

# Integration of Price-Based Demand Response in DisCos' Short-Term Decision Model

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**Abstract**—Real-time electricity prices along with demand-side potentials can provide distribution companies (DisCos) with considerable financial and technical benefits compared to the conventional flat prices. This paper incorporates demand response in DisCos' short-term decision model in a real-time pricing (RTP) environment wherein consumers are charged based on hourly varying prices. Besides the hourly RTP sale prices, the established model deals with other DisCo's short-term activities including hourly purchases from the grid, commitment of distributed generation (DG) units, dispatch of shunt compensators, and invocation of load curtailments (LCs). The stochastic nature of wholesale market prices and customers load is also considered in the model. The model is a mixed integer linear programming (MILP) problem which can be easily solved via commercial software packages. The objective is to maximize the DisCo's expected profit while its revenue is limited by regulating bodies. A typical Finnish 20 kV urban distribution network is used to demonstrate the effectiveness of the established model. Simulation results are presented and discussed to investigate the impacts on both financial and technical aspects of using of RTP sale prices.

**Index Terms**—Demand response, distribution company, real-time electricity pricing, short-term operation.

## NOMENCLATURE

### Indices and Sets

$f, F$	Index and set of distribution feeders.
$g, G$	Index and set of DG units.
$G_i$	Set of units connected to bus $i$ .
$i, i', I$	Indices and set of buses.
$h, h', H$	Indices and set of time periods, e.g., hours.

$t, T$	Index and set of consumer types.
$c, C$	Index and set of likely contingencies.
$\omega, \Omega$	Index and set of likely future scenarios.

### Parameters and Constants

$\alpha_g, \beta_g, \gamma_g$	Cost function coefficients of unit $g$ .
$E_{h,h'}^t$	Price elasticity of load type $t$ indicating how a change in price at hour $h'$ affects the demand at hour $h$ .
$P_{i,t,\omega,h}^{D0}, Q_{i,t,\omega,h}^{D0}$	Active and reactive loads of type $t$ at bus $i$ , hour $h$ , and scenario $\omega$ before response to time-varying sale prices.
$R_g^{UP}, R_g^{DN}$	Ramp-up/down limits of unit $g$ .
$SU_g, SD_g$	Start-up and shut-down costs of unit $g$ .
$T_g^{UP}, T_g^{DN}$	Minimum up/down times of unit $g$ .
$g_{ii'}^f, b_{ii'}^f$	Real and imaginary parts of feeder $f$ admittance.
$r_{ii'}^f, x_{ii'}^f$	Real and imaginary parts of feeder $f$ impedance.
$p_\omega$	Probability of scenario $\omega$ .
$p_c, T_c$	Probability and duration of contingency $c$ .
$\rho_{\omega,h}^{active}$	Wholesale price of active energy at scenario $\omega$ and hour $h$ .
$\rho_{\omega,h}^{reactive}$	Wholesale price of reactive energy at scenario $\omega$ and hour $h$ .
$\rho_{i,t}^{LC}$	VOLL of load type $t$ curtailed by LC at bus $i$ .
$\lambda_t^{average}$	Maximum allowed average service price offered to consumers of type $t$ .

### Functions and Variables

$EDNS$	Expected demand not supplied.
$EENS$	Expected energy not served.
$ECDC$	Expected consumers damage cost.
$s_{g,\omega,h}$	Commitment state of unit $g$ at scenario $\omega$ and hour $h$ .
$P_{ii',\omega,h}^f, Q_{ii',\omega,h}^f$	Active and reactive powers of feeder $f$ from bus $i$ to bus $i'$ at scenario $\omega$ and hour $h$ .

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$P_{ii',\omega,h}^{loss^f}$	Active power loss of feeder $f$ from bus $i$ to bus $i'$ at scenario $\omega$ and hour $h$ .
$Q_{ii',\omega,h}^{loss^f}$	Reactive power loss of feeder $f$ from bus $i$ to bus $i'$ at scenario $\omega$ and hour $h$ .
$S_{ii',\omega,h}^f$	Apparent power of distribution feeder $f$ from bus $i$ to bus $i'$ at scenario $\omega$ and hour $h$ .
$S_{i,\omega,h}$	Apparent power of bus $i$ at scenario $\omega$ and hour $h$ .
$P_{i,t,\omega,h}^{LC}, Q_{i,t,\omega,h}^{LC}$	Active and reactive powers of type $t$ reduced by LC at bus $i$ , scenario $\omega$ , and hour $h$ .
$P_{i,t,c}^{LC}$	Active power of type $t$ reduced by LC at bus $i$ and contingency $c$ .
$P_{g,\omega,h}^{DG}, Q_{g,\omega,h}^{DG}$	Active and reactive powers of unit $g$ at scenario $\omega$ and hour $h$ .
$P_{\omega,h}^{grid}, Q_{\omega,h}^{grid}$	Active and reactive powers imported from the external grid at scenario $\omega$ and hour $h$ .
$Q_{i,\omega,h}^{sh}$	Reactive power of shunt compensators at bus $i$ , scenario $\omega$ , and hour $h$ .
$S_{\omega,h}^{grid}$	Apparent power imported from the external grid at scenario $\omega$ and hour $h$ .
$P_{i,t,\omega,h}^D, Q_{i,t,\omega,h}^D$	Active and reactive loads of type $t$ at bus $i$ , scenario $\omega$ , and hour $h$ after response to time-varying sale prices.
$P_{i,\omega,h}^D, Q_{i,\omega,h}^D$	Active and reactive loads at bus $i$ , scenario $\omega$ , and hour $h$ after response to time-varying sale prices and LC.
$V_{i,\omega,h}, \delta_{i,\omega,h}$	Voltage magnitude and phase angle at bus $i$ , scenario $\omega$ , and hour $h$ .
$X_{g,\omega,h}^{on}, X_{g,\omega,h}^{off}$	Binary variables denoting the start-up and shut-down times of unit $g$ at scenario $\omega$ and hour $h$ .
$\lambda_{t,\omega,h}$	Sale price offered to consumers of type $t$ under RTP strategy at scenario $\omega$ and hour $h$ .
$\lambda_t^{flat}$	Sale price offered to consumers of type $t$ under flat pricing strategy.
$\lambda_{t,\omega,h}^{service}$	Service price offered to consumers of type $t$ at scenario $\omega$ and hour $h$ .

## I. INTRODUCTION

### A. Aim

SINCE restructuring of the power system industry, DisCos are responsible to serve electricity requirements of consumers within their territories [1]. They procure electricity from wholesale markets and DG units, sell it to their end-user customers, and simultaneously operate distribution networks. In order for a DisCo to achieve the maximum possible profit, it is essential to take the advantage of all available opportunities.

Nowadays, thanks to recent advancements in smart grid technologies, RTP programs can be employed to use existing resources more efficiently.

Within a daily time framework, this paper aims to establish a short-term decision model for a DisCo in which consumers are charged based on RTP sale prices. The objective of the model is to maximize the DisCo's expected profit while its revenue is capped by regulating bodies.

