# Optimized *FTR* Portfolio Construction Based on the Identification of Congested Network Elements

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***Abstract*—This paper focuses on the construction of an optimized financial transmission rights (*FTR*) or congestion revenue rights portfolio for an *FTR* market participant given his assessment of the frequency and economic impacts of binding constraints in the transmission network. We overcome the data handling and heavy computing demands of locational marginal price (*LMP*)-difference-based methods for *FTR* selection by recasting the problem into one that focuses on the underlying product of "binding constraints", which are physically observable phenomena, based on the mathematical insights into the structural characteristics of the model used for the clearing of the hourly day-ahead markets. Differentials in the *LMP*s are due to system congestion and so are merely manifestations of binding constraints in the transmission network. In addition, we exploit extensively the salient topological characteristics of large-scale interconnections. The market participant specifies the subset of "focus" constraints and the position he is willing to take on them. Our approach builds on the mathematical insights and topological characteristics with the effective deployment of the orthogonal matching pursuit algorithm to construct the optimized *FTR* portfolio characterized by the minimum number of node pairs for the specification of the *FTR* elements. We apply the proposed approach to a test system based on the *PJM ISO* network and markets to illustrate its capabilities for solving the *FTR* market participant's problem in realistic large-scale systems.**

***Index Terms*—Congestion management, congestion revenue rights, contingency information, financial transmission rights, power transfer distribution factors, transmission usage charges.**

## I. Introduction

CONGESTION has major impacts on electricity markets, since it may restrict the amounts of some transactions and not allow others to take place. As such in the presence of congestion, sellers are unable to sell wherever they wish and buyers are unable to buy from whomever they desire [1]. The result is higher electricity prices at various locations in the grid. Our paper is concerned with congestion in the day-ahead electricity markets or *DAM*s. In each hourly *DAM*, each seller (buyer) receives (pays) the locational marginal price (*LMP*) at the point of power injection (withdrawal) into the grid. Whenever there is a difference between the seller *LMP* and the buyer *LMP*, congestion rents are collected by the independent grid operator (*IGO*) [2]. Uncertainty in *LMP*s results in uncertain congestion rents that create a demand by market players for the financial transmission rights (*FTR*), congestion revenue rights or flow gate rights (*FGR*) hedging instruments. *FTR* entitle the holder to receive the value of congestion as established by the *LMP* difference of each *DAM* during the holding period. *FGR* entitle the holder to be reimbursed the value of congestion determined by the transmission usage charges on a congested network element in a specified direction. In this paper, we focus on *FTR*, since *FTR* are the hedging instruments that are widely used in the *IGO*s. The holder of the *FTR* for a specified *to* and *from* node pair with a physical transaction with the identical injection and withdrawal node pair is not impacted financially by the *LMP* difference between the *to* and *from* nodes as long as the *FTR* for that node pair is in a *MW* amount at or above his physical delivery. The *FTR* reimburse the holder the amount collected by the *IGO* in congestion rents. *FTR* are strictly directional in nature as specified by the source and sink node pair. *FTR* are further characterized by the holding period, which is defined by the start and the end times, and the class. The class refers to the coverage subperiods and, typically, comes in three categories: on-peak, off-peak, and around the clock. The *FTR* tool is further categorized as either a contract or an option type. The *FTR* contracts provide reimbursements to the holder whenever the congestion is in the direction specified by the *FTR*. However, the contracts turn into a liability whenever the *LMP* difference of the source and the sink nodes is negative, i.e., the congestion is in the opposite direction. The *FTR* options are only exercised when the reimbursements are beneficial to the holder.

The *IGO* runs periodic auctions where it sells *FTR* holdings to buyers who bid for the offered quantities. The buyers may either be hedgers or speculators. Hedgers buy *FTR* to ensure reimbursements for all congestion rents incurred for their transactions as a result of grid congestion. Speculators purchase *FTR* even in the absence of physical flows in order to make profits.

The basic concept of *FTR* was first introduced by Hogan in a paper that set out the mathematical framework for the analysis of the *FTR* tool [3]. A more detailed treatment of *FTR* issues was developed later in 2002 [4]. The implementation of *FTR* in various *IGO*s was accompanied by specific rules for each jurisdiction [5]–[8]. A comparative analysis of the *FTR* implementations around the world provides additional insights into different procedures and rules [9]. While the original intent in the introduction of *FTR* was to provide insurance to entities with physical transactions, the use for speculation has been common

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industry practice [10]. The *IGO*s are keen to promote the liquidity of the *FTR* markets and include the participation of entities without physical flows, so as to increase competition in the *FTR* auctions. The salient mathematical aspects of *FTR* auction clearing mechanisms and analysis of the outcomes have been investigated in [11]–[14] as has the utilization of *FTR* in transmission expansion planning and investment decisions [15]–[18]. A very useful and comprehensive survey paper assessing the *FTR* literature is [19]. So far, the issue of the systematic construction of *FTR* portfolios for speculative or hedging purposes has not been studied. This paper aims to study such a methodology for use by either speculators or hedgers. We refer to such entities as market players. Our focus is on the insights from the analysis of the *DAM*s that the participants can effectively deploy in the construction of *FTR* portfolios.

The *LMP* differentials across the system are simply the manifestations of system congestion.[1] While a binding constraint may have a positive impact on the sink-source *LMP* differential of an *FTR* holding, another binding constraint may have the opposite effect. Therefore, the purchase of *FTR* holdings entails the buy of a set of constraints. So in a way, *FTR* provide market participants the opportunity to express their views on the transmission usage charges arising from network element constraints. The main thrust of this paper is to exploit these specific characteristics of *FTR* and the salient topological features of large-scale interconnections in the systematic construction of an *FTR* portfolio for a market participant. We propose a practical methodology to construct an optimized *FTR* portfolio through the judicious consideration of a set of "focus" constraints and the specification of the positions the participant is willing to take on those constraints. In this way, we shift our paradigm away from the computationally intensive, historical *LMP* differences of node pairs to that of binding constraints in the transmission network. In our proposed scheme, we explicitly consider the participant's ability to model the transmission usage charges of a specific constraint, and classify all the "focus" constraints into three non-overlapping subsets referred to by the "specified congestion participation", the "zero congestion participation", and the "do-not-care congestion participation" labels. Based on the specified flows on each constraint, we select a subset of nodes, taking into consideration topological characteristics, for possible *FTR* source or sink nodes and construct the portfolio with the minimum number of *FTR* positions. We note that the optimized *FTR* portfolio induces the desired real power flows on the specified constraints. The recasting of the problem in terms of the underlying constraints, rather than the *LMP* differences of node pairs, results in a methodology that allows participants to judiciously take advantage of their views on the system transfer capability limitations.

The remainder of the paper consists of four additional sections. In Section II, we describe the *FTR* market participant's problem and discuss the key challenges. We gain valuable insights into the key explanatory factors of congestion rents collected by the *IGO* and the *FTR* reimbursements to the holders, by using the optimality conditions of the *IGO* market clearing problem. In Section III, we use these insights in the development of the solution approach to the *FTR* portfolio construction problem faced by the market participant. In Section IV, we discuss the salient aspects of the solution approach with an illustrative example, using a large-scale system based on the *PJM ISO*. In Section V, we provide concluding remarks and discuss the direction for future research. We devote Appendix A to the listing of the nomenclature and Appendix B to the mathematical statement of the *DAM* problem.

