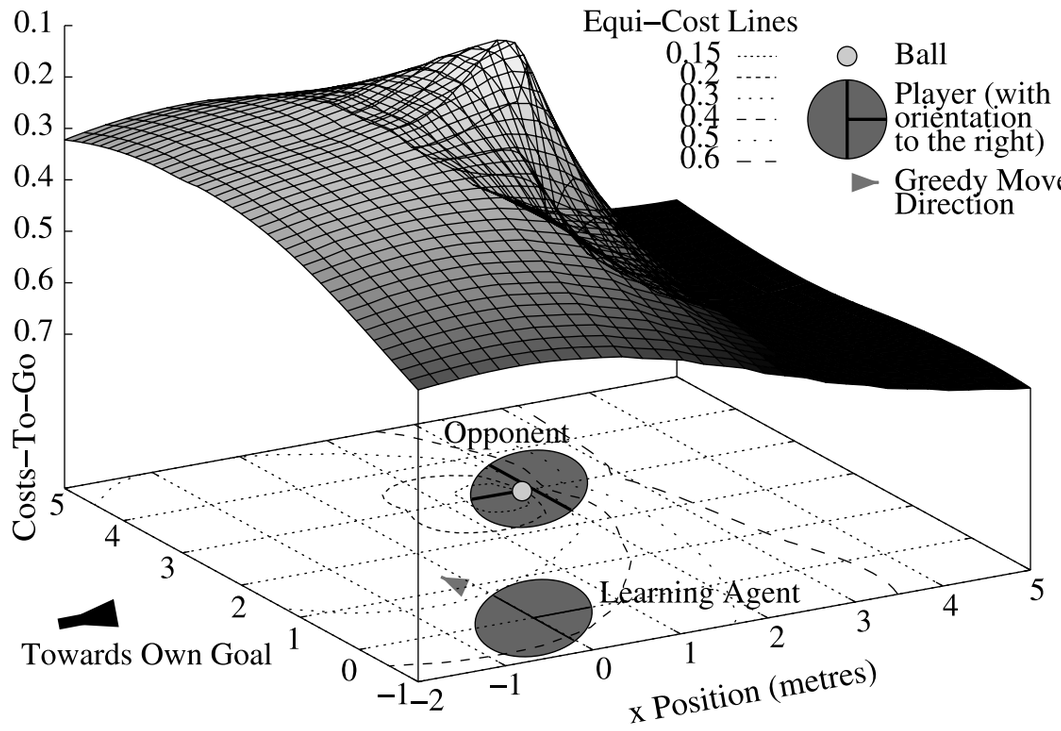


V. Minh et al., Playing Atari with Deep Reinforcement Learning, 2013
Two Minute Papers,Google DeepMind's Deep Q-learning playing Atari Breakout, Youtube,, 2015

Robot Example

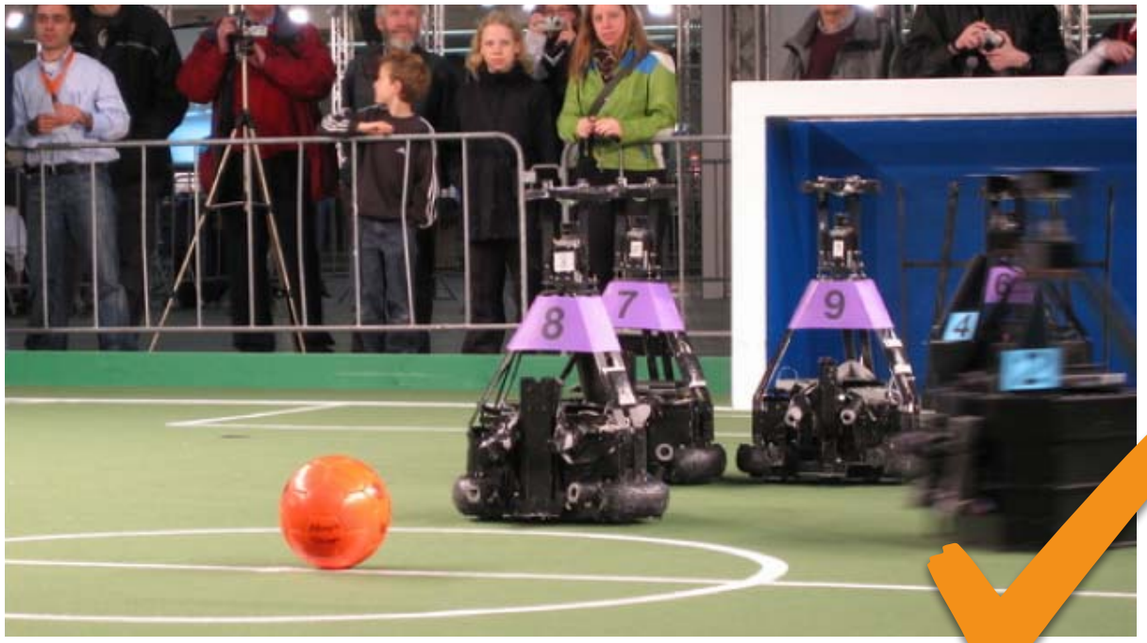
- Q-Learning for learning soccer skills

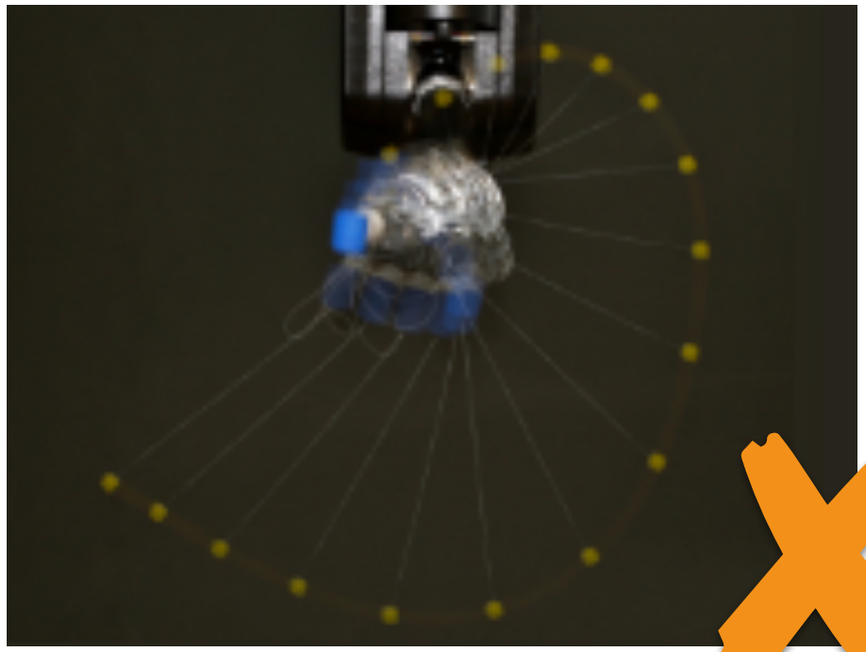




- Robot learns skills such as dribbling and duelling
- Figure shows slice of 9D-input value function

Reinforcement Learning for Robotics





- Value function methods are powerful, but can be difficult to apply to high-dimensional state spaces
- Complex skills often learned using policy search methods

Policy Search Methods

Parameterized Policies

- Policies often have a parameterized form
  - Gaussian linear feedback policy

$$a_t \sim \mathcal{N}(-\Theta s_t, \sigma^2)$$

  - Desired trajectory basis functions

$$s_t^d = \sum_i \theta_i \mathcal{N}(\mu_i - t, h^2)$$

  - Neural network controller

$$a_t \sim \mathcal{N}(f_{NN}(s_t; \theta), \sigma^2)$$

- May be deterministic or stochastic
- Define a family of potential controller solutions

Problem Statement

- Executing a policy results in a sample trajectory

$$\tau = \{s_0, a_0, \ldots s_T, a_T, s_{T+1}\}$$

- Each trajectory provides the robot with a return

$$R(\tau) = \sum_{t=0}^{T} r(s_t, a_t)$$

- Goal: find policy that maximizes the expected return

$$J(\theta) = \int_{\mathbb{T}} p(\tau \,|\, \pi_\theta) R(\tau) \mathrm{d}\tau$$

Find policy parameters that maximize the expected return

Problem Statement

- The goal is to compute $\arg\max_\theta J(\theta)$
- Finding the global optimum is intractable
  - Complicated dynamics and policies
  - Non-convex reward landscape
  - High-dimensional state space
  - No closed-form solutions
- Search (locally) for good policy parameters
- Can use gradient ascent to find (good) local optimum

$$\theta_{i+1} = \theta_i + \beta \nabla_\theta J(\theta)$$

- How to compute estimate of the gradient?