### B. Literature Review

Thanks to smart grids, increasing attention has been devoted to RTP-based demand response programs. The related works have mostly focused on either consumers' perspectives or utilities' point of view. In [2], [3], consumers' opportunity to alter their load profiles in response to RTP prices, in the hope of achieving savings in their electricity bills, has been shown. However, studies in [4] demonstrated that the lack of effective automation systems can be one of the major barriers for achieving cost reductions. In this regards, most of the recent studies have focused on schemes to simplify consumers' active participation in RTP-based demand response programs. In [5], [6], home energy management systems and the associated decision models have been discussed. However, decision models for DisCos under RTP programs have been rarely touched in the literature. General short-term operation frameworks for DisCos that seek either the minimum cost or the maximum profit have even addressed in a few papers. In [7], most important benefits provided by customer-owned back-up generators were quantified. In [8], potential effects of intermittent energy resources on distribution network losses were evaluated. A qualitative survey on potential opportunities and challenges of demand response programs for distribution grid operations was provided in [9]. In [10], potential benefits of demand response on operation of a distribution network were investigated. In [11], a distributed algorithm was developed for managing demand response in a smart grid such that the most flatten load profile is achieved.

A nodal pricing scheme for DG units reflecting their share in reduced network losses was proposed in [12]. The results demonstrated significant price discrepancies caused by high marginal losses. The incremental contribution of DG units in network losses was integrated into the short-term operation of a DisCo that wishes to minimize its hour-ahead energy cost [13]. Reference [14] developed an energy acquisition model for a DisCo that has multiple electricity sources including grid purchases, investor-owned DGs, DisCo-owned DGs, and LC options. In [15], the model developed in [14] was extended to include the dynamic behavior of other DisCos. The extended model is a bi-level optimization problem wherein upper sub-problems maximize individual DisCos' profits while lower sub-problem maximizes total social welfare. In [16], the model developed in [14] and extended by [15] was extended again to incorporate the ac power flow constraints. The impact of different time varying electricity pricing schemes on energy acquisition of a DisCo was studied in [17]. A unified operation model was proposed in [18] wherein the network topology and hourly marginal prices are determined such that social welfare is maximized. A two-stage hierarchical framework was developed in [19] to optimize DisCos' decisions within a short-term horizon. In the first stage, electricity purchases from a day-ahead market and commitment of DG units are

determined, whereas dispatch of online DG units, real-time market exchanges, and LC invocations are the decisions made in the second stage. The model developed in [19] is based on a nonlinear programming that may face convergence problems. The model was extended in [20] to incorporate stochastic nature of real-time market prices and consumers load. Moreover, the extended model is in a MILP format which ensures the global optimal solution of the problem.

Beside the reviewed papers, there are a few other works focused on distribution network operation such as network reconfiguration for loss reduction purposes [21], charging coordination of electric vehicles for loss reduction and voltage profile improvement purposes [22], and the control of energy storage systems for voltage regulation purposes [23].

### C. Contributions

The main contribution of this paper is to incorporate demand response in DisCos' short-term decision making. A DisCo, besides the common short-term activities, has to decide on hourly RTP sale prices. The established model enables DisCos to realize demand side potentials and thereby, providing them with significant technical and financial benefits. A set of regulatory restrictions on sale prices imposed by regulating bodies are considered in the established model to ensure the fairness of offered prices. The uncertainty associated with wholesale market prices as well as customers demand is considered in the decision making process. The developed model is formulated in a MILP format which can be efficiently solved via commercial software packages. The new model can be easily incorporated into a DisCo's decision making since its computational behavior is robust. In addition, impacts of demand side potentials enabled by the proposed model on technical aspects of distribution network operation such as load and voltage profiles and the service reliability are investigated. Finally, a sensitivity analysis is conducted to highlight the impact of demand behavior uncertainties.

### D. Approach

The proposed model is to optimize short-term activities for a DisCo with natural monopoly. In order for the sale prices to be fair towards consumers, revenue cap constraints are assumed to be imposed by regulatory bodies. The demand behavior in response to time-varying prices is captured via price elasticity coefficients. Power flow equations are based on an approximated linear model whose accuracy has been approved in the literature. The uncertainty associated with wholesale market prices and consumers load is modeled using a scenario-based stochastic approach. Quadratic expressions in the model are linearized using the well-accepted piecewise linear approximation technique. The accuracy of the approximated expressions is guaranteed by selecting a large number of linear pieces. Finally, the model is formulated as a MILP optimization problem which can be effectively solved via commercial software packages.

### E. Paper Organization

The rest of the paper is organized as follows. Section II presents the modeling and formulation of the established short-term operation model faced by a DisCo that acts in a RTP environment. Section III provides and thoroughly discusses the results from a realistic case study. Section IV draws relevant conclusions and concludes the paper.

## II. THE DEVELOPED DECISION MODEL

This section provides a short-term decision model for a price-taker DisCo that offers RTP sale prices to consumers incorporating potential benefits of demand side resources. The complete mathematical formulation of the model, including the objective function and constraints, is described in the following sub-sections.

### A. Objective Function

The objective function is to maximize total expected profit of the DisCo. The profit is equal to the DisCo's revenue earned by selling electricity to the consumers minus electricity procurement and operating costs. The DisCo can procure electricity from the main grid and the DG units connected to the distribution network. It can also invoke a LC when it is unavoidable, but it has to pay for the damages inflicted to the customers due to an electric power interruption. Hence, the DisCo's cost consists of the cost of purchasing energy from the external grid, the production cost of DG units, and the compensation cost of LCs. Therefore, the objective function can be formulated as follows:

$$\begin{aligned}
 \text{Max} & \sum_{\omega \in \Omega} \sum_{i \in I} \sum_{t \in T} \sum_{h \in H} p_{\omega} \lambda_{t,\omega,h} P_{i,t,\omega,h}^D - \sum_{\omega \in \Omega} \sum_{i \in I} \sum_{t \in T} \sum_{h \in H} p_{\omega} \rho_{i,t}^{LC} P_{i,t,\omega,h}^{LC} \\
 & - \sum_{\omega \in \Omega} \sum_{h \in H} p_{\omega} \rho_{\omega,h}^{active} P_{\omega,h}^{grid} - \sum_{\omega \in \Omega} \sum_{h \in H} p_{\omega} \rho_{\omega,h}^{reactive} Q_{\omega,h}^{grid} \\
 & - \sum_{\omega \in \Omega} \sum_{g \in G} \sum_{h \in H} p_{\omega} X_{g,\omega,h}^{on} SU_g \\
 & - \sum_{\omega \in \Omega} \sum_{g \in G} \sum_{h \in H} p_{\omega} \times \left[ \alpha_g s_{g,\omega,h} + \beta_g P_{g,\omega,h}^{DG} + \gamma_g P_{g,\omega,h}^{DG^2} \right] \\
 & - \sum_{\omega \in \Omega} \sum_{g \in G} \sum_{h \in H} p_{\omega} X_{g,\omega,h}^{off} SD_g
 \end{aligned} \tag{1}$$

In (1), the first term is the total revenue earned by selling energy to different types of consumers. The second term represents the cost of compensating LCs. The third and fourth terms denote the cost of active and reactive energies imported from the external grid, i.e., purchased from the wholesale market. Note that the cost of reactive power can be due to wholesale market purchases or penalties imposed by transmission network operators. Also, it can be simply dropped where no price or penalty is allocated for absorbing reactive power. The three last terms respectively represent the start-up, production, and shut-down costs of DG units. Note that the hourly revenue earned by the DisCo is equal to the product of the sale price and load. On the other hand, as will be explained later in (34), elasticity coefficients used for modeling demand response establish a linear relationship between the price and load. So, the revenue is proportional to the square of the sale price. However, this non-linearity does not make any concern because it can be converted into linear expressions employing the piecewise linear approximation method [24]. A brief description on the approximation method is given in Appendix.