## II. *FTR* Market Participant Problem

The market participants consist of speculators and hedgers. A hedger has physical flows on the network and potentially has to pay congestion rents for utilizing the network. He pursues to construct an *FTR* portfolio so that its revenues during the holding period are at least as much as the congestion rents he has to pay to the *IGO* during that period. A speculator, on the other hand, purchases *FTR* holdings in *FTR* auctions as an "investment". In light of the uncertainty and the large number of possible combinations of *FTR* he may have to purchase, the market participant faces a challenging problem in constructing an *FTR* portfolio that meets his needs and objectives. Unless additional specific constraints are introduced, the problem is rather unmanageable, particularly since there is inadequate information about the future stream of revenues. An exhaustive evaluation of all the possible combinations in a large-scale network is computationally too demanding a task, particularly when the consideration of the wide variations in the behavior of the *LMP* differences of nodes over the many hours of the holding period is taken into account.

We focus on solving the portfolio construction problem under certain simplifying assumptions. We consider the *FTR* acquisition in a single auction for contracts of identical class for a specified holding period, where we assume that the market participants do not procure significant *MW* amounts of *FTR* so as to impact auction clearing prices. In the analysis for a particular participant, we ignore all other *FTR* held by that market participant. We recast the problem into a form that allows us to exploit the salient characteristics of the topological and physical nature of the underlying network and make effective use of the historical data. In this way, we gain mathematical insights that we apply to reduce the solution state space and construct the proposed solution approach.

We start out by considering congestion in a network. A grid has certain network elements that are congested, i.e., their transfer capability limits are reached and the associated constraints become binding. The *IGO* assesses from every transaction that flows on a congested element charges for its use, which are set at the marginal benefit of the last *MW* of the flow that makes the constraint binding. These so-called transmission usage charges of the binding constraint are the basis for computing the congestion rents collected by the *IGO*. A hedger, who holds *FTR* in a given *MW* amount from a source to a sink node and who has a transaction in the same *MW* amount and with the identical node pair as the *FTR*, receives reimbursement for the transmission usage charges from the *IGO* whenever congestion occurs during the holding period. Similarly, a speculator, who holds *FTR* for the same node

[1]Without any loss of generality, we assume that each market clears using a lossless system network model.

pair and amount, does not incur congestion charges due to the absence of physical flows and the reimbursements by the IGO constitute his revenues.

We make use of the insights into the salient structural characteristics of power system network to recast the market participants problem into a more convenient form. We introduce our notation to obtain a mathematical statement of the problem formulation. We denote the *FTR* with source (sink) node $i$ $(j)$ in the *MW* amount $\gamma$ by the ordered triplet

$$\Gamma = \{i, j, \gamma\} \tag{1}$$

and the set of *FTR* in the portfolio by

$$\mathscr{T} = \{\Gamma_k : k = 1, 2, \ldots, K\}. \tag{2}$$

For each $\Gamma_k$ in $\mathscr{T}$ we must specify the triplet elements $i_k, j_k$ and $\gamma_k$, $k = 1, \ldots, K$. The *FTR* holding period is denoted by $\mathscr{T} = \{h_1, \ldots, h_{|\mathscr{T}|}\}$. We make use of the framework in [20] to model the grid for *FTR* analysis.

We use the mathematical formulation of the hourly *DAM* given in Appendix B and, to simplify the presentation, we consider only lines as network elements. We define the subset $\tilde{\mathscr{L}}|_h = \{\ell_i : \ell_i \in \mathscr{L}|_h, \mu_{\ell_i}|_h \neq 0\}$ of congested lines for hour $h$. Uncongested lines make no contribution to the *LMP* differences, since the complementary slackness conditions ensure that their $\underline{\boldsymbol{\mu}}|_h$ component is zero. We derive the relationship between the *LMP* difference of an arbitrary node pair and the dual variables of the transmission constraints by making use of the Lagrangian function and the stationarity and complementary slackness conditions. We can show that

$$\lambda_{n'}|_h - \lambda_n|_h = \sum_{\ell \in \tilde{\mathscr{L}}|_h} \phi_\ell^{\{n,n'\}}|_h \left(\mu_\ell^M|_h - \mu_\ell^m|_h\right). \tag{3}$$

Here, $\phi_\ell^{\{n,n'\}}|_h$ is the line $\ell$ *power transfer distribution factor* (*PTDF*) with respect to an injection/withdrawal at the node pair $\{n, n'\}$ in the hour $h$ [21]. We interpret $\phi_\ell^{\{n,n'\}}|_h$ as the fraction of the transaction from node $n$ to node $n'$ that flows on the line $\ell$ in hour $h$.

The hour $h$ market outcomes determine the *FTR* revenues $\eta|_h$ for the *FTR* $\Gamma = \{i, j, \gamma\}$ and are given by

$$\eta|_h = (\lambda_j|_h - \lambda_i|_h)\,\gamma = \sum_{\ell \in \tilde{\mathscr{L}}|_h} \left(\mu_\ell^M|_h - \mu_\ell^m|_h\right) \phi_\ell^{\{i,j\}}|_h \gamma. \tag{4}$$

But, the injection $\gamma$ at node $i$ and the withdrawal $\gamma$ at node $j$, give rise to the change $\Delta f_\ell|_h$ in the line $\ell$ flow in hour $h$

$$\Delta f_\ell|_h \approx \phi_\ell^{\{i,j\}}\Big|_h \gamma. \tag{5}$$

We assume the approximation in (5) holds and thus we set

$$\eta|_h = \sum_{\ell \in \tilde{\mathscr{L}}|_h} \left(\mu_\ell^M|_h - \mu_\ell^m|_h\right) \Delta f_\ell|_h. \tag{6}$$

Clearly, (6) indicates that for every hour $h \in \mathscr{T}$ the revenues $\eta|_h$ are purely a function of the transmission usage charges for the congested lines $\ell \in \tilde{\mathscr{L}}|_h$. We therefore focus on a portfolio construction strategy that includes in the *FTR* portfolio $\mathscr{T}$ those *FTR* that for transactions, with the same node pairs and in the same amounts, flows $\Delta f_\ell|_h$ on congested lines $\ell$ of interest are induced. We may view the flows as a weighting factor of the transmission usage charge of each such binding constraint. Our decision variables in the *FTR* selection are the source and sink nodes and the *FTR MW* amounts. Therefore, the *FTR* selection entails the determination of the node pairs $\{i, j\}$ and the amounts $\gamma$ that induce flows $\Delta f_\ell|_h$ on lines $\ell$ of interest in every hour $h$ of the holding period.

Given the expression in (6), we no longer need to be concerned with the *LMP* differences between the node pairs, rather we focus on the binding constraints. We benefit from such an approach because the number of binding constraints in a system is considerably smaller than the number of possible node pairs. We propose a methodology to solve the market participant's problem based on the selection of the *FTR* in $\mathscr{T}$ such that the transactions with same node pairs and amounts induce real power flows on the congested network elements of interest. A hedger is concerned only with the flows that his transactions induce on the congested lines. He wishes to purchase *FTR* such that the *FTR* reimbursements cover the congestion charges. A speculator sees *FTR* as "investments". The estimated revenues have to be sufficient to cover the costs of *FTR* acquisition and produce profits commensurate with the perceived risks. The speculator may wish to acquire *FTR* such that corresponding transactions with the same node pairs and in the same amounts induce flows in all congested lines of the system during the entire holding period. But, he would need to buy a huge number of *FTR*, for which he would incur the associated premiums. Therefore, a speculator or a hedger limits his selection of the subset of congested lines and specifies the level of participation $\Delta f_\ell|_h$ for each line $\ell$ in the selected subset of lines. Instead of the terms congested elements and level of participation, we use the market participant's selection of the binding constraint subset and the *MW* position on each binding constraint.

