

Risk-Constrained Multi-Stage Wind Power Investment

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Abstract—When deciding on wind power investments, three major issues arise: the production variability and uncertainty of wind facilities, the eventual future decline in wind power investment costs, and the significant financial risk involved in such investment decisions. Recognizing the above important issues, this paper proposes a risk-constrained multi-stage stochastic programming model to make optimal investment decisions on wind power facilities along a multi-stage horizon. The proposed model is illustrated using a clarifying example and a case study.

Index Terms—Mathematical program with equilibrium constraints (MPEC), risk management, stochastic programming, wind power investment.

NOTATION: The main notation used throughout this paper is stated below, while other symbols are defined as needed. A subindex ν in the constants and variables below refers to the values in the ν th load demand/wind power production condition, a superscript (t) refers to the values in the t th period and (γ) refers to the values in the γ th scenario.

Constants:

$a^{(t)}$	Amortization factor.
B_k	Susceptance of line k .
c_{ib}	Price offered by the b th block of the i th generation unit.
$c_{\max}^{(t)}$	Budget for investment in wind power.
$c_n^{(t)}(\gamma)$	Investment cost of wind power at bus n .
$d_j^{\max, (t)}(\gamma)$	Peak load of the j th demand.
f_k^{\max}	Transmission capacity of line k .
g_{ib}^{\max}	Upper limit of the b th block of the i th generation unit.
$k_{j,\nu}^D$	Load level of the j th demand.
$k_{n,\nu}^W$	Wind power capacity factor at bus n .
$o(k)$	Sending-end bus of line k .
$r(k)$	Receiving-end bus of line k .

X_n^{\max}	Maximum wind power capacity that can be installed at bus n .
α	Confidence level used to compute the CVaR.
β	Weighting parameter modeling the tradeoff between expected profit and CVaR.
ϑ_ν	Number of hours comprising the ν th load demand/wind power production condition.
$\tau(\gamma)$	Weight of scenario γ .

Variables:

$f_{k,\nu}^{(t)}(\gamma)$	Power flow through line k .
$g_{ib,\nu}^{(t)}(\gamma)$	Power produced by the b th block of the i th generation unit.
$P_{n,\nu}^{W, (t)}(\gamma)$	Wind power produced at bus n .
$X_n^{(t)}(\gamma)$	Wind power to be installed at bus n .
$\delta_{n,\nu}^{(t)}(\gamma)$	Voltage angle at bus n .
$\eta(\gamma), \zeta$	Auxiliary variables to compute the CVaR.

Indices and Sets:

$\Delta_{LL,\nu}^{(t)}(\gamma)$	Set of variables of the lower-level problem in the t th period, γ th scenario, and ν th load demand/wind power production condition.
$\Delta_{UL}^{(t)}(\gamma)$	Set of variables of the upper-level problem in the t th period and γ th scenario.
Ψ_n^D	Set of indices of the demands located at bus n .
Ψ_n^G	Set of indices of the units located at bus n .
Ω_i	Set of indices of the blocks of the i th generation unit.
Ω^G	Set of indices of generation units.
Ω^K	Set of indices of transmission lines.
Ω^N	Set of indices of buses.
Ω^T	Set of indices of periods.
Ω_γ	Set of indices of scenarios.
Ω_ν	Set of indices of load demand/wind power production conditions.
$\Upsilon^{(t)}(\gamma)$	Set of parameters defining the γ th scenario in the t th period.

Manuscript received November 08, 2011; revised February 29, 2012 and May 15, 2012; accepted June 16, 2012. Date of publication July 23, 2012; date of current version January 17, 2013. The work of L. Baringo and A. J. Conejo was supported in part by the Ministry of Economy and Competitiveness of Spain through CICYT Project DPI2009-09573. Paper no. TPWRS-01067-2011.

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Digital Object Identifier 10.1109/TPWRS.2012.2205411

I. INTRODUCTION

A. Motivation and Approach

THREE major elements impact the decision-making process pertaining to wind production facilities, namely:

- 1) Once a wind power facility is built and ready to operate, its actual production is both variable and uncertain, as it depends of how much the wind blows at the particular location of the wind facility. To make informed investment decisions, most production conditions (wind generation and demand levels) of the wind power facilities should be described via scenarios within the decision framework.
- 2) As the technology required to harness wind energy (and to produce electricity out of it) matures, investment costs in wind power facilities are expected to decrease. But to which extend such decrease will materialize is uncertain. To make informed investment decisions, a multi-period (dynamic) framework is thus required to represent uncertain investment costs in different time points along the decision horizon.
- 3) Since both production levels and future investment costs are uncertain, the volatility of the profit distribution for a wind investor is high and thus risk management is a must to make informed decisions.

This paper describes a risk-constrained multi-stage stochastic programming model to make optimal investment decisions on wind production facilities.

The stochastic framework allows representing via scenarios most operating conditions involving wind power productions, as well as their statistical correlation with the demand levels. It also allows representing the uncertainty in future investment costs. The multi-stage arrangement allows representing investment in different points in time, which makes possible to account for the progressive maturity of the wind production technology and its decreasing investment costs. Risk management is realized through the conditional value at risk (CVaR), which is incorporated into the decision-making model via linear expressions rendering a computationally tractable linear problem.

Structurally, the proposed model is a stochastic mathematical program with equilibrium constraints (a stochastic MPEC) that seeks to maximize expected profit while minimizing the risk of profit variability. It is subject to investment constraints and market equilibrium conditions for each scenario representing wind production, load level and future investment cost.

B. Literature Review and Contributions

The generation investment problem has been extensively analyzed in the technical literature [1]–[6]. Most of these references tackle the investment problem in conventional generation sources [1]–[3]. However, wind power investment needs models addressing the uncertain character of wind power generation. Some examples are [4]–[6]. In [4], the authors provide an optimization model to establish the maximum wind power penetration in an electric system. Reference [5] proposes a model to design incentive policies to promote wind power investment. In [6], a stochastic MPEC model is proposed to solve the wind power investment within a market environment.

References above propose static models, i.e., models targeting a single future year. Dynamic models, e.g., [7]–[9],

allow representing a multi-stage framework. However, these references tackle conventional investment problems and thus cannot be directly applied to the wind power investment problem. Additionally, [7]–[9] do not incorporate any risk control; and [7] and [9] do not represent the network. To the best of our knowledge, no wind power investment model using a multi-stage framework and incorporating risk control has been proposed in the technical literature.

