Catalan Numbers

$$C_0 = C_1 = 1$$

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}, n \geq 2$$

```
const int MOD = ....
const int MAX = ....
int catalan[MAX];
void init() {
    catalan[0] = catalan[1] = 1;
    for (int i=2; i<=n; i++) {
        catalan[i] = 0;
        for (int j=0; j < i; j++) {
            catalan[i] += (catalan[j] * catalan[i-j-1]) % MOD;
            if (catalan[i] >= MOD) {
                catalan[i] -= MOD;
            }
        }
    }
}
```

$$C_n = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1}\binom{2n}{n}, n \geq 0$$

Stirling Number of Second Kind

Stirling Numbers of the Second Kind

The **Stirling number of the second kind** $S(n, k)$ is the number of partitions of an n-element set into exactly k non-empty subsets.

For example, $S(4, 2) = 7$ because we have the partitions $\{1|234, 2|134, 3|124, 4|123, 12|34, 13|24, 14|23\}$.

Stirling numbers of the second kind satisfy the recursion

$$S(n + 1, k) = kS(n, k) + S(n, k - 1).$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^j\binom{k}{j}(k-j)^n.$$

$$S(n,k) = [k^n - {}^kC_1*(k-1)^n + {}^kC_2*(k-2)^n - \ldots + {}^kC_n*(k-n)^n]/k!$$

Stirling Numbers of the First Kind

The **Stirling number of the first kind** $c(n,k)$ is the number of permutations of an n-element set with exactly k cycles.

For example, $c(4,2)=11$ because (writing all our permutations in cycle notation) we have the permutations $\{(1)(234),(1)(243),(134)(2),(143)(2),(124)(3),(142)(3),(123)(4),(132)(4),(12)(34),(13)(24),(14)(23)\}$.

The total number of partitions of an n-element set is $B_n=\sum_{k=1}^{n} S(n,k)$ and is called the nth Bell number.