

Unified Stochastic and Robust Unit Commitment

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Abstract—Due to increasing penetration of intermittent renewable energy and introduction of demand response programs, uncertainties occur in both supply and demand sides in real time for the current power grid system. To address these uncertainties, most ISOs/RTOs perform reliability unit commitment runs after the day-ahead financial market to ensure sufficient generation capacity available in real time to accommodate uncertainties. Two-stage stochastic unit commitment and robust unit commitment formulations have been introduced and studied recently to provide day-ahead unit commitment decisions. However, both approaches have limitations: 1) computational challenges due to the large scenario size for the stochastic optimization approach and 2) conservativeness for the robust optimization approach. In this paper, we propose a novel unified stochastic and robust unit commitment model that takes advantage of both stochastic and robust optimization approaches, that is, this innovative model can achieve a low expected total cost while ensuring the system robustness. By introducing weights for the components for the stochastic and robust parts in the objective function, system operators can adjust the weights based on their preferences. Finally, a Benders' decomposition algorithm is developed to solve the model efficiently. The computational results indicate that this approach provides a more robust and computationally trackable framework as compared with the stochastic optimization approach and a more cost-effective unit commitment decision as compared with the robust optimization approach.

Index Terms—Benders' decomposition, mixed-integer linear programming (MILP), robust optimization, stochastic optimization, unit commitment.

NOMENCLATURE

Indices and Parameters

B	Index set of all buses.
\mathcal{E}	Index set of transmission lines linking two buses.
Λ_b	Set of thermal generators at bus b .
T	Time horizon (e.g., 24 h).
SU_i^b	Startup cost of thermal generator i at bus b .
SD_i^b	Shutdown cost of thermal generator i at bus b .
$F_i(\cdot)$	Fuel cost of thermal generator i .

MU_i^b	Minimum up-time for thermal generator i at bus b .
MD_i^b	Minimum down-time for thermal generator i at bus b .
UR_i^b	Ramp-up rate limit for thermal generator i at bus b .
DR_i^b	Ramp-down rate limit for thermal generator i at bus b .
L_i^b	Lower bound of electricity generated by thermal generator i at bus b .
U_i^b	Upper bound of electricity generated by thermal generator i at bus b .
C_{ij}	Capacity for the transmission line linking bus i and bus j .
K_{ij}^b	Line flow distribution factor for the transmission line linking bus i and bus j , due to the net injection at bus b .
π_t^b	Weight of the load at bus b in time t .
$\bar{\pi}^t$	Budget parameter to describe the uncertainty set for the total load in time t .
$\bar{\pi}$	Budget parameter to describe the uncertainty set for the total load for the whole operational horizon.
α	Weight for the expected total generation cost in the objective function.
D_t^{b+}	Upper bound of the load at bus b in time t .
D_t^{b-}	Lower bound of the load at bus b in time t .
D_t^{b*}	Forecasted load at bus b in time t .
d_t^b	Random parameter representing the load at bus b in time t for the robust optimization part.
$d_t^b(\xi)$	Load at bus b in time t corresponding to scenario ξ for the stochastic optimization part.
γ_{it}^{jb}	Intercept of the j th segment line for the generation cost for generator i at bus b in time t .
β_{it}^{jb}	Slope of the j th segment line for the generation cost for generator i at bus b in time t .

First-Stage Variables

y_{it}^b	Binary decision variable: "1" if thermal generator i at bus b is on in time t ; "0" otherwise.
u_{it}^b	Binary decision variable: "1" if thermal generator i at bus b is started up in time t ; "0" otherwise.
v_{it}^b	Binary decision variable: "1" if thermal generator i at bus b is shut down in time t ; "0" otherwise.

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Second-Stage Variables

$q_{it}^b(\xi)$	Electricity generation amount by thermal generator i at bus b in time t corresponding to scenario ξ for the stochastic optimization part.
x_{it}^b	Electricity generation amount by thermal generator i at bus b in time t for the robust optimization part.
$\phi_{it}^b(\xi)$	Auxiliary variable for the stochastic optimization part representing the fuel cost of thermal generator i at bus b in time t corresponding to scenario ξ .
ϕ_{it}^b	Auxiliary variable for the robust optimization part representing the fuel cost of thermal generator i at bus b in time t .

I. INTRODUCTION

DUE to increasing penetration of intermittent renewable energy and introduction of demand response programs, uncertainties occur in both supply and demand sides in real time for the current power grid system. To prevent load shedding and blackout, most ISOs/RTOs forecast the real time net load (e.g., the actual load minus the intermittent renewable generation if the renewable generation curtailment is not allowed) information and then perform reliability unit commitment runs after the day-ahead financial market closes to ensure enough generation capacity available to accommodate real time load and supply fluctuations. A traditional approach is to consider several reserve constraints to achieve the reliability objective. Recently, two-stage stochastic and robust optimization approaches have been studied to enhance this reliability unit commitment run process.

Two-stage stochastic optimization fits well to solve the unit commitment problem under uncertainty for a deregulated electricity market that includes day-ahead and real-time operations. For instance, for the two-stage stochastic optimization approach, the day-ahead unit commitment decision is made in the first stage before the uncertain problem parameter representing the real time information is realized, and the economic dispatch amount is made in the second stage after the uncertain parameter is realized. The objective is to minimize the total expected cost and the uncertain problem parameter (e.g., wind power) is captured by a number of scenarios. In recent works, significant contribution has been made by using the stochastic optimization models to solve unit commitment under uncertainty problems, in particular, under wind power output uncertainty. For instance, recently in [1]–[3], a stochastic unit commitment model is introduced for short-term operations to integrate wind power in the Liberalised Electricity Markets (WILMAR). This model has been successfully implemented and used in several wind power integration studies. In addition, a stochastic formulation, which allows the explicit modeling of the sources of uncertainty in the unit commitment problem, is proposed in [4], a two-stage security-constrained unit commitment (SCUC) algorithm that considers the unit commitment decision in the first stage and takes into account the intermittency and volatility of wind power generation in the second stage is introduced in [5], and

a stochastic unit commitment model, considering various wind power forecasts and their impacts on unit commitment and dispatch decisions, is proposed in [6]. Significant research progress has also been made to solve security-constrained stochastic unit commitment models. For instance, security-constrained stochastic unit commitment formulations addressing market-clearing are described in [7] and the corresponding case studies are performed in [8]. In [9], a scenario-tree based stochastic security-constrained unit commitment is studied. Most recently, two-stage stochastic programming approaches have been studied to consider both slow-start and fast-start generators in [10], in which the slow-start generators are committed in the first stage and fast-start generators are committed in the second stage. These approaches have also been studied to estimate the contribution of demand flexibility in replacing operating reserves in [11] and ensure high utilization of wind power output by adding additional chance constraints in [12].