A key challenge is to identify which constraints are binding for the holding period. A constraint binds when the transfer capability of the system is reached while meeting the demand with the available generation resources in an economic manner. Therefore, the underlying reasons for system congestion intimately depend on the physics of the situation and take into account many additional considerations. The latter include: the economics of the available generation resources and the self-scheduling practices; the maintenance schedules and forced-outage events; the demand requirements, whether fixed or price responsive; and the ever-changing nature of the network topology due to maintenance, forced outages, new transmission equipment, and the way the system and markets are operated. A development of system congestion models along with effective *FTR* portfolio construction is a daunting challenge. Modeling each constraint's transmission usage charge may not be meaningful. Instead, we may wish to consider a set of "focus" constraints that we examine in depth and try to construct a probabilistic model of the transmission usages charges in terms of conditional probability distributions of the transmission usage charges by using historical market outcomes together with values of observable conditioning drivers, such as generation availability, demand levels, system topology, fuel prices, generation outputs of intermittent resources, commitment behavior, and other appropriate explanatory factors. Analysis of past *IGO* data indicates that some constraints bind under "similar" conditions—for instance, during peak demand periods or under certain fuel-price regimes. Under certain conditions for these constraints, the analysis of historical binding frequency as well as of its economic impacts provides some relevant insights into the future behavior. An example of the use of historical data to construct an approximation of the cumulative distribution function of the transmission usage charges is to classify the explanatory drivers of the constraints, which bind under "similar" conditions, into $Q$ non-overlapping system pattern classes, $\mathscr{C}_j, j = 1, \ldots, Q$. Each system pattern class is associated with specific ranges of the values of the explanatory drivers. For each class, we collect the hourly $\mu_\ell$ values and construct the conditional transmission usage charge duration curve (*TUCDC*) with the conditioning being such that the transmission usage charges are for that class of driver range values. We interpret the conditional *TUCDC* to be the complement of the cumulative distribution function of the transmission usage charges $\mu_\ell$ conditioned on the event that the driver values are in that class. The *TUCDC*s are one possible way of modeling transmission usage charges by using historical data.

We note that having a historical transmission usage charge model has certain limitations. System topology, available generation resources, as well as their economics, and the way that power systems are operated are continually changing. Therefore, the historical system congestion pattern may fail to hold in the future. In addition, some constraints that are rather driven by "events", such as forced outages of multiple lines coupled with planned generation outages or by system voltage support capabilities or by system stability limitations, may not be appropriately represented by the transmission usage charges based on historical data.

## III. Proposed Solution Approach

We use the models for the transmission usage charges to categorize the constraints of the system into three classes—"specified congestion participation" ($\mathscr{R}_1$), "zero congestion participation" ($\mathscr{R}_0$) and "do-not-care congestion participation" ($(\mathscr{R}_0 \cup \mathscr{R}_1)^c$). We use the constraints in $\mathscr{R}_1$ and $\mathscr{R}_0$ to identify the subset of node pairs from which the market participant determines the *FTR* in the portfolio that are used to generate the revenues or the needed hedges. The membership of the "do-not-care congestion participation" subset is established in a straightforward manner since this subset is the complement of the union of the two specified subsets. Based on a "threshold" level for a confidence metric, we decide to "accept" or "reject" the transmission usage charge model. In particular, the estimates provided by an "accept" transmission usage charge model are considered to be reliable and close to the actual outcomes over the *FTR* holding period, in contrast with the "reject" ones that do not. For every "accept" transmission usage charge model, we may choose to take a position on a "focus" constraint. In fact, in this work, we denote the $\mathscr{R}_1$ subset to include those constraints in whose transmission usage charge model we have confidence. We specify the $\mathscr{R}_1$ subset to include the constraints on those network elements for which we know that the transmission usage charges are non-zero during the *FTR* holding period. The constraints in the $\mathscr{R}_1$ subset are candidates for the portfolio construction because their transmission usage charges can contribute to the *FTR* portfolio revenues or provide hedges. In the case of a "reject" model, either such an outcome indicates that our modeling approach is not capable of appropriately describing the frequency and economic impacts of the binding constraint or we lack important information on the causal factors. Inability to understand the market outcomes in some reliable manner leads to the conclusion that we are better off by not taking a position on that specific constraint. We include such constraints in the zero congestion participation subset. Since the future occurrences of such cases may result in the reduction of the *FTR* portfolio revenues or incur payments, such network elements are included in the *FTR* portfolio construction in a way that ensures that their associated transmission usage charges cannot impact the revenues or the provision of hedges. The constraints in the do-not-care congestion participation subset are associated with lines whose transmission usage charges are zero for the majority of the hours in the *FTR* holding period so as to have little impact on the *FTR* portfolio revenues or hedging ability. The constraints classified as elements in the do-not-care congestion participation subset are particularly useful in that they serve to augment the solution space of candidates for the *FTR* portfolio construction. Based on purely topological considerations, it is possible to show that there is a limit on the number of lines on which we may specify the flows [22, pp. 56–58]. Even if we specify the market participant's participation so as to have a feasible system, we over-constrain our problem by specifying the flows on lines that are not congested in the *FTR* holding period and, therefore, do not affect the *FTR* revenues. The market participant specifies his requirements in terms of the quadruplets

$$\zeta = \left\{\delta, \ell, z, \mathscr{L}^{[c]}\right\} \tag{7}$$

that state the line $\ell$, its *MW* position $z$, the categorical variable $\delta$, whether it belongs in $\mathscr{R}_0(\delta = 0)$ or $\mathscr{R}_1(\delta = 1)$ and the subset of outaged lines $\mathscr{L}^{[c]}$. For $\delta = 0$, line $\ell$ is an element of $\mathscr{R}_0$ and $z = 0$, since $\ell$ has zero impact on the revenues collected by the market participant. For $\delta = 1$, line $\ell$ is an element of $\mathscr{R}_1$ and the value of $z$ states the *MW* position desired by the market participant under the contingency case $[c]$. For those constraints in $\mathscr{R}_1$, we use predetermined quantities, based on budget constraints along with cost expectations of the auction results, to specify the *MW* amount positions.

The next step is to construct the transactions that satisfy the market participant's requirements, i.e., that induce the desired *MW* position on the lines in the $\mathscr{R}_0$ and $\mathscr{R}_1$ subsets under the specified outages. We describe a transaction $\Omega$ as the triplet

$$\Omega = \{m, n, a\}, \tag{8}$$

where $m$ is the *from* node, $n$ is the *to* node, and $a$ is the *MW* amount. We propose a practical approach for a market participant to determine the number of transactions $U$ that can ensure that the requirements of the $V$ specifications $\zeta^1, \zeta^2, \ldots, \zeta^V$ can be met. The determination of the node pairs $\{m,n\}$ is too large a problem if we consider all the possible nodes pairs in $\mathscr{N}$, since the number is of the order of $\binom{N+1}{2}$. Instead, we select a subset of the nodes by taking into consideration the physical characteristics of the network. This selection explicitly considers the "electrical proximity" of each line in $\mathscr{R}_0$ and $\mathscr{R}_1$ with respect to a node to be selected. We use the *injection shift factors* (*ISF*s) as the measure of the "electrical proximity" [21]. An injection at, or a withdrawal from, a specified node impacts the real power flows on the network lines to a different extent, as indicated by the *ISF*s of such a node. If the value of the *ISF* of a line with respect to a node is close to 1 p.u. in magnitude, then the line is affected markedly by an injection at/withdrawal from that node. We say that the line has a close "electrical proximity" with respect to that node. Our aim is to induce flows on the lines in the subsets $\mathscr{R}_0$ and $\mathscr{R}_1$, without impacting to a great extent the other constraints of the system and, to do so by constructing a subset with as few nodes as possible. From our extensive testing, one practical scheme is to select the terminal nodes of the lines in $\mathscr{R}_0$ and $\mathscr{R}_1$ to construct the subset of nodes. We define the set

$$\mathscr{G} = \{g : g \text{ is either a } from \text{ or a } to \text{ node of line } \ell \text{ in the specification } \zeta^v, v = 1, \ldots, V\}. \quad (9)$$

