A Downsian Spatial Model with Party Activism

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A unidimensional spatial model is proposed in this article. Although its formal structure parallels the spatial model of electoral competition, this model examines the decisions of individuals as they choose whether or not to become activists in one of two political parties. An individual "calculus of participation" is developed that is similar to the spatial interpretation of the "calculus of voting." This calculus is then generalized by examining conditions that may hold for aggregate activism probabilities, and the relationship between the two forms is investigated. Some results are then presented which concern the distributions of activists in the two parties. These results in general conform to the existence of "party cleavages," in which there are two stable (equilibrium) distributions of activists, such that the two parties' activists are relatively cohesive internally and relatively distinctive externally. Finally, some suggestions are offered about how this model can be combined with the spatial model of candidate competition to provide a more complete model of elections.

The spatial model of electoral competition proposed by Downs in 1957 has generated a great deal of subsequent analysis. Davis and Hinich (1965) were the first to formalize Downs's model and extend it to include more than a single competitive dimension; they, Ordeshook, McKelvey, and many others have provided numerous further extensions and modifications (for reviews, see Davis, Hinich, & Ordeshook, 1970; Riker & Ordeshook, 1973). In short, there has been a cumulation of knowledge about these sorts of models. Throughout this development, there has been virtually unwavering attention to a closely intertwined set of questions: a) How do rational citizens decide how and whether to vote in two-candidate (or two-party) elections?; b) What spatial positions would rational candidates choose to offer citizens?; and c) What position(s), if any, would win and hence become the social choice?

I propose to use the spatial framework to analyze a different set of questions. Although these questions do bear on the traditional voting/social choice questions of the spatial model of electoral competition, and although I hope to explore the relationships in other articles, my questions here are different.

The questions I ask concern political parties and actual or potential activists in one or the other of the two parties. Specifically, given a two-party system: a) Who is more or less likely to become involved with a political party, and in which party are they likely to be active? b) What are the consequences of these choices; that is, how are the partisan activists distributed across the space? Are these distributions stable and enduring? and c) What consequences might the answers to questions (a) and (b) have for parties, policies, and elections in the short and long runs? In this article, I propose a model to answer questions (a) and (b). In the concluding section (and more briefly below), I will suggest some of the ways in which question (c) might be explored and answered.

Keeping in mind that the one long-term goal is to develop a "three actor" spatial model (i.e., one with candidates, citizens, and activists), the model of partisan activists I propose will parallel the usual spatial model of elections as closely as possible. The connections between the model of activists and the electoral model will not be made here. Still, it is important to note that there might be some fairly straightforward connections between the two models.

The principal conclusions I will reach here are that the distribution of activists in the two parties will be relatively cohesive within each party, relately divergent between parties, and generally in stable equilibrium. These conclusions seem to be close to the description of the long-observed party cleavages in the United States. At least since the time of McClosky, Hoffman, and O'Hara (1960), research has shown consistently that Democratic partisans and activists differ from

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their Republican peers in predictable, enduring ways, especially along the line of the New Deal cleavage. Page (1978) links these findings with the positions of presidential nominees and partisan cleavages in the electorate. Aranson and Ordeshook (1972; see also Coleman, 1972) illustrate a more specifically spatial modeling relationship. There they showed that if partisan activists are distributed in ways rather like those derived here, candidates who might want to converge to each other (and the policy center) to win the general election might need to diverge along the line of party cleavage to obtain the support of partisan activists in order to receive the nomination in the first place. In effect, I answer in this article the question of why partisan activists might be distributed as these authors assumed them to be.

I call the model a "Downsian spatial model" because, like Downs's own work, the model assumes a single dimension. I also use the by-now-standard Davis, Hinich, and Ordeshook (1970) formalization of Downs. In the next section, I propose a spatial "calculus of participation" that parallels the Riker-Ordeshook (1968) calculus of voting (see especially Riker & Ordeshook, 1973, or Davis, Hinich & Ordeshook, 1970, for its spatial interpretation). In this calculus, the question is whether or not one becomes an activist. It does not ask how many resources, such as time, effort, and money, any activist contributes; it might best be thought of as modeling either the decision to initiate (or maintain) a partisan affiliation or perhaps to contribute some small amount without worrying about how much. This activism calculus is then generalized to a probabilistic formulation, paralleling McKelvey's generation (1975) of the Davis, Hinich, and Ordeshook spatial model of elections. This formulation also permits an interpretation in terms of how much the individual contributes as an activist. With this individual-choice machinery, I then turn to such macrolevel questions as aggregate distributions and stability.

The Definition of the Downsian Spatial Model

What makes these models "spatial" is that choices are based on the evaluation of points along one or more continua, usually interpreted as policy (or ideological) dimensions.[1] The Downsian model assumes a single such dimension, as assumed here:

> A1: The space is a unidimensional continuum, $X \epsilon R^1$.

For mathematical convenience, I assume that there are an infinite number of citizens, with a typical citizen being denoted as *i*. All *i* have preferences defined over all points in *X*, and each *i* has a point of maximum preference or utility, *i*'s "ideal point" in *X*, denoted x_i:

> A2: There are an infinite number of citizens;
>
> A3: For all *i*, there exists a well-defined utility function over *X*, say $U_i(x)$: $R^1 \rightarrow R^1$, and, for all *i*, there is some point of maximum utility, x_i, $x_i \in X$.

The set of all x_i defines the distribution of ideal points, $f(x_i)$. I assume that all x_i are real numbers (i.e., no one most prefers an infinitely positive or negative amount or degree of *X*), so that there is some smallest and largest ideal point value of *X* (say, points *a* and *b*, respectively).[2] I assume that $f(x_i)$ is nonzero, continuous, and differentiable over the range (a,b):

> A4: $f(x_i)$, the distribution of ideal points in *X*, is a continuous, nonzero, differentiable function over the range (a,b), $a,b \in R^1$.

At this point the description of a "Downsian spatial model" is complete. The model of electoral competition would add such variables as candidates, their preferences, and voting rules. The model of party activism, however, will turn on different variables.

The Individual "Calculus of Participation"

Davis, Hinich, and Ordeshook (1970) put their assumptions about citizens' voting behavior in terms of the Riker-Ordeshook calculus of voting. Riker and Ordeshook (1973) argue that their calculus ought to extend to political participation more generally (see especially chap. 3). I will follow their suggestion, but first add a bit more context.

The sort of activist I seek to model is that of the mass activist, the party's rank and file. In par-

[1]Ordinarily a spatial dimension is interpreted as a dimension of public policy (although Downs's own example was more of an ideological than a specifically policy dimension). I will follow this convention, but note that the formal definition specifies no particular interpretation.

[2]The assumption that ideal points are real numbers (i.e., noninfinite) is actually rather common (cf., Kramer, 1977, who uses this assumption, indicates its common usage, and calls the multidimensional generalization "Type I utility functions").

ticular, I am *not* modeling the behavior of the elite activist, the career party official or the actual or potential candidate. Rather, I am considering the person who might perform any of the usual measures of campaign participation for the party (from identification to contributing money, attending rallies, and stuffing envelopes). One crucial presumption, therefore, is that the potential activist is similar to what economists call a "price taker." That is, they take the positions and goals of the party as given, independent of their behavior, just as consumers take the prices of goods as fixed (and, just as the actions of consumers do affect prices infinitesimally, so, too, do the actions of activists affect the positions of the parties infinitesimally). I am *not* studying the behavior, therefore, of those activists whose behavior may be predicated on their potential for changing the party materially. I also ignore the careerist, whose cost-benefit calculations are so vastly different. Activists here are partisans because they desire to see the party realize its goals; they want to support the party, not change it.

I also assume that activists contribute to only one party, not to both simultaneously. Finally, the decision to become an activist is a potentially long-term commitment, a "standing decision." To be sure, the decision to become involved in a specific campaign will depend upon the particular set of candidates standing for election at that particular time and place. But, the "partisan" part of the decision emphasizes that the "standing decision" depends on where and for what the party stands.