The model presented in this paper is an extension of that reported in [6] and solves some of its caveats, namely, a multi-stage framework is considered, risk management is carried out and different sources of uncertainty are taken into account.

Within this context, the main contributions of this paper are twofold:

- 1) to recognize the need of using a multi-stage framework to make informed decisions pertaining to wind power investments, as well as the need to adequately represent the risk involved in such investment decisions;
- 2) to provide a stochastic MPEC model that allows multi-stage decision making considering many wind production and demand level scenarios, and representing the risk of profit variability through the CVaR.

C. Paper Organization

The remainder of this paper is organized as follows. Section II describes the main features of the proposed model. Section III provides the mathematical formulation of the considered model consisting in a bilevel model which can be recast as a MILP problem. Sections IV and V present the main results of applying the proposed model to an illustrative example and a case study, respectively. Section VI concludes the paper providing some relevant remarks. Finally, Appendices A and B summarize the procedure to transform the proposed bilevel model into a MILP problem and the Benders algorithm, respectively.

II. PROBLEM DESCRIPTION

A. Model Assumptions

For the sake of clarity, the main model assumptions are summarized below:

- 1) Each wind power producer offers its production at zero price and is paid the LMP of the bus at which it is located.
- 2) We consider a dc model without losses for the transmission network.
- 3) We assume that the system incorporates enough flexible production units (e.g., CCGTs) to cope with wind power variability.
- 4) Uncertainty only affects the investment costs and the load demand/wind power production conditions.
- 5) Investments are allowed only at the beginning of each time period.
- 6) The potential locations to build wind power facilities are assumed to be input data for the proposed model, i.e., the wind power investor has to decide where to build the wind power facilities among a subset of buses, particularly, those buses characterized by the best wind conditions and, among them, those where it is possible to build wind power facilities.

B. Time Framework

The planning horizon comprises a specific number of time periods, each one spanning a known number of years.

The wind power investor can make its investment decisions (the wind power capacity to be built at each bus of an existing electric energy system) at the beginning of each of these time periods. Note that the wind power capacity built at a time period remains available in the following periods.

In order to characterize the load and wind power conditions, each of the time periods comprising the planning horizon is represented using a single year (the last year of each period) which is considered the reference year of the whole period.

For the sake of clarity, the explanation below is given for the case of two periods but the model can be easily extended to consider a higher number of periods.

C. Sources of Uncertainty

The aim of the wind power investor is maximizing its own profit. This profit is computed as the revenue obtained by selling the wind power in the electricity market minus the wind power investment costs, [6]. Both the revenues and costs are subject to uncertainty as explained below.

The revenue of selling wind power in the market is computed as the LMP of the bus at which the wind power is generated times the wind power generation. This revenue depends on two factors. First, the load of the system: the higher the load of the system is, the higher the LMPs. On the other hand, the revenue depends on the actual wind power generation.

As other renewable sources, the wind power generation is not controllable and depends on the wind power capacity factors. Both the load of the system and the wind power capacity factors are subject to uncertainty and their adequate modeling is crucial to obtain optimal investment decisions.

On the other hand, we face the uncertainty associated with the wind power investment costs. At the beginning of the planning horizon we know the actual values of the investment costs but not their values in the future, when these investment costs could change affecting future investment decisions.

Note that additional sources of uncertainty can be included in the model, e.g., fuel prices, equipment outages, etc.

The uncertainty characterization of the load and wind power levels as well as the wind investment costs is described below.

D. Load Demand/Wind Power Production Uncertainty Characterization

The load consumption and wind power generation in an electric energy system are correlated since low demands usually correspond to high wind capacity factors. Thus, the uncertainty characterization of both parameters has to be addressed jointly.

We use available hourly historical data of load consumption and wind power generation during a whole year in the electric energy system under study. This data is used to model the load consumption and wind power generation in the reference year of each period of the planning horizon.

The historical data consists of 8760 pairs of values of load consumption and wind power generation at each bus of the system (one for each of the 8760 hours of a year). These

values divided by the peak load and the wind power capacity, respectively, provide the load and wind levels. Each of these 8760 load and wind levels represent an operating condition of the electric energy system that needs to be taken into account in order to obtain the optimal wind power investment decisions. However, considering 8760 different operating conditions in a realistic energy system may entail intractability. Thus, we use the K-means clustering mechanism [13], [14] to reduce the original data set into a tractable set maintaining the information and correlation of load consumption and wind power generation. This K-means clustering technique allows classifying the historical operating conditions in a pre-specified number of clusters (groups) according to similarities.

Note that each historical operating condition is defined by a wind production and a load level at each bus of the electric energy system. We define the centroid of each cluster as the mean value (of wind production and load level per bus) of all the historical operating conditions in such cluster.

The K-means clustering technique is based on the iterative algorithm described below:

- 1) Select the number of clusters to allocate the historical operating conditions. Randomly assign to each cluster one historical operating condition. These conditions are the initial cluster centroids.
- 2) Compute the quadratic distances (or any other distance) between each historical operating condition and each cluster centroid.
- 3) Assign each historical operating condition to the closest cluster, i.e., the cluster with the lowest distance between the historical operating condition and the cluster centroid.
- 4) Recalculate the cluster centroids using the historical operating conditions in each cluster.

Steps 2)–4) are repeated iteratively until the historical operating conditions within each cluster remain unaltered within two consecutive iterations.

The reduced data set consists of a lower number of operating conditions, each one represented by a value of load level at each bus, a value of wind level at each bus and the number of hours of the original data set corresponding to each operating condition of the reduced data set. Note that the reduced data set maintains the uncertainty and correlation between load and wind conditions. We use Ω_r to denote the set of indices of load and wind conditions.