Clearly, $|\mathscr{G}| \ll \binom{N+1}{2}$. Since not each network node is traded in *FTR* auctions as either a source or a sink node—a so-called *FTR* pricing node—we need a default option whenever a terminal node is not a pricing node. In such a case, we verify whether there is a directly connected pricing node to the terminal node of a line $\ell$ in $\mathscr{R}_0$ and $\mathscr{R}_1$. Whenever two or more such nodes exist, we select the node with the largest magnitude *ISF* for that line $\ell$. In case none of the directly connected nodes is a pricing node, we repeat the search to include additional nodes that are directly connected to the non-pricing nodes found and continue until a pricing node is reached. Examples of such a selection may be found in [22, p. 27, pp. 34–38].

Starting from the subset $\mathscr{G}$, we construct the set $\mathscr{U}$ of $U$ ordered node pairs $\{m,n\}$ with

$$\mathscr{U} = \{\{m,n\} : m,n \in \mathscr{G}, m < n\} \quad (10)$$

and $U = |\mathscr{U}| = \binom{|\mathscr{G}|}{2}$. The node pairs of possible transactions are given by $\{m_u, n_u\} \in \mathscr{U}$, for $u = 1, \ldots, U$ and the transactions by the triplet $\Omega_u = \{m_u, n_u, a_u\}$. The construction requires the determination of the amounts $a_u$ for $u = 1, \ldots, U$. To do so, we write an equation for each $\zeta^v$, $v = 1, \ldots, V$. The transactions must induce the flows $z^v$ in line $\ell^v$ in the contingency case $[c]$ specified by $\zeta^v$, $v = 1, \ldots, V$. We approximate the effect that a transaction has on a line $\ell$ by its *PTDF* for the specified topology [23]. Similarly, we approximate the effect of a transaction $\Omega = \{m,n,a\}$ and with the line of contingency case $[c]$ outaged, on the real power flow on a non-outaged line $\ell'$ by $(\phi_{\ell'}^{\{m,n\}})^{[c]}a$. Therefore, for the outage case $[c]$, the specification $\zeta^v$, $v = 1, \ldots, V$ sets up the requirement

$$\left[ \left(\phi_{\ell}^{\{m_1,n_1\}}\right)^{[c]} \quad \ldots \quad \left(\phi_{\ell}^{\{m_u,n_u\}}\right)^{[c]} \right] \begin{bmatrix} a_1 \\ \vdots \\ a_u \end{bmatrix} = z^v. \quad (11)$$

The set of $V$ requirements thus results in

$$\underline{\tilde{\mathbf{\Phi}}}\underline{\boldsymbol{a}} = \underline{\boldsymbol{z}}, \quad (12)$$

where row $v$ of $\underline{\tilde{\mathbf{\Phi}}}$ is constructed from the *PTDF*s of the network topologies specified in each quadruplet $\zeta^v$. We determine the amounts $a_u$ of the $U$ transactions by solving for $\underline{\boldsymbol{a}}$ in (12). The rank of a matrix is bounded above from $min\{\text{number of rows, number of columns}\}$. The dimension of the matrix $\underline{\tilde{\mathbf{\Phi}}}$ is $V \times U$. Since $V < U$ and the lines in $\mathscr{R}_0 \cup \mathscr{R}_1$ do not form a loop, we note that

$$U > rank(\underline{\tilde{\mathbf{\Phi}}}) = rank\left(\left[\underline{\tilde{\mathbf{\Phi}}} \vdots \underline{\boldsymbol{z}}\right]\right). \quad (13)$$

The equality in (13) shows that there exists at least one solution to the system of equations and the inequality that the system in undetermined, since the number of unknowns is greater than the linearly independent vectors of $\underline{\tilde{\mathbf{\Phi}}}$.

Therefore a unique solution is possible only if additional constraints are imposed. We formulate the optimization problem of minimizing the $p$-norm of the vector $\underline{\boldsymbol{a}}$ subject to the constraints described by the system (12).

$$\begin{aligned} &min && \|\underline{\boldsymbol{a}}\|_p \\ &subject\ to && \\ & && \underline{\tilde{\mathbf{\Phi}}}\underline{\boldsymbol{a}} = \underline{\boldsymbol{z}}. \end{aligned} \quad (14)$$

For the choice $p = 2$, the solution may result in a large number of elements of $\underline{\boldsymbol{a}}$ that are non-zero, leading to incurred transaction costs for the purchase of the corresponding *FTR* in the portfolio. Such a choice is thus impractical.

The choice $p = 0$ minimizes the $\ell_0$ norm of $\underline{\boldsymbol{a}}$, $\|\underline{\boldsymbol{a}}\|_0 = \lim_{p \to 0} \sum_{u=1}^{U} |a_u|^p$, and determines the minimum number of non-zero transactions amounts that satisfy the constraints. There are two main reasons we choose to construct an *FTR* portfolio with the minimum number of nodes. If we have an *FTR* portfolio with minimum number of node pairs, the *FTR* revenues are primarily influenced by the transmission usage charges of constraints "around" those nodes. In addition, it is highly impractical to participate in the *FTR* auction for the purposes to procure a large number of *FTR*. Such an effort increases the uncertainty of actually having the submitted bids cleared and incurs the premium payment for all acquired *FTR*. Instead, we choose the minimum number of *FTR* to lessen unintended consequences. Computationally, the $\ell_0$ norm optimization problem—referred to as the sparse approximation problem [24]—is hard because of its highly nonlinear nature. A more practical approach is the orthogonal matching pursuit (*OMP*) algorithm that solves (14) using a "greedy" scheme that constructs an approximation of the solution by an iterative process [25], [26]. For the *LMP*-difference-based methods, given the large number of possible node pairs of the order $O((N+1)^2)$, the prediction of a corresponding

TABLE I
SPECIFICATION OF SUBSET $\mathscr{R}_0$ OF THE MARKET PARTICIPANT FOR THE SEPTEMBER 2010 PERIOD

| line | $(2682, 2727)$ | $(2682, 2802)$ | $(700, 1544)$ | $(2710, 2761)$ |
|---|---|---|---|---|
| contingency case | $\{(2511, 2512)\}$ | $\{(2439, 2452)\}$<br>$\{(2511, 2512)\}$ | only for base case | $\{(2439, 2511)\}$ |