In practice, a significant amount of available data for ISOs/RTOs makes it possible to take samples and generate scenarios for the stochastic optimization approach. However, it is always challenging for the stochastic optimization approach to deal with large-sized instances when the scenario size increases significantly. Therefore, different scenario reduction approaches have been proposed to select important scenarios. In this way, the small sample size may lead to the feasibility issues, that is, the day-ahead unit commitment decision might not be feasible for some scenarios which are not selected. Recently, robust optimization approaches have been proposed to ensure the robustness and make the day-ahead unit commitment feasible for most outcomes of the real time uncertain input parameter. For the robust optimization approach, the uncertain parameter is described within a given deterministic uncertainty set and the objective is to minimize the worst-case cost that includes the first-stage unit commitment and the second-stage economic dispatch costs. Recent research works include two-stage robust unit commitment model and Benders' decomposition algorithm developments to ensure system robustness under load uncertainties introduced in [13] and [14], two-stage robust optimization models with slightly different uncertainty sets to provide robust day-ahead unit commitment decisions under wind power output uncertainties presented in [15] and [16], a robust bidding strategy in a pool-based market by solving a robust mixed-integer linear program and generating a bidding curve described in [17], a robust optimization model to integrate PHEVs into the electric grid to handle the most relevant planning uncertainties proposed in [18], and a robust optimization approach to solve contingency-constrained unit commitment with $N - k$ security criterion and decide reserve amounts to ensure system robustness introduced in [19].

The advantage of the robust optimization approach is that it requires minimal information of the input uncertain parameter (as long as the information is sufficient to generate the deterministic uncertainty set) and ensures the robustness of the obtained unit commitment decision, i.e., the day-ahead unit commitment decision is feasible for most outcomes of the real time uncertain problem parameter. However, this approach always faces the challenges on its over conservatism, due to its objective function of minimizing the worst case cost, because the worst case happens rarely.

To address the above shortages of each proposed approach (stochastic or robust optimization approach), in this paper, we propose an innovative unified stochastic and robust unit commitment model to take advantage of both stochastic and robust optimization approaches, especially when the historical data is available. In this case, we can get a rough estimation on the distribution of the uncertain parameter. In our model, we put the weights for the expected total cost and the worst case cost, respectively, in the objective function. This approach allows the system operators to decide the weight for each objective based on their preferences. In addition, if the weight in the objective function for the robust optimization part is zero, then our model turns out to be a two-stage stochastic optimization problem with additional constraints generated based on the robust optimization approach. The main contribution of our proposed innovative approach can be summarized as follows.

- 1) Our proposed approach takes the advantages of both the stochastic and robust optimization approaches. It can provide a day-ahead unit commitment decision that can lead to a minimum expected total cost while ensuring the system robustness.
- 2) Our proposed approach can generate a less conservative solution as compared with the two-stage robust optimization approach and a more robust solution as compared with the two-stage stochastic optimization approach.
- 3) Our proposed approach can be implemented in a single Benders' decomposition framework. The computational time can be controlled by the system operators. Meanwhile, the system operators can also adjust the weights in the objective function, based on their preferences on stochastic and/or robust optimization approaches.

The remainder of this paper is organized as follows. Section II describes our unified stochastic and robust unit commitment model. In Section III, we develop a Benders' decomposition algorithm to solve the problem. In the proposed algorithm, we apply both feasibility and optimality cuts for the stochastic and robust optimization parts respectively. All these cuts are added into the master problem, which provides the day-ahead unit commitment decision. Section IV provides and analyzes the computational experiments through several case studies. Finally, Section V summarizes our research.

II. MATHEMATICAL FORMULATION

Here, we develop a two-stage unit commitment formulation considering both the expected total generation cost and the worst case scenario generation cost. The first stage is to determine the day-ahead unit commitment decision that includes turn-on/turn-off decisions of thermal generators by satisfying unit commitment physical constraints. The second stage contains the decisions on the economic dispatch for the thermal generators under each scenario for the stochastic optimization part and the worst-case scenario for the robust optimization part. In our model, a parameter $\alpha \in [0, 1]$ is introduced to represent the weight of the expected total generation cost and accordingly $1 - \alpha$ represents the weight of the worst case generation cost. In particular, for the case $\alpha = 1$, only the expected total generation cost is considered in the objective function,

and, for the case $\alpha = 0$, only the worst case generation cost is considered. The detailed formulation is described as follows:

$$\min \sum_{t=1}^T \sum_{b \in B} \sum_{i \in \Lambda_b} (SU_i^b u_{it}^b + SD_i^b v_{it}^b) + \alpha E[\mathcal{Q}(y, u, v, \xi)] \\ + (1 - \alpha) \max_{d \in \mathcal{D}} \min_{x \in \chi(y, u, v, d)} \sum_{t=1}^T \sum_{b \in B} \sum_{i \in \Lambda_b} F_i(x_{it}^b) \quad (1)$$

s.t.

$$-y_{i(t-1)}^b + y_{it}^b - y_{ik}^b \leq 0, \\ \forall k : 1 \leq k - (t - 1) \leq MU_i^b, \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (2)$$

$$y_{i(t-1)}^b - y_{it}^b + y_{ik}^b \leq 1, \\ \forall k : 1 \leq k - (t - 1) \leq MD_i^b, \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (3)$$

$$-y_{i(t-1)}^b + y_{it}^b - u_{it}^b \leq 0, \quad \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (4)$$

$$y_{i(t-1)}^b - y_{it}^b - v_{it}^b \leq 0, \quad \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (5)$$

$$y_{it}^b, u_{it}^b, v_{it}^b \in \{0, 1\}, \quad \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (6)$$

where $\mathcal{Q}(y, u, v, \xi)$ is equal to

$$\min \sum_{t=1}^T \sum_{b \in B} \sum_{i \in \Lambda_b} F_i(q_{it}^b(\xi)) \quad (7)$$

s.t.