The wind power generation depends on both the wind power capacity factor and the installed wind power capacity. While the installed wind power capacity is a decision variable of the proposed model and is subject to changes in the future, we can consider that wind power capacity factors remain unaltered throughout the whole planning horizon, as a result of being a physically based phenomenon. Something similar happens with the load consumption that generally increases. However, the load profile throughout the year (obtained by dividing the load consumption by the peak load of each year) can be considered fixed for the whole planning horizon. Note that, if needed, different wind and load profiles can be included through additional scenarios. Additionally, the correlation between load and wind power is considered to remain unaltered, i.e., the number of hours with high load-low wind, medium load-medium wind,

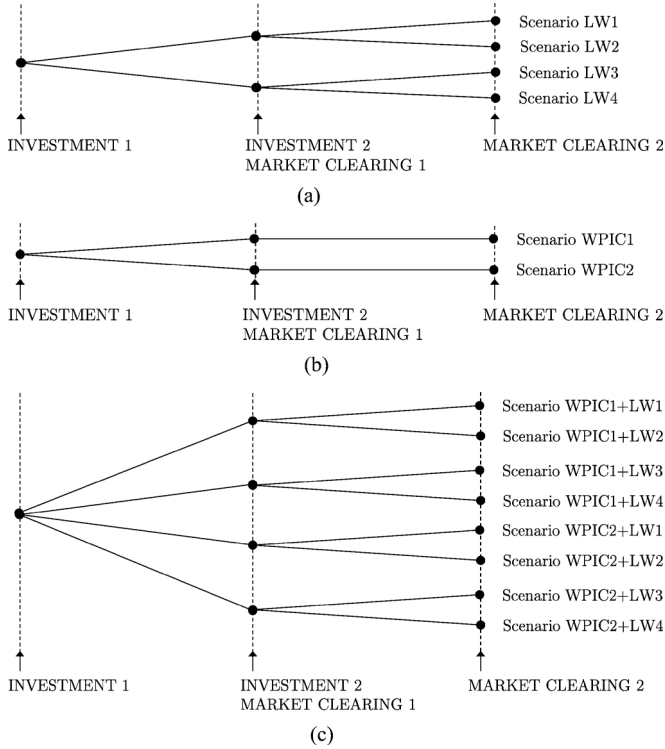


Fig. 1. Scenario trees. (a) Load demand/wind power production uncertainty. (b) Wind power investment cost uncertainty. (c) Load demand/wind power production and wind power investment cost uncertainty.

etc., is considered to be the same for all the years of the planning horizon.

Thus, the uncertainty in load and wind power production is taken into account through the stochastic variable load, in particular the peak load at each bus of the system, and wind power is then computed through appropriate correlation coefficients.

Fig. 1(a) depicts a possible scenario tree considering the uncertainty in the load demand/wind power production of the system. This example considers two possible scenario realizations in the first period and two possible scenario realizations for the second period for each scenario in period 1, i.e., a total of four scenarios for the whole planning horizon (LW1, LW2, LW3, LW4). It is important to note that each scenario in each period comprises a selected number of load and wind power conditions, those obtained by reducing the historical data set using a clustering mechanism. In this case, there is only one possible investment decision at the beginning of the planning horizon and two alternatives at the beginning of the second period depending on the scenario realization in period 1. Accordingly, market clearing (and thus the LMPs computation) is carried out once the load demand/wind power production scenario realization is known, i.e., at the end of each period.

E. Investment Cost Uncertainty Characterization

The renewable source investment costs, and in particular the wind power investment costs, are volatile. Note that we know the wind power investment costs at the beginning of the considered planning horizon but not how these costs change in the

future. If we expect that the investment costs increase in the future we may build a higher wind power capacity now, but if the wind power investment costs decrease, it may be better to wait and invest in the future. Thus, it is necessary to model the uncertainty related to these investment costs using a set of scenarios.

For example, Fig. 1(b) depicts a case in which we have two possible scenario realizations representing two possible wind investment cost realizations in the second period (WPIC1, WPIC2). The main difference with the modeling of the load demand/wind power production uncertainty is that in this case the wind power investor knows the actual value of the investment cost at the moment it makes the investment decisions. The wind power investment cost in the first period does not depend on the scenario realization, since the investment decisions (and thus the payment) are made at the beginning of this period when the wind power investor has a perfect knowledge of the investment costs for this period. Once the first period concludes, the wind power investor knows the actual cost for the second period, and then it decides the optimal investment decisions for period 2.

F. Scenario Tree

As discussed above, there are two sources of uncertainty: the load demand/wind power production uncertainty and the investment cost uncertainty. Both sources of uncertainty are independent, and thus we have to consider all the possible scenario combinations. Symbol Ω_γ denotes the set of indices of load demand/wind power production and investment cost scenarios. For the considered example, the resulting scenario tree is depicted in Fig. 1(c). This scenario tree consists of 8 possible scenario realizations: 4 load demand/wind power production scenarios times 2 investment cost scenarios. Note that there is only one possible investment decision at the beginning of the first period, which does not depend on the scenario realization, and four possible investment decisions at the beginning of the second period depending on the scenario realizations in the first period.

G. Decision Sequence

Within the stochastic framework defined above, decisions are made as follows:

- 1) At the beginning of the first period, the wind power investor determines its investment decisions at this point in time. These are *here-and-now* decisions and do not depend on any scenario realization. These investment decisions affect the two periods since the installed wind power capacity is also available in the second period.
- 2) For each scenario realization in the first period and for each load demand/wind power production condition, the market is cleared obtaining wind power productions, power generations from conventional units, power flows, LMPs, etc.
- 3) Once the first period has finished, the wind power investor knows the investment costs for period 2 and the load demand/wind power production scenario realization in period 1. Then, the investor makes its investment decision for period 2, which affect only the second period (and the following ones, if there were more than two periods) and are *wait-and-see* decisions with respect to the first period

but *here-and-now* decisions with respect to the second period.

- 4) For each scenario realization in the second period and for all load demand/wind power production conditions, the market is cleared.

H. Risk Modeling

The risk of profit variability is considered through the CVaR metric. In a profit maximization problem, as the one we formulate in this paper, the CVaR at the α confidence level is defined as the expected profit of the $(1 - \alpha)$ 100% scenarios that provide the lowest profit. A detailed description of the CVaR can be found, e.g., in [10].