The Riker-Ordeshook calculus is more obviously relevant in this setting. Since they show that no rational citizen would vote for the less preferred candidate, their calculus swings on the turnout decision, leading to their famous equation:

$$R = PB + D - C, \text{ where}$$

R = "Reward" for voting instead of abstaining,
P = (Subjective) probability that one's vote will "count,"
B = Differential benefits from seeing the preferred candidate elected,
D = "Citizen duty" (or positive rewards associated with voting, independent of the outcome), and
C = Costs of voting.

To translate into this context, let us begin with the individual's utility function, $U_i(x)$. As already indicated, I assume that they exist and that they have a single maximum. The Davis, Hinich, and Ordeshook formulation of the Downsian voting model initially specified citizen utility functions as strictly quadratic (cf. Davis & Hinich, 1965). Later work examined "quadratic-based" utility functions. In a unidimensional space, this requirement is that the utility function is unimodal and symmetric about the mode, x_i. At times, the spatial voting model examined unimodal utility functions that were not necessarily symmetric, but were convex (i.e., monotonically and marginally decreasing). Here, I will assume (unless otherwise noted) that all utility functions are quadratic-based. This assumption means that they are unimodal and symmetric, but that they can be any particular form consistent with those constraints (e.g., they could be strictly quadratic or they could be shaped like a normal curve):[3]

U1: For all i, $U_i(x)$ is unimodal and symmetric about x_i, and it is of the same functional form for all i.

The utility function is used to evaluate the comparative offerings of the two parties. In the spatial voting model, the comparison is of the two candidates' positions in X. Here, I assume that there are two parties, say Θ and Ψ, and that all i see and agree upon the position on X that represents the policy goal of each party, say points θ and ψ, $\theta, \psi \in X$. The origin of these two points will be discussed in a later section (briefly, I will assume that the party position is based on the preference of its "typical" activist):

U2: Each party enters into each citizen's utility function as a single point in X, the "goal" or "position" of the party, i.e., as $U_i(\theta)$ and $U_i(\psi)$ for party Θ and Ψ.

The comparable term to the Riker-Ordeshook B term, then, is the difference in the evaluation of the two party positions, as defined in U2.

The P term in the calculus of voting is often intepreted as the probability that one's single vote will make or break a tie between the two candidates. In the context of partisan activity, it is more reasonable to adopt a less specific interpretation of P. After all, there are any number of offices and elections occurring at numerous times. Thus, one might reasonably interpret the P-analog here as the perceived likelihood that one's activity will help the party achieve its goals. In fact, even looking at a single office, one might take a more

[3]Assumptions in this section are denoted by U's (instead of the A's of last section) to emphasize that they concern individual utility functions, separating them from assumptions made in the next section, about "party support functions" (S assumptions) that roughly parallel these.

general interpretation, since it is likely that a winner would be more effective in implementing the party's goals with more votes (or a larger plurality) than with fewer. We can denote these as, for example, $p_{\theta o}$, indicating the probability that party Θ will achieve its goals if individual i "abstains" from activity (chooses alternative action o).

The C and D terms will be divided a bit differently here, although the division is a matter of convenience and not a new approach. There are, of course, real costs associated with being an activist, as well as psychological and decision costs (relevant even for becoming an identifier). The costs of contributing scarce resources are undoubtedly higher than the costs of voting, alone. Here, I differ from many spatial voting studies in permitting them to be variable costs, although it is reasonable to assume that such costs vary independently of i's ideal point, the positions of the two parties, and of X in general.

The D term refers to the benefits of being an activist which are independent of outcome. In the voting calculus, these are called "citizen duty," since many feel it their duty to vote, regardless of who wins or loses. In more general terms, D type benefits can be considered any reward one obtains that comes independent of the outcome. We can think of two classes of these benefits. The first are any of the "duty" type of rewards, i.e., those that are independent of outcome *and* of the goals of the two parties and their evaluation by i. For party activists, such goals include the social, solidary type of benefits (Clark & Wilson, 1961), such as the numerous rewards, mementos, and opportunities to meet the party's public figures. All of these and more can be considered roughly to be independent of the positions of the two parties.

What I assume about these costs and benefits is that "costs" can be considered "net costs"; that is, the usual costs less these social and other benefits, called c_i. The notation serves two points. First, I assume that, for all i, c_i is greater than zero. That is, costs are not outweighed by the "particularized" or "private" benefits of participation. Second, c_i varies from person to person (but is the same for all i considering activity in party Θ as for i being active in party Ψ):

U3: For all i, there exists a "net costs" term, c_i, such that $c_i > 0$, where c_i is equal for activity in party Θ and in party Ψ.[4]

There remains a possible source of intrinsic benefits for participating in party activities that *is* related to the positions of the two parties. Some, possibly many, may feel that they are participating in a good and worthwhile cause. This motivation is political and will be felt even if the activity is unsuccessful (and even if i believes it would be so in advance). But, how good and worthwhile the cause is depends upon how i evaluates that cause. Assume that there is *another* utility function for i with the same ideal point location that is also quadratic-based (but possibly of a different form), say $U_i'(x)$.[5] Then, if i is an activist in, say, party Θ, i receives $U_i'(\theta)$, merely for being involved with that party. But, how good and worthwhile that goal is also may depend in part on the other party's position. After all, working for a good cause may be less worthwhile if no one is seeking anything other than that same end. Since the position of the other party may be relevant, the appropriate formulation would be a comparison of the utility associated with the two parties, i.e., the difference in the evaluation of goal of the party in which one is active, less that of the other party's goal. However, the other party's position may not be as important. Let us weight the other party's goal by a scalar, say a: $0 \leqslant a \leqslant 1$. If a is zero, the position of the second party is irrelevant. If a is one, the position of the second party is as important as the first. To summarize:

U4: For all i, there exists another (unimodal, symmetric about x_i) utility function, $U_i'(x)$, and

U5: For all i, there is an intrinsic utility for being active in party Θ (Ψ) that is $U_i'(\theta) - aU_i'(\psi)$ [or $U_i'(\psi) - aU_i'(\theta)$].

Of course, i receives the utility in U4-U5 if and only if i is active.

Combining our utility assumptions, U1-U5, we can specify the expected utility for being active in party Θ, in Ψ, and in neither. Let A_j denote choosing activity j:

$$1.\quad EU_i(A_\theta) = [(p_{\theta\theta})U_i(\theta) + (p_{\psi\theta})U_i(\psi)] + [U_i'(\theta) - aU_i'(\psi)] - c_i$$

$$2.\quad EU_i(A_\psi) = [(p_{\theta\psi})U_i(\theta) + (p_{\psi\psi})U_i(\psi)] + [U_i'(\psi) - aU_i'(\theta)] - c_i$$

[4]Obviously, the last part of U3 assumes the same level of commitment to the two parties. In effect, then, it also assumes that private benefits provided by the two parties are equally valuable.

[5]For example, the two functions could differ by a constant but be otherwise identical, or the two could be completely different (although unimodal and symmetric) functions (e.g., $U_i(x)$ might be shaped like a normal distribution, whereas $U_i'(x)$ is strictly quadratic).

3. $EU_i(A_o) = [(p_{\theta o})U_i(\theta) + (p_{\psi o})U_i(\psi)]$.

Finally, we assume, of course, that all i are rational expected utility maximizers:[6]

U6: For all i, i will choose the maximum of equations (1-3) above.

The Calculus and Activism

The "calculus of participation" follows as closely as possible the Riker-Ordeshook calculus of voting in a Downsian framework. At this point, we must now connect it with choices. I continue to assume that the individual is choosing whether or not to become an activist and not considering how much the individual contributes.