Finite Difference Method

- Finite difference method procedure:

1. Initialize policy with some initial parameters $\theta_{i=0}$
2. Sample neighborhood of parameters $\tilde{\theta}_{ij} \quad j \in \{1, ..., n\}$
3. Evaluate parameter settings to estimate returns $J(\theta_i) \quad J(\tilde{\theta}_{ij})$
4. Compute finite difference gradient estimate g_{fd}
5. Update parameters using gradient ascent $\theta_{i+1} = \theta_i + \beta g_{\text{fd}}$
6. Repeat from step 2

- Final performance depends on good initial parameters

Finite Difference Method

- Sample n local parameters from grid or stochastically

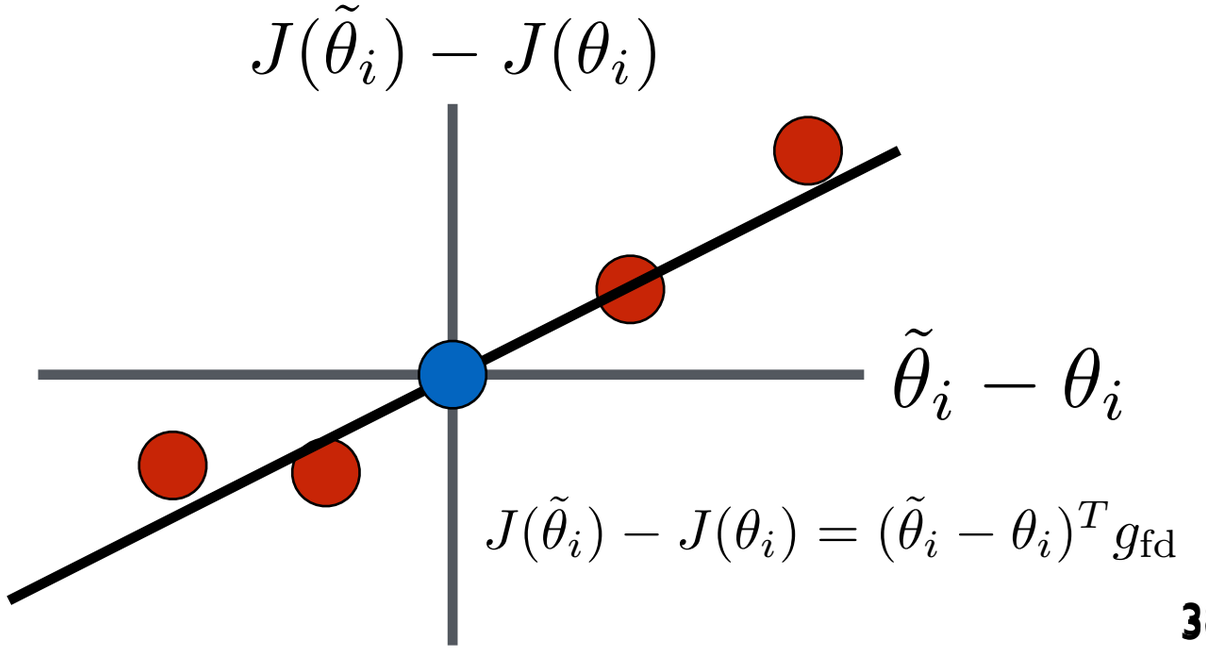
$$\tilde{\theta}_{ij} = \theta_i + \delta \qquad \tilde{\theta}_{ij} \sim \mathcal{N}(\theta_i, \Sigma)$$

- Estimate gradient using linear regression on differences

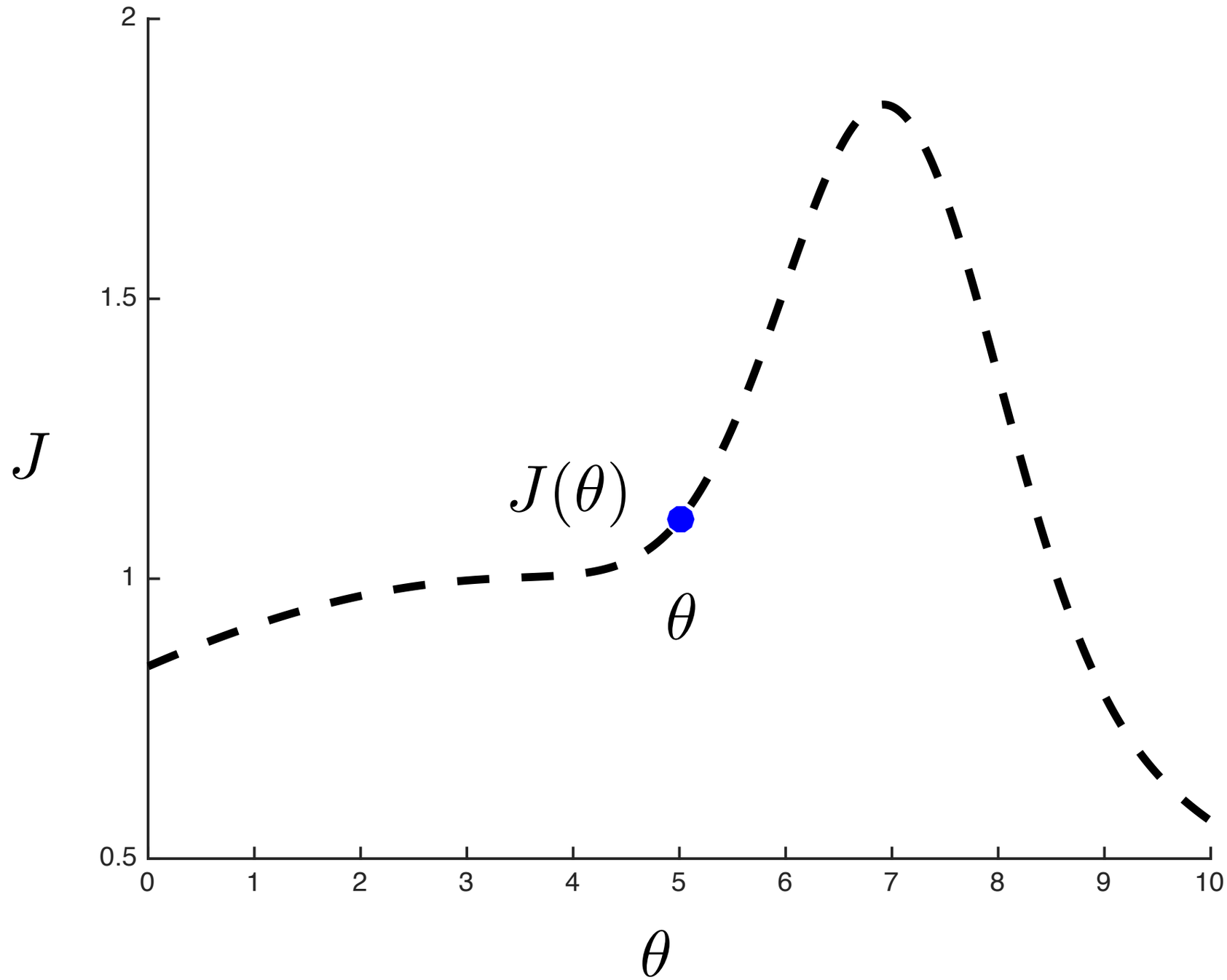
$$\widehat{\Theta} = \begin{bmatrix} | & & | \\ \tilde{\theta}_{i1} - \theta_i & \dots & \tilde{\theta}_{in} - \theta_i \\ | & & | \end{bmatrix} \qquad \widehat{J} = \begin{bmatrix} J(\tilde{\theta}_{i1}) - J(\theta_i) \\ \vdots \\ J(\tilde{\theta}_{in}) - J(\theta_i) \end{bmatrix}$$

$$g_{\text{fd}} = (\widehat{\Theta}\widehat{\Theta}^T + \lambda I)^{-1}\widehat{\Theta}\widehat{J}$$