### B. Problem Optimization Constraints

The DisCo faces a set of technical and financial constraints which are described next.

**Network Equations:** Network constraints ensure that a network operating point satisfies the power flow equations. The active and reactive powers flowing through a feeder between buses  $i$  and  $i'$  at scenario  $\omega$  and hour  $h$  are as follows:

$$P_{ii',\omega,h}^f = g_{ii'}^f V_{i,\omega,h}^2 - g_{ii'}^f V_{i,\omega,h} V_{i',\omega,h} \cos(\delta_{i,\omega,h} - \delta_{i',\omega,h}) - b_{ii'}^f V_{i,\omega,h} V_{i',\omega,h} \sin(\delta_{i,\omega,h} - \delta_{i',\omega,h}); \quad \forall f \in F, \forall \omega \in \Omega, \forall h \in H \quad (2)$$

$$Q_{ii',\omega,h}^f = -b_{ii'}^f V_{i,\omega,h}^2 - g_{ii'}^f V_{i,\omega,h} V_{i',\omega,h} \sin(\delta_{i,\omega,h} - \delta_{i',\omega,h}) + b_{ii'}^f V_{i,\omega,h} V_{i',\omega,h} \cos(\delta_{i,\omega,h} - \delta_{i',\omega,h}); \quad \forall f \in F, \forall \omega \in \Omega, \forall h \in H \quad (3)$$

In order to keep the problem statement linear, the above non-linear equations are replaced by the following equivalent linear expressions as [25]:

$$P_{ii',\omega,h}^f = g_{ii'}^f V_{i,\omega,h} - g_{ii'}^f V_{i',\omega,h} - b_{ii'}^f \delta_{i,\omega,h} + b_{ii'}^f \delta_{i',\omega,h} + \frac{1}{2} P_{loss_{ii',\omega,h}}^f; \quad \forall f \in F, \forall \omega \in \Omega, \forall h \in H \quad (4)$$

$$Q_{ii',\omega,h}^f = -b_{ii'}^f V_{i,\omega,h} + b_{ii'}^f V_{i',\omega,h} - g_{ii'}^f \delta_{i,\omega,h} + g_{ii'}^f \delta_{i',\omega,h} + \frac{1}{2} Q_{loss_{ii',\omega,h}}^f; \quad \forall f \in F, \forall \omega \in \Omega, \forall h \in H \quad (5)$$

The accuracy of the above equations has been approved in different applications [25], [26]. The active and reactive power balance equations at bus  $i$  and hour  $h$  are as follows:

$$\sum_{g \in G_i} P_{g,\omega,h}^{DG} - \sum_{f \in F \& i' \in I} P_{ii',\omega,h}^f = P_{i,\omega,h}^D; \quad \forall i \neq 1, \forall \omega \in \Omega, \forall h \in H \quad (6)$$

$$\sum_{g \in G_i} Q_{g,\omega,h}^{DG} - \sum_{f \in F \& i' \in I} Q_{ii',\omega,h}^f = Q_{i,\omega,h}^D - Q_{i,\omega,h}^{sh}; \quad \forall i \neq 1, \forall \omega \in \Omega, \forall h \in H \quad (7)$$

The above equations are valid for all buses except the main substation. A main substation bus is a connection point between the external grid and the distribution network. Thus, the energy purchased from the external grid should be imported at this point. The active and reactive power balance equations in the main substation are as follows:

$$P_{\omega,h}^{Grid} + \sum_{g \in G_i} P_{g,\omega,h}^{DG} - \sum_{f \in F \& i' \in I} P_{ii',\omega,h}^f = P_{i,\omega,h}^D; \quad i = 1, \forall \omega \in \Omega, \forall h \in H \quad (8)$$

$$Q_{\omega,h}^{Grid} + Q_{i,\omega,h}^{sh} + \sum_{g \in G_i} Q_{g,\omega,h}^{DG} - \sum_{f \in F \& i' \in I} Q_{ii',\omega,h}^f = Q_{i,\omega,h}^D; \quad i = 1, \forall \omega \in \Omega, \forall h \in H \quad (9)$$

The total active and reactive loads at bus  $i$  at scenario  $\omega$  and hour  $h$  can be determined as follows:

$$P_{i,\omega,h}^D = \sum_{t \in T} (P_{i,t,\omega,h}^D - P_{i,t,\omega,h}^{LC}); \quad i \in I, \forall \omega \in \Omega, \forall h \in H \quad (10)$$

$$Q_{i,\omega,h}^D = \sum_{t \in T} (Q_{i,t,\omega,h}^D - Q_{i,t,\omega,h}^{LC}); \quad i \in I, \forall \omega \in \Omega, \forall h \in H \quad (11)$$

The active and reactive power losses associated with feeder  $f$ , from bus  $i$  to bus  $i'$ , are determined as:

$$P_{loss_{ii',\omega,h}}^f = r_{ii'}^f \times \frac{P_{ii',\omega,h}^f{}^2 + Q_{ii',\omega,h}^f{}^2}{V_{i,\omega,h}^2}; \quad \forall f \in F, \forall \omega \in \Omega, \forall h \in H \quad (12)$$

$$Q_{loss_{ii',\omega,h}}^f = x_{ii'}^f \times \frac{P_{ii',\omega,h}^f{}^2 + Q_{ii',\omega,h}^f{}^2}{V_{i,\omega,h}^2}; \quad \forall f \in F, \forall \omega \in \Omega, \forall h \in H \quad (13)$$

The above equations are not in a linear form and they have to be converted into linear expressions using the piecewise linear approximation method. To do this, the square voltage term in the denominator is assumed to be unity and the quadratic terms of active and reactive powers are expressed by a set of variables with a linear relation.

The main substation capacity limit ensures that the power imported from the main grid is limited by capacity of transformers located in the main substation.

$$S_{\omega,h}^{grid} \leq \overline{S^{grid}}; \quad \forall \omega \in \Omega, \quad \forall h \in H \quad (14)$$

which can be replaced by the following constraint:

$$P_{\omega,h}^{grid^2} + Q_{\omega,h}^{grid^2} \leq \overline{S^{grid}}^2; \quad \forall \omega \in \Omega, \forall h \in H \quad (15)$$

Again, the piecewise linear approximation method is used to make the square of active and reactive powers linear.

