Game Theory

Given a pile of n stones. Two players are playing a game of removing 1 or 2 stone from the pile in each turn. The player unable to move will lose the game.

Formula

1. A state is winning if it can reach at least one losing position.
2. A state is losing if all of the states it can reach are winning.

Nim Game

1. Given N piles of stones.

2. Number of stones in each pile can be different.

3. In one move, you can remove any positive number of stones from a single pile.

4. The player unable to move is the loser.

Nim Game

Solution :

if xor == 0 , 2nd player wins

if xor != 0 , 1st player wins

Bogus Nim / Poker Nim

1. Initially player one has x extra stones and player two has y extra stones.

2. In one move a player can add stones from his stock to a single pile or remove some stones from a single pile.

3. The Player not able to make a move is the loser.

Misere Nim

1. Player who pick the last stone is the loser.

2. Same rule as nim game ,

   if xor== 0 , 1st player loser

   if xor != 0 , 1st player winner

3. If every pile has one stone , then corner case will occur.

   if no_of_pile %2 == 0 , 1st player loser

   if no_of_pile %2 == 1 , 1st player winner

Staircase Nim

1. An array of length N is given.

2. Some cell has exactly one coin.

3. In one move you can move a coin to a empty cell in left side.

4. But you can not jump over another coin.

5. The player unable to move is the loser.

Staircase Nim

Solution :

1. Even serial piles are useless , ignore them
2. Xor the odd serial piles
3. if xor == 0 then first player loser
   else First player winner

Sprague-Grundy theorem

Pre Conditions

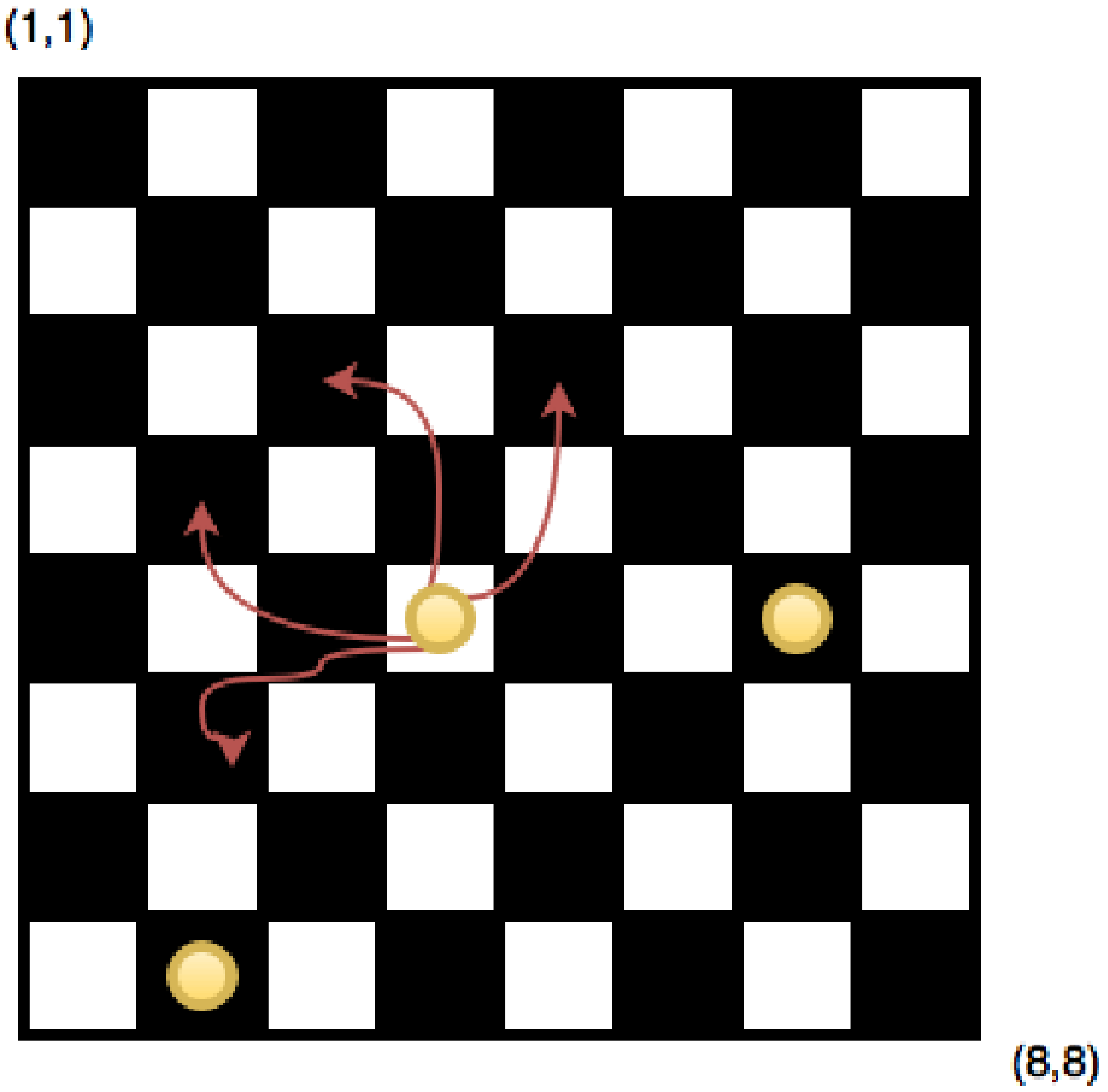
1 . impartial
2 . perfect information
3 . finite
4 . states as either winning or losing
5 . directed acyclic graph