$$\theta_{i+1} = \theta_i + \beta g_{\text{fd}}$$



Finite Difference Example



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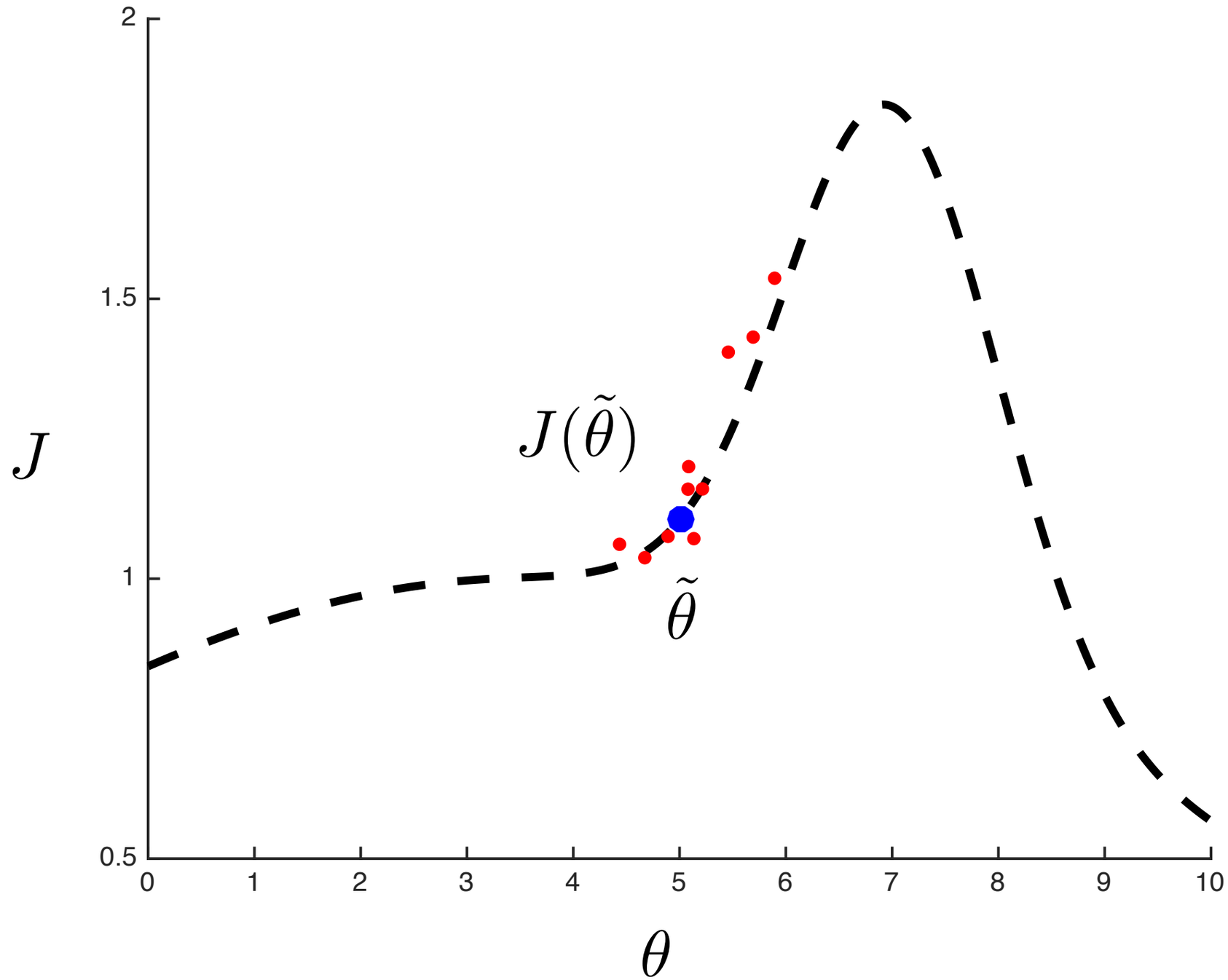

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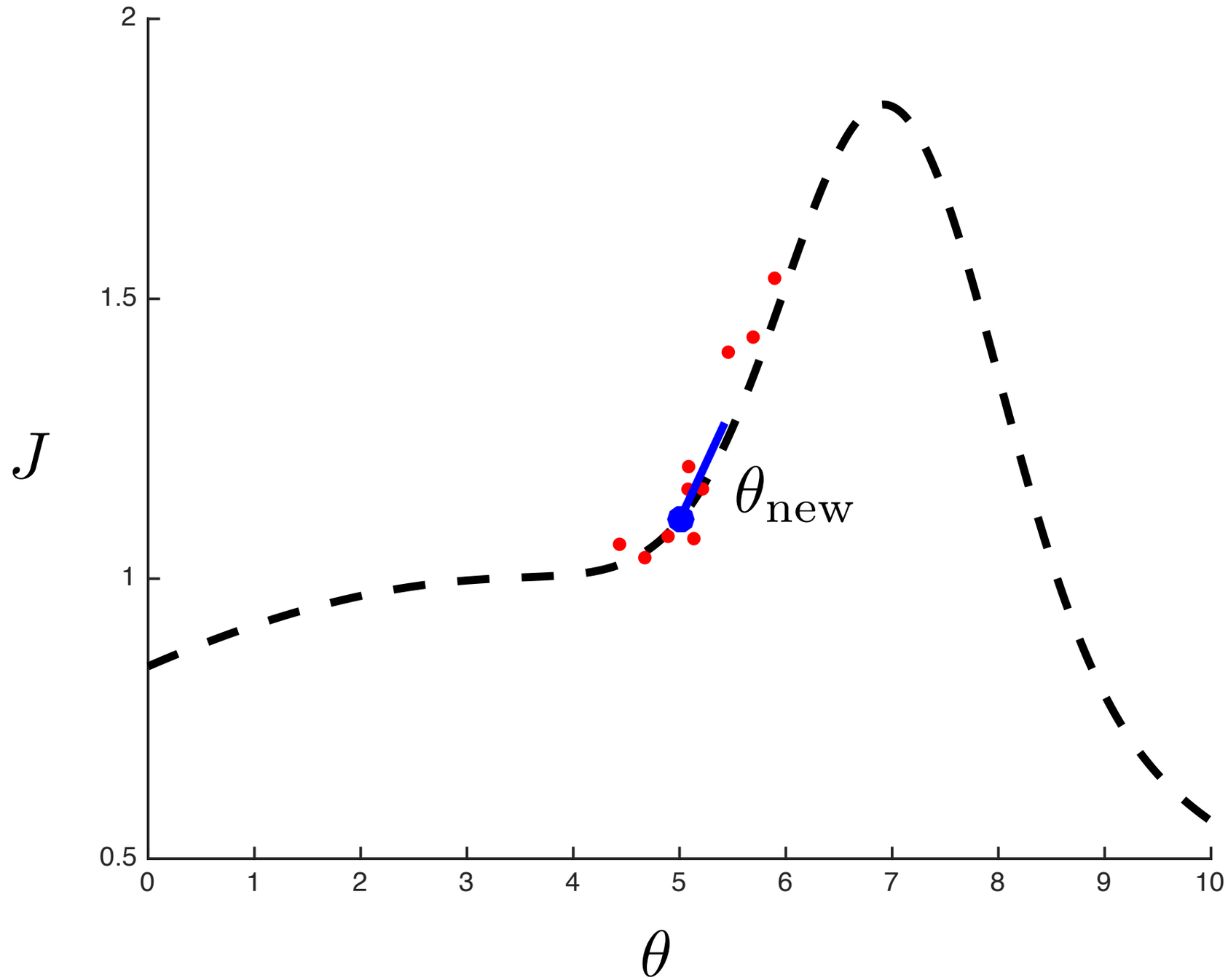

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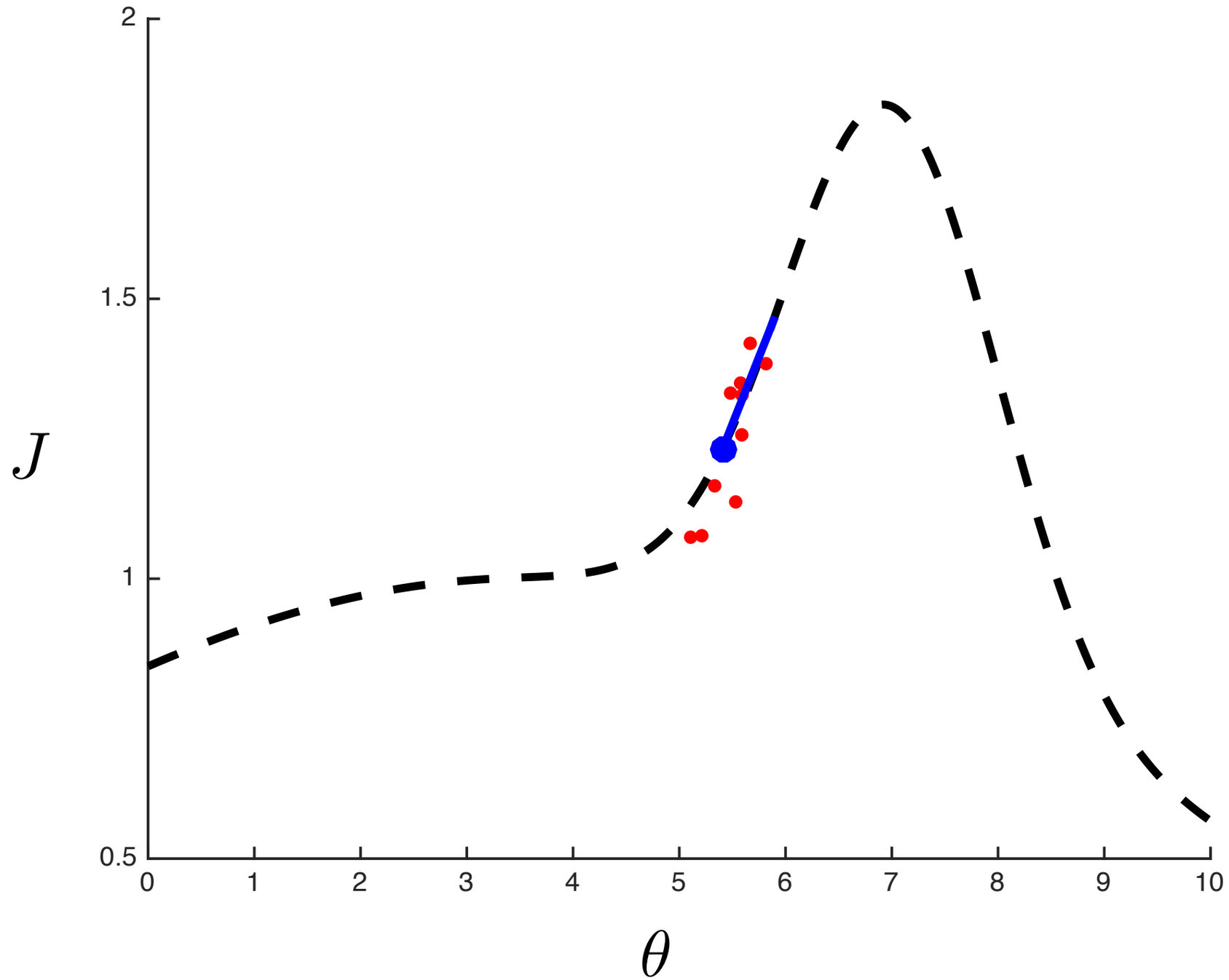