Feeder capacity limits are also considered as follows:

$$-\overline{S_{ii'}^f} \leq S_{ii',\omega,h}^f \leq \overline{S_{ii'}^f}; \quad \forall f \in F, \forall \omega \in \Omega, \forall h \in H \quad (16)$$

which can be replaced by the following constraint:

$$P_{ii',\omega,h}^f{}^2 + Q_{ii',\omega,h}^f{}^2 \leq \overline{S_{ii'}^f}^2; \quad \forall f \in F, \forall \omega \in \Omega, \forall h \in H \quad (17)$$

Similar to (15), the piecewise linear approximation approach is employed to make the expression linear.

Distribution transformer capacity limits are considered to ensure the safe operation of distribution transformers.

$$S_{i,\omega,h} \leq \overline{S_i}; \quad \forall i \in I, \forall \omega \in \Omega, \forall h \in H \quad (18)$$

which can be replaced by the following constraint:

$$P_{i,\omega,h}^D{}^2 + Q_{i,\omega,h}^D{}^2 \leq \overline{S_i}^2; \quad \forall f \in F, \forall \omega \in \Omega, \forall h \in H \quad (19)$$

Similar to (15) and (17), the above expression is made linear using the piecewise linear approximation approach.

Bus voltage limits guarantee an acceptable voltage level at all nodes but the main substation bus. A main substation in a distribution network plays a similar role with voltage controlled buses in transmission networks. Thus, the voltage magnitude at

the main substation bus is held constant. Bus voltage limits are as follows:

$$\underline{V} \leq V_{i,\omega,h} \leq \bar{V}; \quad \forall i \neq 1, \forall \omega \in \Omega, \forall h \in H \quad (20)$$

$$V_{i,\omega,h} = \text{Constant}; \quad i = 1, \forall \omega \in \Omega, \forall h \in H \quad (21)$$

**Constraints on DG Units:** These constraints ensure that the power produced by DGs adhere the respective capacity limits, ramp rates, and minimum up/down times. DG capacity limits guarantee that the power generations of DG units are within their allowed upper and lower limits.

$$s_{g,\omega,h} \underline{P_g^{DG}} \leq P_{g,\omega,h}^{DG} \leq s_{g,\omega,h} \overline{P_g^{DG}}; \quad \forall g \in G, \forall \omega \in \Omega, \forall h \in H \quad (22)$$

$$s_{g,\omega,h} \underline{Q_g^{DG}} \leq Q_{g,\omega,h}^{DG} \leq s_{g,\omega,h} \overline{Q_g^{DG}}; \quad \forall g \in G, \forall \omega \in \Omega, \forall h \in H \quad (23)$$

The following inequalities are taken into account to ensure that the production of DG units obeys to the respective ramp up/down constraints.

$$\begin{aligned} P_{g,\omega,h+1}^{DG} - P_{g,\omega,h}^{DG} &\leq [1 - s_{g,\omega,h+1}(1 - s_{g,\omega,h})] R_g^{UP} \\ &+ s_{g,\omega,h+1}(1 - s_{g,\omega,h}) \underline{P_g^{DG}}; \quad \forall g \in G, \forall \omega \in \Omega, \forall h \in H \end{aligned} \quad (24)$$

$$\begin{aligned} P_{g,\omega,h}^{DG} - P_{g,\omega,h+1}^{DG} &\leq [1 - s_{g,\omega,h}(1 - s_{g,\omega,h+1})] R_g^{DN} \\ &+ s_{g,\omega,h}(1 - s_{g,\omega,h+1}) \underline{P_g^{DG}}; \quad \forall g \in G, \forall \omega \in \Omega, \forall h \in H \end{aligned} \quad (25)$$

Note that the above constraints are not required to be considered for renewable DG units such as wind turbines and solar units. The following inequalities are considered to guarantee that the commitment of DG units respects the associated minimum up/down time constraints. Similar to (24), (25), the following constraints can be simply dropped for small units as well as renewable DG units such as wind turbines and solar units.

$$\begin{aligned} T_g^{UP} \sum_{h'=1}^{T_g^{UP}} s_{g,\omega,h-h'} &\geq T_g^{UP} X_{g,\omega,h}^{off}; \quad \forall g \in G, \forall \omega \in \Omega, \forall h \in H \quad (26) \\ T_g^{DN} \sum_{h'=1}^{T_g^{DN}} (1 - s_{g,\omega,h-h'}) &\geq T_g^{DN} X_{g,\omega,h}^{on}; \quad \forall g \in G, \forall \omega \in \Omega, \forall h \in H \end{aligned} \quad (27)$$

Expressions (24)–(27) are not in a linear format since they contain the product of binary variables. However, they can be easily converted into linear expressions adopting the technique described in [27].

The following constraints are required to prevent a conflicting situation in binary variables denoting the status of DG units.

$$s_{g,\omega,h+1} - s_{g,\omega,h} \leq X_{g,\omega,h+1}^{on}; \quad \forall g \in G, \forall \omega \in \Omega, \forall h \in H \quad (28)$$

$$s_{g,\omega,h} - s_{g,\omega,h+1} \leq X_{g,\omega,h+1}^{off}; \quad \forall g \in G, \forall \omega \in \Omega, \forall h \in H \quad (29)$$

$$\begin{aligned} s_{g,\omega,h+1} - s_{g,\omega,h} &= X_{g,\omega,h+1}^{on} - X_{g,\omega,h+1}^{off}; \\ &\quad \forall g \in G, \forall \omega \in \Omega, \forall h \in H \end{aligned} \quad (30)$$

Finally, it is worthwhile to mention that there are some special DG units with constraints on their output power factor. For example, photovoltaic and fuel cells are capable to inject only active power [28]. As another example, wind turbines with induction generators inject active powers and absorb reactive power [28]. These constraints should be mathematically integrated in the formulations, if there is any.

**Constraints on Shunt Capacitors:** This constraint enforces the reactive power of compensators within their maximum and minimum limits.

$$\underline{Q_i^{sh}} \leq Q_{i,\omega,h}^{sh} \leq \overline{Q_i^{sh}}; \quad \forall i \in I, \forall \omega \in \Omega, \forall h \in H \quad (31)$$

It has to be noted that voltage regulation is among DisCos' responsibilities. They have to install shunt compensators to keep voltage magnitudes throughout the network within acceptable ranges. Note that DisCos do not care about frequency regulation since independent system operators (ISOs) are responsible for that.