$$L_i^b y_{it}^b \leq q_{it}^b(\xi) \leq U_i^b y_{it}^b, \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (8)$$

$$q_{it}^b(\xi) - q_{i(t-1)}^b(\xi) \leq \left(2 - y_{i(t-1)}^b - y_{it}^b\right) L_i^b \\ + \left(1 + y_{i(t-1)}^b - y_{it}^b\right) UR_i^b, \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (9)$$

$$q_{i(t-1)}^b(\xi) - q_{it}^b(\xi) \leq \left(2 - y_{i(t-1)}^b - y_{it}^b\right) L_i^b \\ + \left(1 - y_{i(t-1)}^b + y_{it}^b\right) DR_i^b, \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (10)$$

$$\sum_{b \in B} \sum_{i \in \Lambda_b} q_{it}^b(\xi) = \sum_{b \in B} d_{bt}(\xi), \forall t \quad (11)$$

$$-C_{ij} \leq \sum_{b \in B} K_{ij}^b \left(\sum_{r \in \Lambda_b} q_{rt}^b(\xi) - d_{bt}(\xi) \right) \leq C_{ij}, \\ \forall (i, j) \in \mathcal{E}, \forall t \quad (12)$$

and

$$\chi(y, u, v, d) = \left\{ x : \right. \\ L_i^b y_{it}^b \leq x_{it}^b \leq U_i^b y_{it}^b, \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (13)$$

$$x_{it}^b - x_{i(t-1)}^b \leq \left(2 - y_{i(t-1)}^b - y_{it}^b\right) L_i^b \\ + \left(1 + y_{i(t-1)}^b - y_{it}^b\right) UR_i^b, \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (14)$$

$$x_{i(t-1)}^b - x_{it}^b \leq \left(2 - y_{i(t-1)}^b - y_{it}^b\right) L_i^b \\ + \left(1 - y_{i(t-1)}^b + y_{it}^b\right) DR_i^b, \forall i \in \Lambda_b, \forall b \in B, \forall t \quad (15)$$

$$\sum_{b \in B} \sum_{i \in \Lambda_b} x_{it}^b = \sum_{b \in B} d_{bt}, \forall t \quad (16)$$

$$-C_{ij} \leq \sum_{b \in B} K_{ij}^b \left(\sum_{r \in \Lambda_b} x_{rt}^b - d_{bt} \right) \leq C_{ij}, \\ \forall (i, j) \in \mathcal{E}, \forall t \quad (17)$$

In the above formulation, we denote $F_i(\cdot)$ as the generation cost function of generator i . The objective function (1) is composed of the unit commitment cost in the first stage, and both the expected economic dispatch cost and the worst-case economic dispatch cost in the second stage. Constraints (2) and (3) represent each unit's minimum up-time and minimum down-time restrictions respectively. Constraints (4) and (5) indicate the start-up and shut-down operations for each unit. Constraints (8) and (13) enforce the upper and lower limits of the power generation amount of each unit. The ramping up constraints (9) and (14) require the first-hour minimum generation restriction (e.g., L_i^b) as described in [20] and limit the maximum increment of the power generation amount of each unit between two adjacent periods when the generator is on. Similarly, ramping down constraints (10) and (15) require the last-hour minimum generation restriction (e.g., L_i^b) and enforce the maximum decrement of the power generation amount of each unit between two adjacent periods when the generator is on. Constraints (11) and (16) ensure load balance and constraints (12) and (17) represent the transmission capacity constraints.

III. DECOMPOSITION ALGORITHMS AND SOLUTION FRAMEWORK

A. Scenario Generation

We use Monte Carlo simulation to generate scenarios for the uncertain load. We assume that the load follows a multivariate normal distribution $N(D, \Sigma)$ with its predicted value D and volatility matrix Σ . We can run Monte Carlo simulation to generate N scenarios each with the same probability $1/N$. After generating scenarios, we can replace the second-stage expected total cost objective term by

$$\frac{1}{N} \sum_{n=1}^N \sum_{t=1}^T \sum_{b \in B} \sum_{i \in \Lambda_b} F_i(q_{it}^b(\xi^n)). \quad (18)$$

B. Linearizing $F_i(\cdot)$

The generation cost $F_i(\cdot)$ is usually expressed as a quadratic function, for which we use a J -piece piecewise linear function to approximate. For instance, we have

$$\phi_{it}^b(\xi^n) \geq \gamma_{it}^{jb} y_{it}^b + \beta_{it}^{jb} q_{it}^b(\xi^n), \quad \forall t \in T, \forall b \in B, \forall i \in \Lambda_b, \forall j = 1, \dots, J, \forall n = 1, \dots, N \quad (19)$$

for the stochastic optimization part, and, similarly, we have

$$\phi_{it}^b \geq \gamma_{it}^{jb} y_{it}^b + \beta_{it}^{jb} x_{it}^b, \quad \forall t \in T, \forall b \in B, \forall i \in \Lambda_b, \forall j = 1, \dots, J \quad (20)$$

for the robust optimization part.

