Evolutionary Foundation of Bounded Rationality in a Financial Market

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***Abstract*—This paper provides a foundation for decision making with bounded rationality in economic entities from the viewpoint of evolutionary theory. To this end, first, we conducted an investment test with participants to extract a behavioral learning model for activities with bounded rationality. We found that the decision-making model obtained from this behavioral science approach has characteristics that are frequently seen in the results of observations of instances of bounded rationality. Furthermore, the model presents some well-known biases in decision making, such as profit-and-loss asymmetry in risk avoidance, reference point dependence, and the asset effect. Next, using agent-based simulations, we examined whether our behavioral-learning model for activities had the capacity to become a stable strategy in a market environment where selection pressure exists. When, in response to maximum loss, a drawdown is set as an evaluation criterion for selection, the results of our simulations imply the following: 1) our decision-making model with bounded rationality has the capacity to become a stable evolutionary strategy and 2) entities with bounded rationality can survive in a competitive market. These results are antithetical to the evolutionary explanations used as a basis for rationality in traditional economics, and they indicate the possibility that many well-known biases in decision making can be derived evolutionarily from a single criterion.**

***Index Terms*—Agent-based simulation, artificial market, evolution, experimental economies, sequential investment task.**

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Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

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[1]For example, Benartzi and Thaler [4] and Barberis *et al.* [5] proposed equilibrium models based on prospect theory and clarified that the loss aversion feature was effective in explaining certain stylized facts of a financial return, such as the equity premium puzzle and the volatility puzzle. Shimokawa *et al.* [6] showed that the price process, which is endogenously generated through their model with loss-averse traders, not only has consistencies with the equity premium puzzle and the volatility puzzle but also has great kurtosis, asymmetry of return distribution, autocorrelation of return volatility, and cross-correlation between return volatility and trading volume. We can point out this kind of rethinking and attempt to surpass traditional theory in various fields of finance (please see the studies summarized by Barberis and Thaler [7]).

I. Introduction

Recent developments in behavioral economics and neuroeconomics have forced a re-evaluation of the conventional concept of rationality used in economics (see, for example, [1]–[3]).[1]

The traditional economics model assumes that economic entities are involved in rational decision making; however, developments in the areas of behavioral economics and neuroeconomics have led to advancements in the modeling and identification of bounded rationality in humans by focusing on some of the biases in decision making. Further clarification of these economic behaviors is expected through analyses of studies in these areas. This paper will conduct an examination, from an evolutionary perspective, on whether the typical biases in decision making (or bounded rationality) reported in the studies related to these areas might be the result of economic activities in the marketplace.

According to the famous statement by Friedman [8], which has been frequently used as a basis for the validity of the rationality of economic entities (agents), if an economic entity is not rational, it will eventually lose out to the competition and withdraw from the market, which will lead to the remainder of the market behaving rationally. According to this Lamarckian concept, entities that follow rational decision making exist in the market as a result of the selection of economic entities by the market.

This paper employs an evolutionary perspective and investigates some of the reasons and probable lines for justifying bounded rationality.[2] Specifically, simulations of a market with evolutionary pressures are conducted to show that an agent with observed biases in decision making survives for a long term under market conditions that are close to reality, and that many of the biases in decision making that have been observed in the past can be derived from a single evaluation criterion for entry into, and withdrawal from, the market. In other words, this paper emphasizes that many of the observed biases in decision making have certain evolutionary rationalities. This

[2]The idea of bounded rationality using evolutionary computation was supported by Tsang [9]. It is possible to list several works of the literature in which subjects with bounded rationality survive by presuming some aspects of market imperfection, such as asymmetric information, limitations on trading, or myopic decision making. However, the uniqueness of this analysis lies in its examination of whether decision-making biases obtained from actual experiments using behavioral economics methods can be justified based on the market mechanisms instead of merely showing the possible presence of such biases.

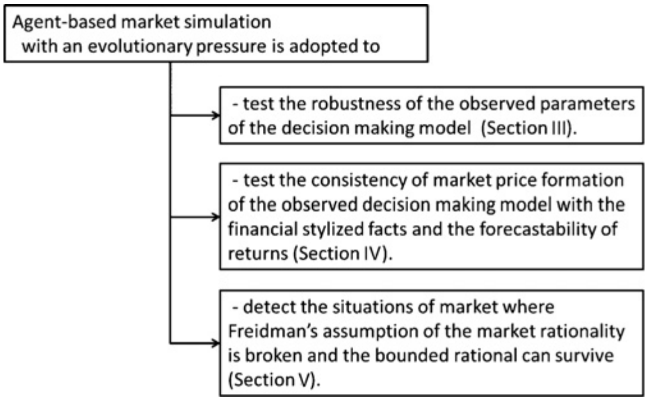

Fig. 1. Objectives of the agent-based simulation.

result contrasts with the basic concept of rationality established by Friedman [8] and relativizes the assumption of rationality conventionally used in economics.

First, we conducted a sequential investment task test (with 159 examinees between the ages of 18–32 years) [10]–[12] in order to describe the bounded rationality of financial markets.[3] Using these data as a basis, we performed a statistical calculation to create a behavioral-learning model from the responses of the examinees. The behavioral-learning model was created by developing reinforcement learning, reflecting the profit-and-loss asymmetry established by Bereby-Meyer and Erey [17]. Our model does not identify the influence of fictive error (addressed by Camerer and Ho [18] and Lohrenz *et al.* [10]), although it can be considered a typical learning model.

In fact, this model is consistent with many of the typical biases observed in decision making and has the following characteristics:

1) adjustable reference point (addressed by Erev and Roth [15] and Bereby-Meyer and Erev [17]);
2) asymmetry of reinforcement learning (related to evidence from neuroscience, e.g., [19] and [20], and prospect theory);
3) disposition effect (addressed by Shefrin and Statman [21] and Odean [22]);
4) market events: deviation, trend reversal, and volatility change (addressed by Lo and Repin [23]).

Next, we conducted an agent-based market simulation [6], [24]–[28]. Our objective is to show whether a human decision-making process derived from the most typical behavioral finance methods or models can be justified based on evolutionary viewpoints, such as market selection (Fig. 1). Therefore, the research conducted performs experiments first, and then specifies the decision-making model using methods of behavioral finance. The evolutionary simulation is adopted not to derive the decision-making model but to justify whether it is valid based on the market selection viewpoints.[4]

A standard equilibrium model in financial theory by Grossman and Stiglitz [29] was used as the market model on the assumption of the existence of rational entities presumed in economics, and of entities with bounded rationality whose decision-making models are based on the behavioral-learning theory observed in the sequential investment task above.[5] We used an equilibrium model that has typically been adopted in the field of agent-based simulation [30]; in addition, we used evolutionary selection pressures that have been a standard for genetic algorithms.

As described above, the results of this simulation affirm decision making with bounded rationality and differ from results obtained by conventional agent-based simulation studies. For example, Chen *et al.* [24] showed that the market becomes effective by adopting the mean absolute prediction error (MAPE) as the fitness function in an artificial market analysis while using genetic programming. The reason our result differs from the conventional result is that our simulation accounts for a drawdown, which has become analogous with the standards for entry into, and withdrawal from, the actual market. This research contributes to financial theory by showing that some of the well-known biases in decision making have certain rationalities that are evident from an evolutionary perspective under this criterion. This paper bridges two important research fields: behavioral economics and agent-based computational economics.

The remainder of this paper is composed as follows. Section II specifies the features of bounded rationality using an experimental approach. We conducted a behavioral investment experiment with 159 subjects to identify their decision rule or biases. Section III shows the robustness of the observed parameters of the bounded rational model in the previous experiment (Section II) from the view of the agent-based market simulation with evolutionary pressure. Section IV indicates the consistency of the agent-based market simulation with evolutionary pressure with the stylized facts of a financial time series and the forecastability of returns. Section V reveals market situations where Friedman's assumption of market rationality is broken by using the agent-based market simulation. Section VI discusses the validity of the evolutionary approach in a market setting and the limitations of our analysis. The final section provides conclusions.

II. Modeling Bounded Rationality

A. Experiment

First, we conducted a behavioral investment experiment to identify the biases that people have in a decision-making activity in order to describe decision making with bounded ra-

[3]A great deal of research has been done on behavioral-learning theory, to analyze how an entity selects its behavior on the basis of information obtained from the environment (see [13] for a good survey on this topic). The reinforcement learning models created by Sutton and Barto [14], Erev and Roth [15], [16], and Bereby-Meyer [17], along with the experience-weighted attraction learning models by Camerer and Ho [18] (which are a hybrid of reinforcement learning and hypothetical behavior), have their own merits and weaknesses.

[4]The reader should keep in mind that the purpose of performing the evolutionary simulations was not to derive decision-making models, but to evaluate whether they are valid based on the perspectives of the market selection process. Therefore, we do not use the evolutionary algorithm to select decision-making models.

[5]A behavioral bias is not akin to bounded rationality. However, we believe that facts observed as a behavioral bias can provide us with important guidelines for modeling bounded rationality in decision making.

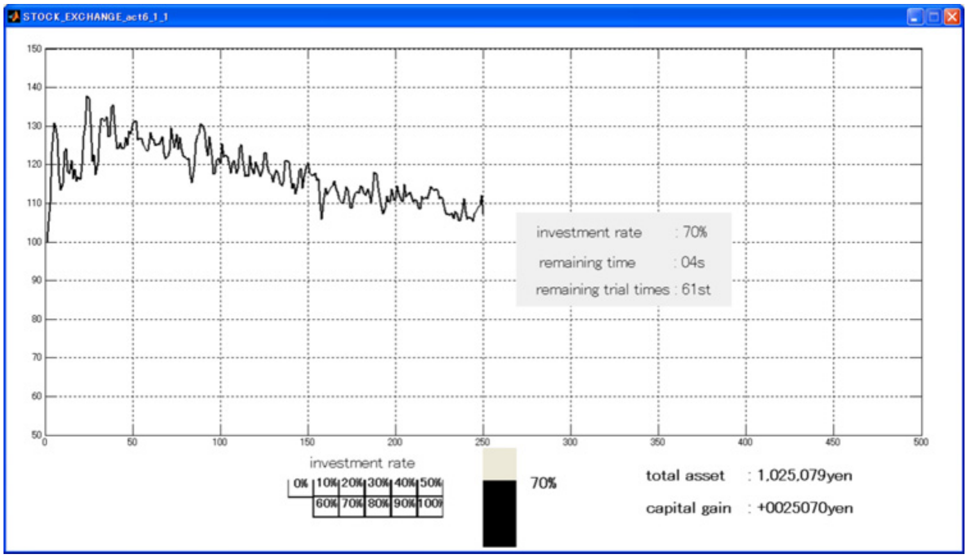

Fig. 2. Price history, current investment rate, time remaining for the next price update, and the remaining number of price updates are displayed in the middle of the screen. When the simulation is started, the price data for 225 periods is displayed, and then the subject makes a decision. At the bottom right of the screen, the current asset price and the gain up until now are displayed. In this figure, the subject has assets worth 1 025 070 yen, has just won 25 070 yen, and has 70% of his assets invested in the market. In order to change the investment rate, the subject either pushes the up or down arrow keys on the keyboard (the up key for increasing the rate and the down key for decreasing it), or clicks the "investment rate" button on the bottom left of the screen display. The slider bar at the bottom middle of the screen indicates the current investment rate. The investment rate moves in increments of 10%.

tionality. We conducted sequential investment task experiments by referring to [10]. Many studies have been conducted to describe bounded rationality according to behavioral-learning theory [13]. This paper is also based on the standard model in this field. The details of the experiment are as follows.