III. MATHEMATICAL FORMULATION

The risk-constrained multi-stage wind power investment problem is formulated using the following bilevel model:

$$\begin{aligned} & \text{Maximize}_{\Delta_{UL}^{(t)}(\gamma), \Delta_{LL,\nu}^{(t)}(\gamma)} \\ & \sum_{\gamma \in \Omega_\gamma} \tau(\gamma) \left\{ \sum_{t \in \Omega^T} \left[\sum_{\nu \in \Omega_\nu} \vartheta_\nu \sum_{n \in \Omega^N} \lambda_{n,\nu}^{(t)}(\gamma) P_{n,\nu}^{W,(t)}(\gamma) \right. \right. \\ & \quad \left. \left. - a^{(t)} \sum_{n \in \Omega^N} c_n^{(t)}(\gamma) X_n^{(t)}(\gamma) \right] \right\} \\ & + \beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\gamma \in \Omega_\gamma} \tau(\gamma) \eta(\gamma) \right) \end{aligned} \quad (1a)$$

subject to

$$P_{n,\nu}^{W,(t)}(\gamma) \leq k_{n,\nu}^W \sum_{m \leq t} X_n^{(m)}(\gamma), \quad \forall t, \forall n, \forall \nu, \forall \gamma \quad (1b)$$

$$\sum_{n \in \Omega^N} c_n^{(t)}(\gamma) X_n^{(t)}(\gamma) \leq c_{\max}^{(t)}, \quad \forall t, \forall \gamma \quad (1c)$$

$$0 \leq \sum_{t \in \Omega^T} X_n^{(t)}(\gamma) \leq X_n^{\max}, \quad \forall n, \forall \gamma \quad (1d)$$

$$X_n^{(1)}(\gamma) = X_n^{(1)}, \quad \forall n, \forall \gamma \quad (1e)$$

$$X_n^{(t)}(\gamma_l) = X_n^{(t)}(\gamma_{\tilde{l}}), \quad (1f)$$

$$\forall n, \forall t \neq 1, \forall l, \tilde{l} : \Upsilon^{(m)}(\gamma_l) = \Upsilon^{(m)}(\gamma_{\tilde{l}}), \quad \forall m < t$$

$$P_{n,\nu}^{W,(t)}(\gamma_l) = P_{n,\nu}^{W,(t)}(\gamma_{\tilde{l}}), \quad (1g)$$

$$\forall n, \forall \nu, \forall t, \forall l, \tilde{l} : \Upsilon^{(m)}(\gamma_l) = \Upsilon^{(m)}(\gamma_{\tilde{l}}), \quad \forall m \leq t$$

$$\begin{aligned} & \zeta - \sum_{t \in \Omega^T} \left[\sum_{\nu \in \Omega_\nu} \vartheta_\nu \sum_{n \in \Omega^N} \lambda_{n,\nu}^{(t)}(\gamma) P_{n,\nu}^{W,(t)}(\gamma) \right. \\ & \quad \left. - a^{(t)} \sum_{n \in \Omega^N} c_n^{(t)}(\gamma) X_n^{(t)}(\gamma) \right] \leq \eta(\gamma), \quad \forall \gamma \end{aligned} \quad (1h)$$

$$\eta(\gamma) \geq 0, \quad \forall \gamma \quad (1i)$$

where $\lambda_{n,\nu}^{(t)}(\gamma) \in \arg \left\{ \right.$

$$\begin{aligned} & \text{Minimize}_{\Delta_{LL,\nu}^{(t)}(\gamma)} \\ & \sum_{i \in \Omega^G} \sum_{b \in \Omega_i} c_{ib} g_{ib,\nu}^{(t)}(\gamma) \end{aligned} \quad (2a)$$

subject to

$$\begin{aligned} & \sum_{i \in \Psi_n^G} \sum_{b \in \Omega_i} g_{ib,\nu}^{(t)}(\gamma) - \sum_{k|o(k)=n} f_{k,\nu}^{(t)}(\gamma) + \sum_{k|r(k)=n} f_{k,\nu}^{(t)}(\gamma) \\ & + P_{n,\nu}^{W,(t)}(\gamma) = \sum_{j \in \Psi_n^D} d_j^{\max,(t)}(\gamma) k_{j,\nu}^D, \quad \forall n \end{aligned} \quad (2b)$$

$$f_{k,\nu}^{(t)}(\gamma) = B_k \left(\delta_{o(k),\nu}^{(t)}(\gamma) - \delta_{r(k),\nu}^{(t)}(\gamma) \right), \quad \forall k \quad (2c)$$

$$-f_k^{\max} \leq f_{k,\nu}^{(t)}(\gamma) \leq f_k^{\max}, \quad \forall k \quad (2d)$$

$$0 \leq g_{ib,\nu}^{(t)}(\gamma) \leq g_{ib}^{\max}, \quad \forall i, \forall b \quad (2e)$$

$$-\pi \leq \delta_{n,\nu}^{(t)}(\gamma) \leq \pi, \quad \forall n \setminus n : \text{ref.} \quad (2f)$$

$$\delta_{n,\nu}^{(t)}(\gamma) = 0, \quad n : \text{ref.} \quad (2g)$$

$$\left. \vphantom{\sum} \right\} \forall t, \forall \nu, \forall \gamma$$

$$\begin{aligned} & \Delta_{LL,\nu}^{(t)}(\gamma_l) = \Delta_{LL,\nu}^{(t)}(\gamma_{\tilde{l}}), \\ & \forall \nu, \forall t, \forall l, \tilde{l} : \Upsilon^{(m)}(\gamma_l) = \Upsilon^{(m)}(\gamma_{\tilde{l}}), \quad \forall m \leq t. \end{aligned} \quad (2h)$$

The risk-constrained multi-stage wind power investment problem above comprises an upper-level problem (1) and a collection of lower-level problems (2). The optimization variables of each of the lower-level problems (2) are $\Delta_{LL,\nu}^{(t)}(\gamma) = \{g_{ib,\nu}^{(t)}(\gamma), \forall i, b; f_{k,\nu}^{(t)}(\gamma), \forall k; \delta_{n,\nu}^{(t)}(\gamma), \forall n\}$, $\forall t, \nu, \gamma$; while the upper-level problem (1) includes these optimization variables and the additional variables $\Delta_{UL}^{(t)}(\gamma) = \{X_n^{(t)}(\gamma), \forall n; \zeta; \eta(\gamma); P_{n,\nu}^{W,(t)}(\gamma), \forall n, \nu\}$, $\forall t, \gamma$.