Here, I will show that this formulation leads to results similar to those obtained by Davis, Hinich, and Ordeshook. Before doing so, we must make some kind of assumption about the particular probability terms in equations 1-3. Specifically, I assume that there is a symmetry between the two parties; contributions are equally valuable to the recipient parties. I also assume that being active in party Θ helps that party to achieve its goals more than "abstaining," which in turn helps party Θ more than being active in the other party:

U7:

a) $(p_{\theta\theta} - p_{\theta\psi}) = (p_{\psi\psi} - p_{\psi\theta})$;

b) $(p_{\theta\theta} - p_{\theta o}) = (p_{\theta o} - p_{\theta\psi})$;

$(p_{\psi\psi} - p_{\psi o}) = (p_{\psi o} - p_{\psi\theta})$;

c) $p_{\theta\theta} > p_{\theta o} > p_{\theta\psi}; p_{\psi\psi} > p_{\psi o} > p_{\psi\theta}$.

With these assumptions, it follows that no one becomes an activist in the party offering the less preferred, the more distant, position:

Prop. 1: For all i, if assumptions U1-U7 hold, and if $|x_i - j| < |x_i - k|$ *for* $\{j,k\} = \{\theta,\psi\}$, then $EU_i(A_j) > EU_i(A_k)$, and i does not choose A_k.

Proof: See appendix (for all proofs).

This proposition is, of course, the same sort of result that Riker and Ordeshook obtained. It says, simply, that one is active in the closer party or is not active in either party. As with the calculus of voting, the interesting question is when one is active at all.

One is active in the closer party rather than not active at all when the following condition holds, which is shown to parallel the famous Riker-Ordeshook formula:

Prop. 2: One is active in the party whose position one prefers when $EU_i(A_j) > EU_i(A_o)$ or (where $p = (p_{jj} - p_{jo})$): $p[U_i(j) - U_i(k)] + [U_i'(j) - aU_i'(k)] - c_i > 0$ or: $PB + D - C > 0$.

The two forms of abstention that have received the most study in the spatial model of electoral competition are termed "indifference" and "alienation." Indifference is the basis of abstention if turnout declines as the utility differential between the two candidates decreases. This basis of abstention is usually tied to the B term, which measures that differential. In this model of partisan activity, there are two sources of indifference, the B term and (possibly) the D term.

Abstention is said to be due to alienation if

> *a citizen's probability of voting decreases as the [utility] he associates with his preferred candidate [decreases] and that his probability of voting increases as this [utility increases].* (Davis, Hinich, & Ordeshook (1970), p. 437, emphasis in original)

Here we can define the two pure types of "abstention" as:

U8: "Abstention" from activism is said to be policy related if it is due to either indifference, alienation, or both:

a) Abstention is due to indifference alone, given that party j is closer to x_i than is k, if either:

(1) $U_i'(x) = 0$ for all i, so that: $EU_i(A_j) - EU_i(A_o) = P[U_i(j) - U_i(k)] - c_i$ or

(2) $a = 1$ for all i, so that: $EU_i(A_j) - EU_i(A_o) = P[U_i(j) - U_i(k)] + [U_i'(j) - U_i'(k)] - c_i$.

b) Abstention is due to alienation alone if $P = 0$ and $a = 0$ for all i, so that $EU_i(A_j) - EU_i(A_o) = U_i'(j) - c_i$.

c) Abstention is due to alienation and indifference combined if $P \neq 0$ or $a \neq 0$, and if $a \neq 1$, and $U_i'(x) \neq 0$ for all i, so that the full equation, above, is relevant.

A More General Model

The Davis, Hinich, and Ordeshook quotation presents the act of abstention in probabilistic

[6]For those who prefer the Ferejohn and Fiorina (1974) minimax regret formulation to the expected utility calculus, note that the minimax regret decision rule could be substituted into all two-party results without changing any of the results obtained here.

fashion, yet the terms of the model are deterministic. It is possible to change the formulation to a probabilistic model, which also yields a more general model. Richard McKelvey did so for the spatial model of electoral competition (1975).[7] Here, I will parallel his formulation for the spatial model of partisan activity.

Consider the set of people who share the same ideal point position, e.g., x_i. These people face the same choices of actions, the same utility function and evaluation of the two parties, and yet, some will choose to become involved and others will not. Why? In the model the answer is that people evaluate the costs and non-policy-related benefits differently. That is, the net costs term, c_i, varies even with all else held constant (and, by construction, that variance is independent of policy-related evaluations). This leads naturally to a probabilistic interpretation, the probability that any given citizen with any given ideal point and pair of party positions will become involved with a party or "abstain,' or the probability that the "*PB* and *D*" terms outweigh the variable c_i. (I will provide a different interpretation later in this section.)

There are three such probability functions, the probability that one is active in party Θ, in party Ψ, and in neither. Call these $P(\Theta)$, $P(\Psi)$, and $P(A)$, respectively. These are functions of the ideal point location (and, more generally, the utility function), the positions of the two parties, and of c_i. Since c_i is an independent random variable, we can write $P(\Theta/x_i, \theta, \psi)$, etc. Finally, since these are probability functions, and since these are assumed to be mutually exclusive and exhaustive acts, the probability terms must sum to one at each x_i: $P(\Theta) + P(\Psi) + P(A) = 1$. Given this identity, we can place conditions on, say, the two "party support functions" [$P(\Theta)$ and $P(\Psi)$], which implicitly constrains the abstention function.

The key here, of course, is to examine conditions on the probability functions, indicating that if they meet certain of these conditions (or take on an appropriate shape or distribution), then certain results can be proved. The definition of them actually contains enough constraints to be set as an assumption:

S1: There exist three probability functions, $P(\Theta)$, $P(\Psi)$, and $P(A)$, representing a probability distribution defined over the three mutually exclusive and exhaustive sets of behavior, participation in party Θ, party Ψ, and neither party, respectively. These three distribution functions are dependent on $U_i(x)$, θ, ψ, and c_i, and all three must sum to one at all x_i for all pairs (θ,ψ). In addition, the two party support functions must be of the same functional form, as oriented by θ and ψ.[8]

Although S1 defines and constrains these functions, the constraints are not very great. Other conditions must be imposed. There are five other conditions (offered below) that they may satisfy either singly or in combination. Propositions will be offered to tie these conditions to the assumptions of the last section.

The first condition on $P(\theta)$ and $P(\psi)$ I call the "Rational Choice Condition." This condition is that the probability of supporting the farther-removed party is always zero. It is obviously closely tied to the rational choice assumption of the last section (U6) and Prop. 1:

S2: The two party support functions, $P(\theta)$ and $P(\psi)$, are said to meet the Rational Choice Condition if, for all x_i and for $\{j,k\} = \{\theta,\psi\}$: $P(j/x_i j,k) = 0$ if $|x_i - j| > |x_i - k|$ and >0 if $|x_j - j| \leqslant |x_i - k|$.

Figure 1 illustrates this, and the following, conditions.