The study used 159 subjects (95 males and 64 females, aged 18–32) in experimental sessions conducted at the Tokyo University of Science, Tokyo, Japan. All subjects were amateur investors with limited trading experience; professional traders were not included. Specifically, 24 subjects had investment experience (19 males, 5 females). Thirteen subjects had investment experience of one year or less, 10 had between one and two years of experience, and 1 had over two years but less than five years. The investment amounts for experienced participants averaged about 825 000 yen, and the highest amount was 3 500 000 yen. Each experimental session lasted approximately 90 min, including instruction time. As a reward, subjects were offered 1500 yen (about 12 U.S. dollars according to the exchange rate at the time of the experiment) as a payment for participating, and this was adjusted ±1500 yen in accordance with earnings. Each subject was assigned a computer terminal.

Our experiment is similar to that of Lohrenz *et al.* [10]; however, there are some differences between their design and ours that are described in Appendix A. Our experimental setting is as follows. All traders started out with 1 000 000 yen. The traders allocated their funds between stocks and risk-free assets (i.e., deposits). They chose their investments for stocks or the allocation rate of their assets, which they could change throughout the experiment. A certain amount of money was calculated according to their investment rate and invested in stocks, and the remaining assets were invested in deposits. We set the deposit rate at 0%.[6] The stock allocation changed depending on fluctuations in stock prices, which were automatically updated every 10 s for 75 iterations. Fig. 2 shows a typical screen display. When we ran the simulator, the price data for 225 periods were displayed, after which the subject made a decision. In Fig. 2, the subject has assets worth 1 025 070 yen, has just gained 25 070 yen, and has 70% of his assets invested in the market. Each subject performed this experiment twice with different price data each time. In our experiment, there were ten price data points that constitute the actual price history. The summary statistics for the price data, method used to assign the price data to the subjects, and other relevant information are summarized in Appendix A.

B. Model

The most suitable learning model related to the subjects' investment behavior was selected from 22 candidate models constructed using six factors considered to have a crucial impact on decision making under uncertainty using the Akaike information criterion (AIC) and the Bayesian information criterion (BIC)[7] (Fig. 3). The following studies on behavioral finance provide validation and interpretations of characteristic features of the candidate models:

1) rational expectation (adopted in traditional financial theory);
2) adjustable reference point (addressed by Erev and Roth [16] and Bereby-Meyer and Erev [17]);
3) fictive error or fictitious investment (addressed by Camerer and Ho [18] and Lohrenz *et al.* [10]);

[6]Stock price data supplied in the experiment were selected from data observed over the past six years in the Tokyo Stock Exchange. A deposit rate of 0% was used because the Japanese market has had a zero interest rate since 2001.

[7]The statistical significance of the difference between the AIC values was determined using the Kishino–Hasegawa (KH) test [31]–[33]. The bootstrap method was used to calculate the statistics of the KH test. The hypothesis that the AIC value of our model is the same as in other models was rejected with a 99% confidence level.

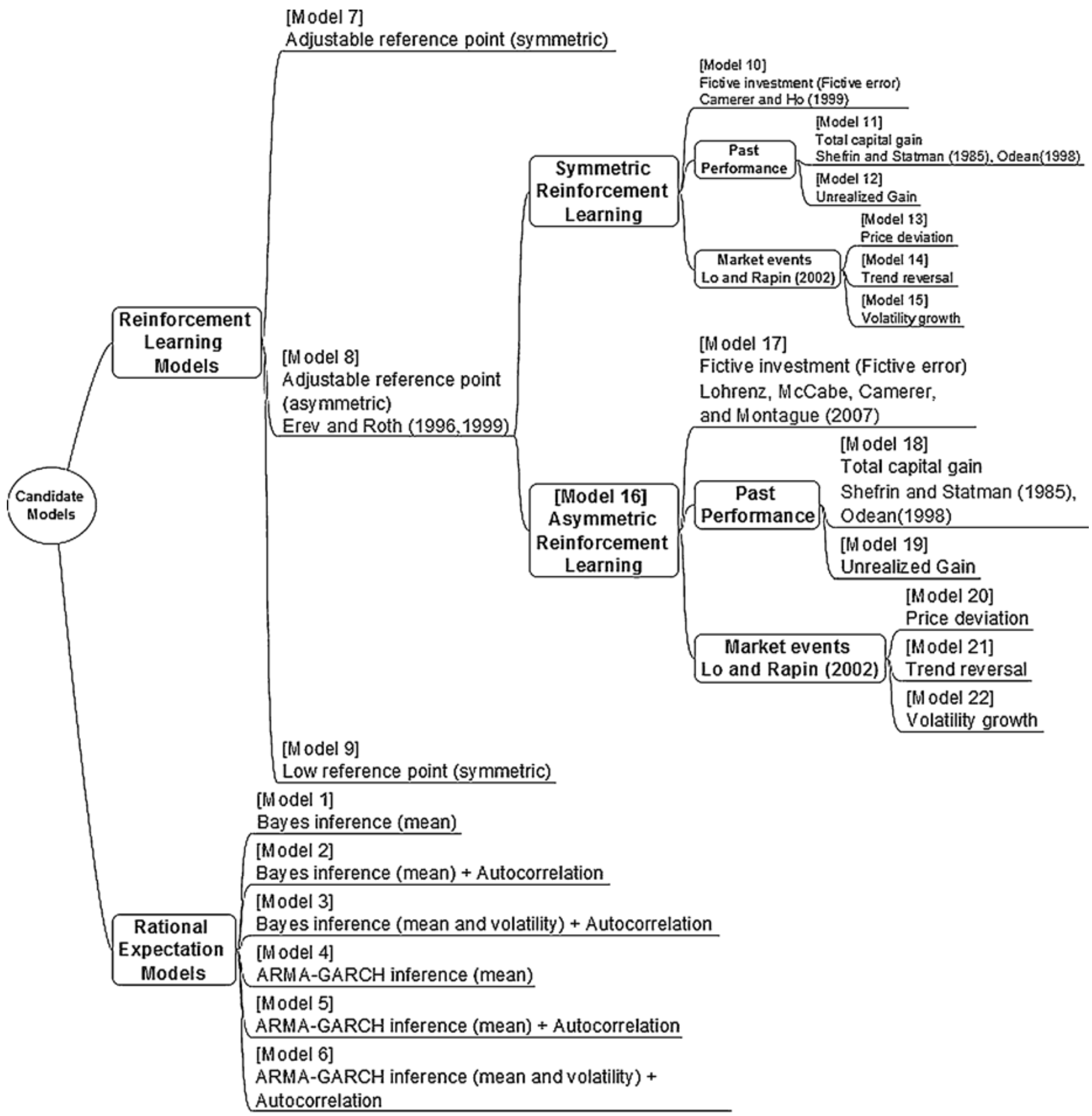

Fig. 3. Candidate models.

4) asymmetry of reinforcement learning (related to the evidence from neuroscience in [19] and [20], and prospect theory);
5) disposition effect (addressed by Shefrin and Statman [21] and Odean [22]);
6) market events such as deviation, trend reversal, and volatility change (addressed by Lo and Repin [23]).

This paper arranged as many models as possible, all of which are considered to be in the field of behavioral economics, to specify the most consistent decision-making model with actual observation data. Therefore, each model should be justified based upon the studies conducted on behavioral economics to date.

We calibrated the parameter values of the candidate models using the maximum-likelihood method with the assumption of a prediction residual having normal distribution. In this subsection, we will only examine the model with the best fit. The other 21 models are introduced in Appendix B. The model was selected using AIC and BIC. The investment rate at period t is denoted by $i\,(t)$, the market price by $p\,(t)$, market return by $r\,(t)$, and invested assets by $S\,(t)$

$$\begin{aligned} i(t) = {} & \phi + \rho \cdot i(t-1) + \delta^{\pm} \cdot (r(t-1) - \text{r.p.}(t-1)) \cdot S(t-1) \\ & + \omega^{\pm} \cdot \text{unrealized gain}(t-1) + \varepsilon \end{aligned} \tag{1}$$

where the reference point, r.p.(t), is defined as

$$\text{r.p.}(t) = (1-\gamma^{\pm}) \cdot \text{r.p.}(t-1) + \gamma^{\pm} \cdot r(t-1) \tag{2}$$

$$\delta^{\pm} = \begin{cases} \delta^{+}, & \text{if } r(t-1) \geq \text{r.p.}(t-1) \\ \delta^{-}, & \text{if } r(t-1) < \text{r.p.}(t-1) \end{cases} \tag{3}$$

$$\omega^{\pm} = \begin{cases} \omega^{+}, & \text{if unrealized gain}(t-1) \geq 0 \\ \omega^{-}, & \text{if unrealized gain}(t-1) < 0 \end{cases} \tag{4}$$

$$\gamma^{\pm} = \begin{cases} \gamma^{+}, & \text{if } r(t-1) \geq 0 \\ \gamma^{-}, & \text{if } r(t-1) < 0 \end{cases} \tag{5}$$

TABLE I
VALUES OF THE PARAMETERS IN THE MOST SUITABLE MODEL

ϕ	ρ	δ^+	δ^-
0.1298	0.6687	1.3862	2.9567
ω^+	ω^-	γ^+	γ^-
−0.0994	−1.4109	0.0822	0.0097

We can observe the following properties: 1) the adjustment of the reference point to the positive return is faster and later than the negative return ($\gamma^+ > \gamma^-$); 2) the reinforcement learning from the excess return, defined as the previous return minus the reference point of return, is highly sensitive to the excess loss ($\delta^+ < \delta^-$); and 3) the disposition effect is relatively strong and has gain-loss asymmetry ($|\omega^+| < |\omega^-|$).

TABLE II
ESTIMATED VALUE AND THE ESTIMATED ERROR (IN PARENTHESES) FOR EACH PARAMETER VALUE, WHICH IS CALCULATED USING THE BOOTSTRAP METHOD WITH 1000 TIMES RESAMPLING

	ϕ	ρ	δ^+	δ^-
Mean Standard deviation	0.1338 (0.0153)	0.665 (0.0324)	1.5941 (0.5218)	2.8808 (0.6114)
	ω^+	ω^-	γ^+	γ^-
Mean Standard deviation	−0.2461 (0.2905)	−1.3016 (0.2746)	0.1279 (0.0609)	0.0261 (0.0641)

Statistically, γ^- and ω^+ cannot be distinguished from 0.

and the unrealized gain rate at period t is calculated as

$$\text{unrealized gain}(t) = \left(\frac{p(t) - \text{purchase price}(t)}{\text{purchase price(t)}}\right)\frac{i(t)\cdot \text{asset}(t)}{\text{asset}(0)} \tag{6}$$

where the term asset(0) is used as the normalized factor and the purchase price is composed of the average price.[8]

In an intuitive sense, the model is a typical reinforcement learning model that additionally takes the asymmetry of a reference point and the disposition effect into consideration. In (1), the second term is an autocorrelation term for investment and the third term is a reinforcement learning term. With the reinforcement learning term, the model learns that if the return on investment (ROI) received in the current period is greater than the reference ROI, then the investment rate for the coming period increases. In decision making, the rate of return that serves as a reference point for profit and loss is determined by weighted moving averages according to (2). The fourth term represents the disposition effect; that is, the investment rate concerning investment assets decreases with greater "unrealized" profit (indicating that profits are locked in). Conversely, investment tends to continue with greater "unrealized" loss (indicating the inability to cut losses). Given (6), "unrealized" profit and loss are calculated as the difference between the purchase price and the current price. In addition, the superscript "" for individual parameters represents asymmetrical learning in the areas of profit and loss. As Bereby-Meyer and Erev [17] noted, the asymmetry of such learning substantially increases a model's descriptiveness.[9]

C. Characteristic Features of the Model

Table I shows the calibrated parameter values of the model generated by applying the maximum likelihood method to ten sample paths. Table II provides the estimated value and estimated error (in parentheses) for each parameter value, which is calculated using 1000 bootstrap resampling iterations. The statistical significance of each parameter estimation can also be confirmed using this estimation. These tables suggest that the features of the decision-making model show the following characteristics: the adjustable reference point, the asymmetry of reinforcement learning, and the disposition effect.