**Constraints on LCs:** The following constraint ensures that the amount of LC invoked by the DisCo at bus  $i$  and hour  $h$  has a positive value less than the total demand at that bus.

$$0 \leq P_{i,t,\omega,h}^{LC} \leq P_{i,t,\omega,h}^D; \quad \forall i \in I, \forall t \in T, \forall \omega \in \Omega, \forall h \in H \quad (32)$$

Since invoking LC lessens both the active and reactive demands of each type of consumers at a given load point, the following constraint is incorporated to keep the power factor constant.

$$\begin{aligned} P_{i,t,\omega,h}^{LC} Q_{i,t,\omega,h}^{D0} - Q_{i,t,\omega,h}^{LC} P_{i,t,\omega,h}^{D0} &= 0; \\ \forall i \in I, \forall t \in T, \forall \omega \in \Omega, \forall h \in H \end{aligned} \quad (33)$$

**Demand Response Model:** The following equations are incorporated to capture the demand behavior in response to time-varying sale prices.

$$\begin{aligned} P_{i,t,\omega,h}^D &= P_{i,t,\omega,h}^{D0} \times \left( 1 + \sum_{h' \in H} E_{h,h'}^t \frac{\lambda_{t,\omega,h'} - \lambda_t^{flat}}{\lambda_t^{flat}} \right); \\ \forall i \in I, \forall t \in T, \forall \omega \in \Omega, \forall h \in H \end{aligned} \quad (34)$$

$$\begin{aligned} Q_{i,t,\omega,h}^D &= Q_{i,t,\omega,h}^{D0} \times \left( 1 + \sum_{h' \in H} E_{h,h'}^t \frac{\lambda_{t,\omega,h'} - \lambda_t^{flat}}{\lambda_t^{flat}} \right); \\ \forall i \in I, \forall t \in T, \forall \omega \in \Omega, \forall h \in H \end{aligned} \quad (35)$$

In the above expressions, a price elasticity coefficient indicates how a change in the respective price affects the associated load [29]. Price elasticity coefficients can be divided into two groups designated as self-elasticity and cross-elasticity. Self-elasticity coefficients relate the electricity price and load associated with the same hour, while cross-elasticity coefficients represent the relationship between the electricity price and load at different hours. A precise estimation of the price elasticity coefficients requires a complex econometric procedure which is beyond the scope of this paper. These coefficients are considered to be known in the problem addressed here.

**Constraints on Sale Prices:** An electricity sale price can be broken up into an energy related part and a service related part.

In RTP environments, retail sale prices fluctuate hourly reflecting the volatility of wholesale market prices. In this paper, wholesale market price is considered as the sale price of the energy related part and is directly passed through to consumers by the DisCo. For the service related part price, called service price hereinafter, the DisCo is allowed to determine hourly service prices by itself. So, the hourly sale price offered to the consumers can be formulated as follows:

$$\lambda_{t,\omega,h} = \lambda_{t,\omega,h}^{service} + \rho_{\omega,h}^{active}; \quad \forall t \in T, \forall \omega \in \Omega, \forall h \in H \quad (36)$$

In order for service prices to be fair towards both the DisCo and consumers, regulatory bodies impose some regulatory restrictions. The following constraint imposes a cap to hourly service prices:

$$\lambda_{t,\omega,h}^{service} \leq \overline{\lambda_t^{service}}; \quad \forall t \in T, \forall \omega \in \Omega, \forall h \in H \quad (37)$$

The following inequality ensures that the average of hourly service prices is limited to a predefined value:

$$\sum_{i \in B} \sum_{h \in H} \lambda_{t,\omega,h}^{service} P_{i,t,\omega,h}^D \leq \lambda_t^{average} \times \sum_{i \in B} \sum_{h \in H} P_{i,t,\omega,h}^D; \quad \forall t \in T, \forall \omega \in \Omega \quad (38)$$

The problem described in the above is based on a MILP format which can be solved via commercial software.

Distribution network data, technical information of DG units and shunt capacitors, VOLL of consumers, wholesale market prices, regulatory limits on sale prices, and price elasticity coefficients are the input data of the model. The optimal decisions from this optimization problem include hourly grid purchases, commitment of DG units, hourly sale prices, reactive power support from shunt compensators, and invocation of LCs.

### III. NUMERICAL STUDY

#### A. Data

To demonstrate the effectiveness of the proposed model, it is applied to a typical Finnish 20 kV urban distribution network [30]. The uncertainty associated with consumers demand and wholesale prices is considered using 30 likely scenarios which are generated based on Finnish historical data. Technical data of the system as well as likely scenarios and their occurrence probabilities are available in [30]. The single-line diagram of the network is depicted in Fig. 1. The main substation, as the only connection point between the distribution network and the external grid, supplies 144 distribution substations (20/0.4 kV) through 6 underground cable feeders. In the simulations, three 500 kW DG units are assumed to be connected to buses 2, 26, and 59.

Fig. 2 shows the expected wholesale price of active energy and electricity demand of the DisCo. It has to be noted that there is a strong correlation between the price and load. Since reactive power prices are not available for the considered market, it is assumed that the wholesale price of reactive energy is equal to the wholesale price of active energy divided by 10. This assumption is according to the conclusions made in [31]. The self-elasticity

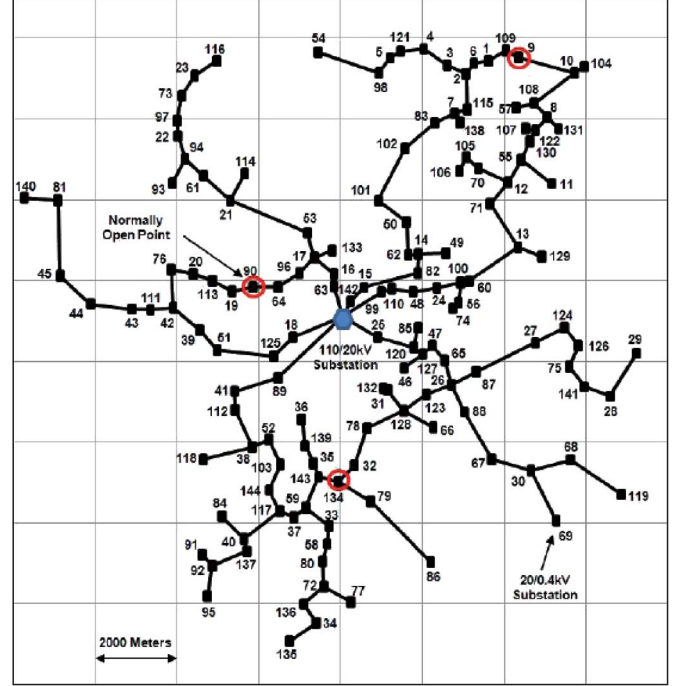


Fig. 1. Single-line diagram of the Finnish 20 kV distribution network.

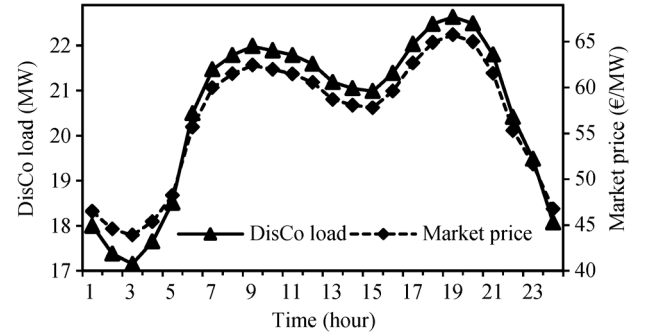


Fig. 2. Hourly expected electricity market prices and DisCo load.

coefficient is set at  $-0.2$  to capture the elastic behavior of the load in response to time-varying prices [32].