TABLE II
SPECIFICATION OF SUBSET $\mathscr{R}_1$ OF THE MARKET PARTICIPANT FOR THE SEPTEMBER 2010 PERIOD

| line | $(5689, 5459)$ | $(2664, 2586)$ | $(2624, 2472)$ | $(2276, 2374)$ |
|---|---|---|---|---|
| contingency case | $\{(4994, 4995)\}$ | $\{(2439, 2511)\}$ | $\{(1622, 8405)\}$ | $\{(2330, 2331)\}$ |
| level of participation | 50 | 10 | 30 | 50 |

number of *LMP* difference variables is computationally highly demanding for a large-scale network. However, the number of congested network elements in the system is, typically, considerably smaller. With the proposed methodology, the prediction of the corresponding transmission usage charges becomes a computation of the order of $O(N+1)$. We further increase the computational efficiency of the proposed methodology *FTR* portfolio selection, by focusing on a judiciously selected subset of possible *FTR* sink or source nodes given by the set $\mathscr{G}$. As the $\ell_0$ and $\ell_1$ norms are closely related, the minimization of the $\ell_0$ norm of $\underline{\boldsymbol{a}}$ results in also reducing the $\ell_1$ norm of $\underline{\boldsymbol{a}}$. A mixed integer linear programming solution approach is an equally valid alternative method for the optimization problem in (14). Once $\underline{\boldsymbol{a}}$ is determined, by minimizing $\|\underline{\boldsymbol{a}}\|_0$, we know the transactions $\Omega_u = \{m_u, n_u, a_u\}, u = 1, \ldots, U$, $a_u \neq 0$, and construct the subset $\mathscr{U}' \subset \mathscr{U}$ with $K$ elements, $K \leq U$, that have $a_u \neq 0$. Since these $K$ transactions satisfy the $V$ market participant specifications, we associate each transaction with the *FTR* for the node pair $\{m_u, n_u\}$ in the amount $a_u$, $u \in \mathscr{U}'$. We construct the portfolio $\mathscr{F} = \{\Gamma_1, \ldots, \Gamma_K\}$, whose element $k$ corresponds to the $\mathscr{U}'$ element given by

$$\Gamma_k = \{m_k, n_k, a_k\},\ k = 1, \ldots, K. \tag{15}$$

The determination of $\mathscr{U}'$ and the quantities $a_u$, $u \in \mathscr{U}'$, allows the identification of each $\Gamma_k$ selected to be in the *FTR* portfolio $\mathscr{F}$ under the condition that all the market participant's requirements are satisfied. The construction is complete. We can take into account the existing *FTR* impacts by evaluating the flows that the transactions, with the same node pairs and in the same amounts as those held in the *FTR* portfolio, induce on the constraints in the two subsets by making detailed use of the appropriate *PTDF*s. Such flows, therefore, modify the specified levels of participation for the constraints in the "specified congestion participation" and "zero congestion participation" subsets. Thus, the incorporation of existing *FTR* requires the modification of the limits so as to explicitly take into account the impacts of the *MW* flows associated with the existing *FTR* in the specification of the participation levels for the constraints in the two subsets.

## IV. NUMERICAL RESULTS

We illustrate the application of the proposed approach on the test system $S$, which is a large-scale system based on the *PJM* network. System $S$ has 14 322 nodes and 19 787 lines. We present the results here that are representative of the extensive tests of the proposed methodology we have carried out on various systems [22, pp. 34–52]. We provide a wide range of sensitivity studies to illustrate the robustness of the optimized portfolio construction methodology. In these studies, we demonstrate the mechanics of the construction algorithm and discuss how we make use of the physical grid characteristics and the insights we gain from the solution. To make the discussion of the numerical results more manageable, we use a single (September 2010) and a three-month (January–March 2011) holding period for *FTR* contracts in the portfolio. The *FTR* market participant makes his *FTR* portfolio selection before the *FTR* auctions, which take place close to a month before the start date of the holding period in the case of monthly auctions. In order to so, the *FTR* market participant constructs models for the transmission usage charges to specify constraints in the $\mathscr{R}_0$ and $\mathscr{R}_1$ subsets, as well as the associated *MW* positions. In these simulation studies, we are given the $\mathscr{R}_0$ and $\mathscr{R}_1$ and the associated *MW* positions and we focus on the analysis of only the *FTR* portfolios revenues, the *LMP* differences of certain node pairs, and the transmission usage charges of the network elements' constraints of the *FTR* holding periods.

The input data for the portfolio construction are in Tables I and II. The contingency cases are selected based on past history of congestion causality. We deliberately chose a small number of elements in the subsets $\mathscr{R}_0$ and $\mathscr{R}_1$ so as to allow the reader to focus on the nature of the results obtained.

We construct the sets $\mathscr{G}$ and $\mathscr{U}$, as defined in (9) and (10). To ensure that the selection of the set $\mathscr{G}$ is sound, we ascertain that the *ISF*s of the each line in the two subsets with respect to their terminal nodes are high. For example, the value of the *ISF* for the line (2682,2727) with respect to an injection at node 2682 (2727) is 0.2120 ($-0.3803$), which are indeed the third and first largest in magnitude for all the nodes in the test system $S$. The magnitude of each *ISF* for every line $\ell$ in $\mathscr{R}_0 \cup \mathscr{R}_1$, with respect to an injection/withdrawal at the majority of the nodes in $\mathscr{G}$, exceeds 0.04.

TABLE III
*FTR* PORTFOLIO $\mathscr{F}_0$ FOR THE SEPTEMBER 2010 PERIOD

| $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\Gamma_5$ | $\Gamma_6$ |
|---|---|---|---|---|---|
| $\{700, 1544, 123\}$ | $\{2624, 1544, 50\}$ | $\{2761, 1544, 3\}$ | $\{2276, 2710, 3\}$ | $\{2276, 5459, 80\}$ | $\{2664, 2586, 14\}$ |

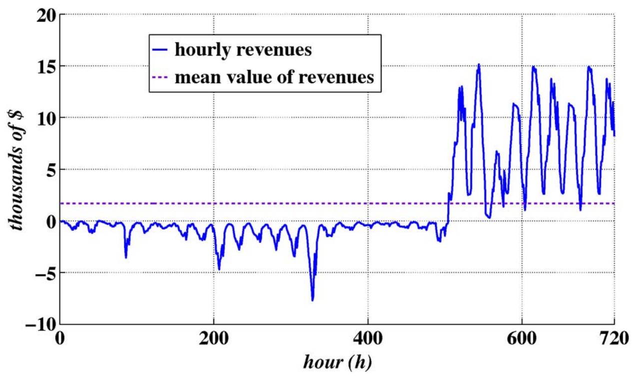


Fig. 1. Hourly revenues for the *FTR* portfolio $\mathscr{F}_0$ for the September 2010 period.

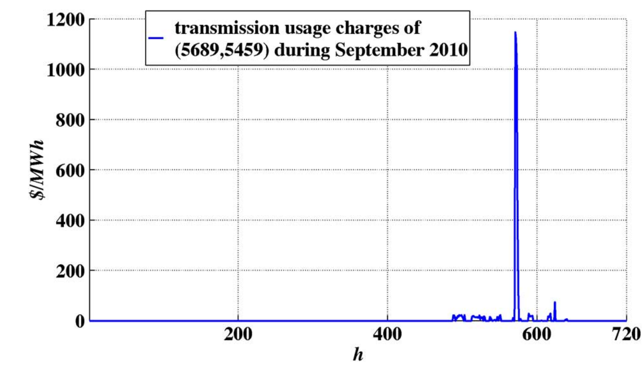


Fig. 2. Hourly transmission usage charges of the line (5689,5459) for the September 2010 period.

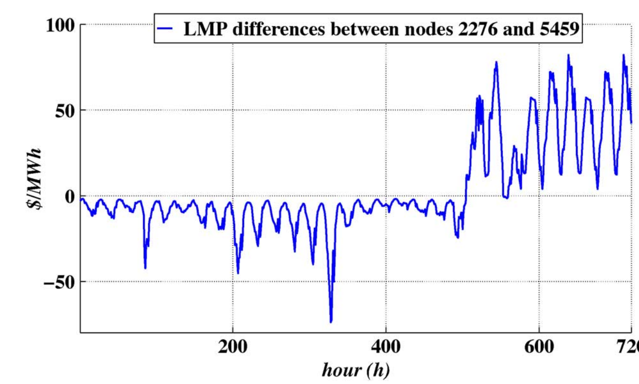


Fig. 3. Hourly *LMP* differences between nodes 2276 and 5459 for the September 2010 period.