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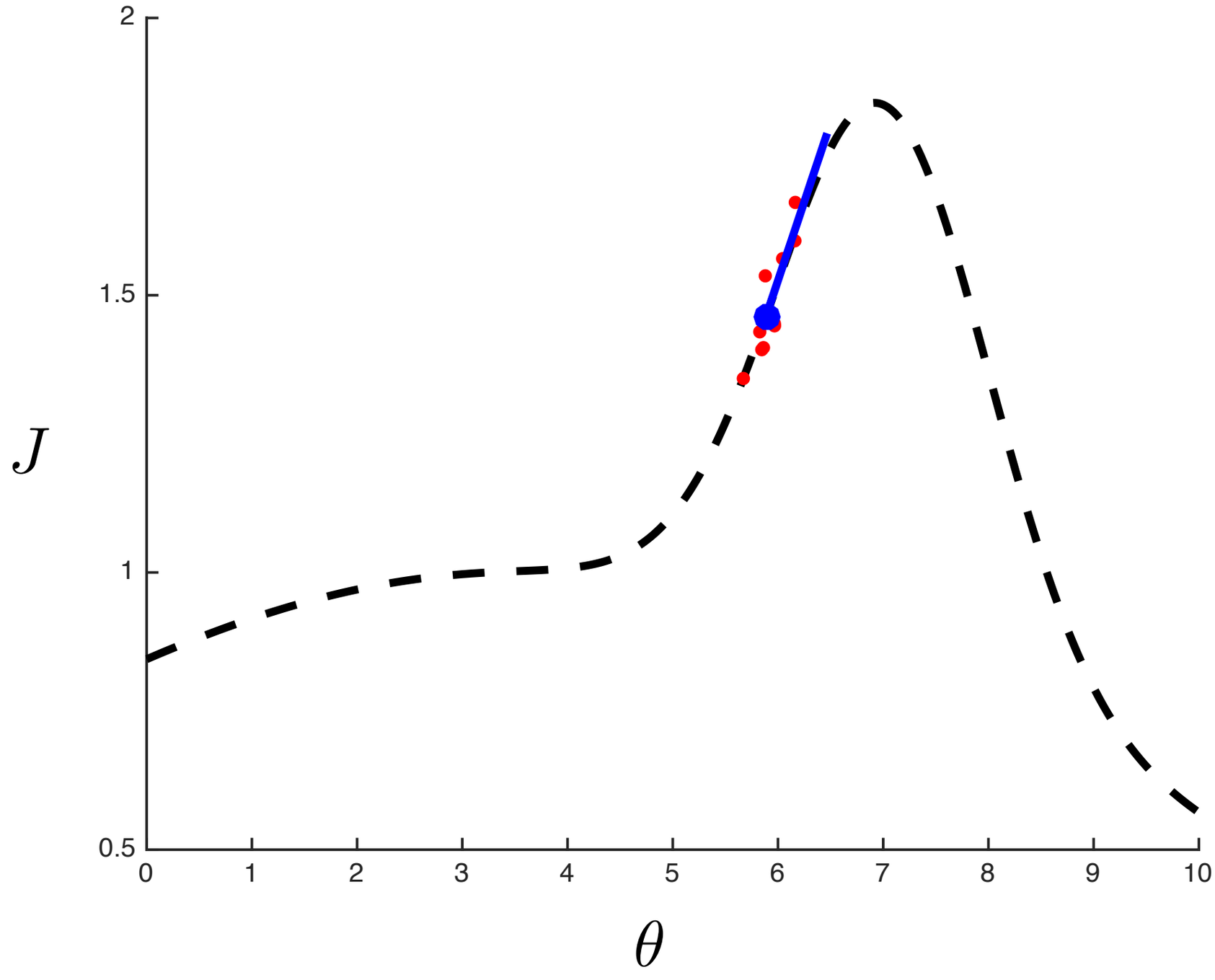