C. The Uncertainty Set of the Load

To generate the uncertainty set for the robust optimization part, we assume the load for each time period t at each bus b is between a lower bound D_t^{b-} and an upper bound D_t^{b+} , which can be decided by the fifth and 95th percentiles of the random load output. In addition, we assume that, for each given time period t , the summation of the weighted loads at all buses is

bounded above by $\bar{\pi}^t$, and the summation of the weighted loads within the whole operational horizon is bounded above by $\bar{\pi}$. Accordingly, the uncertainty set can be described as follows:

$$\mathcal{D} := \left\{ d \in \mathcal{R}^{|B| \times |T|} : D_t^{b-} \leq d_t^b \leq D_t^{b+}, \forall t, \forall b \right\} \quad (21)$$

$$\sum_{b \in B} \pi_b^t d_t^b \leq \bar{\pi}^t, \forall t \quad (22)$$

$$\sum_{t=1}^T \sum_{b \in B} \pi_b^t d_t^b \leq \bar{\pi}. \quad (23)$$

D. Abstract Formulation

For notation brevity, we use matrices and vectors to represent the constraints and variables. For example, we use \mathbf{e} to represent the vector with all components equal to 1. The mathematical model can be abstracted as follows:

$$\min_{y, u, v} (\mathbf{a}^T u + \mathbf{b}^T v) + \alpha \frac{1}{N} \sum_{n=1}^N \mathbf{e}^T \phi(\xi^n) + (1 - \alpha) \max_{d \in \mathcal{D}} \min_{x, \phi} \mathbf{e}^T \phi \quad (24)$$

$$s.t. \quad \mathbf{A}y + \mathbf{B}u + \mathbf{C}v \geq \mathbf{r}, \quad (24)$$

$$\mathbf{F}y - \mathbf{D}q(\xi^n) \leq \mathbf{g}, \quad n = 1, \dots, N \quad (25)$$

$$\mathbf{K}y - \mathbf{P}q(\xi^n) - \mathbf{J}\phi(\xi^n) \leq 0, \quad n = 1, \dots, N \quad (26)$$

$$\mathbf{T}q(\xi^n) \geq \mathbf{S}d(\xi^n) + \mathbf{s}, \quad n = 1, \dots, N \quad (27)$$

$$\mathbf{F}y - \mathbf{D}x(d) \leq \mathbf{g}, \quad (28)$$

$$\mathbf{K}y - \mathbf{P}x(d) - \mathbf{J}\phi(d) \leq 0, \quad (29)$$

$$\mathbf{T}x(d) \geq \mathbf{S}d + \mathbf{s}, \quad (30)$$

$$y, u, v \in \{0, 1\}, x(d), q(\xi^n) \geq 0, \quad (31)$$

$$\phi(d), \phi(\xi^n) \text{ free}, \forall n$$

where

$$\mathcal{D} = \left\{ d \in \mathcal{R}^{|B| \times |T|} : \mathbf{d}^- \leq d \leq \mathbf{d}^+, \mathbf{U}^T d \leq \mathbf{z} \right\}. \quad (32)$$

Constraint (24) represents constraints (2)–(5); constraint (25) represents constraints (8)–(10); constraint (26) represents constraint (19); constraint (27) represents constraints (11) and (12). Constraint (28) represents constraints (13)–(15); constraint (29) represents constraint (20); constraint (30) represents constraints (16) and (17).

E. Benders' Decomposition Algorithm

We can use the Benders' decomposition algorithm to solve the above problem. First, for each scenario ξ^n , $n = 1, \dots, N$, we dualize the constraints (25)–(27) and obtain the following dual formulation for the second-stage economic dispatch for the stochastic part:

$$\psi^{S^n}(y) = \max_{\gamma^n, \lambda^n, \mu^n} (\mathbf{F}y - \mathbf{g})^T \gamma^n + (\mathbf{K}y)^T \lambda^n + (\mathbf{s} + \mathbf{S}d(\xi^n))^T \mu^n \quad (33)$$

$$s.t. \quad \mathbf{D}^T \gamma^n + \mathbf{P}^T \lambda^n + \mathbf{T}^T \mu^n \leq 0 \quad (34)$$

$$\mathbf{J}^T \lambda^n = \mathbf{e} \quad (35)$$

$$\gamma^n, \lambda^n, \mu^n \geq 0$$

where $\gamma^n, \lambda^n, \mu^n$ are dual variables corresponding to the scenario n for constraints (25)–(27) respectively.

Similarly, we dualize the constraints (28)–(30) and obtain the following dual formulation for the second-stage economic dispatch for the robust part:

$$\begin{aligned} \psi^R(y) = & \max_{d \in \mathcal{D}, \gamma, \lambda, \mu} (\mathbf{F}y - \mathbf{g})^T \gamma + (\mathbf{K}y)^T \lambda + (\mathbf{s} + \mathbf{S}d)^T \mu \\ \text{s.t.} \quad & \mathbf{D}^T \gamma + \mathbf{P}^T \lambda + \mathbf{T}^T \mu \leq 0 \end{aligned} \quad (36)$$

$$\mathbf{J}^T \lambda = \mathbf{e} \quad (37)$$

$$\gamma, \lambda, \mu \geq 0 \quad (38)$$

where γ, λ, μ are dual variables for constraints (28)–(30), respectively.

We denote θ^n as the second-stage optimal economic dispatch cost corresponding to scenario $n, n = 1, \dots, N$ and $\bar{\theta}$ as the second-stage optimal economic dispatch cost under the worst case scenario. Then, the master problem can be described as follows, and the problem can be solved by adding feasibility and optimality cuts iteratively

$$\begin{aligned} \min_{y, u, v \in \{0,1\}} & (\mathbf{a}^T u + \mathbf{b}^T v) + \alpha \frac{1}{N} \sum_{n=1}^N \theta^n + (1 - \alpha) \bar{\theta} \\ \text{s.t.} \quad & \mathbf{A}y + \mathbf{B}u + \mathbf{C}v \geq \mathbf{r}, \\ & \text{Feasibility cuts,} \\ & \text{Optimality cuts.} \end{aligned}$$

