Lec6: min sep & hamming distance

keywords: min sep, $\lfloor \frac{t}{2} -1 \rfloor$, Code = $G \cdot b$

Zoom video: https://uchicago.zoom.us/rec/share/OHzcggFgI1cn0blpF3Fly35YtD9nvieWk9MyLCTsLth_MFjfHR7baTZZ-CjyHIJL.47b8Dfyjw8avluWs

---

- Hamming distance:
  # of bits that are different

  $\begin{array}{c} 0\ 1\ 1\ 0 \\ 1\ 1\ 0\ 1 \end{array}$ **3**

$0 \times 0 = 0$
$0 \times 1 = 0$
$1 \times 0 = 0$
$1 \times 1 = 1$

"AND" or "multiplication" as expected

$0 + 0 = 0$
$1 + 0 = 1$
$0 + 1 = 1$
$1 + 1 = \boxed{0}$

"XOR" slightly different than "addition"

$1+1 = 2, \ 2_b = 10$

---

min sep = t, $\underline{\lfloor \frac{t}{2} - 1 \rfloor}$ tolerance

Receiver → coded block (size 4) → find (nearest Hamming distance) admissible block

• Parity Code
2^3 output instead of 2^4

$0010 \begin{array}{c} \nearrow \ ? \\ \searrow \ ? \end{array}$

dist == 1
problem!

$1111 \xrightarrow{\text{okay!}}$

$\begin{array}{cccc} \uparrow & 0\ 0\ 0 & 0 & \longleftarrow 0\ 0\ 0 \\ \updownarrow^{1}_{2} & 0\ 0\ 1 & 1 & \longleftarrow 0\ 0\ 1 \\ \downarrow & 0\ 1\ 0 & 1 & \longleftarrow 0\ 1\ 0 \\ & 1\ 0\ 0 & 1 & \longleftarrow 1\ 0\ 0 \\ & 1\ 0\ 1 & 0 & \longleftarrow 1\ 0\ 1 \\ & 1\ 1\ 0 & 0 & \longleftarrow 1\ 1\ 0 \\ & 0\ 1\ 1 & 0 & \longleftarrow 0\ 1\ 1 \\ & 1\ 1\ 1 & 1 & \longleftarrow 1\ 1\ 1 \end{array}$

egregious false (dist >> 1)

No

Recap:

Src → Encoder → 0110 → Channel → 01101 → Decoder → Dst

"channel coding":
  adding **redundancy** to binary block
  to make code resilient to errors

• replication code

$0\ 1{*}0 \quad 0110 \quad 0110$ 3x rep code
                    could tolerate 1 failure

$0110 \quad 0100$ 2x " "
   ?   ?   can identify 1 failure

wasteful

compare which better?

• parity bit

$0110 \ \rightarrow \ $ XOR over seq of bits

---

block = 3
2^3 input

• Replication Code
  enable the error tolerance 1

$\begin{array}{ccc} 0\ 0\ 0 & 0\ 0\ 0 & 0\ 0\ 0 \\ 0\ 0\ 1 & 0\ 0\ 1 & 0\ 0\ 1 \end{array} \updownarrow 3$

$0\ 0\ 0 \quad 0\ 0\ 1 \quad 0\ 0\ 1$ ↗ unambiguously

minimal hamming dist btw
admissible code words **3**

Can we do better than 3x Rep?

| bit 1 | bit 2 | bit 3 | bit 4 | P(1,2) | P(3,4) | P(1,4) |

block of 4

3 extra bits → $\frac{7}{4}$ **better**

3x rep → $\frac{12}{4}$

$\begin{array}{cc} 0\ 0\ 0\ 0 & 0\ 0\ 0 \ \updownarrow^{\uparrow}_{\downarrow} 3 \\ 1\ 0\ 0\ 0 & 1\ 0\ 1 \\ 0\ 1\ 0\ 0 & 1\ 0\ 0 \\ 0\ 0\ 1\ 0 & 0\ 1\ 0 \\ 0\ 0\ 0\ 1 & 0\ 1\ 1 \end{array}$

min sep 3 here
1 tolerance

Linear Code = $G \cdot b$

block = $[X_1, \dots, X_k] = b$

Code = $G \cdot b$

parity code:

$G = \begin{bmatrix} 1\ 1\ 0\ 0 \\ 0\ 1\ 1\ 0 \\ 1\ 1\ 0\ 0 \\ 1\ 1\ 1\ 1 \end{bmatrix} \begin{array}{l} \\ N > k \end{array}$

$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_1 + X_2 + X_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & & \\ & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

repetition code are bad

Random linear code works quite well