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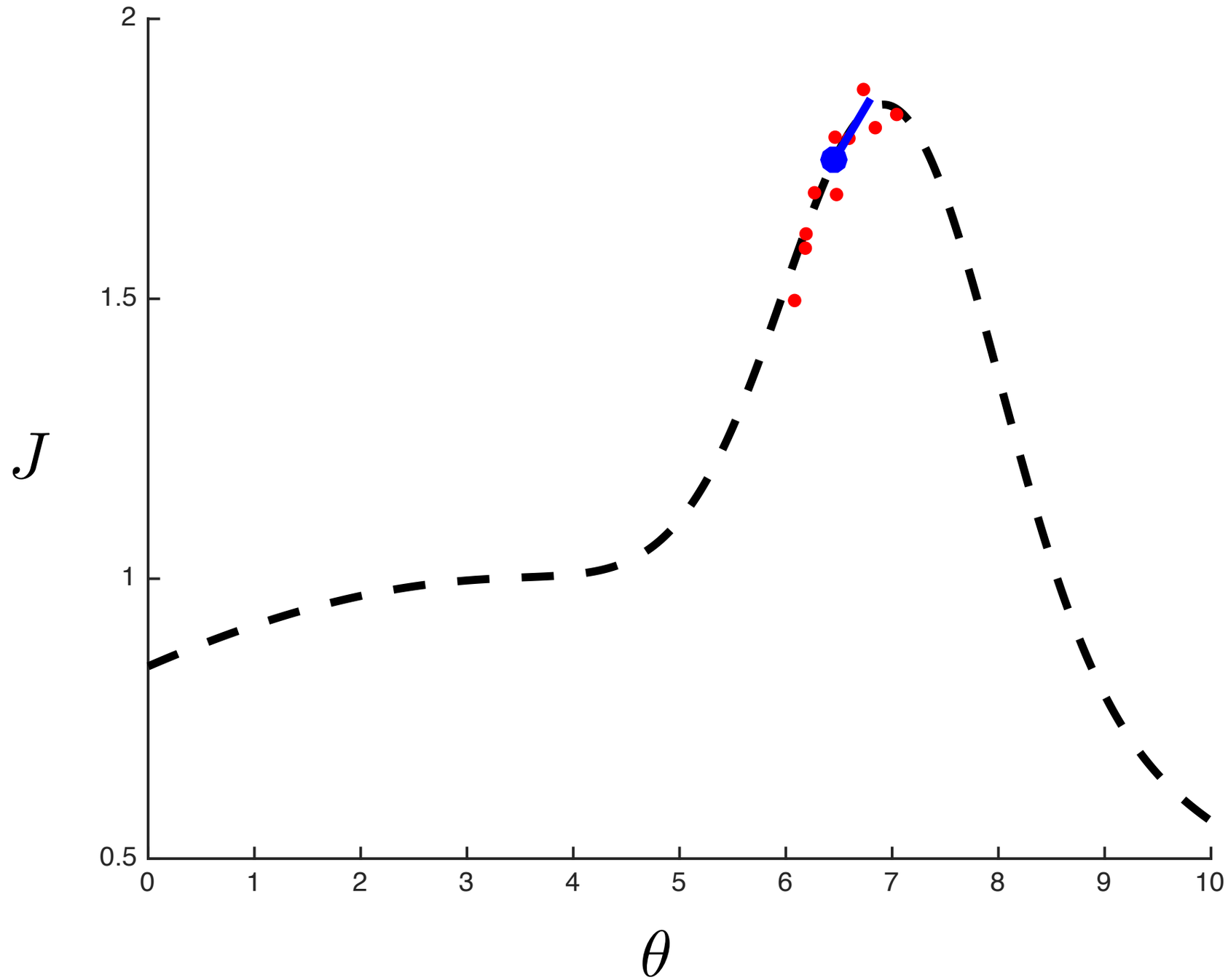
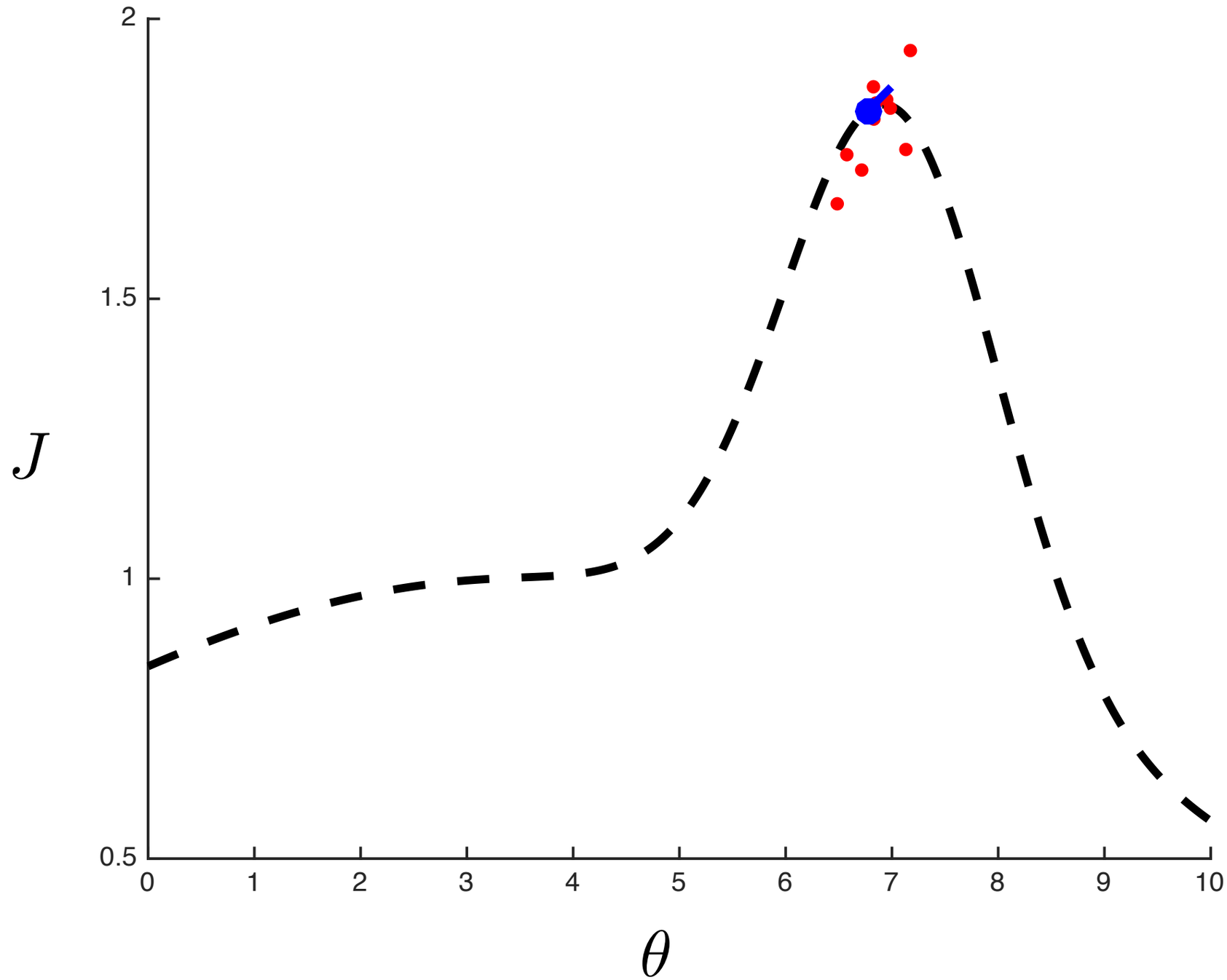
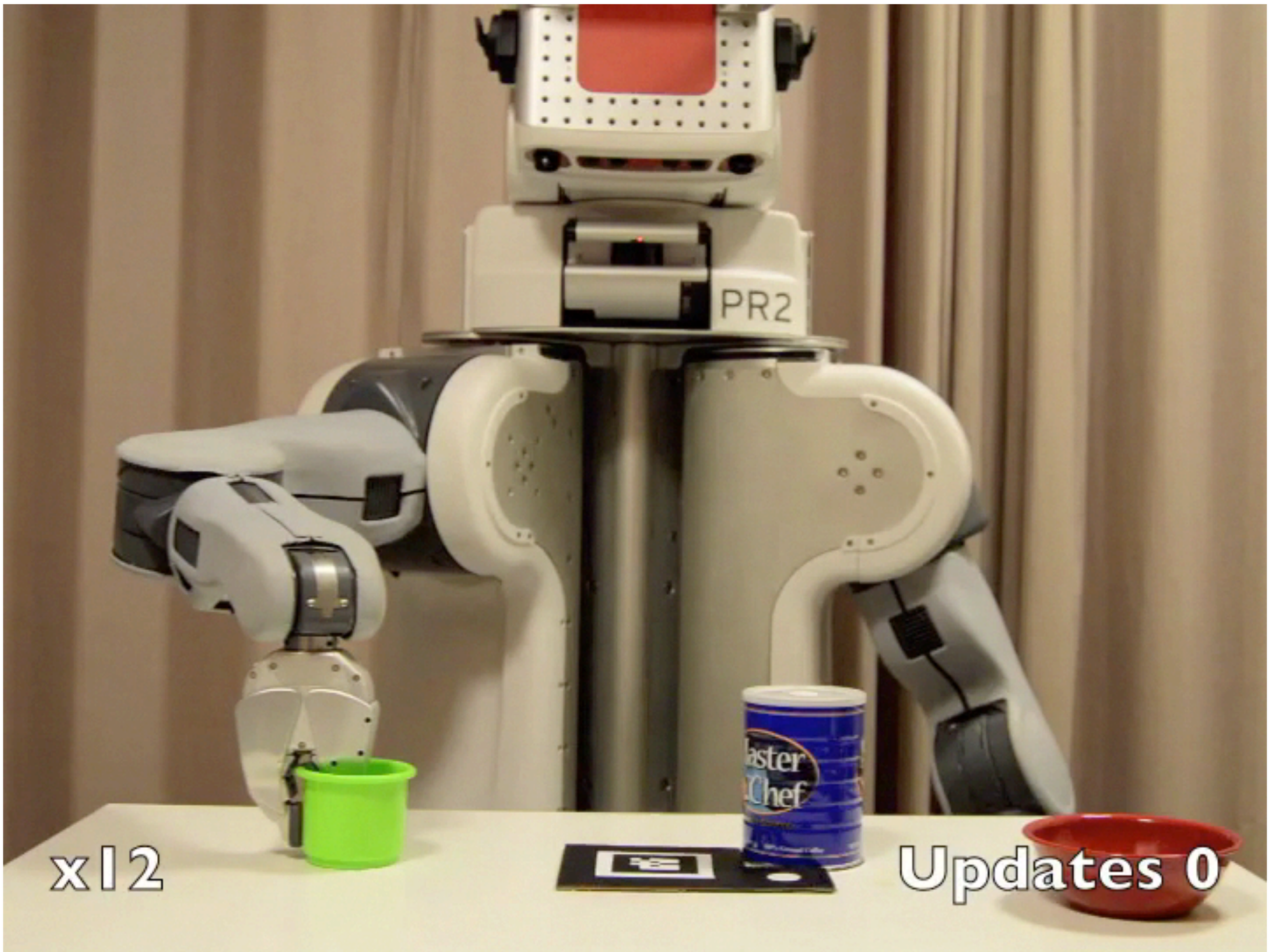