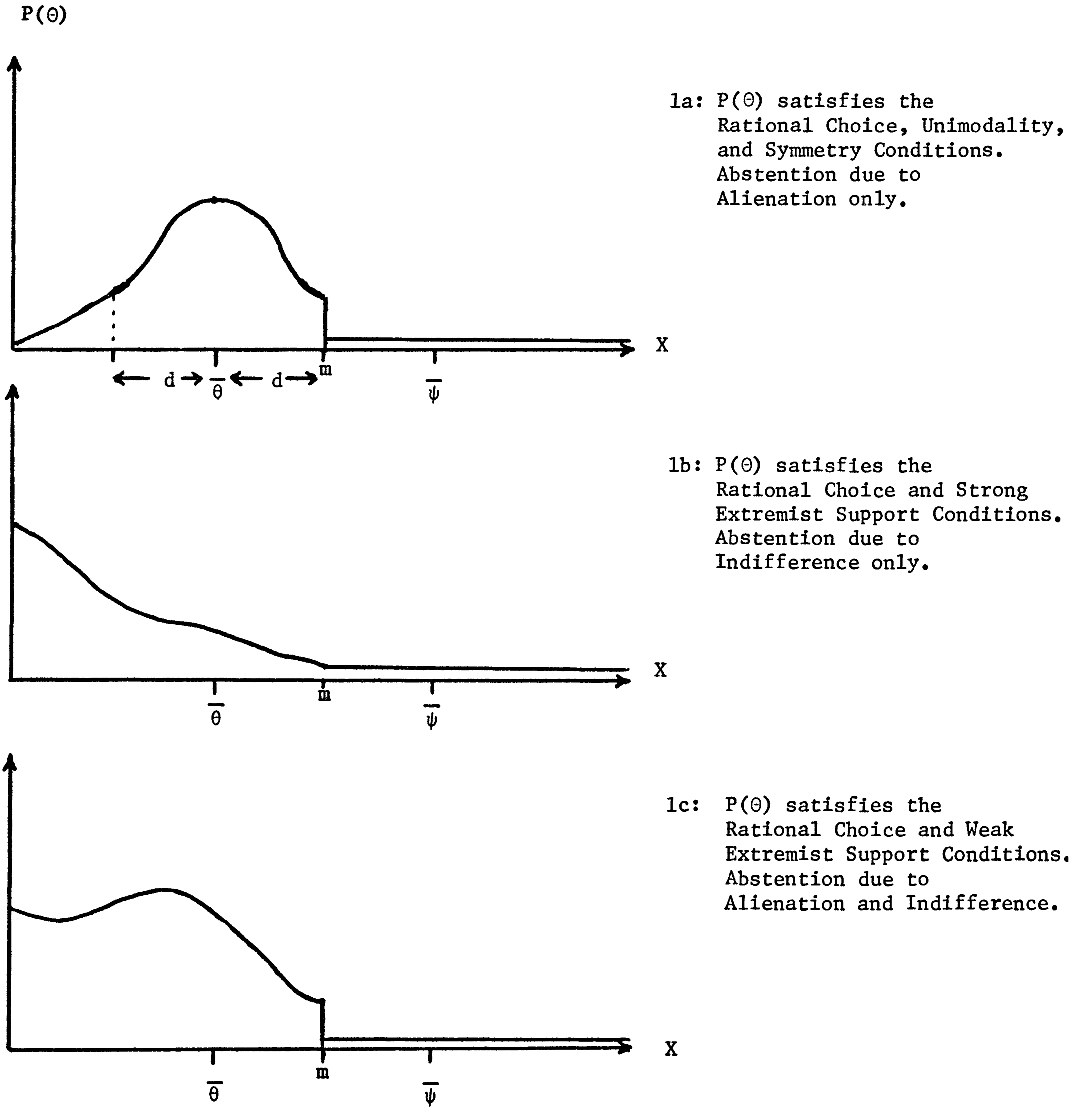
The next two conditions are used together and so will be defined together. They are, simply, that the two-party support functions are unimodal and symmetric about the respective party positions, at least up to the midpoint between the two party positions (at which point, the Rational Choice Condition begins to come into play). The motivation for this condition is the alienation form of "abstention," in which the farther one is from the closer party, the less likely one is to become involved with that party:

S3: The two party support functions meet the Conditions of Unimodality and Symmetry about the party positions if, for all x_i and for $\{j,k\} = \{\theta,\psi\}$,

[7]I use terminology similar to McKelvey's. Some care must be used in comparing the two models, because I define some of the terms differently (and of course put them to a different use). A more formal derivation of the probabilistic model from U1-U7 may be found in Aldrich and McGinnis (1983).

[8]The last sentence of this condition is similar to McKelvey's "metric symmetry" condition. His condition requires that the two (candidate) support functions be of the same functional form, but mirror images of each other. For example, if $P(\Theta)$ is unimodal and symmetric about θ, then $P(\Psi)$ also must be unimodal and symmetric, but about ψ. McKelvey shows that if utility is quadratic based, and voting and abstention are policy related (which will hold here if assumptions U1-U7 apply), then metric symmetry holds.

Figure 1. Examples of Probability Support Functions



a) $P(j)$ is strictly monotone decreasing for all x_i such that $|x_i - j| \leqslant |x_i - k|$, and

b) For any pairs of points, x and y, such that $|x - j| \leqslant |x - k|$, $|y - j| \leqslant |y - k|$, and $|x - j| = |y - j|$, then $P(j/x) = P(j/y)$.

The final two conditions are two versions of extremist support; that is, relatively more extreme individuals are more likely to support the closer party than are their more moderate peers. The "strong" condition is that the party support function is monotonically increasing from the midpoint between the two party positions, through its own position, to the appropriate infinity. This condition is related to abstention due to indifference. The "weaker" extremist support condition is that for any two points equidistant from the closer party position, the more moderate point has a lower (or no higher) probability of supporting that party than its more extreme counterpart. This condition holds if abstention is due to indifference, to alienation, or to both reasons together:

S4: The following two conditions apply to any ideal point positions closer to the position of party j than k.

a) $P(j)$, for $\{j,k\} = \{\theta,\psi\}$ are said to meet the Strong Extremist Support Condition if, for all x_i, $P(j)$ is strictly monotone increasing as $|x_i - [(\theta+\psi)/2]|$ increases.

b) $P(j)$, for $\{j,k\} = \{\theta,\psi\}$ are said to meet the Weak Extremist Support Condition, if for all pairs of points, say $\{x,y\}$ such that if $|x - j| = |y - j|$ and $|x - k| > |y - k|$, then $P(j/x) \geqslant P(j/y)$.

Note that S3 and S4 can hold if the rational choice condition holds. S3 and the strong version of S4 are not compatible, but S3 and the weak version are (if all the weak inequalities are strict equalities in S4b). Also note that if the Strong Extremist Support Condition holds, then the Weak version holds necessarily, but the reverse may not be true.

To this point, the aggregate probability functions have been interpreted as the probability that one "joins" a party or not, similar to the interpretation of McKelvey's probability of voting functions as representing the probability (or relative frequency) of voting for one candidate or the other. Although this is a perfectly justifiable interpretation for party activism understood as, say, identifying with a party, or being active in a party without regard to how many scarce resources an individual contributes, that is not the only, nor the most general possible, interpretation.

Probability density functions are basically distributions that are normalized to sum to one. They need not be interpreted as relative frequencies of behavior. Instead, they could be interpreted as the relative contributions of individuals. This interpretation suggests that a probability of activism term can be less tied to individuals, more closely related to the amount of contributions. For example, the mean of the distribution of $P(\Theta)$ under the first interpretation indicates the ideal point position of the "typical" activist. Under the second interpretation, it represents the average contribution (be it of time, money, or effort). Looked at in terms of individual partisans, it represents the ideal point position of the "typical" activist, *weighted* by the amount of scarce resources he or she contributes.[9] Briefly, the interpretation used here is a special case of a more general class of models. The major (and, admittedly, it is major) restriction is that the opportunity costs forgone by contributing are distributed independently of the ideal point position in X. The amount the individual activist contributes, then, is determined via profit maximizing procedures. If R, in the calculus, is not greater than zero, then i does not become an activist. If, however, R is positive, then i will contribute a positive but finite amount. That amount is measured by the proportion of possible forgone opportunities actually forgone.

[9]A formalization of this interpretation is presented in Aldrich and McGinnis (1983).

Connections between Individual Utility and Party Support Functions

The following propositions tie the assumptions made about individual utility functions and choice to the conditions placed on the aggregate probability of activism functions. It is important to remember that utility assumptions (those denoted by U's) imply the aggregate conditions (those denoted by S's), but not vice versa. That is, the probability conditions are more general and may hold even if the utility conditions do not.

Throughout, the A-type assumptions defining the spatial model are assumed to hold. Then,

Proposition 3: If the conditions of Prop. 1 (U1-U7) hold, then the Rational Choice Condition (S2) holds. In addition, if abstention is policy related, then S1 (specifically, its last sentence) holds.

Essentially, Proposition 3 states that if utility functions are quadratic-based, then the Rational Choice Condition holds, so that the probability of supporting the spatially more distant party is zero. Further, if abstention is due to either or both of the policy-based reasons (indifference or alienation) or is constant in expected value across the policy space, then the two support functions will be mirror images of each other, oriented about the midpoint between the two party positions.