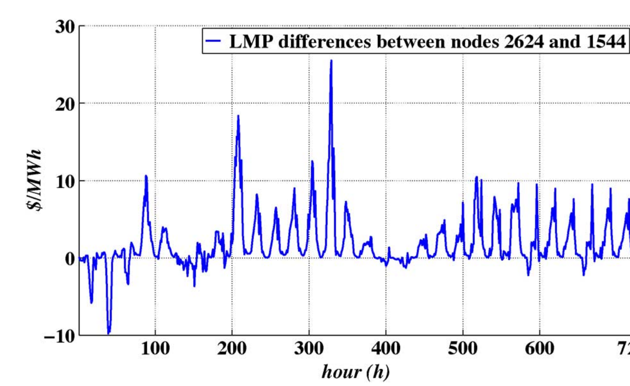


Fig. 4. Hourly *LMP* differences between the nodes 2624 and 1544 for the September 2010 period.

We solve the optimization problem in (14) and the resulting solution consists of the *FTR* portfolio shown in Table III. The market participant purchases the set of *FTR* $\mathscr{F}_0$ and collects the revenues for the month of September 2010. The revenues amount to \$0.3253 million, based on the historical *DAM* outcomes for September 2010. To gain some insights into these revenues we provide in Fig. 1, the hourly revenues of $\mathscr{F}_0$ in September 2010. The negative revenues for the first part of the month are due to the fact that the transactions associated with the *FTR* in $\mathscr{F}_0$ have impacts on the flows on the lines that do not belong in $\mathscr{R}_0 \cup \mathscr{R}_1$, and also that the actual contingencies are different than those in the specifications of $\mathscr{R}_0$ and $\mathscr{R}_1$. The hourly transmission usage charges of the line (5459,5689) specified in $\mathscr{R}_1$ are shown in Fig. 2. While there is a small number of hours with high transmission usage charges, the total revenues are sufficiently high to include line (5459,5689) in $\mathscr{R}_1$, since its inclusion brings a sizeable contribution. Line (5459,5689) plays a key role in the *LMP* difference of the *FTR* $\Gamma_5$, whose *LMP* difference is given in Fig. 3. Given that the line (5689,5459) has the *PTDF* 0.620 for the transaction from node 2276 to node 5459, the result of (3) makes amply clear the relationship between the plots in Figs. 2 and 3.

An additional aspect that makes the solution interesting is that there is some diversity, which demonstrates the advantages of holding a portfolio over a position in single node pair *FTR*. While the total revenues of $\mathscr{F}_0$ are negative in the first days of the month, there are *FTR* in the portfolio $\mathscr{F}_0$ that have positive revenues for those hours. For example, the *FTR* $\Gamma_2$ results in positive revenues for the hours 200 to 400, due to the positive *LMP* differences between nodes 2642 and 1544 in those hours, as seen in Fig. 4. The presence of diversity reduces the negative revenues of those hours.

Next, we focus our discussion on the robustness of the optimized portfolio $\mathscr{F}_0$. We ran a set of sensitivity studies on the base case discussed above. The case studies are concerned with the replacement of the equality constraints for lines that belong in $\mathscr{R}_1$, by inequality constraints with a tunable parameter $\epsilon$:

$$\underline{\boldsymbol{z}}(1-\epsilon) \leq \underline{\tilde{\boldsymbol{\Phi}}\boldsymbol{a}} \leq \underline{\boldsymbol{z}}(1+\epsilon). \tag{16}$$

We vary the parameter $\epsilon$ that belongs in [0, 0.10] in steps of 0.005. As the value of $\epsilon$ increases, we augment the feasibility

TABLE IV
RESULTS OF THE SENSITIVITY CASES FOR THE SEPTEMBER 2010 PERIOD

| **Sensitivity cases** $\epsilon$ | **Symbol** | ***FTR* portfolio elements** | **total *FTR*** $MW$ **amount** | **revenues million ($)** |
|---|---|---|---|---|
| 0.025 | $\mathscr{F}_{0.025}$ | $\{\{700, 1544, 119\}, \{2624, 1544, 48\}, \{2761, 1544, 3\}, \{2276, 2710, 3\}, \{2276, 5459, 78\}, \{2664, 2586, 14\}\}$ | 265 | 0.3187 |
| 0.05 | $\mathscr{F}_{0.05}$ | $\{\{700, 1544, 117\}, \{2624, 1544, 48\}, \{2761, 1544, 3\}, \{2276, 2710, 3\}, \{2276, 5459, 76\}, \{2664, 2586, 14\}\}$ | 261 | 0.3108 |
| 0.075 | $\mathscr{F}_{0.075}$ | $\{\{700, 1544, 113\}, \{2624, 1544, 46\}, \{2761, 1544, 3\}, \{2276, 2710, 3\}, \{2276, 5459, 74\}, \{2664, 2586, 13\}\}$ | 252 | 0.3026 |
| 0.10 | $\mathscr{F}_{0.10}$ | $\{\{700, 1544, 110\}, \{2624, 1544, 44\}, \{2761, 1544, 3\}, \{2276, 2710, 3\}, \{2276, 5459, 72\}, \{2664, 2586, 13\}\}$ | 245 | 0.2944 |

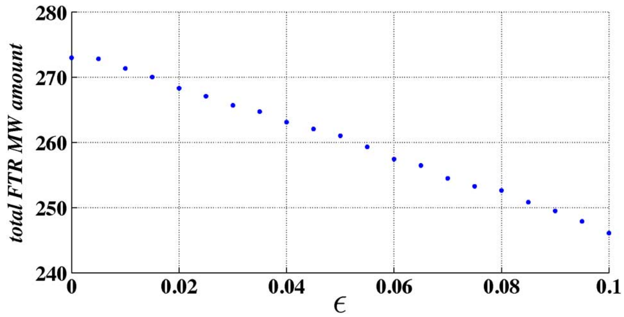


Fig. 5. Variation of the sum of the *FTR* amounts as a function of the tolerance $\epsilon$.

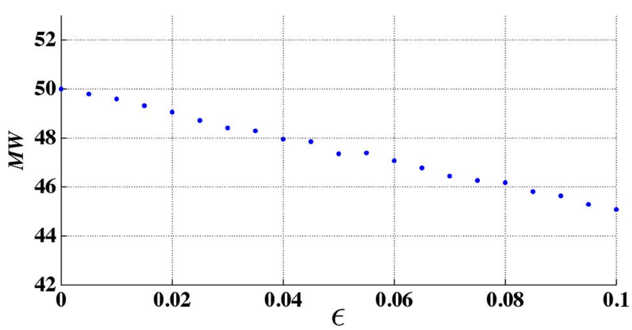


Fig. 6. Participation in *MW* on line (5689,5459), with outaged line (4994,4995), as a function of the tolerance $\epsilon$.

region of the optimization problem. The total amount of the resulting *FTR* portfolios is a monotonically non-increasing function of $\epsilon$, as shown in Fig. 5. This is true because as the feasibility region is augmented, the $\ell_1$ norm of vector $\underline{\boldsymbol{a}}$ decreases, i.e., the total *FTR MW* amount. We display in Table IV some of the portfolios corresponding to different values of $\epsilon$ with the associated revenues. We notice that the proposed methodology is robust, since the elements in the portfolios for the sensitivity cases have the same node pairs. In the sensitivity cases, we note that the revenues decrease with the total *FTR MW* amount. To understand the reasons for the decreasing revenues, we need to focus on the level of participation on the congested lines. The level of participation is no longer defined by the market participant and is a result of the optimization problem. For example the level of participation on line (5689,5459), which belongs to $\mathscr{R}_1$, decreases as $\epsilon$ increases, as shown in Fig. 6. For $\epsilon \neq 0$, the market participant fails to benefit as much from the high values of the transmission usage charges for the line and has lower revenues than for $\epsilon = 0$.