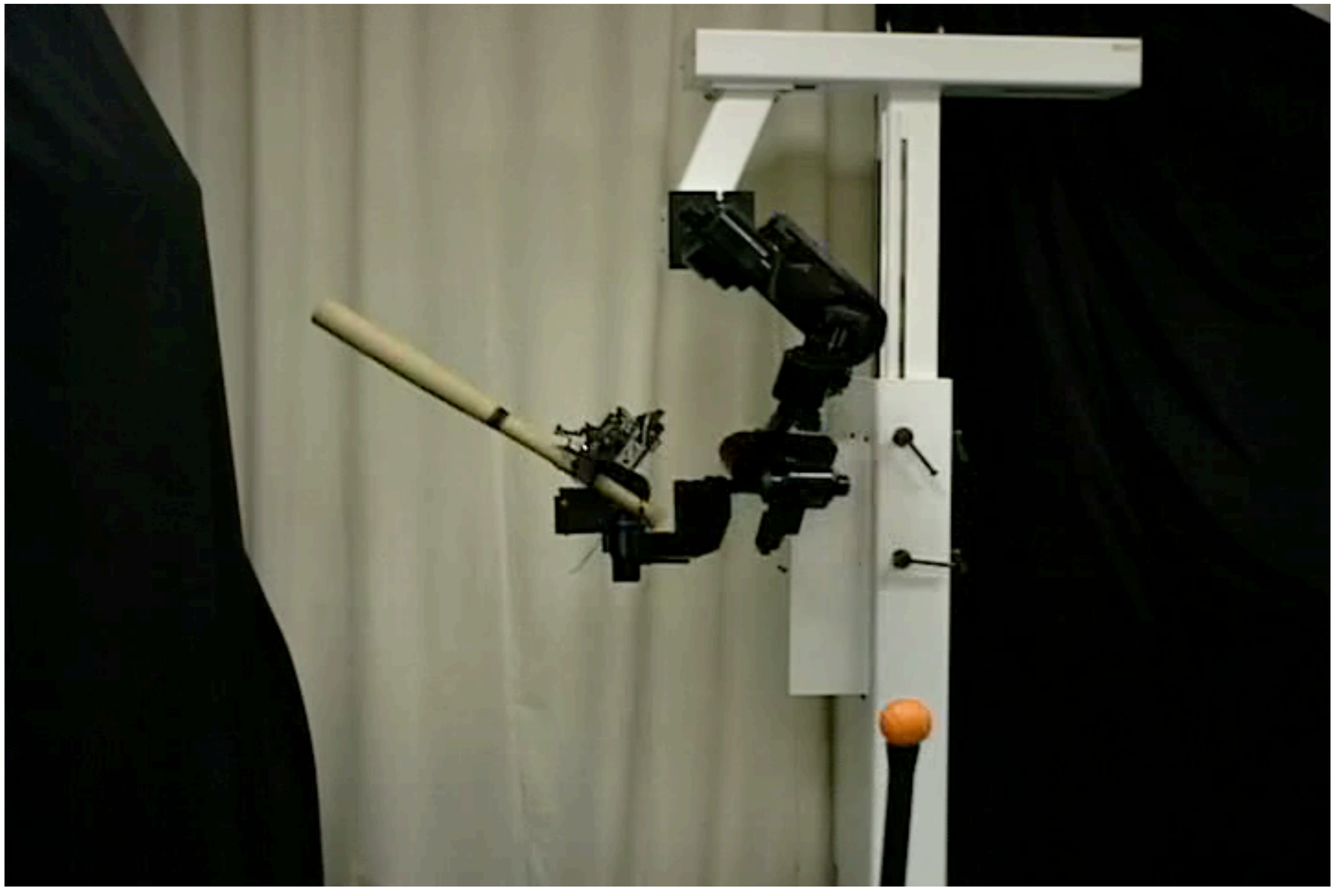
Finite Difference Example



REPS Example



Natural Actor Critic Example



Options and Hierarchical Policies

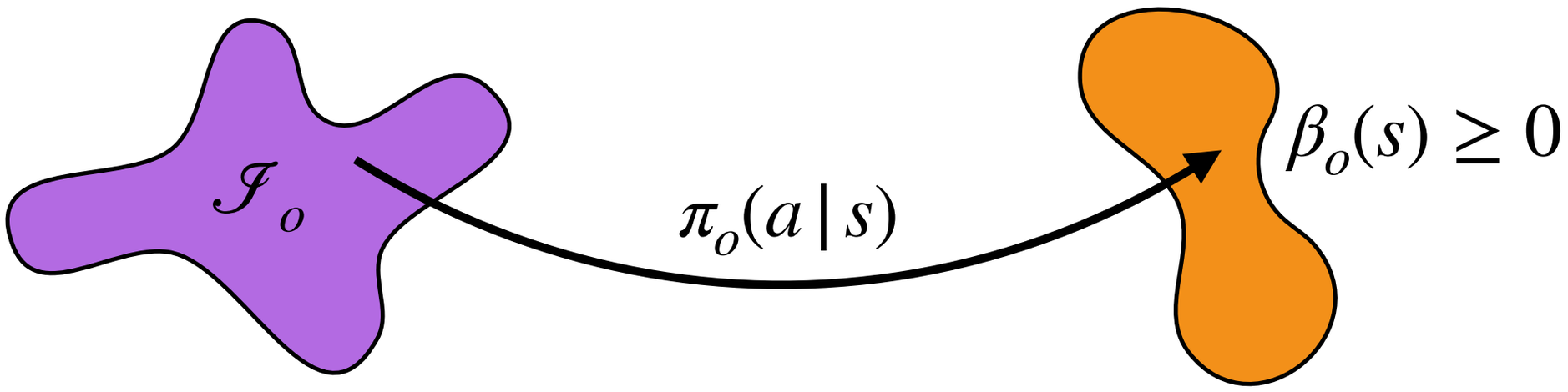
Options

- Often need to perform certain subtasks many times
  - Grasping objects, moving between rooms…
- Always reasoning about lowest-level actions is inefficient
  - Joint torque controller for cleaning an entire building…

- Learn Options as sub-policies for subtasks
  - Option policy $\pi_o(a \mid s)$
  - Initiation set $\mathcal{I}_o \subseteq \mathbb{S}$
  - Termination conditions $\beta_o(s)$

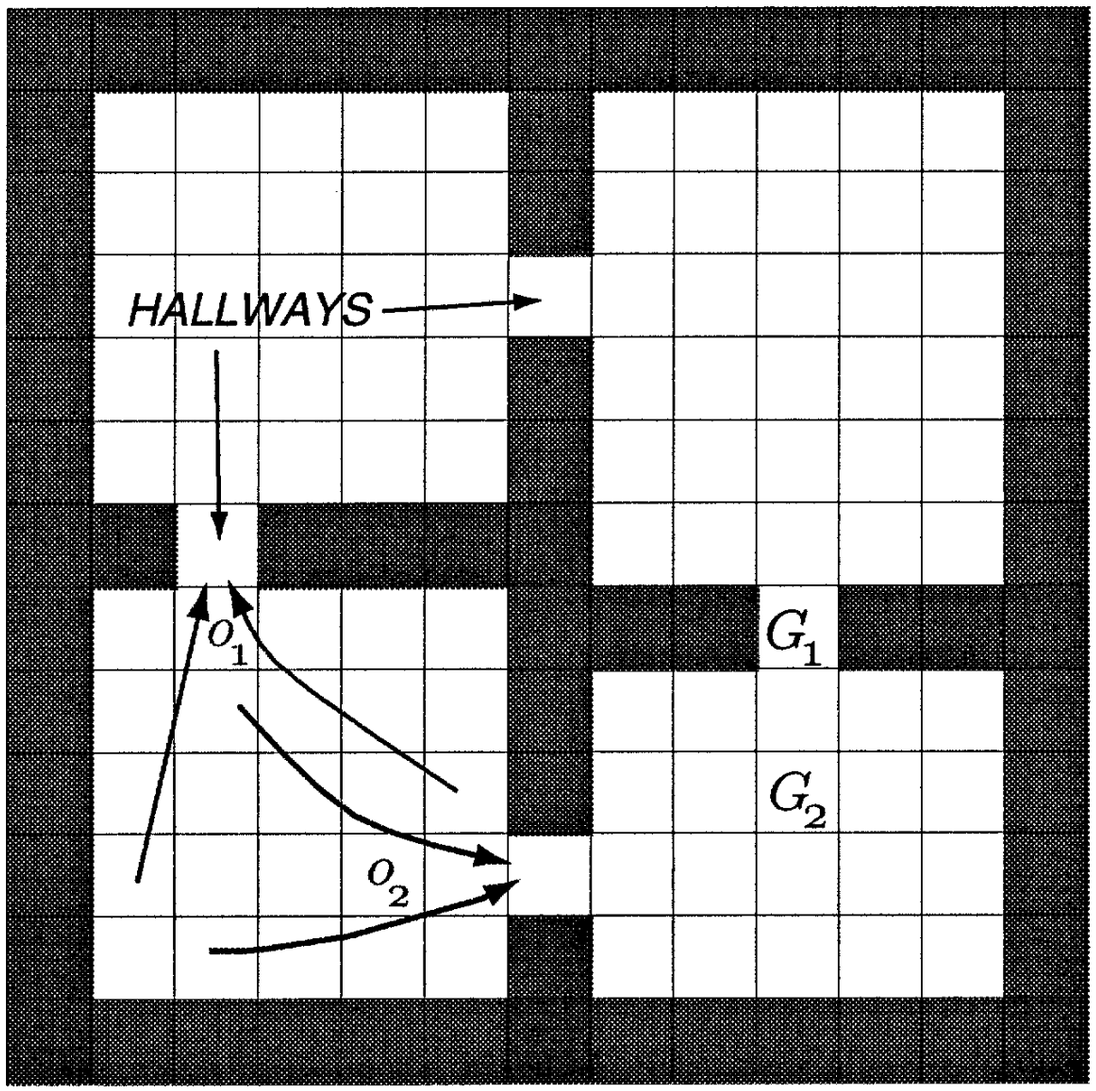
Initiation and Termination

- Initiation set $\mathcal{I}_o$
  - States from which the option may be started
  - Defines scope of states from which policy must work
- Termination conditions $\beta_o(s)$
  - Define probability of terminating for each state
  - Terminate when goal is reached or out of scope for policy