1) The autocorrelation of decision making can be confirmed by the high value of ρ.
2) Reinforcement learning from the excess return, defined as the previous return minus the reference point of return, is highly sensitive to the excess loss ($\delta^+ < \delta^-$).
3) The disposition effect is relatively strong in the loss domain and relatively small in the gain domain. There is gain-loss asymmetry ($|\omega^+| < |\omega^-|$).
4) The adjustment of the reference point to the positive return is faster and later than the adjustment to the negative return ($\gamma^+ > \gamma^-$). Statistically, γ^- and ω^+ cannot be distinguished from 0.

Some of these characteristics are consistent with the results of the psychological experiments on traders carried out by using Kahneman and Tversky's [1] value function in prospect theory and by Lo and Repin [23]. Furthermore, these characteristics explain the over-reaction and under-reaction often reported in price formation in the market [3], [36]–[40], a phenomenon referred to as the disposition effect [41], [42]. Concerning the characteristics of market return sequences, the following features have been confirmed in the relationship between financial stylized facts and the decision biases of the bounded rational traders:

1) the return does not follow a normal distribution and has great kurtosis;
2) there exists an autocorrelation of the volatility of return distribution;
3) the forecastability of returns exists.

Section IV shows these relationships.

III. EVOLUTIONARY FOUNDATION

A. Evolutionary Artificial Market Simulation

We conducted an agent-based simulation using an artificial market with selection pressures to discover the conditions capable of introducing an evolutionary basis into the bounded rationality model (obtained from the observation above) and to determine whether the model can become a stable market

[8]In order to confirm the robustness of the model, we conducted the overfitting test and a χ^2 test. We conducted the over-fitting test as follows: 1) we separated the samples into two groups: in-sample and out-of-sample, and then we used the in-sample data to calibrate the parameter values with maximum log likelihood; and 2) we compared the mean square deviation of the expected values with that of the actual values for investment behavior within the out-of-sample range of each model. As a result, our model achieved the highest prediction accuracy among all models. We also conducted a χ^2 test, the result of which verified our conclusion that the model we have chosen best fits the investment behaviors of the examinees.

[9]We can point out some attempts to refine learning models from the viewpoint of pattern recognition. For example, Shimokawa *et al.* [12] intended to reform the ordinal reinforcement learning model in order to take into account nonlinear decision making by using a Bayesian three-layer perceptron. Kampouridis *et al.* added evolutionary decision-making to the market fraction hypothesis [34] and the dinosaur hypothesis [35]. They implemented reinforcement learning with genetic programming.

strategy. Evolutionary approaches to economic and financial modeling encourage a re-examination of "rationality," as it is assumed in traditional economic and financial models; furthermore, they have helped to describe "the learning process of actors in a market." For example, Chen and Yeh [43], [44] used genetic programming to study traders with rational expectations and the attainment of market efficiency. They also modeled how market participants were retrained using strategies from a strategic pool in what they termed a "business school" and proposed a more realistic learning process [24]. In addition, Markose *et al.* [45], Robson [46], [47], and Jaramillo and Tsang [25] indicated that the "Red Queen principle," which states that one must continue learning in order to retain one's current position in a market, is the result of coevolution in the market.[10]

The design of the artificial market was in accordance with a financial market model [6], [26], [28] with typical heterogeneous trader agents, as in [29]. We assumed that rational traders and bounded rational traders exist in this artificial financial market and that the bounded rational traders make their decisions according to the behavioral-learning model observed in the previous section. The modeling of the rational traders in our study is the same as in [29], which is typically used in traditional economic models. Specifically, the current artificial market has the following features.

1) *Preference*: Rational agents feature a constant absolute risk aversion (CARA) utility function and determine an investment strategy for each period in order to maximize their investments over the long term. In contrast, bounded rational agents feature a decision-making scheme (1)–(6), as was actually observed (and described in Section II-B). Parameters for this scheme are learned coevolutionally.
2) *Price determination*: Market prices for each period are determined by the supply and demand of market participants meeting (the market-clearing condition). A Walrasian price adjustment process is not assumed exogenously.
3) *Information representation*: In accordance with [29], information concerning the true value of assets is provided along with noise to rational agents. This information is communicated to market participants via market prices.
4) *Social learning*: Individual parameters for the decision-making scheme (1) are learned using a genetic algorithm that causes the bounded rational agents to attempt to reduce their own losses. This learning affects market price formation and consequently affects the learning of other actors. Such social learning occurs coevolutionally.

We conducted simulations for markets with selection under the following procedures:

Step 1) initialization;
Step 2) transaction in the market for a certain period;
Step 3) evaluation of agents according to transaction results in the market;
Step 4) manipulation according to evolutionary theory (selection, crossover, and mutation);
Step 5) repetition of Steps 2–4 G times (here, 1000 generations).

The details of these steps are explained as follows.

Step 1: This market has 100 traders in every generation. The traders are divided into rational traders and bounded rational traders at a ratio of 3:7 (this condition is eased later). The chromosomes of the bounded rational traders are set to the parameter values of the model $(\phi, \rho, \delta^+, \delta^-, \omega^+, \omega^-, \gamma^+, \gamma^-)$. To examine whether the model obtained from the experiment is valid, the initial parameters are given in a normal distribution, with a mean calibrated from the experiment results shown in Table I and a variance of 1.

Step 2: Trader agents in each generation conduct transactions in the artificial market according to their own decision-making model for a certain period (here, 50 periods[11]), and the prosperity of the next generation is evaluated on the basis of their transaction results.[12] The details of the artificial market, the demand function for each type of trader, and the pricing model are described as follows.

a) *Design of the artificial market*: The basic economic structure of this market simulation draws on the traditional rational expectation model in which heterogeneous agents have different information with CARA utility functions [29].

We consider a finite horizon (N-period) model. Two assets (risky assets and risk-free assets) exist in this economy. For simplicity, the dividend rate and the risk-free rate r_f are assumed to be 0. The fundamental value of the risky assets, denoted by v, is settled at period 0 and cleared with the value in period $N + 1$. The value v is determined according to a normal distribution $N(\bar{v}, 1/\tau_v)$. All the traders cannot directly observe value v; they infer the fundamental value from the available information, past prices, and private signals at each period. Following the tradition of heterogeneous agent models, our model is assumed to have rational traders (informed traders) and bounded rational traders who use our learning model (1) to determine investments.

In each period, some agents are selected from trader groups to participate in the trade market. To simplify the analysis, we suppose that the number of market participants in both trader groups is constant in every period. We denote the number of rational and irrational traders as P^R and P^B, respectively. Each trader is assumed to

[10]Our study does not aim to re-evaluate rationality, but instead aims to examine whether decision-making biases obtained from actual experiments, using behavioral economics methods, can be justified from market mechanism perspectives.

[11]The evaluation period (50 periods) is determined *ad hoc*. There is no corroboration for this criterion based on observations, but the effects of relaxing this assumption are shown in Fig. 12. Generally, decision-making bias is more apparent when the criteria for evaluation are more short-term (i.e., investors make decisions more myopically).

[12]Here, the market price is determined endogenously. This means that the strategies of all the traders have mutual effects on their performances. Coevolution represents the process wherein the strategies of each agent have an influence on the others; consequently, there is a change in their decisions on issues such as fitness and the adaptive landscape. Kauffman [48] pointed out that coevolution applies to economic literature. Our simulation is also a process of coevolution.

be selected and to have participated in trading only once in each period, although he or she can observe the past price sequence of the risky assets, and hold the traded assets for $N + 1$ periods. This setting is similar to the short-term investment model in [49], in which investors myopically make decisions, although they have a long memory. The timing of the participation in the trade market is exogenously given; a trader's strategic decision about the timing of participation is not considered in this paper. Let the rational traders have demand functions corresponding to their limit prices. The price in each period is determined such that demand and supply can be matched.

b) *Modeling of the rational traders*: In any period t, the rational traders predict the values of their assets using private signals and the past price sequence $P^{t-1} = (p(0), p(1), ..., p(t-1))$, and then they determine their demand so as to maximize expected utility, given a limit order price. Traders offer a demand function corresponding to each limit price. The rational traders receive different signals. The private signals that trader i receives in period t are expressed as $s_i(t) = v + \epsilon_s(t)$, where $\epsilon_s(t)$ represents the white noise of a signal following the normal distribution $N(0, 1/\tau_\epsilon)$. When new information is gained, the traders update their predictions with respect to the asset value v using Bayes's rule. Since it is assumed that the investors trade in the market only once, their decision becomes similar to a static optimization problem. More precisely, as shown below, the expected value of the exponential utility function is maximized under the usable information in the current period. The initial wealth possessed at t by a rational trader is represented as $W_i(t)$, the value of the risk-free assets held at t is represented as $S_i(t)$, and the value of the risky assets held at t is represented as $x_i^R(t)$. The rational trader's optimization problem is

$$\begin{aligned} &\underset{x_i^R(t)}{Max}\ E\{-\exp[-\eta W_i(t)] \,|\, s_i(t), P^{t-1}\} \\ &s.t. \quad W_i(t) = S_i(t) + P^0(t)x_i(t) \end{aligned} \tag{7}$$

where $P^0(t)$ is the limit price given at the time of decision making and η is the degree of risk aversion. Under the assumption that the risk-free rate is 0, the term-end assets are given as

$$\begin{aligned} W_i(t) &= (1 + r_f)^{N-t} S_i(t) + v x_i(t) \\ &= S_i(t) + v x_i(t) \end{aligned} \tag{8}$$

since risky assets should be liquidated by the fundamental value v. To solve this problem, we first rewrite the trader's demand $x_i^R(t)$ given a limit price $P^0(t)$ at t as

$$x_i^R(t) = \frac{E\{v \,|\, s_i(t), P^{t-1}\} - P^0(t)}{\eta Var\{v \,|\, s_i(t), P^{t-1}\}} \tag{9}$$

where $E\{v \,|\, s_i(t), P^{t-1}\}$ represents the conditional expectation of v by trader i in period t, and $Var\{v \,|\, s_i(t), P^{t-1}\}$ represents the conditional variance.[13]

[13]We referenced the introduction of [50] to derive the demand function.

c) *Modeling of the bounded rational traders*: The bounded rational traders are assumed to have observable decision biases in our experiment, i.e., their investment decisions exhibit tendencies such as adjusted reference points, asymmetric response, and disposition effects. The bounded rational traders are assumed to make decisions according to our model given in Section II-B. Simply put, the bounded rational traders are assumed to be homogeneous and their holding assets are normalized as 1. Given that the investment rate is $i(t)$, the bounded rational traders' demand is

$$x_i^B(t) = i(t) \tag{10}$$

where i(t), which is the behavior model obtained in Section II-B, is defined as

$$\begin{aligned} i(t) = \quad & \phi + \rho \cdot i(t-1) + \delta^{\pm} \cdot (r(t-1) \\ & -\text{r.p.}(t-1)) \cdot S(t-1) \\ & +\omega^{\pm} \cdot \text{unrealized gain}(t-1) + \varepsilon \end{aligned} \tag{11}$$

where r.p.(t), which is the reference point, is defined as

$$\text{r.p.}(t) = (1-\gamma^{\pm}) \cdot \text{r.p.}(t-1) + \gamma^{\pm} \cdot r(t-1). \tag{12}$$

The unrealized gain rate at period t is calculated as

$$\begin{aligned} &\text{unrealized gain}(t) = \\ &\left(\frac{p(t) - \text{purchase price}(t)}{\text{purchase}(t)}\right) \frac{i(t) \cdot \text{asset}(t)}{\text{asset}(0)}. \end{aligned} \tag{13}$$

Since the rational traders are informationally superior to the bounded rational traders and our model is supposed to be common knowledge, the bounded rational traders' decision does not create informational problems for the rational traders' decision problems.

d) *Endogenous determination of the equilibrium price by the market*: The equilibrium price is determined at the point where the supply of assets matches demand. Hence, the equilibrium price is determined in period t so as to satisfy

$$\sum_i^{P^R} x_i^R(t)(p(t)) + \sum_i^{P^B} x_i^B(t) = 0. \tag{14}$$

Proposition:
Under these circumstances, the equilibrium price is determined as follows:

$$p(t) = \frac{\tau_\varepsilon \tilde{s}(t) + \tau_v \bar{v} + \tau_\varepsilon P^R \sum_{j=1}^{t-1} \tilde{s}(j)}{\tau_t} + \left(\frac{P^B}{P^R}\right) \frac{i(t)}{\tau_t} \eta \tag{15}$$

where $\tau_t = \tau_\epsilon + \tau_v + (t-1)P^R \tau_\epsilon$ and $\tilde{s}(t) = \sum_i^{P^R} \frac{s(t)}{P^R}$.