It is assumed that the average of offered service prices should be less than € 8 per megawatt hour and hourly offered service prices must not be more than € 16 per megawatt hour.

#### B. Study Results

In order to demonstrate the performance of the RTP sale prices designed by the established model, a comparison benchmark is required. For this reason, a situation is considered in which consumers pay a flat price for the amount of electricity they consume. Clearly, in such a situation, consumers have no monetary incentive to reduce or shift their electricity usage when the wholesale market prices are higher. This situation is called base case hereinafter. Fig. 3 compares the resulted time-varying prices and the flat price offered in the base case. As can be seen, the sale prices resulted from the developed model vary over the time. These fluctuations motivate consumers to modify their usage.

Fig. 4 compares the expected load profile resulted from the proposed model with that of the base case. According to the re-

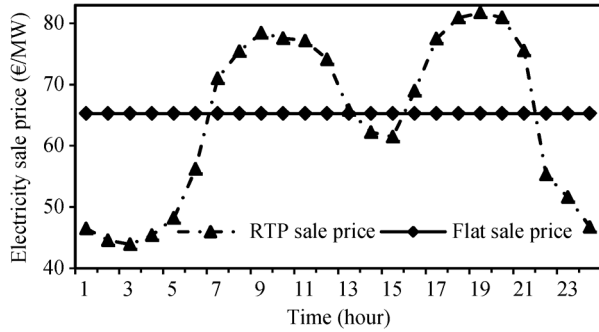


Fig. 3. Hourly expected electricity sale prices.

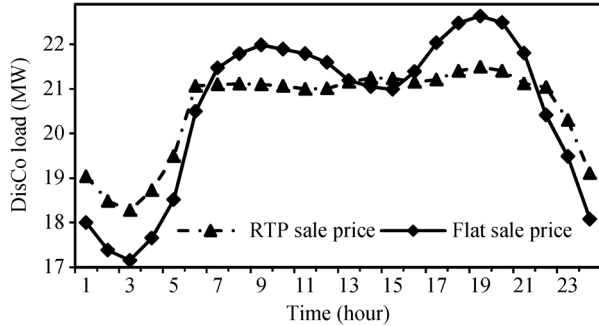


Fig. 4. DisCo expected load profile.

TABLE I  
LOAD PROFILE CHARACTERISTICS FOR THE NEW MODEL AND BASE CASE

Load characteristic	Base case (Flat price)	Established model (RTP price)	Enhancement
Peak [MW]	22.6337	21.4933	1.1404
Valley [MW]	17.151	18.2758	1.1247
Load factor [%]	90.89	95.63	4.74

sults, the expected RTP sale prices designed by the established model would improve load profile characteristics dramatically. In the case of RTP sale prices, there is a more evenly distributed load across different hours of the day. The detailed improvements in the load profile caused by the RTP prices are given in Table I. The new model leads to nearly 5% peak reductions, 6.6% increments in the valley, and 5.2% increments in the load factor. The resulted enhancements reveal that the developed model and the suggested sale prices are effective.

The load profile modifications described earlier would provide significant financial and technical benefits. Table II provides the financial aspects of the new model and base case. Owing to the results, the established model is beneficial for both the DisCo and consumers. It can be seen that the DisCo's profit increases by € 66.39 (i.e., 2.78%) as the RTP sale prices are offered. Such an increment in the DisCo's profit besides the potential technical benefits provides great incentives for DisCos to offer RTP sale prices. In addition, as mentioned earlier, applying time-varying sale prices provides consumers with the opportunity to pay less money for their electricity consumption. Utilizing the RTP prices, although they are designed such that the DisCo's profit is maximized, the total consumers' payment decreases by € 738.53 (i.e., 2.29%) in comparison with the base case.

As mentioned heretofore, application of RTP prices can reduce demand procurement and system operation cost. The ex-

TABLE II  
FINANCIAL ASPECTS OF THE NEW MODEL AND BASE CASE

Financial aspect	Base case (Flat price)	Established model (RTP price)	Enhancement
Expected profit earned by DisCo [€]	2,384.45	2,450.84	66.39
Expected money paid by consumers [€]	32,214.63	31,476.10	738.53

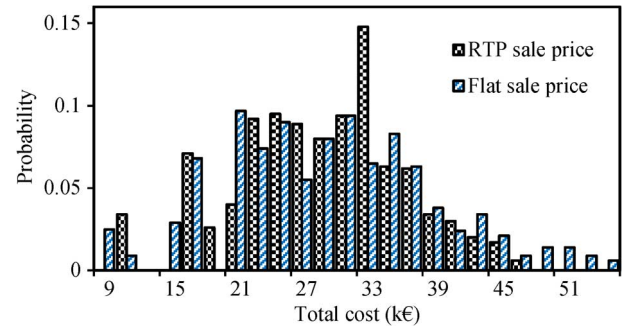


Fig. 5. Histogram of total system cost.

TABLE III  
STATISTICAL INFORMATION ABOUT DISCO'S PROFIT IN DIFFERENT SCENARIOS

Case study	Best scenario	Standard deviation	Worst scenario
Base case (Flat price)	€ 23,164	€ 9,483	€ -22,407
Established model (RTP price)	€ 37,160	€ 324	€ 21,916

pected total cost reduces from € 29 830 in base case to € 29 025 in the case with RTP prices. Synonymously, 2.7% reduction in the expected total cost is achieved when the proposed approach is utilized. The application of RTP prices can also reduce total cost volatility. Fig. 5 compares histogram of total system cost for the two cases. As can be seen, volatility of the costs when the flat price is offered is larger than that of the case in which RTP prices are offered by the new approach. It is worthwhile to mention that the standard deviation of the costs in cases with RTP and flat prices are respectively € 7548 and € 9534.

Tables III and IV provide statistical information about DisCo's profit and consumers' payment associate with different scenarios. According to the results, application of RTP prices significantly reduces volatility in the DisCo's profit. The standard deviation decreases by 29 times when RTP prices are offered to the consumers. The profit in both the worst and best scenarios is increased significantly when the proposed model is utilized. As it can be seen in Table IV, RTP prices also provide consumers with the opportunity to reduce their payment by more than two times in the best scenario (i.e., from € 30 878 to € 14 002). This, however, is in expense of a great volatility in consumers' bills. The standard deviation increases by more than 7 times when RTP prices are offered to the consumers. Moreover, their payment in the worst scenario is more than three times larger than that of the best scenario. This can be a big barrier in front of RTP strategies in capturing the attention of conservative consumers.