The objective function (1a) of the upper-level problem represents the maximization of the expected profit plus a coefficient times the CVaR. The first line of (1a) is the revenue obtained by selling wind power in the pool, computed as the wind power production for each load demand/wind power production condition (ν) times the LMP of the bus at which the wind plant is located. LMPs are computed as the dual variable associated with the balance constraints (2b). Zonal pricing (as in Nord Pool [11]) is an alternative to LMPs in the case of systems with a very large number of buses. The revenue for each load demand/wind power production condition is multiplied for the corresponding number of hours ϑ_ν , thus obtaining the expected yearly revenue. The second line of (1a) is the wind investment cost, which is multiplied in each period by an amortization factor $a^{(t)}$, [12]. The amortization costs represent the equivalent amount of money to be paid in each period. Thus, since we are not considering the value of the wind plants once the planning horizon has finished, the amortization factor in the first period will be higher than in the successive ones since the installed wind capacity in the first period is used (and thus amortized) during a longer time. We assume that all costs (for all periods) are expressed in the same money values so it is not necessary to multiply by a discount rate. Revenues and costs are computed for each scenario and thus are multiplied by the weight of the scenario $\tau(\gamma)$. We assume that the wind producer fully recovers its investment costs by selling its wind power production in the pool. However, note that if subsidies or incentive programs to wind producers are available, they can be easily incorporated in the model. Finally, the third line of

the objective function (1a) is the CVaR multiplied by a factor β to materialize the tradeoff between profit and risk, so that the higher the value of β , the more risk averse the wind power investor is.

Equations (1b)–(1d) model the wind power operation and investment constraints for all scenarios γ . Constraints (1b) limit the wind power production to the installed wind capacity times a factor $k_{n,\nu}^W$ modeling the wind power capacity factor for each operating condition. For the sake of simplicity, we consider that wind capacity factors do not depend on the installed wind power capacity. Note that the installed wind capacity at each period consists of the wind capacity built in that period and in the previous ones. Constraints (1c) imposes a cap on the investment budget for each period. Constraints (1d) limit the total wind capacity to be installed at each bus of the system throughout the whole planning horizon. Wind curtailment is allowed if needed to satisfy transmission capacity limits. This is embedded within the proposed model through the right-hand side of (1d). Constraints (1e)–(1g) are non-anticipativity constraints, i.e., constraints that prevent anticipating information. Constraints (1e) impose that the investment decisions at the beginning of the planning horizon do not depend on any scenario realization, while constraints (1f) impose that, for periods others than the first, the investment decision variables depend on the scenario realization on the previous periods but they are unique for all the possible scenario realizations in the future. Constraints (1g) are non-anticipativity constraints for the wind power generation. Finally, constraints (1h) and (1i) allow incorporating the CVaR risk metric. ζ is an auxiliary continuous variable further characterized below and $\eta(\omega)$ is an auxiliary non-negative continuous variable equal to the difference between ζ and the profit of scenario ω if this difference is non-negative, and equal to zero if this difference is negative. The optimal value of ζ is the value at risk (VaR), which is the largest value of the profit such that the probability of the profit being lower than or equal to this value is lower than or equal to $(1 - \alpha)$. We refer the reader interested in a detailed description of the CVaR to [10] and [15].

The upper-level problem (1) is constrained by a collection of lower-level problems (2), which represent the market clearing for each scenario, for each period and for each load demand/wind power production condition within each scenario and period.

The objective function (2a) represents the minimization of the generation cost, equivalent in this case to the maximization of the social welfare since loads are considered constant within each operating condition, period and scenario.

Equation (2b) represent the power balance at each bus of the system. Constraints (2c) define the power flows through lines, which are limited to the transmission capacities by constraints (2d). Note that we use a dc model without losses for the sake of simplicity. Constraints (2e) enforce power bounds for blocks of generation units other than wind power units. Constraints (2f) and (2g) limit the voltage angle and fix the voltage angle at the reference bus, respectively. Finally, constraints (2h) are non-anticipativity constraints that enforce that variables $\Delta_{LL,\nu}^{(t)}$ depend on the scenario realization of previous periods but they do not depend on the possible scenario realizations in future periods.

TABLE I
CONVENTIONAL PRODUCTION UNIT OFFER DATA
AND PEAK LOADS FOR THE THREE-BUS SYSTEM

Bus #	Units		Peak load (MW)
	Offer size (MW)	Offer price (\$/MWh)	
1	60, 50, 40	65, 75, 84	150
2	60, 50, 40	61, 72, 81	120
3	50, 40, 40	63, 80, 86	120

The bilevel model (1)–(2) is easily transformed into an MPEC which can be recast as a single-level MILP problem following the procedure described in Appendix A.

IV. THREE-BUS ILLUSTRATIVE EXAMPLE

A. System Data

The proposed model is illustrated using a three-bus system. This system comprises 3 generation units, 3 loads, and 3 transmission lines. The data defining the size and price of power blocks offered by the conventional units, as well as the reference peak loads at each bus of the system, is provided in Table I. We consider that conventional producers offer at marginal cost, being this offering price fixed throughout the planning horizon. Transmission lines, connecting buses 1–2, 2–3 and 1–3, have a susceptance of 5 p.u. and a maximum transmission capacity of 100 MW.

We consider that it is possible to install wind power capacity only at bus 2, being the maximum wind power capacity that can be installed throughout the planning horizon 300 MW. The value of wind power investment cost at the beginning of the first period is equal to \$1 million per MW and the investment budget is not limited.

The planning horizon comprises 10 years divided in 2 periods of 5 years. The wind investment decisions can be made at the beginning of each of the two periods, i.e., at the beginning of the first and sixth years. The amortization rates are considered equal to 28% and 14% for the first and second periods, respectively, i.e., $a_n^{(1)} = 0.28$ and $a_n^{(2)} = 0.14$, $\forall n$.

B. Load and Wind Power Uncertainty Characterization

As described in Section II, we use hourly historical data of load consumption and wind power generation of year 2007 in the electricity market of the Iberian Peninsula [16], [17], which is reduced to a tractable set using a clustering algorithm [13]. The reduced data consists of 20 operating conditions, each one defined by a load level, a wind power capacity factor and the number of hours of the original data set comprising this operating condition. The resulting load and wind conditions are provided in Table II. For simplicity, we assume that load demand/wind power production conditions are the same at all buses of the system.

C. Scenario Characterization

In order to illustrate the working of the proposed model, we consider and analyze the following cases.