Please see Appendix C or refer to [6] and [11] for a verification of this equilibrium price formula.

In the following simulations, the parameter values $\bar{v} = 100$, $\tau_v = 0.01$, $\tau_\epsilon = 0.004$, and $\eta = 2$ are adopted.

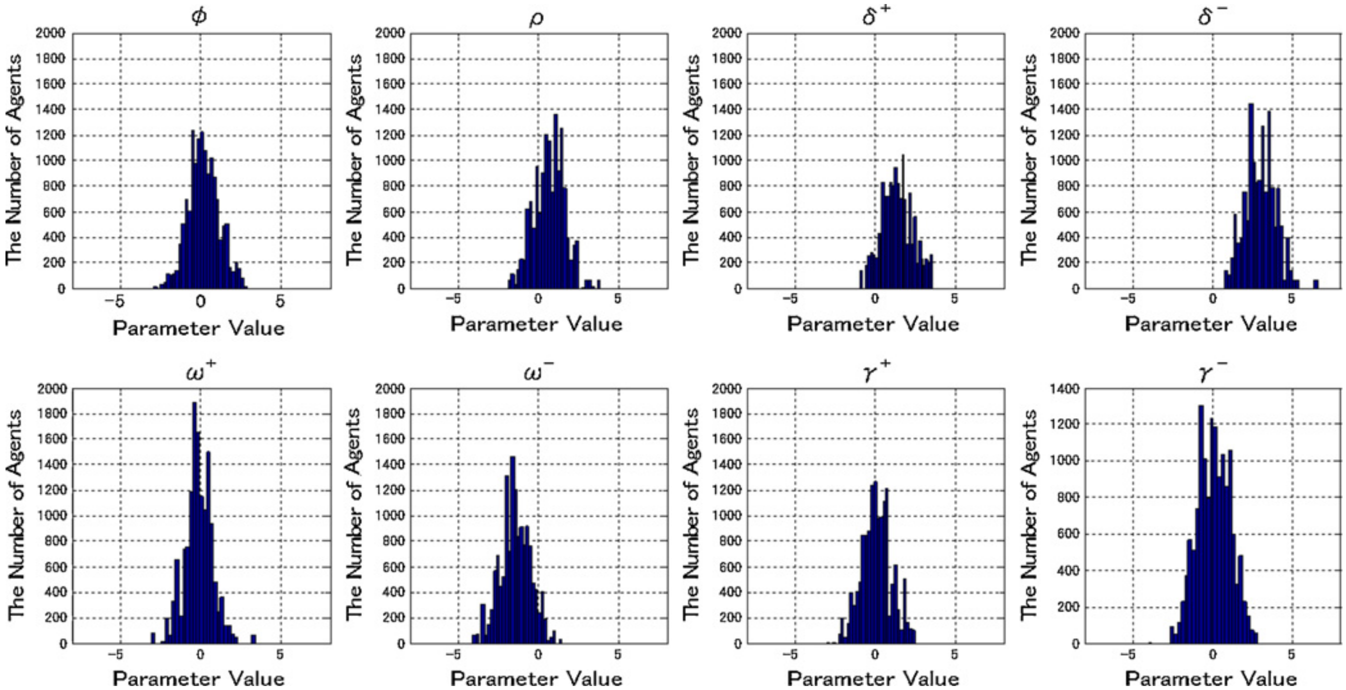

Fig. 4. Main result: histogram of the parameter values of the agents in the last generation.

Step 3: As a result of the market transactions described above, the value of the assets possessed by each trader is endogenously determined.

To characterize each bounded rational agent, the chromosome randomly assigned in Step 1, which is composed of the parameter values of the investment function, is used. To assess a trader's fitness, the performance of the trader in the artificial market defined in Step 2 during the evaluation period is adopted. We use a drawdown as a function for selection. Here, the drawdown implies the minimum amount of assets held (i.e., the maximum amount of loss) within the evaluation period. This evaluation standard forces a trader to withdraw from the market once he or she makes a huge loss, despite the large profits he or she may have made in the past. For example, general traders are forced to withdraw from the market under their own credit restrictions if their loss exceeds a certain amount, while professional traders usually have limits on the loss allowable over a certain period of time. The drawdown has become analogous with the standard for entry into and withdrawal from the actual market.[14] Moreover, the drawdown represents a situation that our evolutionary analogy defines as a fatal situation. The use of this kind of evaluation condition is a feature of this paper. As we demonstrate later, this evaluation method can define many of the biases in decision making, explaining them as results of evolutionary selection.

Step 4: Following the evaluation, the next generation (offspring) is generated by applying three operators: selection, two-point crossover, and mutation.

We adopt an elite selection strategy, in which genes ranked in the top 70% (we refer to this factor as the "eselection rate") of the evaluation are selected for the next generation. This selection strategy implies that the trader whose drawdown is very large should leave the market and it provides a natural analogy for the selection pressure in the real market. Professional traders would be selected using this kind of evaluation in the real world. The crossover rate is set to 0.15 and the mutation rate is set to 0.01.

[14] De Long *et al.* [51] and Shleifer and Vishny [52] stated that more realistic evaluation standards than those set above can block arbitrage and consequently affect the distortions of the market price.

B. Results of the Simulation

1) *Primary Result:* We conducted a simulation of artificial markets according to the parameter values in Table III. Fig. 4 shows the distribution of each chromosome of the bounded rational traders in the last generation of the simulations. The horizontal axis indicates the parameter values and the vertical axis shows the corresponding number of agents in each histogram; the total number of agents was 14 000 (100×0.7 agents $\times$ 200 simulations).

Table IV shows the average and deviation of each histogram. By comparing the results of actual trading experiments in Section II-B with these results, it is clear that all the characteristics match. Therefore, the decision-making model of (1) can be represented as a stable strategy endogenously derived from the market simulations, and the bounded rationality obtained from the experiments can be said to have certain kinds of evolutionary rationality.

This is because the use of a drawdown in the evaluation rules promotes the survival of traders with decision-making biases. Such biases are represented by the following three characteristics of (1):

1) $\delta^+ < \delta^-$;
2) $|\omega^+| < |\omega^-|$;
3) $\gamma^+ > \gamma^-$.

First, $\delta^+ < \delta^-$ indicates the tendency to reduce investment to avoid risk when the price falls below a certain reference point. More specifically, the first characteristic shows the existence

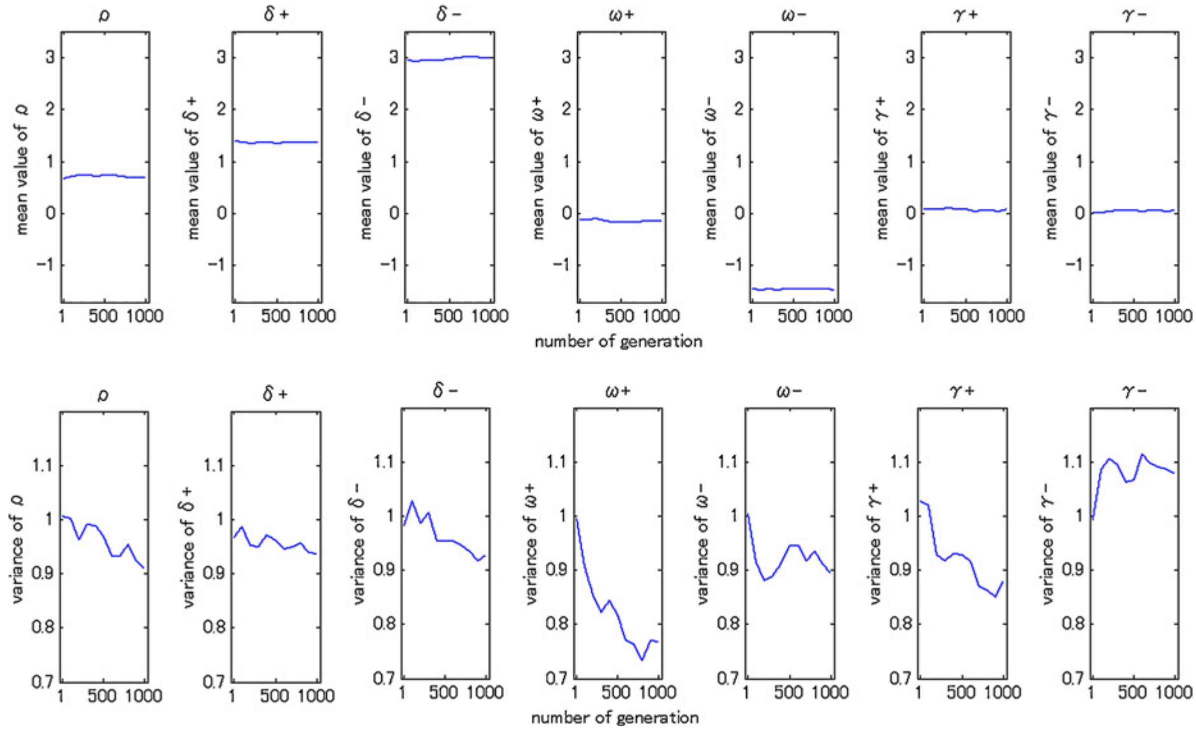

Fig. 5. Independence on the initial distribution. These figures show the relation between the number of generations and the parameter values. The upper row represents the mean estimate values of parameters, and the lower row shows the variance estimate values of parameters.

TABLE III

Parameters Used in the Above Simulation

Name of the Parameter	Explanation	Value
S	Number of simulations	200
G	Number of generations	1000
N	Number of agents	100
P^B/N	Ratio of the bounded rational traders	0.7
T	Evaluation period	50
$Prob_s$	Selection probability (rate)	0.7
$Prob_c$	Crossover probability (rate)	0.15
$Prob_m$	Mutation probability (rate)	0.01
$Asset(1)$	Initial asset value	1
$\bar{v}$	Average of the true fundamental values of the risky assets	100
τ_v	Accuracy of the true fundamental values of the risky assets	0.01
τ_ϵ	Accuracy of private signal noise	0.004
η	Degree of risk aversion	2
ϕ, ρ, δ^+, δ^-, ω^+, ω^-, γ^+, γ^-	Initial parameter values	Normal distribution of Distribution 1 whose average is set to the values in Table I

TABLE IV

Results of the Simulations: The Average and the Distributions of the Parameter Values

	ϕ	ρ	δ^+	δ^-	ω^+	ω^-	γ^+	γ^-
Mean	0.1774	0.6913	1.3630	2.9963	−0.1416	−1.4529	0.0806	0.0645
Standard deviation	0.9432	0.9529	0.9680	0.9638	0.8766	0.9459	0.9382	1.1038

of asymmetry in the risk aversion measurements in profits and losses, on the basis of the reference point in addition to the dependence on the reference point. The reason these characteristics are derived under the evaluation criterion using a drawdown is because the biases in decision making help decrease the maximum loss through excessive risk avoidance behavior in the area where loss is generated.