F. Benders' Cuts for the Stochastic Optimization Part

1) *Feasibility Cuts:* We use the L-shaped method to generate feasibility cuts. In this case, we do not need to consider constraints (26) since we can always choose $\phi(\xi^n)$ to make these constraints satisfied. Thus, they will not affect the feasibility. For constraints (25) and (27) in scenario $\xi^n, n = 1, \dots, N$, the feasibility check problem is shown as follows:

$$\min_{q(\xi^n), \kappa} \sum_{j=1}^4 \mathbf{e}^T \kappa^j \quad (39)$$

$$\text{s.t.} \quad \mathbf{D}q(\xi^n) + \kappa^1 - \kappa^2 \geq \mathbf{F}y - \mathbf{g} \quad (40)$$

$$\mathbf{T}q(\xi^n) + \kappa^3 - \kappa^4 \geq \mathbf{S}d(\xi^n) + \mathbf{s} \quad (41)$$

$$q(\xi^n) \geq 0, \kappa^j \geq 0, j = 1, \dots, 4. \quad (42)$$

The dual of the above formulation can be described as follows:

$$\omega^{S_n}(y) = \max_{\hat{\gamma}^n, \hat{\mu}^n} (\mathbf{F}y - \mathbf{g})^T \hat{\gamma}^n + (\mathbf{S}d(\xi^n) + \mathbf{s})^T \hat{\mu}^n \quad (43)$$

$$\text{s.t.} \quad \mathbf{D}^T \hat{\gamma}^n + \mathbf{T}^T \hat{\mu}^n \leq 0, \quad (44)$$

$$\hat{\gamma}^n, \hat{\mu}^n \in [0, 1], \quad (45)$$

where $\hat{\gamma}^n$ and $\hat{\mu}^n$ are dual variables corresponding to the n th scenario for constraints (40) and (41), respectively. Then, we can perform the following steps to check feasibility.

- 1) If $\omega^{S_n}(y) = 0$, y is feasible for the n th scenario.
- 2) If $\omega^{S_n}(y) > 0$, generate a corresponding feasibility cut $\omega^{S_n}(y) \leq 0$.
- 2) *Optimality Cuts:* At each iteration, after solving the master problem, we obtain $\theta^n, n = 1, \dots, N$, and y . If we

substitute y into the subproblem and get $\psi^{S_n}(y)$, we should have $\psi^{S_n}(y) \leq \theta^n$. If $\psi^{S_n}(y) > \theta^n$, we can claim that y is not an optimal solution and we can generate a corresponding optimality cut as follows:

$$\psi^{S_n}(y) \leq \theta^n.$$

G. Benders' Cuts for the Robust Optimization Part

After solving the master problem, we apply the bilinear approach to get $\psi^R(y)$. The bilinear approach is proved to converge to optimality with a small gap (less than 0.05%) in a reasonable and much shorter time than the exact separation algorithm does [13]. We solve the following two linear programs iteratively:

$$\begin{aligned} \psi^{R1}(y, d) = & \max_{\gamma, \lambda, \mu} (\mathbf{F}y - \mathbf{g})^T \gamma + (\mathbf{K}y)^T \lambda + (\mathbf{s} + \mathbf{S}d)^T \mu \\ \text{(SUB1)} \quad \text{s.t.} \quad & \mathbf{D}^T \gamma + \mathbf{P}^T \lambda + \mathbf{T}^T \mu \leq 0 \end{aligned} \quad (46)$$

$$\mathbf{J}^T \lambda = \mathbf{e} \quad (47)$$

$$\gamma, \lambda, \mu \geq 0 \quad (48)$$

$$\begin{aligned} \psi^{R2}(y, \gamma, \lambda, \mu) = & \max_d \mu^T \mathbf{S}d + (\mathbf{F}y - \mathbf{g})^T \gamma + (\mathbf{K}y)^T \lambda + \mathbf{s}^T \mu \\ \text{(SUB2)} \quad \text{s.t.} \quad & \mathbf{d}^- \leq d \leq \mathbf{d}^+ \end{aligned} \quad (49)$$

$$\mathbf{U}^T d \leq \mathbf{z}. \quad (50)$$

Next, we discuss the feasibility and optimality cuts based on the proposed approach.

1) *Feasibility Cuts:* If the first-stage solution y is infeasible, then we should have $\psi^R(y)$ infeasible or unbounded. Note that $\psi^{R2}(y, \gamma, \lambda, \mu)$ is feasible and bounded from above because the feasible region of d is a polyhedra (e.g., bounded), and the objective function is continuous. Therefore, we only need to check the feasibility of (SUB1). The feasibility check for a given d is the same as that for the stochastic case

$$\omega^{R1}(y, d) = \max_{\hat{\gamma}, \hat{\mu}} (\mathbf{F}y - \mathbf{g})^T \hat{\gamma} + (\mathbf{s} + \mathbf{S}d)^T \hat{\mu} \quad (51)$$

$$\text{(FEA1)} \quad \text{s.t.} \quad \mathbf{D}^T \hat{\gamma} + \mathbf{T}^T \hat{\mu} \leq 0 \quad (52)$$

$$\hat{\gamma}, \hat{\mu} \in [0, 1]. \quad (53)$$

Note here that, in order to check the feasibility under the worst case scenario, we need to solve (FEA1) and the following (FEA2) iteratively to find the worst case load d :

$$\begin{aligned} \omega^{R2}(y, \hat{\gamma}, \hat{\mu}) = & \max_d \hat{\mu}^T \mathbf{S}d + (\mathbf{F}y - \mathbf{g})^T \hat{\gamma} + \mathbf{s}^T \hat{\mu} \\ \text{(FEA2)} \quad \text{s.t.} \quad & \text{Constraints (49) and (50).} \end{aligned} \quad (54)$$

This procedure stops when we have $\omega^{R2}(y, \hat{\gamma}, \hat{\mu}) \leq \omega^{R1}(y, d)$. The detailed algorithm is shown as follows.

- Step 1) Pick an extreme point d in \mathcal{D} .
- Step 2) Solve (FEA1), and store the optimal objective value $\omega^{R1}(y, d)$ and the optimal solution $\hat{\gamma}$ and $\hat{\mu}$.
- Step 3) Solve (FEA2), and store the optimal objective value $\omega^{R2}(y, \hat{\gamma}, \hat{\mu})$ and the optimal solution d^* .
- Step 4) If $\omega^{R2}(y, \hat{\gamma}, \hat{\mu}) > \omega^{R1}(y, d)$, let $d = d^*$ and go to step 2). Otherwise, go to step 5).
- Step 5) If $\omega^{R1}(y, d) = 0$, terminate the feasibility check for the robust part, and go to the feasibility check for

the stochastic part. Otherwise, add the feasibility cut $\omega^{R1}(y, d) \leq 0$ to the master problem.