1) *Case 1. Investment Cost Uncertainty:* In this case we consider uncertainty only in the investment costs. We consider that

TABLE II
DEMAND AND WIND CONDITIONS

ν	k^D (pu)	k^W (pu)	ϑ (h)	ν	k^D (pu)	k^W (pu)	ϑ (h)
1	0.4679	0.4573	214	11	0.6536	0.5401	236
2	0.3945	0.2343	246	12	0.7545	0.4580	381
3	0.7216	0.1957	578	13	0.6452	0.2692	695
4	0.5374	0.3101	507	14	0.6208	0.0760	532
5	0.8680	0.3966	154	15	0.8241	0.2438	332
6	0.5229	0.0943	646	16	0.6224	0.3986	452
7	0.7166	0.0829	686	17	0.3455	0.4876	288
8	0.7286	0.3239	493	18	0.5380	0.2077	670
9	0.3113	0.3289	311	19	0.8337	0.1046	347
10	0.3146	0.0892	377	20	0.6286	0.1644	615

TABLE III
SCENARIO DATA FOR CASE 2

Scenario #	Peak Load 1st period	Peak load 2nd period	τ
1		H (1.4)	0.09
2	H (1.2)	M (1.2)	0.12
3		L (1)	0.09
4		H (1.2)	0.12
5	M (1)	M (1)	0.16
6		L (0.8)	0.12
7		H (1)	0.09
8	L (0.8)	M (0.8)	0.12
9		L (0.6)	0.09

there are three possible realizations (scenarios) of the investment cost in the second period, namely high (H), medium (M) and low (L), which mean that the investment cost in the second period is equal to the investment cost in the first period multiplied by factors 1.3, 1 and 0.7, respectively. The weights of H, M and L scenarios are 0.3, 0.4 and 0.3, respectively.

2) *Case 2. Load Demand/Wind Power Production Uncertainty*: In this case we assume that investment costs are the same for the first and second periods. The uncertainty only affects the system load and wind production. As described in Section II, we consider that wind power capacity factors do not change in the planning horizon, i.e., we consider the uncertainty in the load of the system and assume that wind capacity factors maintain the correlation pattern provided in Table II.

The scenario data are provided in Table III. We consider three possible scenarios realizations in the first period and three scenarios in the second period for each of the scenarios in period 1. The number between parenthesis in the second and third columns represents a factor that multiplies, in periods 1 and 2, the reference peak loads of the system provided in Section IV. A rendering the actual value of peak load for each period and scenario. The fourth column gives the weight of each scenario.

3) *Case 3. Load Demand/Wind Power Production and Investment Cost Uncertainty*: In this case we consider uncertainty in both load demand/wind production and investment cost. We consider two realizations of the investment costs in the second period, medium (M) and low (L), which mean that investment costs in the second period are equal to and 30% lower than the investment costs in the first period, respectively. The probability

TABLE IV
SCENARIO DATA FOR CASE 3

Scenario #	Period 1 Load	Period 2		τ
		Cost	Load	
1	H (1.1)	M (1)	H (1.21)	0.252
2	H (1.1)	M (1)	L (1.045)	0.168
3	L (0.95)	M (1)	H (1.045)	0.168
4	L (0.95)	M (1)	L (0.9025)	0.112
5	H (1.1)	L (0.7)	H (1.21)	0.108
6	H (1.1)	L (0.7)	L (1.045)	0.072
7	L (0.95)	L (0.7)	H (1.045)	0.072
8	L (0.95)	L (0.7)	L (0.9025)	0.048

associated to these scenarios is 0.7 and 0.3, respectively. Additionally, we consider two realizations of the load of the system in period 1, high (H) and low (L), which mean that the peak load in period 1 is 10% higher and 5% lower than the reference peak load, respectively. For each of these load scenario realizations, we consider two load scenario realizations in the second period, H and L, which mean an increase of 10% and a decrease of 5% in the peak load of period 2 with respect to the peak load in period 1, respectively. In each period, the probabilities associated to the H and L scenarios are 0.6 and 0.4, respectively. These scenarios belong to the scenario trees depicted in Fig. 1. As stated before, the wind power is obtained through appropriate correlation coefficients. Since the investment costs and the load/wind of the system can be considered independent, we obtain the eight scenarios provided in Table IV. Numbers between parenthesis in the second and fourth columns are factors that multiply the reference peak load and provide for each scenario the peak load of the system in periods 1 and 2.

The factor between parenthesis in the third column multiplies the investment cost in the first period and provides the investment cost in period 2 for each scenario. The weight of each scenario in Table IV is obtained by multiplying the probabilities associated to the corresponding wind power investment cost and load demand/wind production scenario realizations. For example, the weight of scenario 1 (0.252) is computed as 0.7 (H investment cost) times 0.6 (H load in period 1) times 0.6 (H load in period 2).

D. Results

All the results for this Illustrative Example and for the Case Study in Section V are obtained using CPLEX 11.2.1 [18] under GAMS [19] on a Linux-based server with four processors clocking at 2.9 GHz and 250 GB of RAM.

Problem (1)–(2) is solved considering different values of the weighting parameter β . Figs. 2(a)–2(c) depicts the efficient frontier for cases 1, 2 and 3, respectively. The efficient frontier shows how the expected profit decreases as the CVaR increases. We assume a confidence level $\alpha = 0.95$.

It is relevant to analyze how the optimal investment decisions change as the considered level of risk changes through the weighting parameter β . We consider two situations. First, a risk-neutral investor, i.e., $\beta = 0$. Second, a risk-averse investor with $\beta = 10$. Note that for the considered problem, a value of β equal to 10 is realistic as it allows identifying a risk-averse solution sufficiently different from the risk-neutral one. The optimal

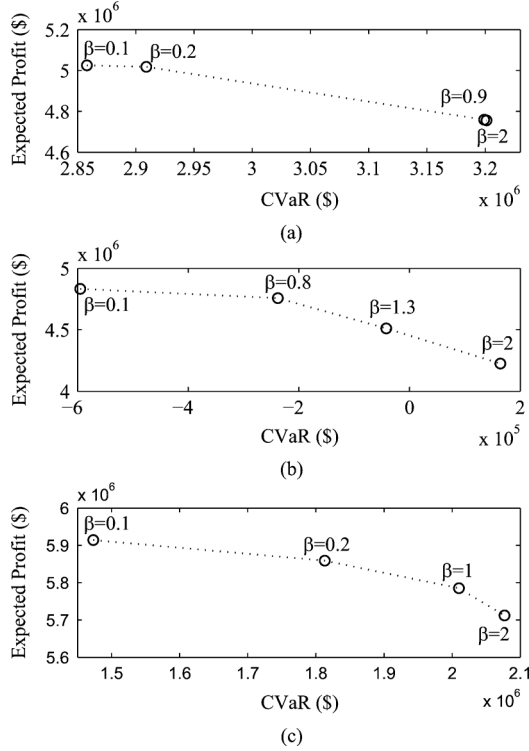


Fig. 2. Expected profit versus CVaR.