Next, $\gamma^+ > \gamma^-$, which is the reference point factor, represents the tendency for the quick adjustment of reference points in a period of rising prices. Furthermore, it represents the slow adjustment of reference points in a period of falling prices. Tversky and Kahneman [1] reported that subjectivity to the value of the gain was 2–2.5 times greater than subjectivity to the value of the loss. Furthermore, in our simulation, the sensitivity (which is represented by the parameter delta) to a loss is approximately 2.2 times larger than the sensitivity to a gain. As shown in $\delta^+ < \delta^-$ in (1), the quick adjustment of reference points in a period of rising prices immediately reduces the influence of the price increase on the investment ratio, while the slow adjustment of reference points in a period of falling prices results in the continued influence of the price decline on the investment

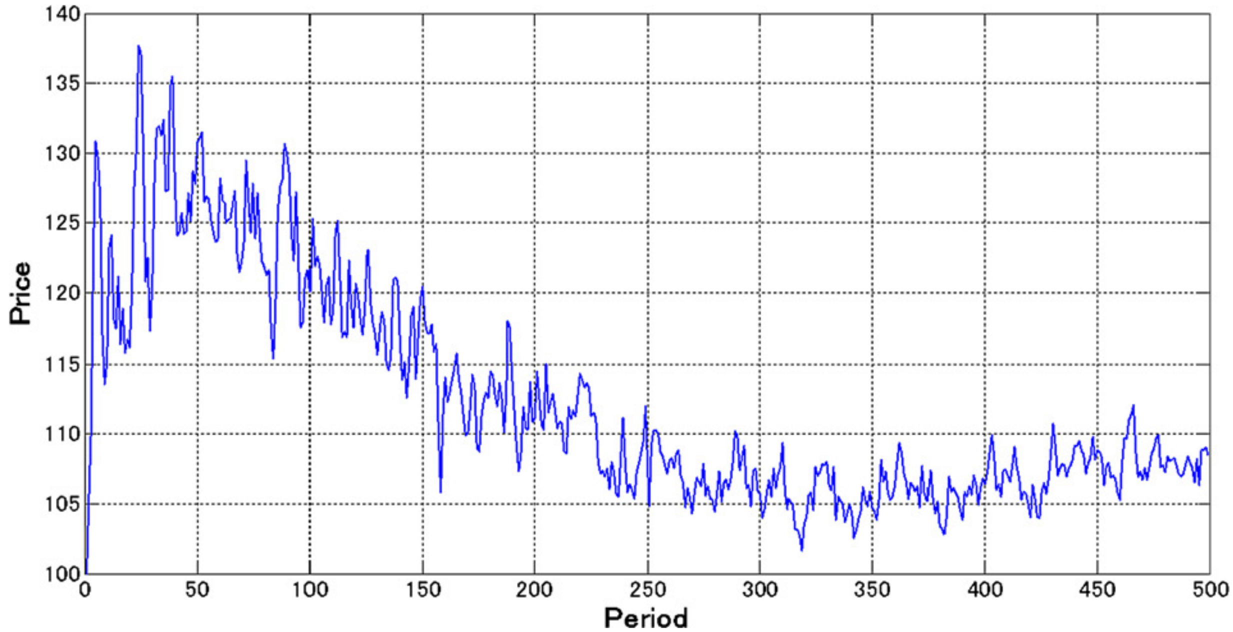

Fig. 6. Market price sequence of the benchmark case.

ratio for a relatively long period of time. These characteristics promote a careful attitude toward a price decline, which leads to a smaller drawdown. In other words, when prices are below the reference point, a number of risk-avoidance behaviors set in, as seen in the characteristics of $\delta^{+} < \delta^{-}$.

Finally, $|\omega^{+}| < |\omega^{-}|$ represents the tendency to avoid risks when there are latent profits (unrealized returns), and the tendency to prefer risk when there are latent losses (unrealized losses). In other words, these are disposition effects that promote the disposal of assets with hidden profits (unrealized gains), since the disposal of risky assets with latent losses cannot be implemented. The reason why this tendency is consistent with the drawdown is that a trader who has obtained latent profits at an early stage tries to suppress the drawdown by assuming risk-avoidance behaviors in the later stages in order to survive; furthermore, a trader who has suffered latent losses and who intends to launch a come-from-behind bid for victory with one last effort has a better chance of survival. For these reasons, our decision-making model (1) is expected to emerge from our market simulations as a stable, endogenously derived solution. This implies that evolutionary simulation with a drawdown as an evaluation criterion can explain dependence on reference points, asymmetry in the measure of risk aversion, and asset effects, which are the representative biases in prospect theory.

2) *Robustness Test:* In evolutionary simulations, such as genetic algorithm (GA), the dependence of the results on the initial values is always a matter of concern. In this section, we conduct two types of examinations to check the robustness of the results in Section III-B1. We increase the number of generations and observe the changes in order to further examine dependence on the initial distribution. If the results depend on the initial values, a transition to other reasonable and more stable solutions should be observed as the number of generations increase. Fig. 5 shows the relationship between the number of generations and the parameter values. The vertical axes of the graphs on the top represent the average parameter values mentioned in the captions, while the vertical axes of the graphs on the bottom represent the standard deviation of the parameters mentioned in the captions. The horizontal axes indicate the number of generations.[15]

The results indicate that the average parameter values are constant during the simulations and that variance values other than the γ^{-} values converge to the mean estimate value. This is consistent with our experiment, where the γ^{-} values could not be distinguished from zero. These results confirm that the characteristics obtained from our experiments can provide a stable solution. As mentioned above, it is clear that the characteristics of the investment behaviors obtained from our experiments can be used to develop stable strategies for the adaptive landscape in the market environment we assumed.

IV. Statistical Features of the Equilibrium Price Sequence

A. Financial Stylized Facts

The price sequence of our model has the following features.

1) When the rational traders dominate the market: The first term of the market price (15), which represents a weighted average of information gain, converges with the fundamental value v and the second term, which represents the effect of the bounded rational agents to the price, goes to 0 as information is accumulated or precision rises. Thus, the market price converges to the

[15]The parameters are determined based on the mean value of a stationary distribution. To confirm the robustness of the main results, simulations with 1000 generations are performed and the Geweke's convergence test [53] is applied to the latter 500 generations to identify the convergence.

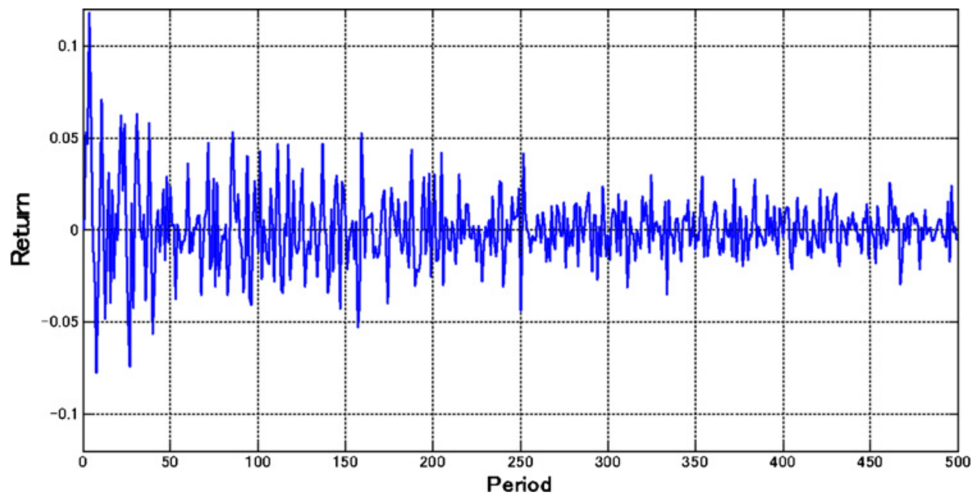

Fig. 7. Market return sequence of the benchmark case.

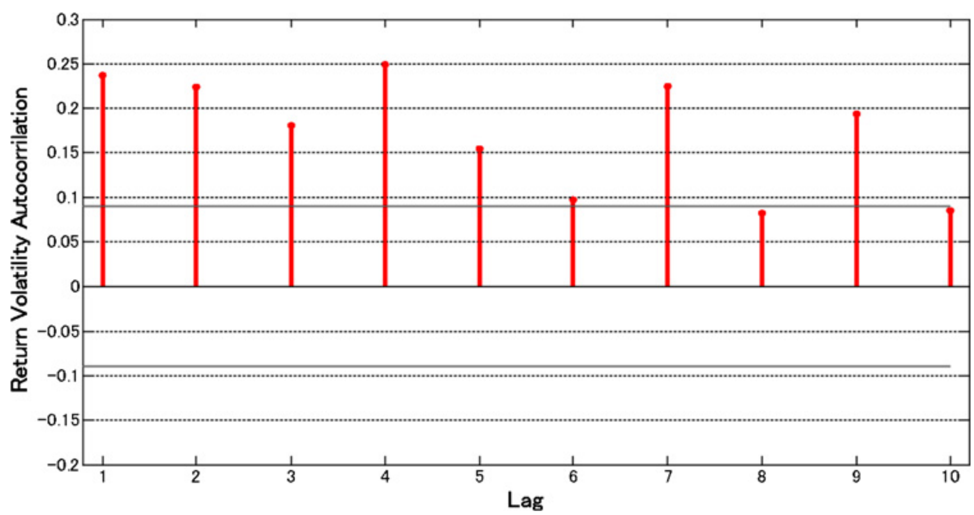

Fig. 8. Autocorrelation function on volatility of returns: the horizontal axis represents lag order and the lines at plus and minus 0.09 represent 5% confidence interval with the presupposition that the process of returns follows the MA process of lag order.

fundamental value, and if noise traders are considered, the price sequence completely becomes a random walk.

2) When the bounded rational traders survive (or under the convergence process of case 1): In this case, the decision biases of the bounded rational traders have some influence on market price formation. Figs. 6 and 7 represent sample sequences of price and return in the benchmark case, respectively (the parameter values of investment are defined in Table IV, and other parameter values are defined in Table III). Table V[16] shows the moments of return distribution. Obviously, the return does not follow a normal distribution and has great kurtosis. This is because the boundedly rational decision defined by (1) has strong autocorrelation, which causes traders to overshoot the market price. Fig. 8 represents the autocorrelation function of return volatility. The vertical axis represents lag order and a solid line represents a 5% confidence interval with the presupposition that the process of returns follows the MA process of lag order. The autocorrelation of volatility of return seems to continue for the long term. By using Ljung and Box's Q-statics test, we confirmed the existence of autocorrelation of volatility with first, fourth, eighth, and 12th lags, respectively (5% significance level). The autocorrelation of volatility is consistent with previously observed facts [54], [55]. Model selection using AIC shows that the GARCH model where an autocorrelation of the volatility of return distribution is allowed, is most suitable for describing the return sequence. Of the ARMA, GARCH, and EGARCH models [56], [57], GARCH(1, 1) is the most suitable for our purposes.

B. Forecastability of Returns

The features of the bounded rationality of market participants can enable the forecasting of market returns that are frequently observed in empirical analysis.

Fig. 9 shows the average response (1000 simulations) of market price to fundamental shocks and displays changes

[16]The reader can find Tables V–VII in a supplementary file, which is available for download through the *IEEExplore* website.