The other propositions deal with particular forms of abstention. Both propositions, therefore, assume U1-U7 and that abstention is policy related (U8). They differ over what part of U8 applies.

Proposition 4: If abstention is due solely to *alienation* (i.e., U8b holds), then the party support functions will be unimodal and symmetric (S3 holds).

Proposition 5: a) If abstention is due solely to indifference, and if $U_i(x)$ is convex as well as quadratic based, then the Strong Extremist Support Condition (S4a) holds.

b) If abstention is due to indifference or alienation or both (U8a, b, or c holds), then the Weak Extremist Support Condition holds.

The basic relationships, then, are that if utility is quadratic based, then the Rational Choice Condition follows. If, in addition, abstention is policy related, the two support functions are of a similar form, and the Weak Extremist Support Condition follows. If the policy-related form of abstention is alienation, then the support functions will be unimodal and symmetric. If abstention is due instead to indifference (and if the utility functions are also convex), then the support functions satisfy the Strong Extremist Support Condition.

These propositions tie the support function approach to the more common rational choice framework. Nonetheless, the probability approach is more general. For example, it could be the case that extremists are more likely to be involved in partisan activity than moderates and, thus, the extremist support condition applies, even though the utility function approach is wrong.

Dynamics of Party Activism

The model to this point has focused on individual choices only. I think it reasonable to suggest that the formulation of the problem is not especially novel, nor the results particularly unexpected. In this section, I add the assumption that provides a social aspect to the problem and pose the social-dynamic question to which the choice "machinery" of the last sections can provide answers.

The key assumption for these questions is the determination of the two-party positions that enter the individual utility functions. What I assume is this: The individual evaluates the position of the party, as he or she considers whether or not to become actively involved in party affairs, by the positions of those who are already active in the party. After all, the goals of a party are what its members make them. Further, many of the most important rewards of activity are those that involve working for and with others to achieve (or even to attempt to achieve) valued outcomes. In short, the social solidary rewards and rewards from participating in a good and worthwhile cause with others whom you like, admire, and respect weigh heavily for the activist considered here (i.e., the non-careerist and non-candidate).

Now, what position is taken by the "typical" activist is less obvious. Here I assume that it is the mean ideal point of current activists (but demonstrate that the median could be substituted without changing the basic conclusions). If the probability functions are interpreted as the probability of joining or not, then the mean ideal point is a simple, unweighted average. In the alternative interpretation, the mean is the weighted average, where the weights are based on the extent of the contribution made. In either case, the entire distribution is summarized by a single parameter. In effect, the assumption implies that the degree to which the party contains consensus or dissension does not matter. Similarly, at this point, there is no separate accounting of the position of the candidates and the parties. Candidates are simply party activists (although they may be weighted more heavily, if that interpretation is followed). Certainly, such calculations are or at least may be quite important, and I hope to generalize the model along these lines. For now, this assumption must be taken as a first approximation.

Even if that assumption is a first approximation, it opens up some theoretically interesting questions. Consider, in particular, the question of dynamics. Let us say an individual decides to become an activist in one of the parties for the first time. He or she does so because he or she prefers the position of the average activist in that party, in comparison to the other party's position, sufficiently to outweigh the costs of activism. By becoming involved in the party, he or she changes the position of the party, even if so slightly as to be all but unobservable. But that change (especially if combined by similar decisions made by "people like him or her") may be big enough to make it worthwhile for still others to join, perhaps drive some out of the party, and perhaps cause similar changes in the other party. Thus, the assumption that people become involved in party affairs based on the preferences of others already involved implies a dynamic to the distribution of activists in the two parties:

> A5: For all i, the points θ and ψ, as they enter into i's utility function, are defined as the mean ideal point position of those currently active in party Θ and Ψ, respectively.

The rest of the article examines this dynamic, especially by focusing on such factors as the stability and location of the mean ideal point of activists (because if these two means do not change, then the full distributions will stabilize). The questions asked are whether such activism calculations lead to any stability in the distributions of the two parties' activists in the policy space, and where in the space would we expect to find the typical activist in the two parties. As we will see, the answers conform roughly to the general view of party cleavages.

The terms such as "equilibrium," "convergence," and "divergence" appear in this model, just as they do in the spatial voting model. There are important differences, however. In the voting models, "equilibrium" refers to positions that candidates might want to take to ensure the best possible outcome, and such points are in equilibrium if the candidates have no incentive to change their positions, once adopted. Here "equilibrium" refers to the behavior of citizens as potential activists. Since activists get involved in a party due to the preferences of those already in the party, a decision by some citizens may affect the decisions of others. The two parties are said to be in equilibrium if no new citizens want to become involved in either political party, nor do any current activists want to change their decision. As a result of these decisions, the party centers are also in equilibrium, in the sense that the location of the typical activist will not change, since the individuals who make up the parties do not change their behavior. To say that the parties are "converging" means that the party centers are getting closer together, i.e., more people in between the two parties are becoming active or more extreme activists are dropping out or both, and similarly for "divergence."

Some Equilibrium Results

The basic approach here is to impose certain conditions on the party support functions and to look at various distributions of preferences in the electorate as a whole. This approach is useful, for the kinds of distributions of $f(x_i)$ that are examined are fairly general (unimodal, bimodal, and multimodal), and also because once the probability of activism functions and $f(x_i)$ are specified, the full distributions of activists (say, $F(\Theta)$ and $F(\Psi)$, respectively in the electorate can be specified by solving equations like:

$$F(\Theta/x_i, \theta, \psi) = \int_a^b P(\Theta_i)\ f(x_i)\, dx_i$$

The first two results deal with a one-party system. They are listed as lemmas, because the one-party system is a useful fiction in that it imposes limits on the possible location of equilibria in the two-party case. Let Θ be the one party.

LEMMA 1. *If* $f(x_i)$ *is a continuous function over the interval* (a,b) *and* $P(\Theta/x_i, \theta)$ *is a continuous function not always equal to zero, then there exists at least one equilibrium value,* θ^**, interior to the interval* [a,b].[10]

LEMMA 2. *If, in addition,* $f(x_i)$ *is also strictly unimodal, with mode at* $y \in X$*, and if* $P(\Theta/x_i, \theta)$ *satisfies the conditions of unimodality and symmetry about* θ*, then:*[11]

a) Any equilibria, θ^**, will be "near" the mean of* $f(x_i)$*, and hence "near"* y.

b) If $f(x_i)$ *is also symmetric,* $\theta^* = y$*, the mean and mode of* $f(x_i)$*, is the unique equilibrium.*

The two-party case is, of course, the more interesting and relevant. The first proposition is a very general existence result. It states that if the Rational Choice Condition holds (i.e., no one is active in the further removed and hence less preferred party), then there is at least one equilibrium pair of party positions (let m denote the midpoint between the two party centers ($m = (\theta + \psi)/2)$).

PROPOSITION 6. *If* $f(x_i)$ *is continuous over the interval* [a,b], *and if* $P(\Theta)$ *and* $P(\Psi)$ *are continuous over* [a,m] *and* [m,b], *respectively, not everywhere zero, and meet the Rational Choice Condition, then at least one equilibrium pair of party positions,* (θ^*, ψ^*)*, exists.*[12] *Further, these equilibria positions will find the two party centers interior to* [a,b] *and separated from each other, so that* $\theta^* \neq \psi^*$.

This proposition imposes only the Rational Choice Condition, which therefore examines choices between the two parties only and does not impose any conditions on abstention. In addition, the conclusion is fairly general; it states that the decisions of the individual citizens, as they choose whether or not to become party activists based on where current activists in the two parties are located, will lead to a stable distribution of activists in the two parties. Further, the two parties will be located such that the center of each party will not be at an extreme, and the two parties will be distinctive from each other. That, by

[10] All equilibria analyzed in the Appendix are stable equilibria unless otherwise noted.