As noted earlier, the load profile modifications realized by the established model would provide significant benefits in technical aspects of the network operation. Fig. 6 shows the hourly



TABLE IV  
STATISTICAL INFORMATION ABOUT CONSUMERS' PAYMENT IN  
DIFFERENT SCENARIOS

Case study	Best scenario	Standard deviation	Worst scenario
Base case (Flat price)	€30,878	€984	€33,486
Established model (RTP price)	€14,002	€7,325	€49,128

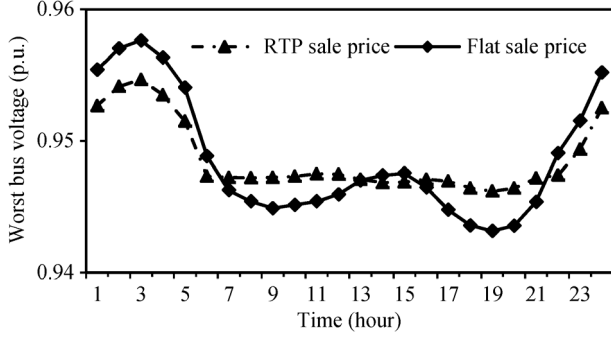


Fig. 6. Hourly expected worst bus voltages.

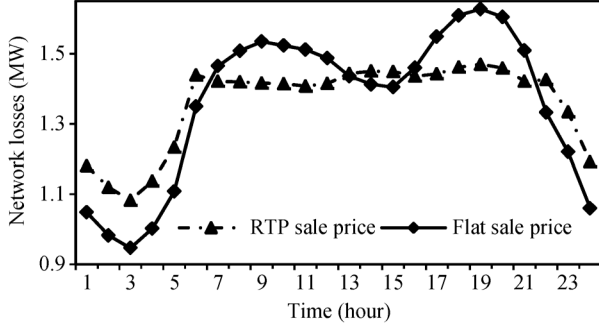


Fig. 7. Hourly expected distribution network losses.

expected worst bus voltage of the network for both the established model and base case. It is observed that a significant improvement in bus voltage magnitudes is achieved when the demand side potentials are enabled by the RTP sale prices offered in the new model. The voltage profile obtained by the developed model is a more evenly distributed curve rather than that of the base case. Fig. 7 portrays the hourly expected distribution network losses for both the developed model and base case. It can be seen that a major portion of the network losses occurred at hours 8–10 and 17–21 in the base case. The established model, however, reduces the system losses during the peak periods.

Table V gives the expected total network loss and the associated cost for the developed model and base case. It is observed that the expected total network loss is reduced by 2.28% when the established model is implemented and as such the total cost of network losses is decreased by € 93.67 (i.e., 5%) too. An interesting observation is that the mentioned loss reduction leads to the much higher percentage of a cost reduction. This is because of the fact that the loss reduction is mainly occurred within the peak hours when the wholesale market prices are higher.

So far, the financial benefits of the developed model as well as the enhancements in the network losses and voltage levels have been investigated. Besides, the demand side potentials enabled by the established model would be beneficiary in enhancing the network reliability during unpredictable contingencies [33]. Synonymously, the developed model encourages consumers to

TABLE V  
DISTRIBUTION NETWORK LOSSES FOR THE NEW MODEL AND BASE CASE

Item	Base case (Flat price)	Established model (RTP price)	Enhancement
Expected network loss [MW]	32.7103	31.9648	0.7455
Expected cost of network loss [€]	1,868.79	1,75.11	93.67

TABLE VI  
SYSTEM ORIENTED RELIABILITY INDICES FOR  
THE NEW MODEL AND BASE CASE

Reliability index	Base case (Flat price)	Established model (RTP price)	Enhancement
EDNS [kW]	6.3205	6.1840	0.1365
EENS [kWh]	151.693	148.416	3.277
ECDC [€]	3028.092	2943.277	84.815

reduce their load during peak hours when the market prices are high and therefore, reduces the likelihood of an involuntary emergency LC. To quantify the reliability benefits, several system oriented reliability indices including EDNS, EENS, and ECDC are calculated for the considered distribution network. In order to calculate the indices, first, the set of likely contingency states is constituted. A contingency state denotes the status of network facilities and load. Then, the states are investigated to judge whether any violation in operating conditions, e.g., network capacities or voltage levels, is observed. For states with violations, remedial actions such as network reconfiguration are applied to alleviate the violations where load curtailment is the last resort. Having calculated the amount of required load curtailment at different load points and system states, the indices are calculated using the following formulas:

$$EDNS = \sum_{c \in C} \sum_{i \in I} \sum_{t \in T} p_c P_{i,t,c}^{LC} \quad (39)$$

$$EENS = \sum_{c \in C} \sum_{i \in I} \sum_{t \in T} T_c P_{i,t,c}^{LC} \quad (40)$$

$$ECDC = \sum_{c \in C} \sum_{i \in I} \sum_{t \in T} \rho_{i,t}^{LC} T_c P_{i,t,c}^{LC} \quad (41)$$

The above procedure is applied to the network under study and the reliability indices are calculated. Table VI gives the indices for the developed model and base case. The results shown in this table clearly portray the great impacts of the developed model on the reliability performance of the network. By applying the new model, EENS and ECDC are reduced by 2.16% and 2.80%, respectively. Note that the achieved reliability improvement does not benefit only the consumers, but also decreases the DisCo's compensation cost of emergency interruption events.

The reviewed financial and technical advantages would provide both the DisCos and consumers with great incentives to enroll in demand side management programs. It must be emphasized that the preceding discussions pertain to the short-term benefits enabled by the RTP sale prices and the developed model. In long-term, the benefits provided by the developed model are much higher. For example, the peak reduction accomplished by the model can defer significant investment costs



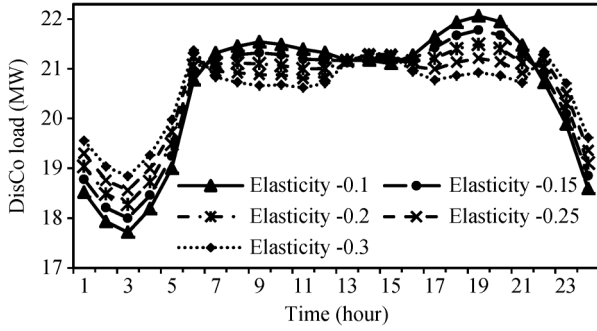


Fig. 8. DisCo expected load profile for different price elasticity of demand.

TABLE VII  
FINANCIAL BENEFITS OF THE NEW MODEL FOR  
DIFFERENT SELF ELASTICITY COEFFICIENTS

Elasticity coefficient	Increment in DisCo profit [€]	Decrement in consumers bill [€]	Decrement in consumers damage cost [€]
-0.10	24.50	372.98	45.34
-0.15	46.21	553.68	65.70
-0.20	66.39	738.53	84.82
-0.25	85.05	925.86	102.01
-0.30	102.15	1163.46	117.40

or the voltage regulation realized by the model can avoid costly reactive power planning projects.

### C. Impact of Price Elasticity of Demand

In the previous simulations, the elasticity coefficient was set to  $-0.2$ . However, the complex behavior of consumers makes the precise estimation of the coefficients impossible. On the other hand, the optimum sale prices and even other operating decisions are not constant for different price elasticity coefficients of demand. Hence, the simulations are repeated for a diverse range of elasticity coefficients. The obtained load profiles are shown in Fig. 8. It is observed that a more evenly distributed load profile is achieved as the price elasticity of demand increases. From this figure, the DisCo encounters higher peak loads if demand response is either too small or too large. In the case of smaller responses, e.g.,  $-0.1$ , the peak hours remain the same as those of the base case. However, in the case of larger responses, e.g.,  $-0.3$ , the peak load is occurred at the hours which were the mid-peak in the base case. The main reason behind this observation is that consumers tend to adjourn their non-critical loads to the nearest point of time when the sale prices are lower.