2) *Optimality Cuts*: At each iteration, we solve the master problem and obtain $\bar{\theta}$ and y . Similarly, if $\psi^R(y) > \bar{\theta}$, we can generate a corresponding optimality cut as follows:

$$\psi^R(y) \leq \bar{\theta}.$$

The detailed algorithm to obtain optimality cuts is as follows.

- Step 1) Pick an extreme point d in \mathcal{D} .
- Step 2) Solve (SUB1), and store the optimal objective value $\psi^{R1}(y, d)$ and the optimal solution γ and μ .
- Step 3) Solve (SUB2), and store the optimal objective value $\psi^{R2}(y, \gamma, \lambda, \mu)$ and the optimal solution d^* .
- Step 4) If $\psi^{R2}(y, \gamma, \lambda, \mu) > \psi^{R1}(y, d)$, let $d = d^*$ and go to step 2). Otherwise, go to step 5).
- Step 5) If $\psi^{R2}(y, \gamma, \lambda, \mu) > \bar{\theta}$, generate the corresponding optimality cut $\psi^{R2}(y, \gamma, \lambda, \mu) \leq \bar{\theta}$ to the master problem. Otherwise, terminate the optimality check for the robust part and go to the optimality check for the stochastic part.

Finally, the flow chart of our proposed algorithm is shown in Fig. 1.

H. Special Cases and Discussions

The robust optimization approach helps ensure the robustness of the first-stage unit commitment decision. We can consider a special case in which we only use the constraints provided by the robust optimization approach to guarantee the solution robustness without considering the worst-case economic dispatch cost in the objective function (i.e., $\alpha = 1$). In addition, if we only take a small number of scenarios, then we don't need to use the Benders' decomposition approach to generate cuts for the stochastic part. Instead, we can put the stochastic part in the master problem, and add feasibility cuts by considering the constraints in the robust part. Therefore, the special case can be reformulated as follows:

$$\min_{y, u, v} (\mathbf{a}^T u + \mathbf{b}^T v) + \frac{1}{N} \sum_{n=1}^N \mathbf{e}^T \phi(\xi^n)$$

s.t. Constraints (24) – (27),

$$\mathbf{F}y - \mathbf{D}x(d) \leq \mathbf{g}, \mathbf{T}x(d) \geq \mathbf{S}d + \mathbf{s}, \forall d \in \mathcal{D}$$

$$y, u, v \in \{0, 1\}, x(d) \geq 0, \phi(\xi^n) \text{ free.}$$

For the above formulation, the decision variable $x(d)$ is the auxiliary variable representing the generation amount corresponding to the case in which the load is d . Constraints (24)–(27) are put in the master problem. We apply the robust feasibility check described in Section III-G to find the worst-case load d^* . Then we can add the Benders' feasibility cuts in the form $\omega^{R1}(y, d^*) \leq 0$ to the master problem.

Note here that we provide a general framework to solve the unified stochastic and robust unit commitment problem. There are several deviations we can explore for each part in the algorithm. For instance, for the stochastic part, we can choose the Benders' decomposition approach or solve the deterministic equivalent formulation (as above for the special case); for the robust optimality part, we can consider including or not including

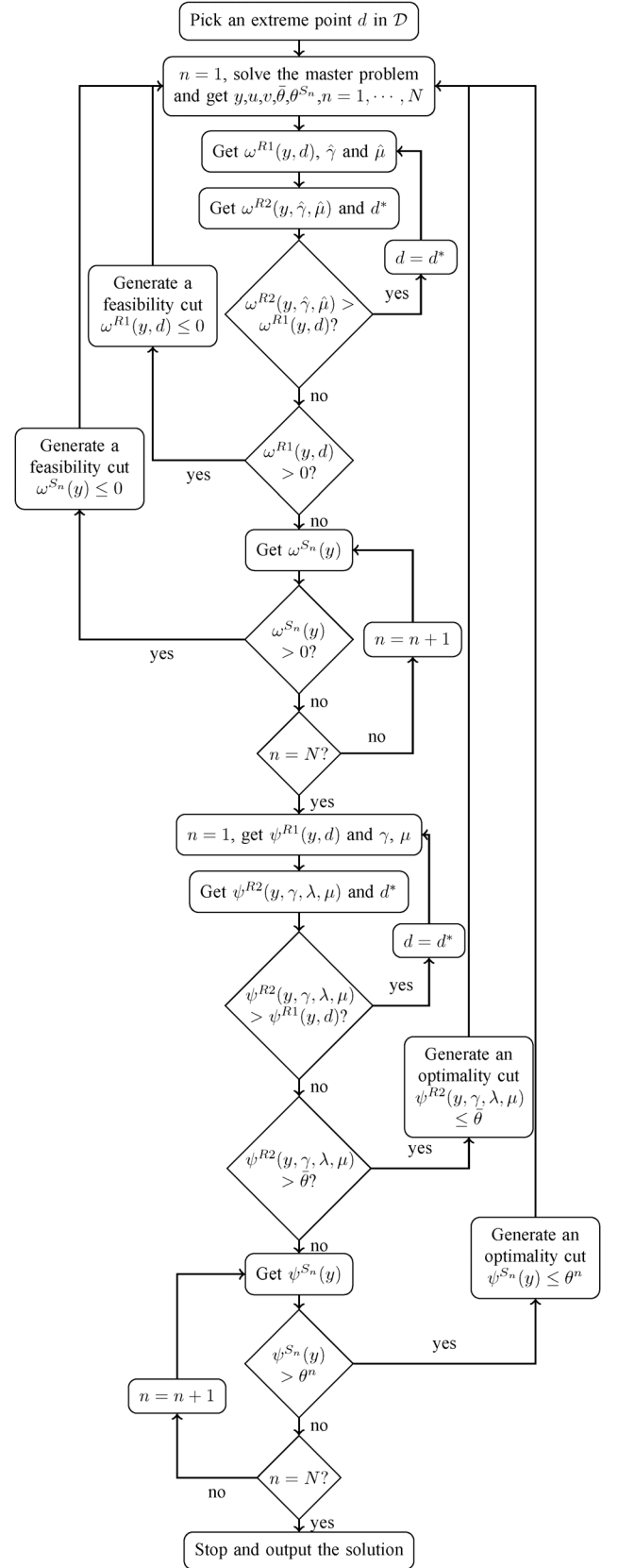


Fig. 1. Flow chart of the proposed algorithm.