TABLE V
SPECIFICATION OF SUBSET $\mathscr{R}'_0$ OF THE MARKET PARTICIPANT FOR THE JANUARY TO MARCH 2011 PERIOD

| | | |
|---|---|---|
| line | (2472, 2624) | (169, 200) |
| contingency | {(8405, 8518)} | {(167, 168)} |

In order to illustrate the application of the proposed methodology to a longer holding period, we construct the optimized *FTR* portfolio for the three-month holding period of January–March 2011. The $\mathscr{R}'_0$ and $\mathscr{R}'_1$ subsets are given in Tables V and VI. We construct the sets $\mathscr{G}'$ and $\mathscr{U}'$, as defined in (9) and (10), respectively. We solve the optimization problem (14) and the resulting solution consists of *FTR* as shown in Table VII. The revenues amount to \$6.3891 million, based on the historical *DAM* outcomes for January to March 2011. To gain some insights into these revenues, we provide in Fig. 7 the hourly revenues of $\mathscr{F}'$ during the three-month holding period. We depict in Fig. 8 the *LMP* difference between nodes 2727 and 2682. As we may see, the *LMP* difference is large at two points in time. This is due to the fact that another line outage, that of line (2511,2512), caused the real power flow on line (2727,2682) to reach its limiting value. As in the one-month holding period, we run sensitivity cases that indicate that the proposed methodology is robust, since the elements in the portfolios for the sensitivity cases have the same node pairs and decreasing amounts as we vary $\epsilon$. The results are shown in Fig. 9.

The results of the representative cases presented here indicate the effectiveness of the proposed approach in the construction of robust portfolios over a broad range of conditions.

## V. CONCLUDING REMARKS

In this paper, we present a practical methodology for a market participant to systematically construct his *FTR* portfolio, making ample use of historical data and mathematical insights we garnered from the analysis of the underlying problem. This *FTR* portfolio construction approach allows a market participant to express his views on the transmission usage charges of a set of focus network element constraints. The categorization of the system constraints into three non-overlapping sets takes explicitly into account each participant's capability to model the uncertainty in the transmission usage charges. Given the specified *MW* participation levels of the focus constraints, the proposed approach selects a subset of nodes for possible *FTR* source or sink nodes, and constructs the portfolio with the

TABLE VI
SPECIFICATION OF SUBSET $\mathscr{R}'_1$ OF THE MARKET PARTICIPANT FOR THE JANUARY TO MARCH 2011 PERIOD

| line | (9230, 9229) | (2444, 2443) | (2727, 2682) | (5664, 2482) |
|---|---|---|---|---|
| contingency case | base case | base case | $\{(2511, 2512)\}$ | $\{(2883, 8286)\}$ |
| level of participation | 100 | 40 | 20 | 20 |

TABLE VII
*FTR* PORTFOLIO $\mathscr{F}'$ FOR THE JANUARY TO MARCH 2011 PERIOD

| $\Gamma'_1$ | $\Gamma'_2$ | $\Gamma'_3$ |
|---|---|---|
| $\{2682, 9229, 32\}$ | $\{2472, 2444, 219\}$ | $\{2682, 2472, 21\}$ |

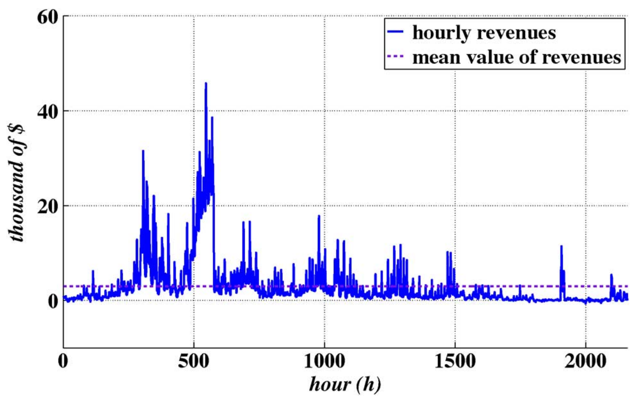


Fig. 7. Hourly revenues for the *FTR* portfolio $\mathscr{F}'$ for the period January to March 2011.

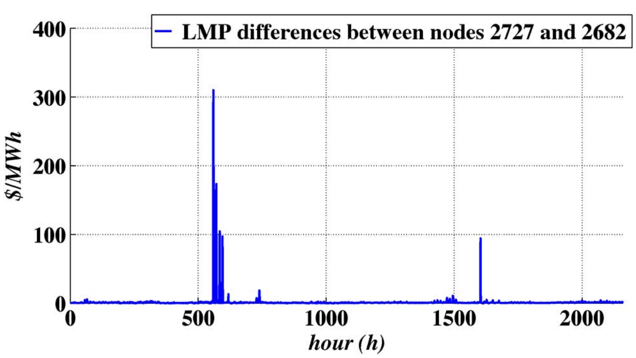


Fig. 8. *LMP* differences between nodes 2727 and 2682 for the January to March 2011 period.

minimum number of node pairs by determining the minimum number of transactions that induce the desired real power flows on the specified constraints. We illustrate the application of the proposed methodology to *PJM*, the largest electricity market in North America. The representative cases clearly demonstrate the capabilities of the proposed methodology.

There are natural extensions of the work presented here. The portfolio definition may be generalized to include additional *FTR* holdings of different periods, classes, and types. As the system topology changes from hour to hour, the actual system deviates from the model used in the *FTR* auction representation, that is based on a "fixed" topology. In the future work, we will consider such deviations in our optimization framework

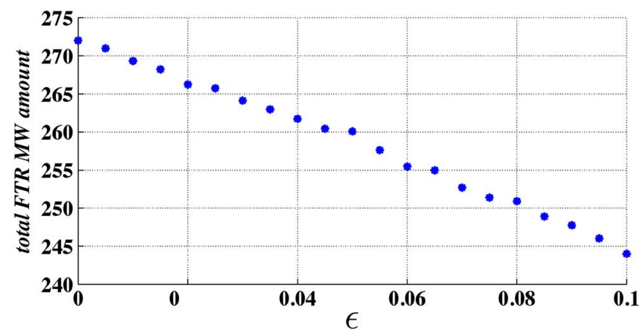


Fig. 9. Variation of the sum of the *FTR* amounts as a function of the tolerance $\epsilon$.

to improve on the effectiveness of the portfolio for the specified constraints. In the current approach, we specify the *MW* amounts of the *FTR* holdings without considering the ability to procure them through the auction. In our future studies, we will change the deterministic approach to a stochastic one so as to include the uncertainties in the *FTR* auction by incorporating appropriate bidding strategies. We will report on these developments in future papers.