TABLE V
COMPARISON OF RISK-NEUTRAL AND RISK-AVERSE
RESULTS FOR THE THREE-BUS SYSTEM

Case	Scenario #	Risk-neutral		Risk-averse	
		$X_2^{(1)}$ (MW)	$X_2^{(2)}$ (MW)	$X_2^{(1)}$ (MW)	$X_2^{(2)}$ (MW)
1	1		0		0
	2	114.3	39.0	155.3	0
	3		185.7		146.7
2	1-3		108.3		108.3
	4-7	152.3	0.8	60.7	92.3
	7-9		0		0
3	1-2		198.8		136.0
	3-4	67.2	92.4	130.0	78.7
	5-6		232.8		170.0
	7-8		232.8		170.0

investment decisions for the risk-neutral and the risk-averse investors are provided in Table V.

In case 1, the risk-averse solution involves a higher wind power capacity to be installed in the first period than that pertaining to the risk-neutral solution. The investment cost is subject to uncertainty in the second period and thus, installing a higher wind power capacity at the beginning of the first period reduces the profit variability since the investment cost for period 1 is known at the time the investment decisions are made.

In case 2 the uncertainty is in the load demand/wind power production of the system so the risk-averse solution consists in installing a lower wind power capacity in period 1 than that pertaining to the risk-neutral solution, because the risk-averse investor prefers to wait until the second period when it has a better knowledge of the load of the system.

TABLE VI
DEMAND AND WIND CONDITIONS FOR THE IEEE 118-BUS TEST SYSTEM

ν	k_t^D (pu)	k_{t+2}^D (pu)	k_{north}^W (pu)	k_{south}^W (pu)	ϑ (h)
1	0.6601	0.6620	0.1637	0.1794	4711
2	0.3902	0.3908	0.3023	0.3346	678
3	0.4079	0.4058	0.4690	0.5161	549
4	0.4075	0.4048	0.1052	0.1193	832
5	0.7038	0.6996	0.3953	0.4346	1990

Finally, in case 3 we consider the uncertainty in both the load demand/wind power production of the system and the investment costs. In this case, the risk-averse solution proposes installing a higher wind power capacity in the second period than in the risk-neutral solution. Investment cost is expected to decrease in the second period with a probability of 30%. A risk-neutral investor prefers to wait and take advantage of this possible reduction of the investment costs, while a risk-averse investor prefers to invest a higher amount at the beginning of the planning horizon and reduce the profit variability. Finally, note also that the optimal solutions involve a higher wind power capacity in those scenarios with higher load.

V. CASE STUDY

To further analyze the proposed model, results from a case study based on the IEEE 118-Bus Test System [20] are provided in this section.

The IEEE 118-Bus Test System comprises 54 generating units, 99 loads, and 186 transmission lines. Data defining this system can be found in [20].

The planning horizon comprises 15 years divided in five-year periods. The investment decisions can be made at the beginning of the first, sixth, and eleventh years.

We consider that wind power can be installed at buses 7 and 89, with a maximum of 800 MW at each bus throughout the planning horizon. Investment cost at the beginning of the planning horizon is equal to \$1 million per MW and amortization rates are considered equal to 42, 28, and 14% for the first, second, and third periods, respectively, i.e., $a_n^{(1)} = 0.42$, $a_n^{(2)} = 0.28$, and $a_n^{(3)} = 0.14$, $\forall n$. Investment budgets are not limited.

We consider that the IEEE 118-Bus Test System is divided in two demand zones, namely t and $t+2$. Historical load levels in these zones are assumed to be the same but with two hours delay. On the other hand, we consider two wind zones in the system, north and south, and assume that wind power capacity factors in the south zone are 10% higher than in the north, and that a two-hours wind power delay exists between the north and south zones. Note that this is considered due to the lack of data and to show that the model can incorporate different load demand/wind power generation conditions at different buses. We reduce the historical data set into the five operating conditions provided in Table VI.

In each period, we consider two possible scenario realizations for the load demand/wind power production of the system, which imply an increase of 10% (H scenario) and a decrease of 5% (L scenario) in the peak load with respect to the previous period, respectively. The probabilities of these

TABLE VII
SCENARIO DATA FOR THE IEEE 118-BUS TEST SYSTEM

Scenario #	Period 1 Load	Period 2 Cost	Period 2 Load	Period 3 Cost	Period 3 Load	τ
1	H	M	H	M	H	0.10584
2	H	M	H	M	L	0.07056
3	H	M	H	L	H	0.04536
4	H	M	H	L	L	0.03024
5	H	M	L	M	H	0.07056
6	H	M	L	M	L	0.04704
7	H	M	L	L	H	0.03024
8	H	M	L	L	L	0.02016
9	H	L	H	M	H	0.04536
10	H	L	H	M	L	0.03024
11	H	L	H	L	H	0.01944
12	H	L	H	L	L	0.01296
13	H	L	L	M	H	0.03024
14	H	L	L	M	L	0.02016
15	H	L	L	L	H	0.01296
16	H	L	L	L	L	0.00864
17	L	M	H	M	H	0.07056
18	L	M	H	M	L	0.04704
19	L	M	H	L	H	0.03024
20	L	M	H	L	L	0.02016
21	L	M	L	M	H	0.04704
22	L	M	L	M	L	0.03136
23	L	M	L	L	H	0.02016
24	L	M	L	L	L	0.01344
25	L	L	H	M	H	0.03024
26	L	L	H	M	L	0.02016
27	L	L	H	L	H	0.01296
28	L	L	H	L	L	0.00864
29	L	L	L	M	H	0.02016
30	L	L	L	M	L	0.01344
31	L	L	L	L	H	0.00864
32	L	L	L	L	L	0.00576