[11] These proofs assume that the mode is clearly interior to the range of ideal points (and that both modes are well interior to the interval $[a,b]$ in the bimodal case). This is assumed for convenience. For Lemma 2, for example, if the mode of $f(x_i)$ is at point a, then it is relatively straightforward to show that the party center will converge as close to point a as is possible, consistent with the equilibrium condition in the Appendix.

[12] Although the proposition as written assumes that $\theta < \psi$ at the outset, it obviously holds as well if $\theta > \psi$.

itself, is a long way toward establishing the sorts of distributions one would expect to find if party cleavages exist.

The above proposition includes the result that the two party centers will be separated from each other; that is, there will be some distinction between the positions of the "typical" activists in the two parties. In a sense, this result is not too surprising, as one of the major motivations for activism is joining like-minded peers, especially when there are differences between the two parties. Proposition 6, however, does not indicate how distinctive the two parties may be. If people do become activists in part to be with like-minded peers, then it would seem that the case of least divergence would be when the distribution of ideal points in the electorate is unimodal. The following proposition, then, deals with that case, where it is to be understood that the mode of $f(x_i)$ is interior to the interval (a,b); that is, $f(x_i)$ is strictly increasing from a to the mode, say point y, and then strictly decreasing from y to b:

PROPOSITION 7. *If the conditions of Proposition 6 hold, and if* $f(x_i)$ *is unimodal with mode at* y; $a < y < b$, *then for* $\{j,k\} = \{\theta,\psi\}$:

a. If P(j) *and* P(k) *satisfy S3, or S4a, or S4b, then the two party means, at equilibrium, will be divergent with* $j^* \leqslant$ *mean of* $f(x_i) \leqslant k^*$.

b. (1) If P(j) *and* P(k) *meet the Conditions of Unimodality and Symmetry (S3), the distance separating* j^* *and* k^* *will be least.*

(2) If P(j) *and* P(k) *meet Weak Extremist Support (S4b) and not S3 and not S4a (Strong Extremist Support), the distance separating* j^* *and* k^* *will be greater than case (1).*

(3) If P(j) *and* P(k) *meet Strong Extremist Support (S4a), the distance separating* j^* *and* k^* *will be greater than in case (2).*

The point of this proposition, in general, is that any form of policy-related abstention will lead to "divergence" of the two sets of party activists. If abstention is the result of alienation, the two sets of party activists will be the most similar (although still divergent). If abstention is the result of indifference, however, this will lead to sets of party activists whose preferences are more sharply distinguished on the policy dimension.

One final proposition about a unimodal $f(x_i)$ concerns the special case of it being symmetric as well as unimodal.

PROPOSITION 8. *If the conditions of Proposition 7 hold and if* $f(x_i)$ *is also symmetric, then an equilibrium pair of positions of the party centers exists such that* θ^* *and* ψ^* *are equidistant about the mode of* $f(x_i)$.

Presumably, the case of a unimodal distribution of ideal preferences in the electorate is the most "consensual" and therefore the most likely to lead to relatively similar party centers. As it happens, however, a *bimodal* preference distribution may have at least three pairs of equilibrium positions.

PROPOSITION 9. *If the conditions of Proposition 7 hold, except that* $f(x_i)$ *is bimodal, with modes, say* y *and* z, *such that* $a < y < z < b$, *then at least one, and possibly all three, of the following equilibrium pairs* $\{j^*, k^*\} = \{\theta^*,\psi^*\}$ *exist:*

a) A pair exists about the mode y,

b) A pair exists about the mode z, *and/or*

c) A pair exists such that j^* *is near* y, k^* *is near* z.

Note that, like Proposition 7, this proposition holds for any sort of policy-related abstention. Note that the increased divergence of the party centers, as in Proposition 7, follows straightforwardly. That is, abstention resulting from indifference ordinarily will lead to party centers that are further apart than if abstention results from alienation.

The three possible pairs of equilibria found in Proposition 9 only indicate that there are local equilibria possibilities. And the logic of joining or withdrawing from activism implies a marginal adjustment process, so that the party activists' decisions may lead to, and remain stable at, local equilibria.

The most interesting part of Proposition 9 is the third set of equilibria outcomes. Here, rather as we would expect, the parties are dominated by sets of activists at each mode; that is, there would be a dominantly liberal and dominantly conservative party, rather more clearly so than in the unimodal case. Still, even in the unimodal case, where most people are moderate, one party would consist mostly of moderately liberal to liberal activists. Thus, the addition of the bimodal case gives only a sharpened sense of party cleavages.

Finally, the analysis could be extended to multimodal distributions by repeated application of the last proposition. Of more interest would be an extension of the logic to the multiparty context. For example, we could redefine the Rational Choice Condition to be that the probability of activism is zero for all parties but the closest one and assume that the party support function is unimodal and symmetric about its own party center, within the constraints set by the redefined choice condition. Then, it is relatively easy to show that there is an equilibrium set of party positions, with each party activist distribution being centered near one of the modes.[13]

Although the analysis has assumed that the citizen evaluates the position of the party by reference to the *mean* ideal point location of current activists, the reliance on the mean per se is not very restrictive. As the following proposition shows, the results are changed very little (specifically, only in terms of the exact location of equilibria) if some other definition of "the party center" is used.

PROPOSITION 10. *All results above continue to hold if the median ideal point location is substituted for the mean ideal point position of current party activists in* $U_i(x)$.

Summary

The three basic conclusions are:

1. Equilibria (and locally stable ones at that) ordinarily exist in this model of party activist distributions, based only on the Rational Choice Condition which concerns only the choice between parties, not the choice of whether or not to be an activist.
2. At equilibrium, it appears that the parties are relatively cohesive internally and relatively distinctive from each other, with party centers, or typical partisan activists, distributed in a way that resembles party cleavages (and the distributions of activists assumed in, for example, Aranson & Ordeshook, 1972).
3. These conclusions seem rather robust, for they hold in a generally similar way for either or both types of spatially related abstention and for unimodal, bimodal, and multimodal $f(x_i)$.

Discussion

Two topics merit further discussion. The first topic is the limitations of the model at this point, and the second is the way in which this model might be combined with the spatial voting model.

The most obvious limitation of the model presented in this article is that it is unidimensional. A unidimensional model is empirically limited and, if past experience with spatial modeling is any guide, may be quite different from its multidimensional equivalent. An early version of this model was generalized to the multidimensional case (Aldrich, in press). There, stable, cleavage-like equilibria were found, and although that model is much less general than the current one, the principal results obtained there can be shown to be special cases of the decision calculus used here. Also, Aldrich and McGinnis (1983) provide a preliminary extension to n-dimensions and find that the results do generalize in cases studied so far. Thus, it seems that the most obvious limitation is not a major problem.

There are other limitations to the current model. For example, the citizen's decisions rest on evaluating the party as only the policy preference of its typical activist. There is no measure of such factors as the importance of dispersion of preferences among activists, variable probabilities of potential success, and the role of relative advantages one party may have in terms of "selective incentives." There is no study of leadership motivation, although I am currently working on a model with a party leader seeking to attract activists. Further, there is no institutional structure here. At this point, a party is no more than a distribution of its activists. There are no rules, norms, or traditions, just as there is as yet no incorporation of differing offices or regional patterns. In short, there is a great deal of room for further research to make a more complete and realistic model of political parties.

The second topic is how this model may be merged with the traditional spatial model of electoral competition. This model was designed with that goal in mind, so its formal structure should make such a combination relatively easy. There seem to me to be three major ways that such a combination may be rewarding. First, party activists may impose constraints on the strategies that its candidates can follow. Aranson and Ordeshook (1972) illustrate one way in which these constraints may operate. There, candidates need to win the support of activists to win nomination, which then serves to constrain their possible general election strategies. Activists also may serve to constrain candidates, if candidates need their support to make their general election strategies effective. For example, adopting a position closer to the center of the party may generate greater contributions from those activists, contributions that may be useful for increasing turnout of the candidate's supporters on election day.