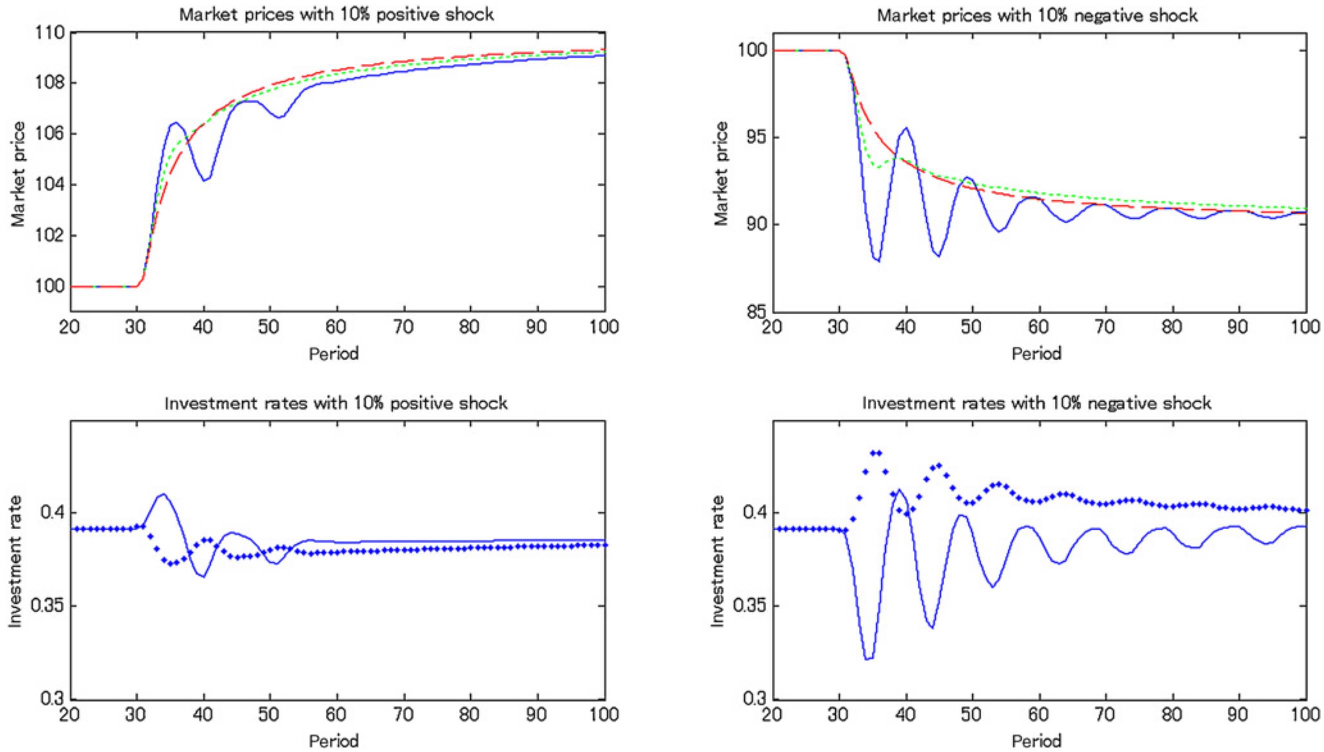

Fig. 9. Responses of the market price and changes in each trader's investment rates to the shocks in which the fundamental value changes by 10% at 30 periods.

in each trader's investment rates, which are generated from the above price formation and learning model, to the shocks wherein the fundamental value changes by 10% over 30 periods. The left column of the figure represents a positive 10% shock (good news) and the right column represents a negative 10% shock (bad news).

The upper row of each column represents the responses of the market price. The broken line represents the case where only the rational investor exists, i.e., the population ratio between rational investors and bounded rational investors is 1:0. It is observed that the market price gradually converges to the new fundamental value as the rational investor's private information is accumulated and the information is transmitted among investors through the price. The dotted line represents a case where the rational traders and bounded rational traders in our learning model exist at a ratio of 0.75:0.25. In this case, an overreaction of price (compared with the dotted line) occurs in the short term and an underreaction is observed in the medium and long terms. This underreaction depends on the adjustment of the reference point and the disposition effect in the learning model. The solid line provides price changes when the population ratio is 0.3:0.7 (as in Table III). Consequently, it can be observed that the abovementioned tendency becomes much stronger. Moreover, a fluctuation is caused. The overreaction is especially remarkable in the case of a negative shock.

In the long run, the market price settles to a fundamental value in each case. This is because rational investors gradually adjust the distortion caused by the decision making of the bounded rational investor.

The lower row of each column displays the changes in investment rates to the exogenous fundamental shock. The solid line represents the reaction of the bounded rational investor and the dotted line represents the reaction of the rational investor. We can observe that the rational investor changes his or her investment rate to absorb the bounded rational behavior of the bounded rational traders. For example, if a negative shock causes bounded rational traders to panic and remain extremely passive, the reduction of investment by the bounded rational traders indicates an overreaction to the market price. Thus, rational traders increase their investment rate as a result.

The predictability of security prices has been one of the most important subjects in financial research. In recent decades, experimental studies in this field have reported the existence of the predictability of stock prices and denied the efficient market hypothesis. There are two types of predictabilities: contrarian, which is based on the investor overreaction hypothesis, and momentum, which is based on underreaction [36]–[40].

In our learning model, overreaction is caused in the relative short term and underreaction is observed in the mid to long term. Moreover, overreaction is remarkable when a price falls, as is underreaction when the price rises, owing to the asymmetrical feature of reinforcement learning (i.e., investment rate is more sensitive to a loss than to a gain). In addition, the asymmetrical adjustment of the reference point (i.e., adjustment of a reference point is much faster for a gain than for a loss) creates crucial decision-making biases in our model. Thus, it can be expected that stock prices

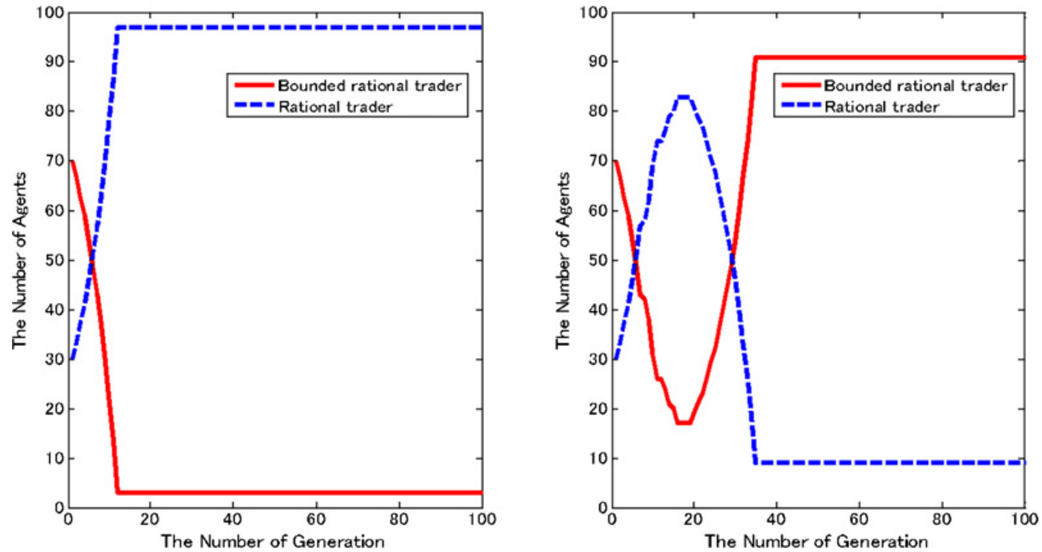

Fig. 10. Changes in the proportion of rational traders and bounded rational traders during simulations.

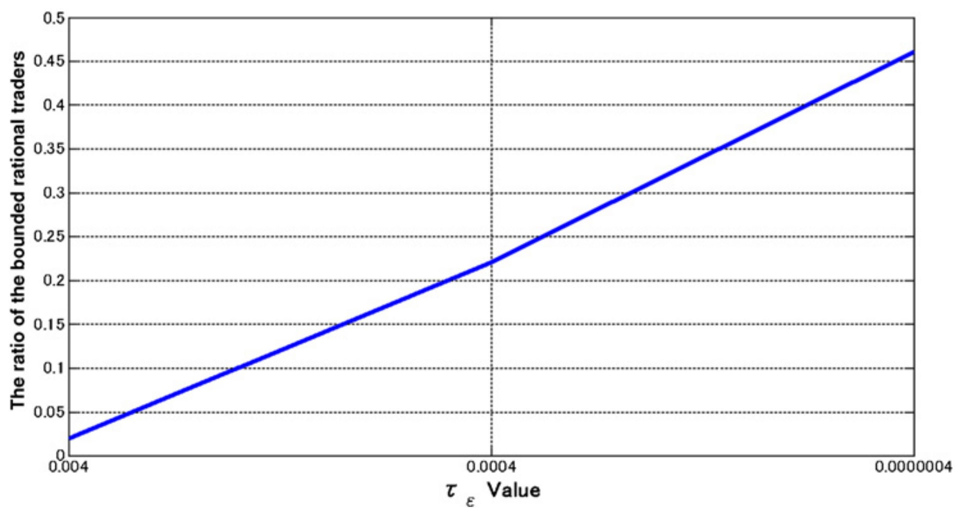

Fig. 11. Decreasing informational superiority of the rational traders increases the bounded rational traders. The vertical axis shows the survival probability of the bounded rational traders, and the horizontal axis represents the precision of the private signal of the rational trader.

will increase comparatively calmly during rising periods and decrease rapidly and considerably during falling periods. This also means that the momentum strategy is effective when the price is rising, and the contrarian strategy is effective when the price is falling.[17]

V. Emergence of an Irrational Market

We examined the survival possibility of bounded rational traders by changing the proportion of rational traders and bounded rational traders in the market. Considering that selection pressure applies to rational traders, our market provides an extremely favourable setting for rational traders. This is because the rational traders have information about the true fundamental values of risky assets, and these values have a great influence on pricing rules. Therefore, if the bounded rational traders can survive in this market environment, it would prompt a serious questioning of the basis of the rationality argument established by Friedman [8] This subsection specifies which situations allow the emergence of an irrational market and investigates whether Friedman's assertion holds.

The vertical axes of the two graphs in Fig. 10 indicate the number of agents and the horizontal axes show the number of generations. The straight line represents bounded rational traders, while the dotted line represents rational traders. The graph on the left depicts the case where rational traders control the market, while the graph on the right represents the case where bounded rational traders dominate the market. While most simulations are akin to those shown in the graph on the left, the cases shown on the right are more similar to reality. Table VI summarizes the survival probabilities of bounded rational agents under three conditions. The table indicates that

[17]Grinblatt and Han [41] and Frazzini [42] examined whether or not the tendency of some investors to hold on to their losing stocks, driven by prospect theory and mental accounting, creates a spread between a stock's fundamental value and its equilibrium price, as well as price underreaction to information. Grinblatt and Han [41] used Fama-MacBeth-type cross-section regression to illustrate the possibility that a variable proxying for aggregate unrealized capital gains is the key to generate the profitability of a momentum strategy. Frazzini [42] also tested whether a disposition effect induces the underreaction to news and suggested that it may do so, leading to return predictability and a postannouncement price drift (momentum effect).