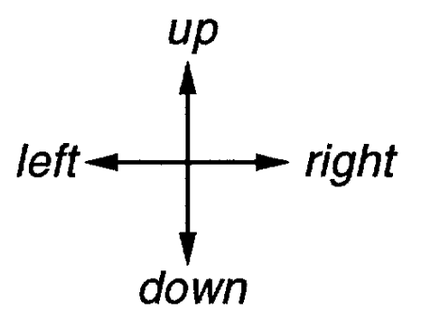


Classic Hallway Example

R.S. Sutton et al. / Artificial Intelligence 112 (1999) 181–211



4 stochastic primitive actions



Fail 33% of the time

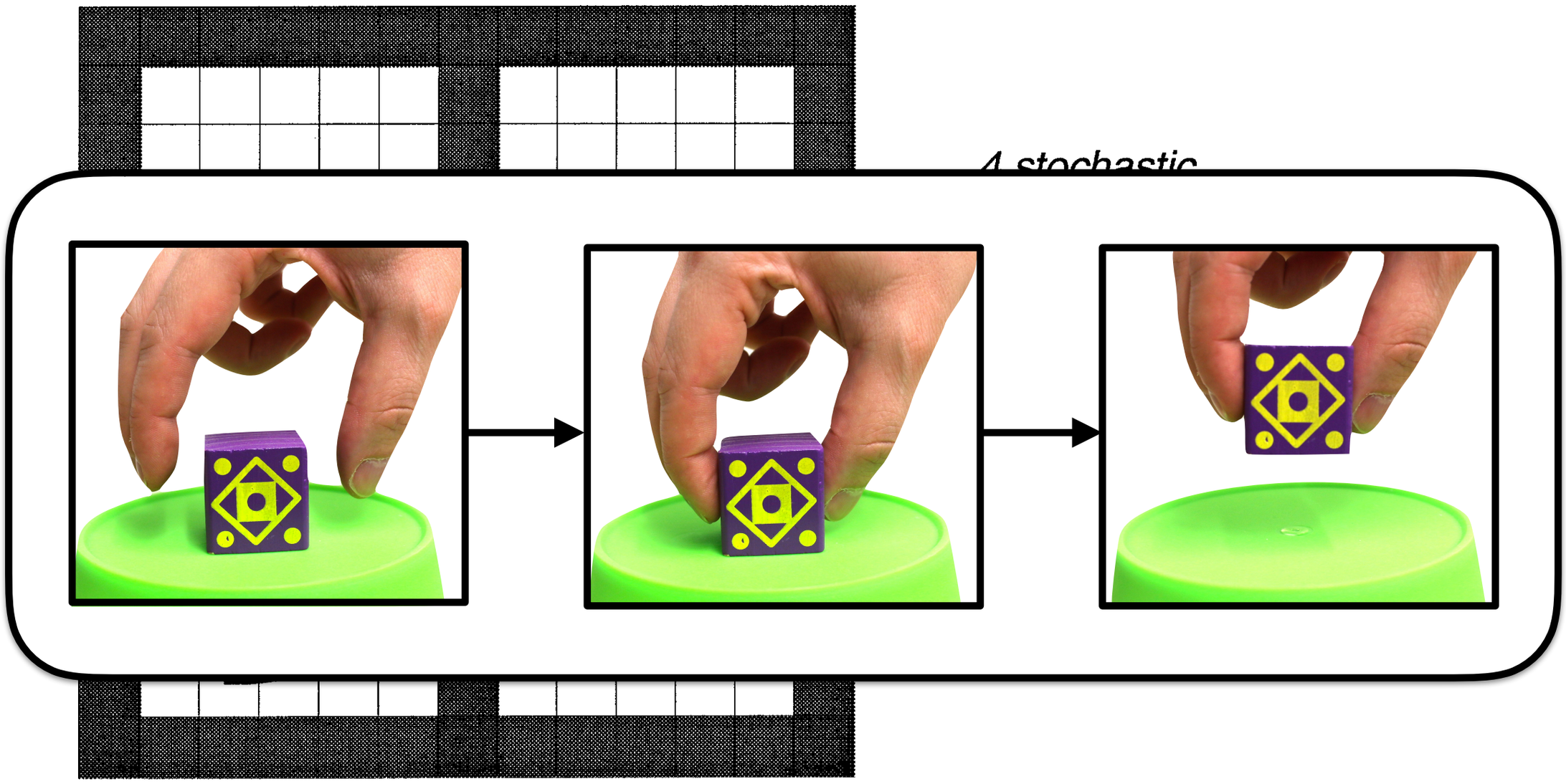
8 multi-step options

(to each room's 2 hallways)

- Learning option policy can be seen as solving smaller MDP

Classic Hallway Example

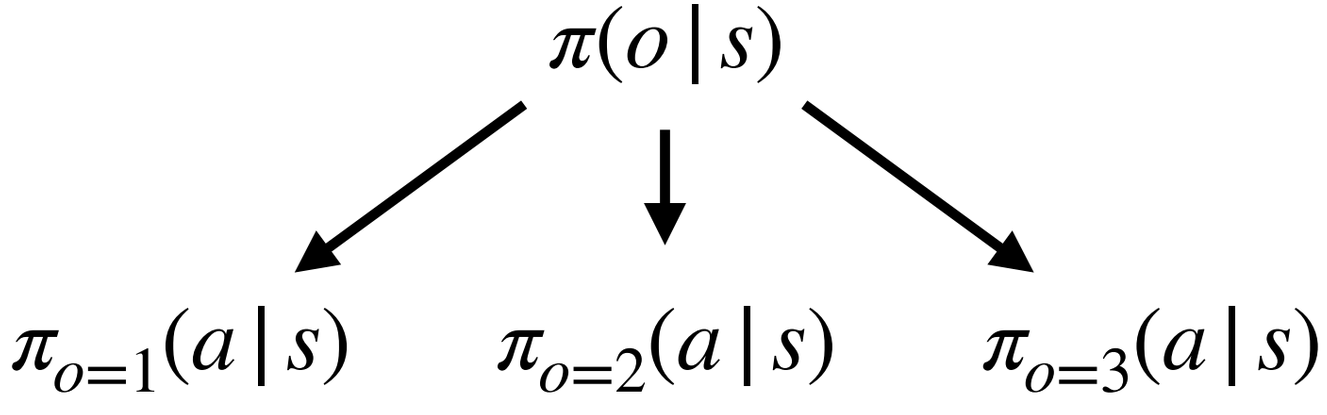
R.S. Sutton et al. / Artificial Intelligence 112 (1999) 181–211



- Learning option policy can be seen as solving smaller MDP

Hierarchical Policies

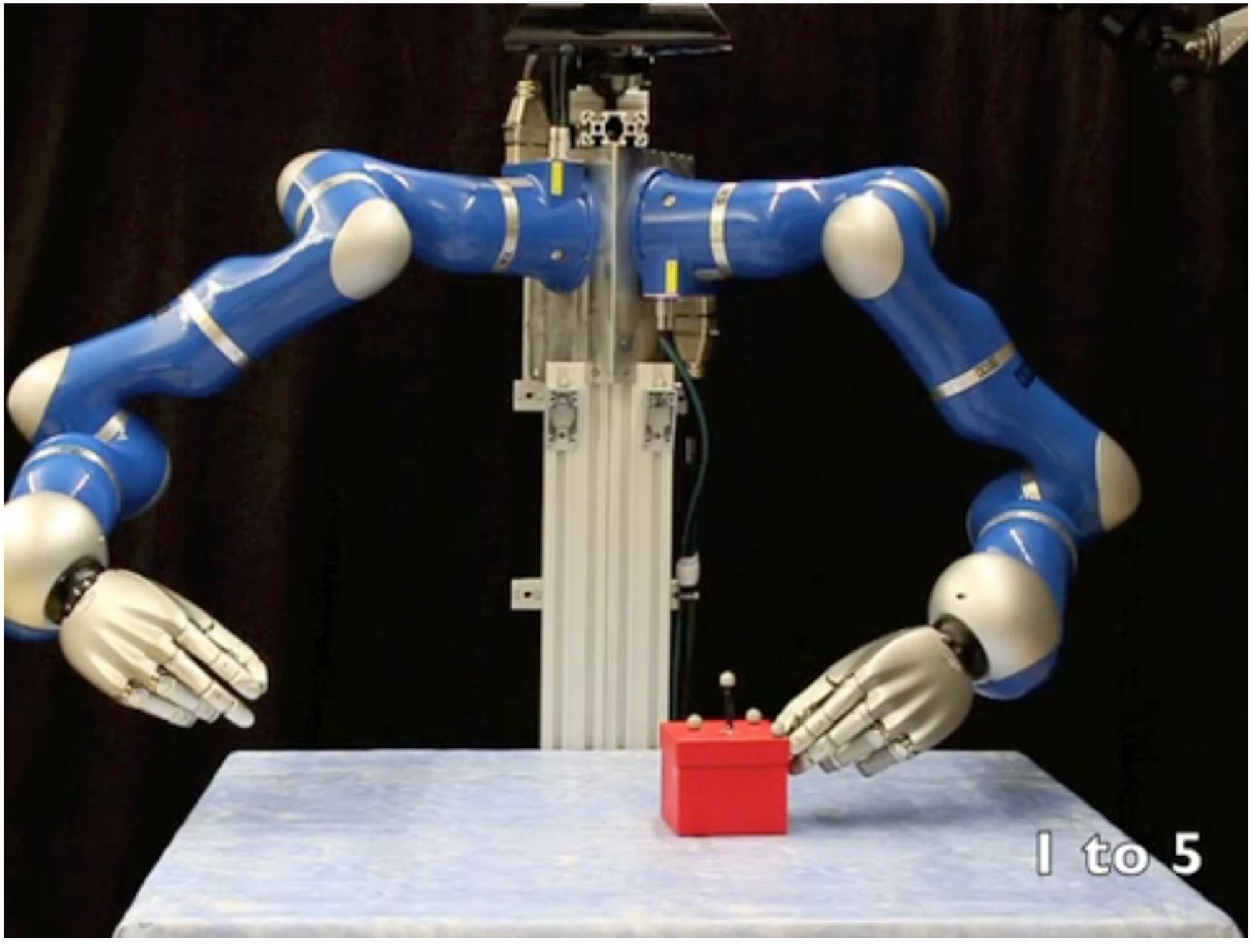
- Learn policies that select options in same way as actions