## APPENDIX A
## NOMENCLATURE

| | |
|---|---|
| $\|\underline{\boldsymbol{a}}\|_p$ | $p$-norm of vector $\underline{\boldsymbol{a}}$. |
| $\mathscr{F}$ | *FTR* portfolio. |
| $\mathscr{G}$ | Subset of nodes selected to construct $\mathscr{F}$. |
| $K$ | Number of *FTR* in $\mathscr{F}$. |
| $\mathscr{L}\|_h$ | Set of connected lines in the network in hour $h$. |
| $\tilde{\mathscr{L}}\|_h$ | Set of congested lines in hour $h$. |
| $\mathscr{L}^{[c]}$ | Subset of outages lines. |
| $\mathscr{C}_j$ | System pattern class $j$. |
| $\mathscr{R}_0$ | "Zero congestion participation" subset. |
| $\mathscr{R}_1$ | "Specified congestion participation" subset. |
| $(\mathscr{R}_0 \cup \mathscr{R}_1)^c$ | "Do-not-care congestion participation" subset. |
| $\mathscr{T}$ | *FTR* holding period. |
| $\mathscr{U}$ | Set of ordered node pairs that belong in $\mathscr{G}$. |
| $U$ | Cardinality of the set $\mathscr{U}$. |
| $z$ | *MW* position on a constraint. |
| $\Gamma = \{i, j, \gamma\}$ | *FTR* with the source (sink) node $i$ ($j$) in the amount $\gamma$ *MW*. |
| $\delta$ | Categorical variable, determines if a line belongs to $\mathscr{R}_0$ or $\mathscr{R}_1$. |

| | |
|---|---|
| $\Delta f_\ell\|_h$ | Change in the real power flow at line $\ell$ in hour $h$. |
| $\zeta$ | Quadruplet that specifies the market participant's requirements. |
| $\eta\|_h$ | $\Gamma$ *FTR* revenues in hour $h$. |
| $\lambda_n\|_h$ | *LMP* at node $n$ in hour $h$. |
| $\mu_\ell\|_h$ | Transmission usage charge for line $\ell$ in hour $h$. |
| $\mu_\ell^M\|_h$ | Dual variable for the transmission constraint in the forward direction in hour $h$. |
| $\mu_\ell^m\|_h$ | Dual variable for the transmission constraint in the reverse direction in hour $h$. |
| $\phi_\ell^{\{n,n'\}}\|_h$ | Line $\ell$ *PTDF* with respect to node pair $\{n,n'\}$ in hour $h$. |
| $\underline{\tilde{\boldsymbol{\Phi}}}$ | Matrix of the *PTDF*s with respect to the node pairs in $\mathscr{W}$ of the lines and for the network topologies specified by $\zeta$s with row $v$ corresponding to $\zeta^v$. |
| $\Omega=\{m,n,a\}$ | Transaction from node $m$ to node $n$ in the amount $a$ *MW*. |

## APPENDIX B
## *DAM* CLEARING MECHANISM

We consider a power system with the set of $(N+1)$ nodes $\mathscr{N} = \{0,1,\ldots,N\}$, with the slack bus at node 0, and the set of $L$ lines $\mathscr{L}=\{\ell_1,\ldots,\ell_L\}$. We denote each line by the ordered pair $\ell=(n,m)$, where $n$ is the *from* node, and $m$ is the *to* node with $n,m\in\mathscr{N}$, with the real power flow $f_\ell\geq 0$ whenever the flow is from $n$ to $m$ and $f_\ell<0$ otherwise. We assume the network to be *lossless*. We denote the diagonal branch susceptance matrix by $\underline{\boldsymbol{B}}_d\in\mathbb{R}^{L\times L}$ and the reduced branch-to-node incidence matrix for the subset of nodes $\mathscr{N}/\{0\}$ by $\underline{\boldsymbol{A}}\in\mathbb{R}^{L\times N}$. The corresponding nodal susceptance matrix is $\underline{\boldsymbol{B}}\in\mathbb{R}^{N\times N}$. For the simplicity of the discussion, we assume the network contains no phase shifting devices. We denote the slack bus nodal susceptance vector by $\underline{\boldsymbol{b}}_0=[b_{01},\ldots,b_{0N}]^T$ and $\underline{\boldsymbol{b}}_0+\underline{\boldsymbol{B}}\underline{\boldsymbol{1}}^N=\underline{\boldsymbol{0}}$, where $\underline{\boldsymbol{1}}^N$ is the $N$-dimensional vector with each element with value 1. The network characteristics are a function of time and we use the notation $|_h$ to denote the value of each parameter, matrix, or variable for the snapshot corresponding to the hour $h$ of the holding period.

We state the hour $h$ *DAM* problem for the set of sellers $\mathscr{S}|_h \triangleq \{s_1|_h,\ldots,s_S|_h\}$ and the set of buyers $\mathscr{B}|_h \triangleq \{b_1|_h,\ldots,b_B|_h\}$. The objective function in the hour $h$ *DAM* clearing is to maximize the social surplus of the buyers in $\mathscr{B}|_h$ and the sellers in $\mathscr{S}|_h$. The hour $h$ power injection $p_n^e|_h$ and withdrawal $p_n^x|_h$ at each $n\in\mathscr{N}$ are given by

$$p_n^e|_h=\sum_{\substack{s_i|_h\in\mathscr{S}|_h \text{ is}\\ \text{at node } n}} p^{s_i|_h} \quad\text{and}\quad p_n^x|_h=\sum_{\substack{b_j|_h\in\mathscr{B}|_h \text{ is}\\ \text{at node } n}} p^{b_j|_h}.$$

The hour $h$ *DAM* optimization problem statement is[2]

$$\begin{aligned} &max && \left\{\sum_{j=1}^{B|_h}\beta^{b_j|_h}\left(p^{b_j|_h}\right)-\sum_{i=1}^{S|_h}\kappa^{s_i|_h}\left(p^{s_i|_h}\right)\right\}\\ &subject\ to && \\ & && \underline{\boldsymbol{p}}^e|_h-\underline{\boldsymbol{p}}^x|_h=\underline{\boldsymbol{B}}|_h\underline{\boldsymbol{\theta}}|_h \quad \longleftrightarrow \underline{\boldsymbol{\lambda}}|_h\\ & && p_0^e|_h-p_0^x|_h=\underline{\boldsymbol{b}}_0^T|_h\underline{\boldsymbol{\theta}}|_h \quad \longleftrightarrow \lambda_0|_h\\ & && \underline{\boldsymbol{f}}|_h=\underline{\boldsymbol{B}}_d|_h\underline{\boldsymbol{A}}|_h\underline{\boldsymbol{\theta}}|_h\leq\underline{\boldsymbol{f}}^M|_h \longleftrightarrow \underline{\boldsymbol{\mu}}^M|_h\\ & && -\underline{\boldsymbol{f}}|_h\leq\underline{\boldsymbol{f}}^m|_h \quad \longleftrightarrow \underline{\boldsymbol{\mu}}^m|_h. \end{aligned} \tag{17}$$

The network topology as well as the line flow limits may vary with time during the *FTR* holding period. Such effects are not captured by the *FTR* auction, but are explicitly represented in the revenues realized as a result of the *DAM* clearing outcomes. In (17), we also indicate the dual variables that correspond to the various constraints. In *FTR* analysis, the *LMP*s $\underline{\boldsymbol{\lambda}}|_h$ and $\lambda_0|_h$ are of particular interest. A non-zero *LMP* difference between two nodes signals that one or more line flows are at their limiting values, i.e., one or more transmission constraints is binding. The dual variables of the transmission constraints are $\underline{\boldsymbol{\mu}}^M|_h$ and $\underline{\boldsymbol{\mu}}^m|_h$. At any point in time, at most one of the limits may be binding and so either $\underline{\boldsymbol{\mu}}^M|_h$ or $\underline{\boldsymbol{\mu}}^m|_h$ is 0 or both are 0, we can write $\underline{\boldsymbol{\mu}}|_h=\underline{\boldsymbol{\mu}}^M|_h-\underline{\boldsymbol{\mu}}^m|_h$ to be the transmission usage charge vector for the $L|_h$ lines that are available in hour $h$.

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