Second, possible candidates for elective office generally emerge from the ranks of party activists. If candidates themselves have policy preferences, then we have some explanation of what sorts of preferences most candidates are likely to have. Further, of course, a candidate's own policy preferences may impose important constraints on the platforms they offer the public (see Wittman, 1983, for theoretical and some empirical accounts; Page, 1978, for more evidence).

[13]These results are available from the author on request.

Third, this model of party activism may be useful for making rational choice models of elections richer and more useful models of electoral behavior. For example, campaign participation is a fairly common act among the mass public, by itself. Often, nearly 20% of the respondents to national election surveys report some form of campaign activism during presidential election years. Further, partisan identification may be considered, at least in part, as a kind of partisan activity like that modeled here.

In sum, although the model presented here is limited in many ways, my hope is that it opens up a wide range of possibilities for improving formal models of electorally relevant behavior, at least in terms of their utility as empirical explanations of many common forms of behavior, and for making these models more consistent with the findings of several decades of empirical findings about electoral behavior.

Appendix. Proofs of Propositions

I. Propositions on Individual Choice

N.B.: The *i* subscript is deleted for convenience.

PROPOSITION 1. If U1-U7 hold, and if $|x - j| < |x - k|$ then $EU(A_j) > EU(A_k)$, for $\{j,k\} = \{\theta,\psi\}$.

1) $EU(A_j) = p_{jj}U(j) + p_{kj}U(k) + U'(j) - aU'(k) - c$ by U1-U5.
2) $EU(A_k) - EU(A_j) = (p_{jj} - p_{jk})U(j) - (p_{kk} - p_{kj})U(k) + (1 - a)[U'(j) - U'(k)]$.
3) $(p_{jj} - p_{jk}) = (p_{kk} - p_{kj}) > 0$ (call this p) by U7.
4) $EU(A_j) - EU(A_k) = p[U(j) - U(k)] + (1 - a)[U'(j) - U'(k)]$.
5) Since p and $(1-a)$ are positive, the sign of the above depends on the sign of the two U-differentials. By U1 and U4, $U(x)$ and $U'(x)$ are strictly negative monotone in distance, therefore both differentials are positive.
6) Therefore, $EU(A_j) - EU(A_k) > 0$. By U6, A_k is never chosen.

PROPOSITION 2. Given that j is preferred, i chooses A_j over A_o if:

1) $EU(A_j) - EU(A_o) > 0$ by U6, or if
2) $p_{jj}U(j) + p_{kj}U(k) + U'(j) - aU'(k) - c - \{p_{jo}U(j) + p_{ko}U(k)\} > 0$ or
3) $(p_{jj} - p_{jo})U(j) + (p_{kj} - p_{ko})U(k) + [U'(j) - aU'(k)] - c > 0$.
4) $(p_{jj} - p_{jo}) = (p_{ko} - p_{kj}) > 0$ *(call it p')* by U7. Therefore,
5) 1) may be written $p'[U(j) - U(k)] + [U'(j) - aU'(k)] - c > 0$.

II. Propositions on Utility and Probability Functions

PROPOSITION 3.

A. If U1-U7 (and hence Proposition 1) hold, then S2 holds. Since $|x - j| < |x - k|$ implies $EU(A_j) > EU(A_k)$, by U6 no one closer to j joins k. Hence $P(k/x,j,k) = 0$.

B .If U1-U7 holds, the last sentence of S1 holds. For proof, see McKelvey (1975, prop. 4.3, p. 840).

PROPOSITION 4. If U1-U7 and U8b holds, $P(j)$ and $P(k)$ are unimodal and symmetric about points j and k, respectively.

1) For x closer to j than k, U8b states that $EU(A_j) - EU(A_o) = R = U'(j) - c$.
2) By U3, $E(c) = C > 0$ for all i, i.e., $E(R) = U'(j) - C$.
3) By U4, $U'(j)$ is maximized at j, so $E(R)$ and $P(j)$ are maximized at j.

4) By U4, $U'(j)$ is symmetric about j, so $E(R)$ and $P(j)$ are, too, since $U'(j)$ is the only variable argument of $E(R)$ and $P(j)$.

PROPOSITION 5.

A. See McKelvey (1975, prop. 4.5, p. 841).

B. If U1-U7 and any part of U8 hold, Weak Extremist Support is met.

1) Let $d(x,j)$ denote the Euclidean distance between x and j, etc.

2) To define Weak Extremist Support, consider two points, e.g., x and y, such that $d(x,j) < d(x,k)$, $d(y,j) < d(y,k)$, $d(x,j) = d(y,j)$, and $d(x,k) > d(y,k)$.

3) By Proposition 3, $P(k/x) = P(k/y) = 0$.

4) By U1 and U4, $U(j/x)$ and $U'(j/x)$ equal $U(j/y)$ and $U'(j/y)$, respectively, whereas $U(k/x) < U(k/y)$ and $U'(k/x) < U'(k/y)$.

5) If U8b holds, then $P(j/x) = P(j/y)$ by Proposition 4, and the Weak Extremist Support Condition holds (trivially).

6) If U8a holds, then either of the two conditions for $EU(A_j) > EU(A_o)$ to hold have utility differentials as the only variable arguments (given U3). By step 4, the differentials are larger for x than y, hence $P(j/x) > P(j/y)$ and Weak Extremist Support holds.

7) If U8a and U8b hold, then $EU(A_j) - EU(A_o)$ is $p'[U(j) - U(k)] + [U'(j) - aU'(k)] - c$. The utility differentials are again the only variable, and the condition holds.

Equilibrium Proofs

Some definitions:

1. The density of activists in party j for $\{j,k\} = \{\theta,\psi\}$ is $F(j,k) = \int_a^b P(j)f(x)dx$
2. The mean ideal point, j^*, is: $j^* = [\int_a^b xP(j)f(x)dx]/F(j,k)$.
3. j is an equilibrium if: $\int_a^j xP(j)f(x)cx = \int_j^b xP(j)f(x)dx$, i.e., if $j^* = j$.
4. (j^*,k^*) are an equilibrium pair if 3 holds for both simultaneously.
5. Let $m = (j+k)/2$, and let $j < k$ initially.
6. By Proposition 3, $P(j) = 0$ for $x > m$, $P(k) = 0$ for $x < m$. Therefore, $F(j,k) = \int_a^b P(j)f(x)dx = \int_a^m P(j)f(x)dx$, and similarly for $F(k,j)$.
7. Since $P(j)$ and $f(x)$ are continuous, $F(j,k)$ is continuous. The LHS and RHS of Def. 3. are also continuous as j varies continuously.
8. Let $h(j) = xP(j)f(x)dx$ and $h(k) = xP(k)f(x)dx$.

Lemma 1. A one party equilibrium (say, for j^*).

1) Let j^* exist and vary continuously from a to b.

2) If $j^* = a$, $\int_a^{j^*} h(j) = \int_a^a h(j) = 0$, while $\int_{j^*}^m h(j) = \int_a^m h(j) = F(j)$.

3) If $j^* = b$, from def. 3 of equilibrium, $LHS = F(j)$, while $RHS = 0$.

4) Consequently, $j^* = a$ and $j^* = b$ cannot be equilibrium values.

5) Since LHS and RHS are continuous, a j that varied continuously from a to b, like j^* above, yielded a LHS that began at zero, ended at $F(j)$, whereas RHS went from $F(j)$ to zero. Thus, there was some value of j in (a,b) such that $LHS = RHS$, hence a j^*, equilibrium value, exists.

Lemma 2.