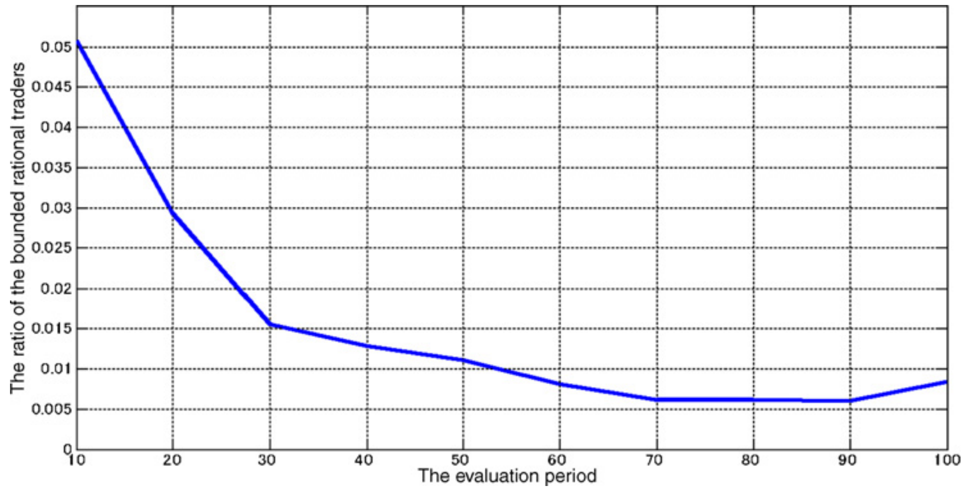

Fig. 12. Myopic assessment increases the bounded rational traders. The vertical axis shows the survival probability of the bounded rational traders, and the horizontal axis represents the evaluation periods.

as the informational advantage of the rational agents is eased, as the evaluation period is shortened, and as the selection becomes loose, the survival probability of the bounded rational traders increases.

Standard economic theory assumes that the market is strictly rational and that irrational entities immediately withdraw from the market. However, our simulation results clearly contradict this hypothesis. We believe that an agent-based simulation in the market is an effective approach to determine which market situation allows bounded rational agents to survive.

Our setting is extremely favourable for rational traders, as they have advance information about the true fundamental value of assets. Fig. 11 shows the reduction of the information advantage and the survival probability of bounded rational traders in relation to the precision of the private signal, τ_ε. The figure indicates that the survival probability of bounded rational traders rises greatly as the information superiority of the rational traders decreases.

The vertical axis of Fig. 12 shows the survival probabilities of the bounded rational traders when the evaluation period is changed. The horizontal axis represents the evaluation period. Note that the data includes nine types of selection probabilities and the initial proportion in order to remove the influence of other parameter values.[18] The results show that the bounded rational traders have a progressively better chance for survival as the evaluation period shortens. This implies that the short-term pursuit of profits in the present helps a large number of bounded rational traders survive in the market.

The vertical axis of Fig. 13 represents the ratio of the bounded rational traders who survived the simulations in relation to change in the selection rate, while the horizontal axis shows the selection probability. This figure indicates that bounded rational traders have an increasingly higher probability of survival as the selection probability increases (since the selection function becomes progressively smaller). This result may imply that the market becomes more rational with increased competition.

The reader may observe that the survival rate of the bounded rational traders is relatively low. However, the low ratio of the bounded rational traders has a sufficiently large effect on market price formation. Table VII shows the relationship between the ratio of the bounded rational agents and the distortions of the market returns, which are endogenously generated by the market simulation. We can see that the market return distribution deviates sufficiently from the normal distribution, which emerges when there is no bounded rational agent; this occurs even if the ratio of the bounded rational agent is under 5%.

VI. Discussion and Limitations

A. Evolutionary Approach in a Market Setting

The reader may question the scientific importance of justifying bounded rationality and decision biases in an evolutionary sense in a market setting. Such a question is relevant when an evolutionary approach is used to validate economic modeling.

In the field of economic research, the theory of the evolutionary approach has been adopted as one of the grounds from which man's decision-making rules and market structure are generated. This paper also adopts this approach on the grounds of bounded rationality and the fact that Friedman used the evolutionary approach to justify market rationality. In other words, we adopted the approach to determine which decision-making tendencies of traders can survive in the market, which market conditions strengthen the survival capability of bounded rationality, and how individual actors develop decision-making rules through adaptive learning (thinking).

Of course, this evolutionary simulation in a market setting is not the same as the evolutionary process in a biological sense; however, we believe that this simulation is an effective analogy to analyse decision-making rules or biases in the market.

Although almost all existing research specifies the bounded rationality of economic decisions from an experimental

[18]For example, when the evaluation period for the given data is 10, the data includes the simulation results for cases where the selection probability lies between 0.1 and 0.9 (stepwise count of 0.1). Furthermore, the simulation results for cases where the initial proportions of bounded rational traders lie between 0.1 and 0.9 (stepwise count of 0.1) are also included. In other words, the data at each point represents the result of 16 200 simulations (nine types of selection probabilities × nine types of initial proportion × 200 simulations).

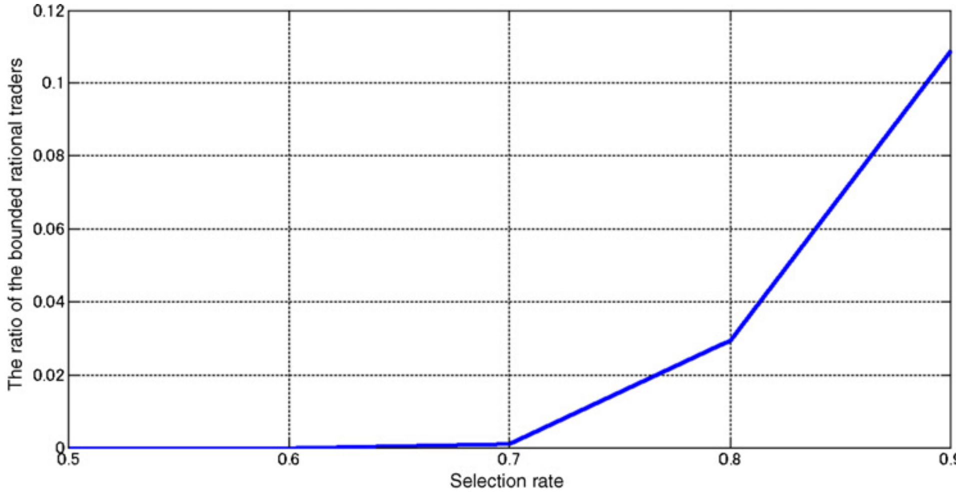

Fig. 13. Loose selection increases the bounded rational traders. The vertical axis shows the survival probability of the bounded rational traders, and the horizontal axis represents the selection rate.

approach, it is also necessary to validate decision-making rules from the viewpoint of survivability through coevolution in a market. In this sense, evolutionary simulation using agent-based modeling can play an important role in decision-making research.

There may also be questions about the evolutionary criteria ("the selection criterion" and "mutation") used in this manuscript, as they obviously do not accurately correspond to evolutionary processes in actual markets. Having said that, those criteria are the most acceptable approximation of an adaptive learning process using a traditional genetic algorithm process.

B. Generalization of Our Experimental Result

In our analysis, we do not pay much attention to the subjects' investment experience. Because of this limitation in subject selection, we have to consider the generalizability of our analysis result for the decision-making model. We believe that our results can be generalized in cases where the levels of investment experience differ. We have come to this conclusion because, first, the same decision-making model is selected for both subject groups, even when the groups are divided according to investment experience. Second, decision-making models in other studies, that comprise a wider range of subject groups have shown similar tendencies as those in our research. For example, the reinforcement learning model including asymmetry was employed by Bereby-Meyer and Erev [17]. Moreover, Weber and Camerer [58] indicated that the disposition effect was robustly observed among professional traders.

However, the relationship between investment experience and the approach to decision making is important and requires further examination [23]. We would like to address this problem in a future study.

C. Price Formation

In our analysis, the price sequence is given exogenously for decision-making experiments using actual stock prices, whereas it is decided endogenously in the evolutionary simulation. In this regard, readers should keep in mind that the experiments and simulation lack consistency. Certainly and ideally, the prices in the experiments should be given endogenously, and they should take the suitable degree of the aggregate effects into consideration. Therefore, experimental designs involving the double auction should be considered in the future.

However, the effects are relatively weak in large markets such as financial markets and therefore tend, instead, to be too strong with only less than ten participants. It is anticipated that experiments involving a few people would create strategic conditions that make it difficult to estimate a suitable decision-making model. An attempt to treat this aggregation effect endogenously in our analysis requires a large-scale experiment (estimating simulations with 100 agents) and a large sum of experiment costs. With such foreseeable limitations, it was felt that adopting competitive markets where the prices are given exogenously would be more appropriate to approximate the weak aggregation effects rather than performing the small-scale double auction.[19]

D. Conditions in the Context of Real Markets

Further discussions are necessary to explain the conditions, in the context of real markets, where bounded rational agents can gain a sizable population. For instance, Fig. 11 indicates that the ratio of the bounded rational agents can increase up to 22%. This occurs when the accuracy of the information that the rational agents obtain declines significantly (this means the information accuracy decreases to one-tenth of the benchmark: $\tau_\varepsilon = 0.004$ to $\tau_\varepsilon = 0.0004$). Realistically, this may be a difficult condition. Additional discussions are essential to evaluate the validity of cases where the information accuracy declines dramatically as such.

This paper anticipates cases where rare events such as a market crash occur. In such circumstances, we assume that obtaining adequate information would generally be difficult

[19]Suzuki *et al.* [59] conducted a market experiment using endogenous prices. Here, the price information method is elected using the double auction. However, this is a small-scale case involving only four market participants. The result of this experiment has also confirmed that "the asymmetric response to profit and loss," "reference point dependency," and "the disposition effect" have significant effects on investment behaviors.

and that defining subjective probability rationally would become challenging. Moreover, if a lack of market liquidity and myopic decision making emerge within markets, the bounded rational agents are expected to obtain sufficient sizes coupled with such conditions. However, further discussions are required to evaluate the validity of these parameters in comparison to actual markets.

VII. Conclusion

Instinctual human traits, including decision biases, can often be explained from the viewpoint of evolution. For example, LeDoux [60] provided the following evolutionary basis for the activity of the amygdala and our emotional reaction to fear.

> *The system is not, strictly speaking, a system that results in the experience of fear. It is a system that detects danger and produces responses that maximize the probability of surviving a dangerous situation in the most beneficial way. It is, in other words, a system of defensive behavior. As noted above, I believe that emotional behaviors, such as defensive green behaviors, evolved independent of conscious feeling, and that we should not assume that animals other than humans feel afraid when they are in danger. We should, in other words, take defensive behaviors at face value: they represent the operation of brain systems that have been programmed by evolution to deal with danger in routine ways* [60, Section 5, p. 128].

This paper provided an evolutionary foundation for the decision biases observed in financial markets. First, we conducted experiments and described boundedly rational trade behaviors to provide an evolutionary foundation for the subject of bounded rationality. The model obtained in this paper explained issues such as the adjustment of reference points, asymmetry in the profit-and-loss response, and the disposition effect; these were consistent with the reports on bounded rationality that were presented by previous studies.

Next, we conducted an evolutionary simulation using an artificial market model with selection pressures in order to check whether the observed bounded rationality could have an evolutionary basis. The results confirmed the following.

1) The decision biases in investment behaviors obtained from our experiments can be made into stable strategies for the adaptive landscape of the market environment presented in this paper. In other words, these decision biases were evolutionarily rational.
2) The observed decision biases were consistent with well-known biases such as dependence on reference points, asymmetry in the measurement of risk aversion, and the asset effect. Furthermore, these biases can be derived using a drawdown as the single evaluation criterion from an evolutionary viewpoint. This drawdown is analogous to the evaluation standards for entry and withdrawal in the actual market.
3) These results were reinforced by the informational inferiority of the rational agent, the myopic evaluation of traders, and loose selection.

For a long time, researchers have been trying to explain the main concept of finance theory from an evolutionary viewpoint, and several studies employing evolutionary approaches have contributed to financial research [61]. These works reexamine the validity of assumptions that were considered to be natural in traditional financial models, and they have succeeded in replicating various financial facts. Our research follows this tradition.

The reader can find Tables V–VII, and Appendixes in a supplementary file. The supplementary file is available for download through the *IEEExplore* website.

References

[1] A. Tversky and D. Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," *J. Risk Uncertainty*, vol. 5, no. 4, pp. 297–323, 1992.