the worst-case cost in the objective function; for the robust feasibility part, once the worst-case load d^* is detected, we can add

TABLE I
RESULTS UNDER DIFFERENT RATIO% AND BUDGET% SETTINGS

Ratio%		Budget%		
		5%	15%	25%
5%	Obj. Val.(\$)	737,134	737,134	737,134
	# of Start-ups	10	10	10
	Time(sec)	48	48	48
15%	Obj. Val.(\$)	737,439	738,372	738,272
	# of Start-ups	10	11	11
	Time(sec)	61	163	163
25%	Obj. Val.(\$)	738,561	739,374	740,175
	# of Start-ups	12	12	13
	Time(sec)	106	392	1193

the dual inequality in the form $\omega^{R_1}(y, d^*) \leq 0$, or a group of primal inequalities in the form $\mathbf{F}y - \mathbf{D}x(d^*) \leq \mathbf{g}, \mathbf{T}x(d^*) \geq \mathbf{S}d^* + \mathbf{s}$ (cf. [15] and [21]).

IV. COMPUTATIONAL RESULTS

Here, we report experimental results for a modified IEEE 118-bus system, based on the one given online,¹ to show the effectiveness of the proposed approach. The system contains 118 buses, 33 generators, and 186 transmission lines. The operational time interval is 24 h. In our experiments, we set the feasibility tolerance gap to be 10^{-6} and the optimality tolerance gap $(\psi^{R2} - \bar{\theta})/\bar{\theta}$ to be 10^{-4} . The MIP gap tolerance for the master problem is the CPLEX default gap. We use C++ with CPLEX 12.1 to implement the proposed formulations and algorithms. All experiments are executed on a computer workstation with four Intel Cores and 8-GB RAM. In our experiment, we first perform sensitivity analysis of our proposed approach in terms of the effects of the uncertainty set and the objective weight α . Then, we compare the performances of our proposed approach with the stochastic and robust optimization approaches.

A. Sensitivity Analysis

1) *Sensitivity Analysis of Uncertainty Set*: For convenience, we normalize the weight parameter $\pi_t^b = 1$. We first let $D_t^{b+} = (1 + \text{Ratio}\%)D_t^{b*}$ and $D_t^{b-} = (1 - \text{Ratio}\%)D_t^{b*}$, $\forall t, \forall b$. Then, for each time t , we let the budget $\bar{\pi}^t = (1 + \text{Budget}\%) \sum_{b \in B} D_t^{b*}$. Finally, we let the overall budget $\bar{\pi} = 0.9 \sum_{t=1}^T \sum_{b \in B} D_t^{b+}$. In our experiment, we allow the sensitivity analysis parameters Ratio% and Budget% to vary from 0 to 25%. In addition, we use a five-piece piecewise linear function to approximate the generation cost function.

We test the performance of our proposed approach under various Ratio% and Budget% settings. Note here that when Budget% > Ratio%, constraint (22) is redundant and accordingly the computational results for these cases are the same as the one in which Budget% = Ratio%. In this experiment, we set the sample size N to be 5, and report the optimal objective value, the number of start-ups, and the computational time for each setting in Table I.

From results shown in Table I, we can observe the following.

- 1) Given an identical Budget%, the objective value and the number of start-ups increase as Ratio% increases, because the problem becomes more conservative.

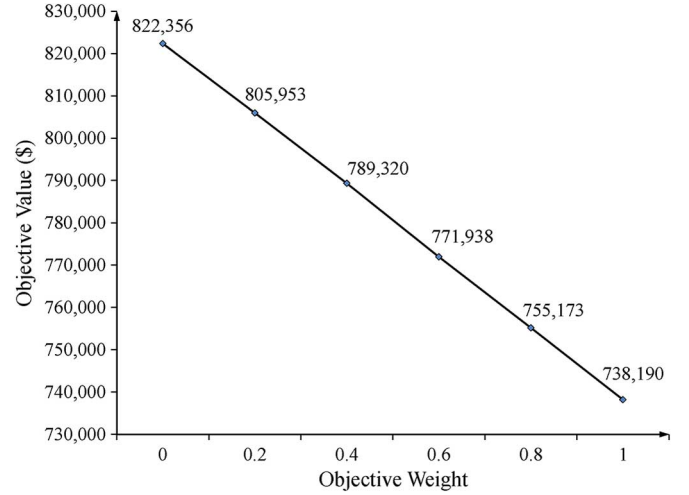


Fig. 2. Relationship between the objective value and the objective weight.

- 2) Similarly, given an identical Ratio%, the objective value and the number of start-ups increase as Budget% increases, because the system allows more load fluctuations.

2) *Sensitivity Analysis of Objective Weight α* : From the model described in Section II, it can be observed that as α increases, the optimal objective value decreases, because the problem becomes less conservative as the worst-case cost component has a smaller weight. In this subsection, we test the model under various objective weight α settings to show how significantly the objective value decreases as α increases. (Note here that if $\alpha = 0$, only the worst-case total cost is considered and if $\alpha = 1$, only the expected total cost is included.) We set Ratio% = 10% and Budget% = 10%. The computational results are shown in Fig. 2.

B. Proposed Approach Versus Stochastic Optimization Approach

Here, we set Ratio% = Budget% and compare the performances of our proposed approach with the traditional two-stage stochastic optimization approach under different Budget% and scenario-size settings. In this experiment, we do not include the worst-case generation cost in the objective function (i.e., $\alpha = 1$). The total costs corresponding to the traditional two-stage stochastic optimization approach (SO) and our proposed unified stochastic and robust optimization approach (SR) are obtained by the following two steps.