  - Take multiple steps into account for discounting rewards
  - Options can also call other options (options all the way down)
- Often call mid-level options "skills", e.g., grasp, pour, place

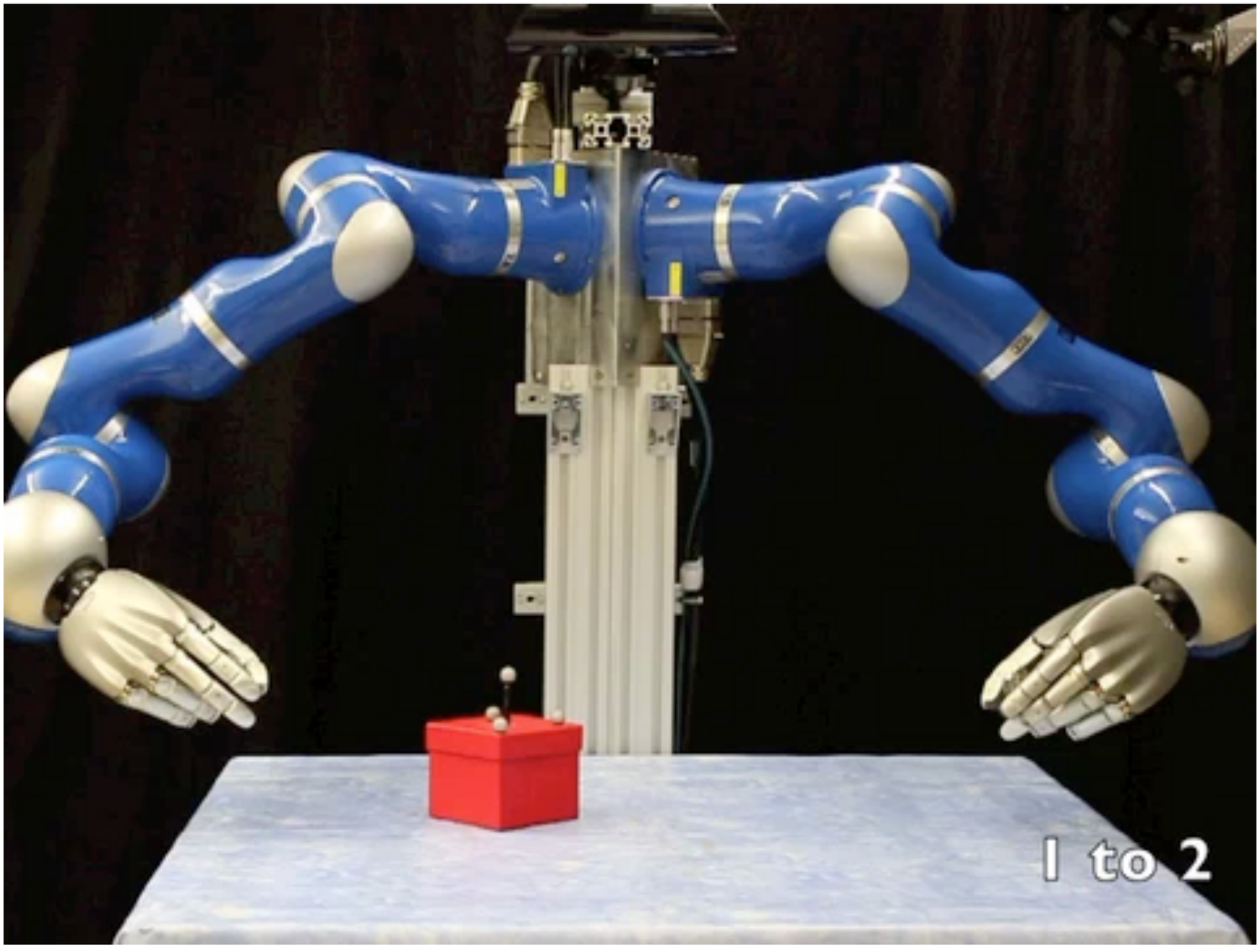
Robot Example

- Select different options based on object position



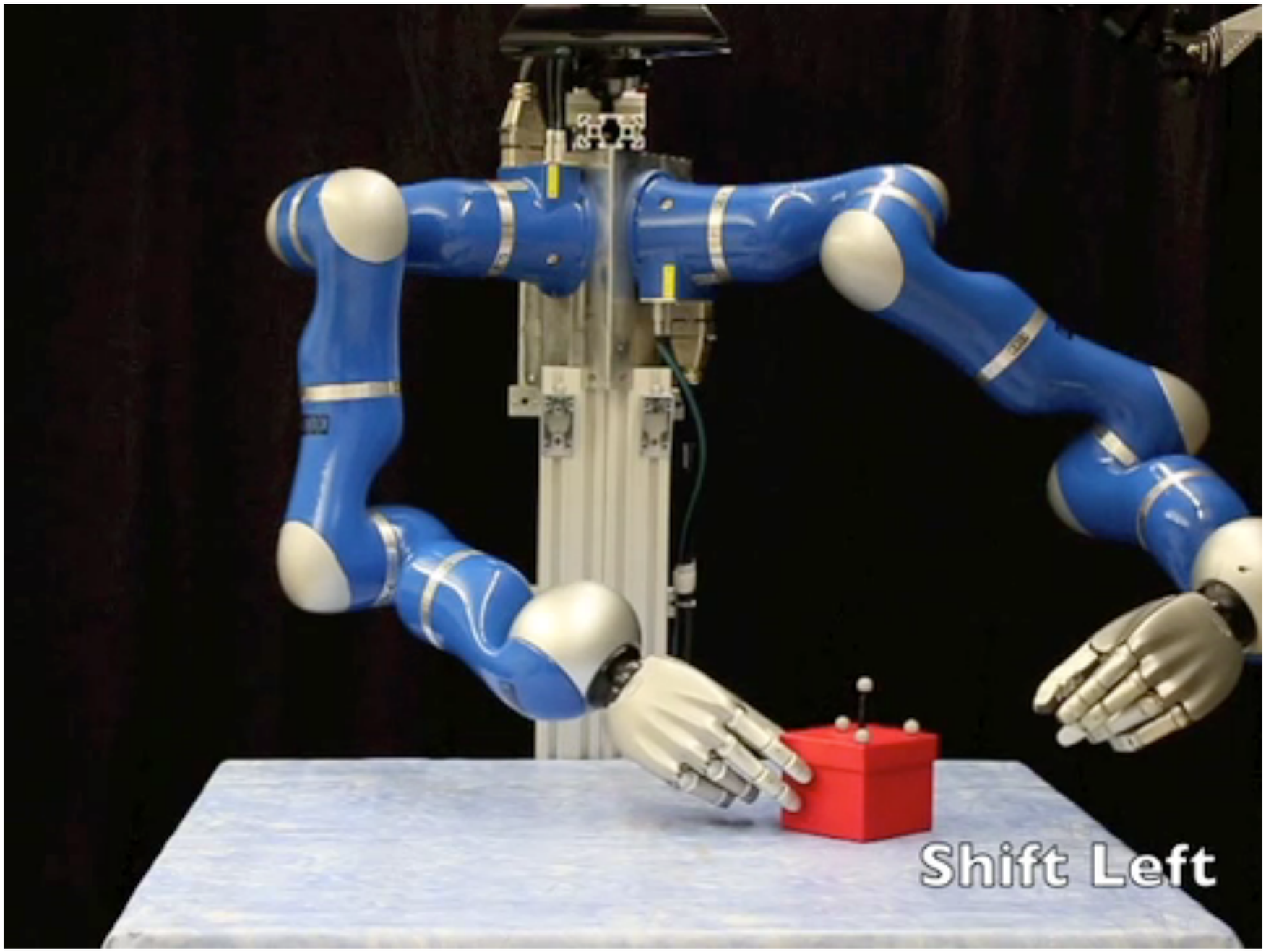
Robot Example

- Select different options based on object position



Robot Example

- Reuse skills/options between tasks



Questions?