The financial and technical aspects of the distribution network operation for various elasticity coefficients are respectively listed in Tables VII and VIII. It can be seen from the results that financial benefits delivered to both the DisCo and consumers increase by increasing the level of demand response. The distribution network voltage profile, energy losses, and network reliability are also improved as the price elasticity of demand increases. This observation reveals the importance of the mechanisms developed to increase the consumers' knowledge about how to respond to time-varying sale prices as well as the protocols proposed to realize automatic load management.

TABLE VIII  
TECHNICAL BENEFITS OF THE NEW MODEL FOR  
DIFFERENT SELF ELASTICITY COEFFICIENTS

Elasticity coefficient	Network voltage profile improvement [p.u.]	Network loss reduction [kWh]	Reduction in EENS [kWh]
-0.10	0.0017	440.4	1.813
-0.15	0.0021	608.1	2.556
-0.20	0.0022	745.5	3.277
-0.25	0.0024	847.0	3.990
-0.30	0.0024	912.9	4.659

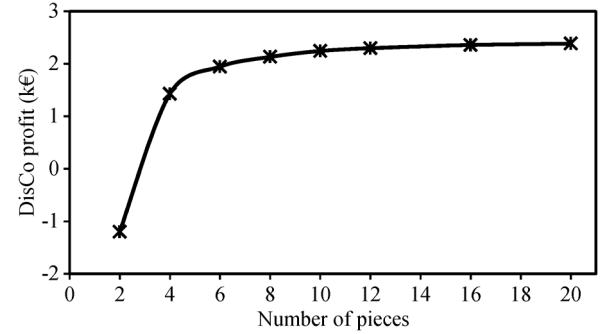


Fig. 9. Evolution of DisCo expected profit versus the number of pieces.

### D. Accuracy Verification

As mentioned earlier, the piecewise linear approximation approach is employed in several non-linear expressions to keep the problem statement linear. However, the determination of the required number of pieces is a challenging issue. Too few pieces might lead to inaccurate results, while too many pieces may make the problem computationally difficult. To overcome this issue, the number of adopted pieces is usually determined based on engineering judgments. The evolution of the DisCo profit versus the number of pieces is plotted in Fig. 9. It is observed that the DisCo profit stabilizes in the vicinity of 10 pieces. Therefore, the accuracy of the results provided in this paper is justified.

## IV. CONCLUSION

This paper established a probabilistic short-term decision model for a DisCo to enable demand side potentials via RTP sale prices. Integration of demand changes in response to time-varying RTP prices into DisCos' decision model is the main contribution of the paper. The model determines hourly grid purchases, hourly sale prices, commitment of DG units, dispatch of shunt capacitors, and invocation of LCs so that the DisCo profit is maximized. The model is in a MILP format that can be effectively solved via commercial solvers. A numerical study on a typical Finnish distribution network revealed the capability of the developed model in using the existing facilities more efficiently. The results indicated that the proposed model delivers substantial benefits to both DisCos and consumers. From the financial point of view, application of the developed model led to an increment in the DisCo's expected profit as well as to dramatic decrements in consumers' expected payment and damage cost of emergency interruptions. It was also

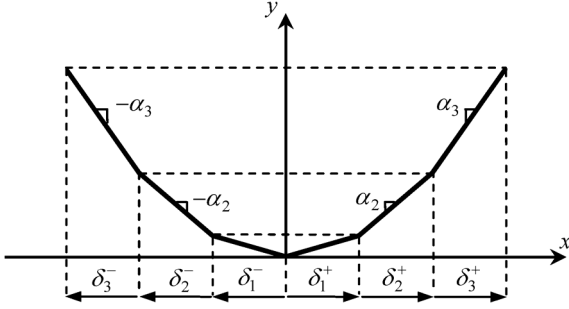


Fig. A.1. Piecewise linear approximation of a quadratic function.

shown that the approach decreases volatility in the DisCo's profit. However, the reduction in consumers' expected payment is in expense of a great deal of volatility in their bills. This can overwhelm part of benefits of the proposed approach when uncertainties are too much. Viewed from the technical perspective, implementing the established model resulted in more evenly distributed load and voltage profiles, smaller network losses and peak load, and better service reliability. The study results also demonstrated that the financial and technical benefits would increase as demand response increases. These drastic advantages provide great incentives for DisCos to apply RTP sale prices as well as to support attempts toward a higher demand response.

#### APPENDIX

This section is to provide a brief discussion over the piecewise linear approximation of a quadratic function. Consider the following quadratic function:

$$y = x^2 \quad (\text{A.1})$$

A linear approximation of  $y$  can be achieved using a set of piecewise linear segments as shown in Fig. A.1. this can be mathematically formulated as follows:

$$y = \sum_{l \in L} \alpha_l \times \delta_l^+ + \sum_{l \in L} \alpha_l \times \delta_l^- \quad (\text{A.2})$$

$$x = \sum_{l \in L} \delta_l^+ - \sum_{l \in L} \delta_l^- \quad (\text{A.3})$$

$$\frac{x}{M} \leq b \leq 1 + \frac{x}{M} \quad (\text{A.4})$$

$$0 \leq \delta_l^+ \leq b \times \bar{\delta}; \quad \forall l \in L \quad (\text{A.5})$$

$$0 \leq \delta_l^- \leq (1 - b) \times \bar{\delta}; \quad \forall l \in L \quad (\text{A.6})$$

$$\frac{\bar{\delta} - \delta_l^+}{M} \leq b_l^+ \leq 1 + \frac{\bar{\delta} - \delta_l^+}{M}; \quad \forall l \in L \quad (\text{A.7})$$

$$\frac{\bar{\delta} - \delta_l^-}{M} \leq b_l^- \leq 1 + \frac{\bar{\delta} - \delta_l^-}{M}; \quad \forall l \in L \quad (\text{A.8})$$

$$0 \leq \delta_l^+ \leq b_{l-1}^+ \times \bar{\delta}; \quad \forall l \in L \quad (\text{A.9})$$

$$0 \leq \delta_l^- \leq b_{l-1}^- \times \bar{\delta}; \quad \forall l \in L \quad (\text{A.10})$$

In the above formulations,  $l \in L$  represents linear segments.  $\delta_l^+$ ,  $\delta_l^-$  are continues auxiliary variables.  $b$ ,  $b_l^+$ , and  $b_l^-$  are binary auxiliary variables.  $\alpha_l$  is the slope of each linear segment.

$\bar{\delta}$  denotes the length of linear segments.  $M$  is an arbitrary large number. Expression (A.2) is the linear approximation of (A.1). (A.3) and states that the sum of the values in linear segments is equal to  $x$ . (A.4)–(A.6) ensure the upper and lower limits of linear segments. (A.7)–(A.10) enforce the adjacency of linear segments.

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