- Step 1) Solve the problem by using the SO and SR approaches, respectively, and obtain the corresponding first-stage unit commitment solutions.
- Step 2) Fix the unit commitment solutions obtained in step 1) and solve the second-stage problem repeatedly for 50 randomly generated instances to obtain the total cost for each approach.

To compare the performances between these two approaches, we introduce a penalty cost at the rate of \$5000/MWh [14], for any power imbalance or transmission capacity/ramp-rate limit violation.

We first test the performances of the SO and SR approaches under various Budget% settings. We set the scenario size to be

¹[Online]. Available: <http://motor.ece.iit.edu/data>

TABLE II
COMPARISON BETWEEN SO AND SR APPROACHES

Budget%	Model	T.C.(\$)	UC.C.(\$)	# of Start-ups	Time(sec)
3%	SO	737,799	49,500	10	62
	SR	737,799	49,500	10	50
5%	SO	738,112	49,500	10	59
	SR	738,112	49,500	10	47
10%	SO	740,878	49,500	10	62
	SR	739,911	52,500	11	126
15%	SO	752,575	49,500	10	63
	SR	741,170	51,000	11	167
20%	SO	782,361	49,500	10	63
	SR	742,866	54,000	12	222

5 and report the computational results in Table II. The Budget% scenarios are given in the first column. The total costs (T.C.) obtained by each approach are reported in the third column. The UC costs (UC.C.) for each approach are given in the fourth column and the numbers of start-ups are given in the fifth column. Finally, the CPU times are recorded in the sixth column.

From Table II, we have the following observations:

- 1) First, we observe that when Budget% $\leq 5\%$, the unit commitment decisions obtained from the SO and SR approaches are the same. This is because the uncertainty set is so small that the unit commitment decisions obtained from the SO approach are robust enough to accommodate the uncertainty, and there is no need to generate feasibility cuts in the SR approach. However, when Budget% $\geq 10\%$, the UC decisions obtained from the SO approach are not feasible to some simulated load scenarios. But the UC decisions obtained from the SR approach are always feasible under different budget levels. Therefore, due to the penalty cost for the violation in the power balance or transmission capacity constraints, the SR approach incurs a smaller total cost than the SO approach when Budget% $\geq 10\%$. Moreover, when Budget% increases, the total cost gap between these two approaches increases. This result verifies that the proposed approach can provide a more robust solution as compared to the SO approach, especially when the system has more uncertainties.
- 2) Second, we observe that, as compared to the SO approach, the SR approach requires more generators committed to provide sufficient generation capacity to guarantee the supply meeting the load. As a result, the SR approach has a larger UC cost than the SO approach. This result also verifies that the proposed SR approach can provide a more robust solution as compared to the SO approach.

Next, we test the system performances of the SO and SR approaches under various scenario-size settings. We set Budget% = 20% and test the scenario size $N = 1, 5, 10$, and 20, respectively. First, we observe that for the SO approach, the number of start-ups increases when the number of scenarios increases (e.g., the number of start-ups is 10 when $N = 1$ and this number increases to 11 when $N = 20$). On the other hand, in our proposed SR approach, the number of start-ups remains the same. Therefore, the proposed SR approach is more robust than the traditional SO approach. Second, we observe that as the scenario size increases, the total cost of the SO approach and the total cost gap between the SO and SR approaches

TABLE III
COMPARISON BETWEEN RO AND SR APPROACHES

Budget%	Model	T.C.(\$)	UC.C.(\$)	# of Start-ups	Time (sec)
3%	RO	737,294	49,500	10	375
	SR	737,275	49,500	10	49
5%	RO	738,514	51,000	11	292
	SR	737,275	49,500	10	48
10%	RO	739,515	54,000	12	375
	SR	738,190	52,500	11	127
15%	RO	739,868	54,000	12	339
	SR	738,506	51,000	11	168
20%	RO	749,064	63,300	15	303
	SR	739,320	54,000	12	223

decrease (e.g., the total cost of SO is \$782,361 and the total cost of SR is \$742,866 when $N = 1$; the total cost of SO is \$758,496 and the total cost of SR is \$742,866 when $N = 20$). This is because as the scenario size increases, the first-stage unit commitment solution obtained by the SO approach becomes more robust so that the system incurs less penalty cost in the second stage. However, there is still a big gap between the two when $N = 20$, which verifies that it is necessary to add the robust uncertainty set to ensure system robustness of the day-ahead unit commitment decision.

C. Proposed Approach Versus Robust Optimization Approach

We also compare the performances of our proposed SR approach with the traditional two-stage robust optimization (RO) approach under various Budget% settings. We set the scenario size for the SR approach to be 5 and summarize the computational results in Table III. The Budget% settings, the total costs, the UC costs, the numbers of start-ups, and the CPU times are reported in the first, third, fourth, fifth, and sixth columns, respectively.

First, from our computational results, we observe that there are no penalty costs incurred for both the SR and RO approaches, which means that the unit commitment decisions for each approach are feasible for all generated scenarios. Second, from Table III we observe that our proposed SR approach commits less number of units in the first stage than the RO approach. That is, as compared to the RO approach, SR leads to smaller unit commitment and total costs. This result indicates that our proposed SR approach can generate a less conservative solution as compared to the RO approach while maintaining system robustness. Finally, we observe that, the CPU times for the RO approach are larger than our proposed SR approach for each tested instance, because the initial solution for the RO approach is worse than that of our proposed SR approach, and more optimality cuts are generated to make the algorithm converge.

V. CONCLUSION

In this paper, we developed a unified stochastic and robust unit commitment model for ISOs/RTOs to perform reliability unit commitment runs, so as to achieve a robust and cost-effective unit commitment solution. Our proposed approach takes the advantages of both the stochastic and robust optimization approaches. The uncertainty set provided by the robust optimization approach ensures the robustness of the unit commitment de-

cision. Meanwhile, the expected total cost in the objective function provides system operators flexibility to adjust the cost-effectiveness of the proposed approach. The proposed model can be solved efficiently by our proposed Benders' decomposition framework, which includes both feasibility and optimality cuts. Finally, computational experiments show the effectiveness of our proposed approach.

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