[2] R. Thaler, "Mental accounting matters," *J. Behavioral Decision Making*, vol. 12, no. 3, pp. 183–206, 1999.

[3] C. Camerer, J. Kagel, and A. Roth, *The Handbook of Experimental Economics*. Princeton, NJ: Princeton University Press, 1995.

[4] S. Benartzi and R. Thaler, "Myopic loss aversion and the equity premium puzzle," *Quart. J. Econ.*, vol. 110, no. 1, pp. 73–92, Feb. 1995.

[5] N. Barberis, M. Huang, and T. Santos, "Prospect theory and asset prices," *Quart. J. Econ.*, vol. 116, pp. 1–53, Feb. 2001.

[6] T. Shimokawa, K. Suzuki, and T. Misawa, "An agent-based approach to financial stylized facts," *Physica A Statist. Mech. Its Applicat.*, vol. 379, no. 1, pp. 207–225, 2007.

[7] N. Barberis and R. Thaler, "A survey of behavioral finance," in *Handbook of the Economics of Finance*, G. Constantinides, M. Harris, and R. M. Stulz, Eds. Amsterdam, The Netherlands: Elsevier, 2003, ch. 18, pp. 1053–1128.

[8] M. Friedman, *Essays in Positive Economics*. Chicago, IL: Univ. Chicago Press, 1953.

[9] E. Tsang, "Computational intelligence determines effective rationality," *Int. J. Autom. Comput.*, vol. 5, no. 1, pp. 63–66, 2008.

[10] T. Lohrenz, K. McCabe, C. Camerer, and P. Montague, "Neural signature of fictive learning signals in a sequential investment task," *Proc. Natl. Acad. Sci.*, vol. 104, no. 22, p. 9493, 2007.

[11] T. Shimokawa, K. Suzuki, T. Misawa, and K. Miyagawa, "Predictability of investment behavior from brain information measured by functional near-infrared spectroscopy: A Bayesian neural network model," *Neuroscience*, vol. 161, no. 2, pp. 347–358, 2009.

[12] T. Shimokawa, K. Suzuki, and T. Misawa, "Argumented reinforcement learning and investment decision making," *Neurocomputing*, vol. 72, nos. 16–18, pp. 3447–3461, 2009.

[13] C. Camerer and R. Foundation, *Behavioral Game Theory: Experiments in Strategic Interaction*. Princeton, NJ: Princeton Univ. Press, 2003.

[14] R. Sutton and A. Barto, *Reinforcement Learning: An Introduction*. Cambridge, MA: MIT Press, 1998.

[15] I. Erev and A. Roth, "On the need for low rationality cognitive game theory: Reinforcement learning in experimental games with unique mixed strategy equilibria," in *Mimeo*. Pittsburgh, PA: Univ. Pittsburgh, 1996.

[16] I. Erev and A. Roth, "Predicting how people play games: Reinforcement learning in experimental games with unique, mixed strategy equilibria," *Amer. Econ. Rev.*, vol. 88, no. 4, pp. 848–881, 1998.

[17] Y. Bereby-Meyer and I. Erev, "On learning to become a successful loser: A comparison of alternative abstractions of learning processes in the loss domain," *J. Math. Psychol.*, vol. 42, nos. 2–3, pp. 266–286, 1998.

[18] C. Camerer and T. Ho, "Experience-weighted attraction learning in games: A unifying approach," *Econometrica*, vol. 67, no. 4, pp. 827–874, 1999.

[19] H. Breiter, I. Aharon, D. Kahneman, A. Dale, and P. Shizgal, "Functional imaging of neural responses to expectancy and experience of monetary gains and losses," *Neuron*, vol. 30, no. 2, pp. 619–639, 2001.

[20] B. Knutson and R. Peterson, "Neurally reconstructing expected utility," *Games Econ. Behavior*, vol. 52, no. 2, pp. 305–315, 2005.

[21] H. Shefrin and M. Statman, "The disposition to sell winners too early and ride losers too long: Theory and evidence," *J. Finance*, vol. 40, no. 3, pp. 777–790, Jul. 1985.

[22] T. Odean, "Volume, volatility, price, and profit when all traders are above average," *J. Finance*, vol. 53, no. 6, pp. 1887–1934, Dec. 1998.

[23] A. Lo and D. Repin, "The psychophysiology of real-time financial risk processing," *J. Cognitive Neurosci.*, vol. 14, no. 3, pp. 323–339, 2002.

[24] S. Chen and C. Yeh, "Evolving traders and the business school with genetic programming: A new architecture of the agent-based artificial stock market," *J. Econ. Dynamics Control*, vol. 25, nos. 3–4, pp. 363–393, 2001.

[25] S. Martinez-Jaramillo and E. Tsang, "An heterogeneous, endogenous and coevolutionary GP-based financial market," *IEEE Trans. Evol. Comput.*, vol. 13, no. 1, pp. 33–55, Feb. 2009.

[26] B. LeBaron, "Agent-based computational finance," in *Handbook of Computational Economics*, vol. 2. Amsterdam, The Netherlands: Elsevier/North-Holland, 2006, pp. 1187–1233.

[27] C. Hommes, "Heterogeneous agent models in economics and finance," in *Handbook of Computational Economics*, vol. 2. Amsterdam, The Netherlands: Elsevier/North-Holland, 2006, pp. 1109–1186.

[28] K. Suzuki, T. Shimokawa, and T. Misawa, "Agent-based approach to option pricing anomalies," *IEEE Trans. Evol. Comput.*, vol. 13, no. 5, pp. 959–972, Oct. 2009.

[29] S. Grossman and J. Stiglitz, "On the impossibility of informationally efficient markets," *Amer. Econ. Rev.*, vol. 70, no. 3, pp. 393–408, Jun. 1980.

[30] B. LeBaron, W. Arthur, and R. Palmer, "Time series properties of an artificial stock market," *J. Econ. Dyn. Control*, vol. 23, nos. 9–10, pp. 1487–1516, 1999.

[31] H. Linhart, "A test whether two AICs differ significantly," *South Afr. Stat. J.*, vol. 22, no. 1, pp. 153–161, 1988.

[32] H. Kishino and M. Hasegawa, "Evaluation of the maximum likelihood estimate of the evolutionary tree topologies from DNA sequence data, and the branching order in hominoidea," *J. Molecular Evol.*, vol. 29, no. 2, pp. 170–179, 1989.

[33] H. Shimodaira, "Assessing the error probability of the model selection test," *Ann. Instit. Statist. Math.*, vol. 49, no. 3, pp. 395–410, 1997.

[34] M. Kampouridis, S. Chen, and E. Tsang, "The market fraction hypothesis under different genetic programming algorithms," in *Information Systems for Global Financial Markets: Emerging Developments and Effects*. Pennsylvania: GIG Global, 2011, pp. 37–54.

[35] M. Kampouridis, S. Chen, and E. Tsang, "Testing the dinosaur hypothesis under empirical datasets," in *Parallel Problem Solving From Nature XI*, LNCS 6239. 2010, pp. 199–208.

[36] W. DeBondt and R. Thaler, "Does the stock market overreact," *J. Finance*, vol. 40, no. 3, pp. 793–805, 1985.

[37] J. Lakonishok, A. Shleifer, and R. Vishny, "Contrarian investment, extrapolation, and risk," *J. Finance*, vol. 49, no. 5, pp. 1541–1578, 1994.

[38] E. Fama and K. French, "The cross-section of expected stock returns," *J. Finance*, vol. 47, no. 2, pp. 427–465, 1992.

[39] N. Jegadeesh and S. Titman, "Returns to buying winners and selling losers: Implications for stock market efficiency," *J. Finance*, vol. 48, no. 1, pp. 65–91, 1993.

[40] A. W. Lo and A. C. MacKinlay, "Stock market prices do not follow random walks: Evidence from a simple specification test," *Rev. Financial Stud.*, vol. 1, no. 1, pp. 41–66, 1988.

[41] M. Grinblatt and B. Han, "Prospect theory, mental accounting, and momentum," *J. Financial Econ.*, vol. 78, no. 2, pp. 311–339, 2005.

[42] A. Frazzini, "The disposition effect and under-reaction to news," *J. Finance*, vol. 61, no. 4, pp. 2012–2046, 2006.

[43] S. Chen and C. Yeh, "On the emergent properties of artificial stock markets: The efficient market hypothesis and the rational expectations hypothesis* 1," *J. Econ. Behavior Organization*, vol. 49, no. 2, pp. 217–239, 2002.

[44] C. Yeh and S. Chen, "Market diversity and market efficiency: The approach based on genetic programming," *Artif. Intell. Simul. Behaviour J.*, vol. 1, no. 1, pp. 147–167, 2001.

[45] S. Markose, E. Tsang, and S. Jaramillo, "The red queen principle and the emergence of efficient financial markets: An agent based approach," in *Nonlinear Dynamics and Heterogeneous Interacting Agents* (Lecture Notes in Economics and Mathematical Systems), vol. 550, part V, T. Lux, S. Reitz, and E. Samanidou, Eds. Berlin/Heidelberg, Germany: Springer, pp. 287–303, 2005.

[46] A. Robson, "The evolution of rationality and the Red Queen* 1," *J. Econ. Theory*, vol. 111, no. 1, pp. 1–22, 2003.

[47] A. Robson, "Complex evolutionary systems and the Red Queen*," *Econ. J.*, vol. 115, no. 504, pp. F211–F224, 2005.

[48] S. Kauffman, *At Home in the Universe: The Search for Laws of Self-Organization and Complexity*. Oxford, U.K.: Oxford Univ. Press, 1995.

[49] X. Vives, "The speed of information revelation in a financial market mechanism," *J. Econ. Theory*, vol. 67, no. 1, pp. 178–204, 1995.

[50] C. Chamley, *Rational Herds: Economic Models of Social Learning*. Cambridge, U.K.: Cambridge Univ. Press, 2003.

[51] J. De Long, A. Shleifer, L. Summers, and R. Waldmann, "Noise trader risk in financial markets," *J. Political Econ.*, vol. 98, no. 4, pp. 703–738, 1990.

[52] A. Shleifer and R. Vishny, "The limits of arbitrage," *J. Finance*, vol. 52, no. 1, pp. 35–55, 1997.

[53] J. Geweke, "Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments," in *Bayesian Statistics*, vol. 4. Oxford, U.K.: Oxford Univ. Press, 1992, pp. 169–193.

[54] Z. Ding and C. Granger, "Modeling volatility persistence of speculative returns: A new approach," *J. Econometrics*, vol. 73, no. 1, pp. 185–215, 1996.

[55] S. Taylor, "Modeling stochastic volatility: A review and comparative study," *Math. Finance*, vol. 4, no. 2, pp. 183–204, 1994.

[56] R. F. Engle, "Autoregressive conditional heteroscedasticity with estimates of variance of united kingdom inflation," *Econometrica*, vol. 50, no. 4, pp. 987–1008, 1982.

[57] T. Bollerslev, "Generalized autorregressive conditional heteroskedasticity," *J. Econometrics*, vol. 31, no. 3, pp. 307–327, 1986.

[58] M. Weber and C. Camerer, "The disposition effect in securities trading: An experimental analysis," *J. Econ. Behavior Organization*, vol. 33, no. 2, pp. 167–184, 1998.

[59] K. Suzuki, K. Kinoshita, K. Miyagawa, S. Shiomi, T. Misawa, and T. Shimokawa, "An investment behavior analysis using by brain computer interface," *Trans. Japan. Soc. Artif. Intell.*, vol. 25, no. 1, pp. 183–195, 2010.

[60] J. LeDoux, *The Emotional Brain*. New York: Simon and Schuster, 1996.

[61] E. Tsang and S. Martinez-Jaramillo, "Computational finance," *IEEE Comput. Intell. Soc. Newsletter*, pp. 3–8, Aug. 2004.

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