TABLE VIII
RESULTS FOR THE IEEE 118-BUS TEST SYSTEM

β	#	$X_7^{(1)}$ (MW)	$X_7^{(2)}$ (MW)	$X_7^{(3)}$ (MW)	$X_{89}^{(1)}$ (MW)	$X_{89}^{(2)}$ (MW)	$X_{89}^{(3)}$ (MW)
0	1-2			179.27			154.4
	3-4			800			154.4
	5-6		0	0		645.6	154.4
	7-8			800			154.4
	9-10			0			0
	11-12			0			0
	13-14		800	0		800	0
	15-16			0			0
	17-18	0		0	0		800
	19-20		0	800		0	800
	21-22			0			800
	23-24			800			800
	25-26			0			0
	27-28		800	0		800	0
	29-30			0			0
	31-32			0			0
1	1-2			622.71			181.17
	3-4			800			181.17
	5-6		0	0		618.83	181.17
	7-8			800			181.17
	9-10			0			0
	11-12			0			0
	13-14		800	0		800	0
	15-16	0		0	0		0
	17-18			0			0
	19-20		0	800		800	0
	21-22			0			0
	23-24			800			0
	25-26			0			0
	27-28		800	0		800	0
	29-30			0			0
	31-32			0			0

scenarios are 0.6 and 0.4, respectively. On the other hand, we consider that wind power investment costs in each period can remain unaltered (M scenario) or can be 30% lower (L scenario) than the investment costs in the previous period, with a probability of 0.7 and 0.3, respectively. Jointly considering load demand/wind power production and investment cost scenarios, we obtain for the three periods the 32 scenarios summarized in Table VII.

Note that considering the above data, there are one possible investment decision at the beginning of the first period, four different investment decisions at the beginning of period 2 and 16 alternative investment decisions at the beginning of the third period depending on the scenario realizations.

The characteristics of the proposed model, an MPEC recast as a MILP problem, make it intractable if a high number of operating conditions and scenarios are considered in systems with a large number of buses. However, the proposed problem has a decomposable structure. If the investment decision variables (i.e., the wind capacity to be built at the beginning of each period) and the auxiliary variable ζ are fixed to given values, problem (1)–(2) can be decomposed by scenario γ . Thus, Benders decomposition (summarized in Appendix B) can be applied [21].

One of the characteristics of the considered problem is its non-convexity. To ensure that the solution attained is not a local maximum, we initialize the Benders' algorithm in different points and select the solution that provides the highest value of the objective function (1a). Note that if the algorithm is initialized in a large enough number of initial solutions, the obtained local optimum will generally be the global optimum or close to it. This is a common practice in non-convex problems like the one proposed in this paper, [22].

Results for a risk-neutral ($\beta = 0$) and a risk-averse ($\beta = 1$) wind power investor are provided in Table VIII.

Bus 89, located in the south, has higher wind power capacity factors than bus 7, located in the north. Thus, it is preferable to install wind power capacity at bus 89.

Note that at bus 89, the available 800 MW of wind power are built for all scenarios and for the two values of β , while at bus 7, the whole 800 MW are only built for specific scenarios, e.g., scenarios 9–16 and 25–32, in which the investment cost in period 2 decreases.

The investment decisions are different depending on the scenario realizations. For example, for $\beta = 0$ and bus 89, the wind investor builds the whole 800 MW at the beginning of the

second period in scenarios 9–16 and 25–32, which are characterized by the reduction of the investment cost with respect to period 1. In scenarios 1–8 and 17–24, the wind power investment cost in period 2 is equal to that in the first period, and the wind power investor decides not to build the whole 800 MW and wait until the third period. However, the solutions for these two set of scenarios are different: in scenarios 1–8, the load of the system in period 1 is higher than in scenarios 17–24, and the wind power investor decides to build part of the 800 MW at the beginning of period 2, while in scenarios 17–24, the wind power investor decides not to build any wind power capacity in this point in time.

Regarding the differences between the risk-neutral and the risk-averse investment decisions, note that the most significant difference is found in scenarios 17–24 at bus 89. While a risk-neutral wind power investor prefers to wait until period 3 and take advantage of a possible reduction of the investment costs, the risk-averse investor prefers to build the wind power capacity at the beginning of the second period, reducing the profit variability.

The average computation time required to obtain the optimal solution applying Benders decomposition is 2.34 hours for $\beta = 0$ and 38.25 hours for $\beta = 1$, which is compatible with the time requirements in investment studies as the one proposed in this paper.

VI. CONCLUSIONS

Considering the analysis and case studies above, the conclusions below are in order:

- 1) A multi-stage modeling is a must as wind investment costs significantly change along the study horizon, and having the option of investing in different points in time makes a significant difference.
- 2) The proposed MILP—derived from a stochastic MPEC—is tractable for systems of realistic size provided that the number of considered scenarios is small enough. If needed, Benders decomposition can be readily used to ease the computational burden.
- 3) As expected, different risk-aversion levels for the wind power investor result in different investment strategies in wind facilities.
- 4) An illustrative example and a case study illustrate the theoretical relevance and practical interest the proposed methodology.
- 5) Future work includes the development of an equilibrium model considering rivals investment decisions.

APPENDIX A

MPEC TO MILP PROBLEM TRANSFORMATION

The bilevel problem (1)–(2) is easily transformed into an MPEC that can be recast as a single-level MILP problem following the procedure below [6]:

- 1) Each one of the lower-level problems (2) is replaced by its Karush-Kuhn-Tucker (KKT) conditions.
- 2) KKT conditions are included as constraints of the upper-level problem (1) rendering a single-level nonlinear problem.

- 3) Nonlinear terms are linearized through exact linear expressions based on the Fortuny-Amat transformation of complementarity constraints [23] and the strong duality equality [24].

APPENDIX B

BENDERS ALGORITHM

The Benders algorithm is summarized below:

- 1) Trial investment decisions and an arbitrary value for the auxiliary variable ζ are provided.
- 2) Problem (1)–(2) is solved with investment variables and auxiliary variable ζ fixed to the given values in the previous step. This way problem (1)–(2) can be solved per scenario. The solutions of these problems (one per scenario) provide sensitivities.
- 3) Sensitivities of step 2) are used to solve a so-called master problem, which constitutes an increasingly accurate approximation of the original problem. The solution of this problem provides updated values for the investment variables and for the auxiliary variable ζ .

Steps 2)–3) are repeated iteratively until investment variables remain unchanged in two consecutive iterations.

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