Theorem

1. Any perfect-information impartial two-player game can be reduced to the game of Nim.

2. The current player has a winning strategy if the xor-sum of the pile sizes is non-zero.

একটা দাবার বোর্ডে k টা কয়েন রাখা আছে। কোনো কয়েন (x,y) ঘরে থাকলে প্রতি চালে কয়েনটিকে (x-2, y+1) অথবা (x-2, y-1) অথবা (x-1, y-2) অথবা (x+1, y-2) ঘরে সরানো যায় , তবে কয়েনটি বোর্ডের বাইরে সরানো যাবে না।



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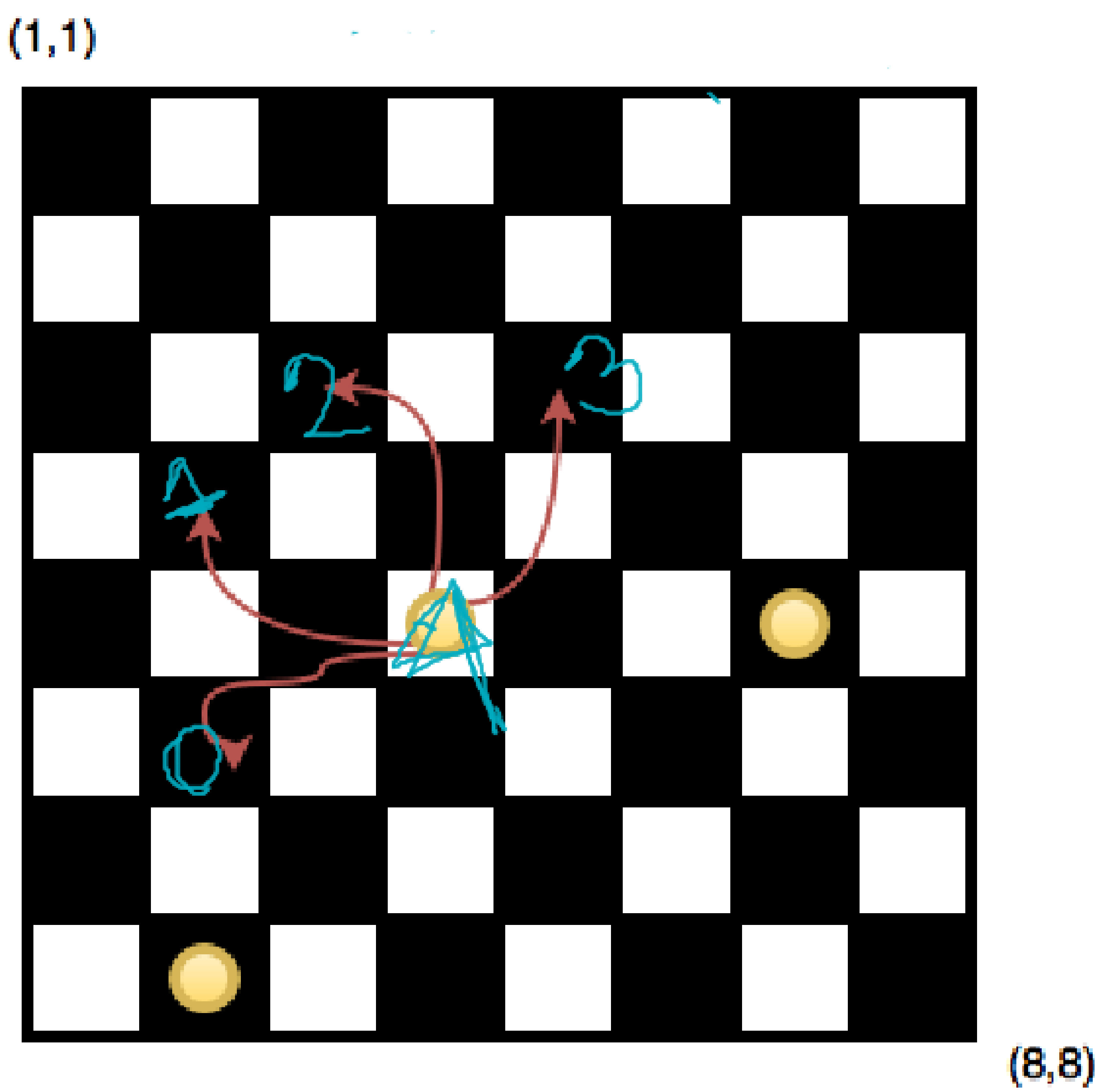

0	0	1	1	0	0	1	1
0	0	2	1	0	0	1	1
1	2	2	2	3	2	2	2
1	1	2	1	4	3	2	3
0	0	3	4	0	0	1	1
0	0	2	3	0	0	2	1
1	1	2	2	1	2	2	2
1	1	2	3	1	1	2	0