A. Unimodal and symmetric $f(x)$ and $P(j)$.

1. Let the mode of $f(x)$ be 0, so $a = -b$, etc.
2. Let $j = 0$. Then, the question is does $\int_a^o h(x) = \int_o^b h(x)$?
3. There is a one-to-one correspondence for the pairs $(-x,x)$ where $-x \epsilon [a,0)$, $x \epsilon (o,b]$, and thus a one-to-one mapping of each point in the integral on *LHS* with that on *RHS*.
4. By conditions of lemma, $P(j/x) = P(j/-x)$ and $f(x) = f(-x)$. Therefore, $h(x) = h(-x)$ for all x in range of integrals. Therefore, $j^* = 0$ is an equilibrium.
5. For uniqueness, suppose $j < 0$. Let x be any point in *LHS* integral. Let y be the point in *RHS* such that $d(x,j) = d(y,j)$. For each x, there is exactly one such y, whereas for some y, there is no x, for the rest there is but one x. By assumption, $P(j/x) = P(j/y)$, but $f(x) < f(y)$ for all such (x,y) by definition of unimodality of $f(x)$. Thus, *LHS* < *RHS*. The same holds in reverse if $j > 0$, hence $j^* = 0$ is unique.

B. Unimodal $f(x)$, $P(j)$, symmetric $P(j)$.

1. Let t be the mean of $f(x)$.
2. By definition of a mean, t is mean if: $\int_a^t xf(x_o)dx = \int_t^b xf(x)dx$.
3. j^* is an equilibrium if: $\int_a^{j^*} xP(j)f(x)dx = \int_{j^*}^b xP(j)f(x)dx$.
4. Since $P(j)$ is unimodal and symmetric, if $j = t$, then the $P(j)$ "weight" applied to the equation for t will make the two equations hold at approximately the same value, i.e., j^* "near" t.
5. Since y, the mode of $f(x)$, is "near" t, j^* will be "near" y.

PROPOSITION 6. Existence of (j^*,k^*) when $P(j)$, $P(k)$ meet S1, S2 and $f(x)$ is unimodal.

1. First j^* cannot equal a, for $LHS(j)$ of the equilibrium condition would be 0, implying that $RHS(j) = 0$, implying that $m = a$, that $k^* = a$, and that $LHS(j) = 0$, but, as in Lemma 1, $RHS(k)$ will not be zero. Similarly, neither j^* nor k^* can be at b. Hence, $(j^*,k^*) \neq \{a,b\}$.
2. By S2, $P(j) = P(k) = 0$, if $j > m$, $k < m$ for $j < k$. So, if $j = m$ (and hence $k = m$) $RHS(j) = LHS(k) = 0$ in the equilibrium equations, so that, since both parties cannot have $F(j) = F(k) = 0$, any equilibria cannot be at m, either. Steps 1 and 2 prove the last sentence of Proposition 6.
3. Existence of an equilibrium pair for *symmetric* $f(x)$.

   a. Set $j = -k$ throughout, so that if one party is at equilibrium, the other will be simultaneously for symmetric $f(x)$.

   b. Let $j = a$, initially, and let it move continuously toward b. Therefore, both sides of the equilibrium condition will be continuous.

   c. At $j = a$ $(k = -a = b)$, $LHS(j) = 0$, $RHS(j) > 0$.

   d. At $j = m = 0$, $RHS(j) = 0$, $LHS(j) > 0$.

   e. Since the changes are continuous, $LHS(j) = RHS(j)$ at some $a < j < 0$. Since this is true for k at the same time, a (j^*, k^*) exists.
4. Existence of equilibrium for *asymmetric* $f(x)$.

   a. Since it has been shown that no part of the equilibrium equations can be zero, we can rewrite the joint equations as:

$$LHS(j)/RHS(j) = RHS(k)/LHS(k) = 1.$$

   For continuous changes in j,k (without crossing m), both ratios are continuous.

b. By starting with $j = a$, $k = b$, both ratios equal zero.

c. By then adjusting j and k (primarily by moving them toward each other), while keeping the equality of the two ratios, as j approaches k (and hence j and k approach m) the ratios can be made to increase toward infinity. At $j = k = m$, both are infinite.

d. By construction, the two ratios were kept equal. Since both went from zero to infinity, they equaled one together, i.e., a (j^*,k^*) exists.

PROPOSITION 7.

A. $P(j)$ is unimodal and symmetric, $f(x)$ is unimodal.

1. Let t be the mean of $f(x)$, and recall the definitions in Lemma 2.
2. Suppose we consider a one-party equilibrium, with $j = t$. Then, $\int_a^{j=t} xP(j)f(x)dx = \int_{j=t}^{b} xP(j)f(x)dx$ if j is an equilibrium.
3. But, there must be a $k \epsilon (j,b)$, with $F(j,k) > 0$, hence $k < b$.
4. By the Rational Choice Condition, $P(j) = 0$ for all $m < x \leqslant b$.
5. Hence, for j^* to be at t, there must be some pairs of points, say (e,g) with $d(e,j) = d(g,j)$ and $e < j < g < m$ with $P(j/e) > P(j/g)$, in violation of Symmetry of $P(j)$.
6. By the same reasoning, $t < k^* < b$.

B. $P(j)$ satisfies Weak Extremist Support, not Strong Extremist Support and not Symmetery.

1. For pairs such as (e,g) defined in step 5 above, by Weak Extremist Support, $P(j/e) \geqslant P(j/g)$, and for at least one such pair, the inequality must be strict (or $P(j)$ satisfies Symmetry).
2. Consequently, $P(j)$ imposes more "weight" on the *LHS* and less on the *RHS* of the equilibrium condition, while $P(k)$ works similarly.
3. Therefore, $d(j^*,k^*)$ must be greater than in case a.

C. $P(j)$ satisfies Strong Extremist Support.

1. Similar reasoning applies here as in comparing cases a and b.
2. Hence, $d(j^*,k^*)$ must be even greater.

PROPOSITION 8. $f(x)$ is unimodal and symmetric.

1. Let the mode of $f(x)$ be zero, constrain j to equal k.
2. Begin with j at point a (hence, k at point b). As before, $j = a$, $k = b$ is not an equilibrium pair.
3. As j continuously approaches $m = 0$, $LHS(j)$ of the equilibrium condition goes continuously from 0 to $F(j,k)$, *RHS* goes continuously from $F(j,k)$ to zero with both $F(j,k) > 0$ (although possibly unequal).
4. Therefore, a j^* exists. By construction a k^* exists at the same time with $k^* = -j^*$.

PROPOSITION 9. $f(x)$ is bimodal.

1. That an equilibrium pair exists about mode y follows almost immediately from prior propositions, since the mode at z may be very "small," relative to the mode at y. Thus, $f(x)$ may be bimodal but "almost" unimodal. The same holds for part b.
2. For part c, suppose $f(x)$ is symmetric with midpoint of 0 (and $y = -z$). Then, constrain $j = -k$. As j moves from point a (or $-b$) to 0, we get the same sort of equilibrium result as earlier. However, as $m = 0$, and by the Rational Choice Condition, $h(j)$ is positive over $[a,0]$ only, over which $f(x)$ is essentially unimodal. Hence via Lemma 1, j^* will be "near" y, as in the "one-party" case. The same will be true of k^* simultaneously.

PROPOSITION 10. The party centers are defined as medians.

1. The definition of j is $j = [\int_a^b P(j)f(x)dx]/F(j,x)$.
2. The new equilibrium condition is that (j^*, k^*) are in equilibrium if:

   j^*: $\int_a^{j^*} P(j)f(x)dx = \int_{j^*}^{b} P(j)f(x)dx$ and

   k^*: $\int_a^{k^*} P(k)f(x)dx = \int_{k^*}^{b} P(k)f(x)dx$.
3. Note that if $j = a$, then $\int_a^{j=a} P(j)f(x)dx = 0$, etc.
4. By similar reasoning, the rest of the above proofs follow.
5. The major change is that, in Proposition 7a, j^* will be less than the *median* of $f(x)$ which is, in turn, less than k^*. That follows, since the comparable expressions are:

   median of $f(x) = s$, such that: $\int_a^s f(x)dx = \int_s^b f(x)dx$,

   j^* is an equilibrium if: $\int_a^{j^*} P(j)f(x)dx = \int_{j^*}^{b} P(j)f(x)dx$, etc.

   The same holds for Lemma 2.
6. Other than these location results, the remainder of the proofs holds for (j,k) defined as the median instead of the mean ideal point location.

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