Financial Storage Rights

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***Abstract*—Should energy storage buy and sell power at wholesale prices like utilities and generators, or should its physical and financial operation be asynchronyous as with transmission lines? In the first case, storage straightforwardly profits through intertemporal arbitrage, also known as load shifting and peak shaving. In this paper, the latter case, referred to as *passive storage*, is examined. Because passive storage does not make nodal price transactions, new mechanisms are necessary for its integration into electricity markets. This issue is addressed by further developing the analogy between energy storage and transmission. Specifically, financial rights are defined for storage, and tracing is extended to multiperiod power flows linked by storage. Like flowgate transmission rights, the new *financial storage rights* redistribute the system operator's merchandising surplus and enable risk-averse market participants to hedge against nodal price volatility resulting from *storage congestion*. Simple examples are given demonstrating the implementation of the new mechanisms. Game-theoretic analysis suggests that financial storage rights mitigate gaming when both the generator and load bid strategically.**

***Index Terms*—Congestion, energy capacity right (ECR), energy storage, financial storage rights (FSR), gaming, market power, power capacity rights (PCR), tracing.**

I. Introduction

ENERGY storage is crucial to renewable integration and can dramatically enhance power system efficiency and reliability [1], [2]. The broad integration of storage into power systems depends on both its physical and economic implementations. The current regulatory framework in U.S. ISOs and RTOs impedes this objective by requiring that storage be classed exclusively either as a wholesale market-based asset or as a transmission asset, which recovers its annual revenue requirement through rate-based payments. While it is currently standard to treat storage as a generation asset, some system operators like PJM are now looking at the alternative [3]. Storage is able to provide diverse services that naturally fit in both classes, so that choosing just one results in its undervaluation and suboptimal utilization; see [4] for a broader discussion of these issues.

These regulatory barriers have necessitated consideration of *passive storage*, which shifts load but does not buy or sell power in wholesale markets, enabling it to be more easily treated as a transmission asset. However, while the concept has been presented (e.g., in [3]), it has not yet received rigorous treatment as a component of wholesale market design and transmission planning. In this paper, mechanisms for consistently linking the economics of passive storage to system operation are motivated by the following physical analogies between storage and transmission.

- Both move power: whereas transmission does so spatially, storage moves power forward in time. A power system with transmission and storage can be drawn as a network with spatial and temporal edges.
- The amount of power moved is constrained by a power capacity. Storage is additionally constrained by an energy capacity.
- Both have large fixed procurement costs and low marginal operating costs.

First, financial rights are defined for storage in the spirit of financial transmission rights, see [5]–[8] and especially [9], wherein financial transmission rights are generalized to more broadly defined transmission assets. Conceptually, the rights are similar to flowgate transmission rights [6], [7], except that they must be defined for both power and energy constraints. These are respectively referred to as power capacity rights (PCR) and energy capacity rights (ECR) and combined in financial storage rights (FSR). Just as financial transmission rights take on nonzero value under transmission congestion, the value of storage rights is realized when storage is used to capacity. This is referred to as *storage congestion*; note that both types of congestion increase the minimum objective value of economic dispatch by constraining system operation. For this reason, transmission has been referred to as a "negative externality." Increasing the capacity of either has been empirically shown to alleviate nodal price volatility [10].

The rights are constructed from the dual multipliers of multiperiod optimal power flow, which can be used to construct nodal prices for markets with storage [11], [12]. For notational concision, the linearized power flow is employed. Details like line losses and reactive power could be incorporated using a convex relaxation [13], [14]. Storage parameters are permitted to vary with time to accommodate flexible load aggregations modeled as virtual energy storage, see [15] and [16].

FSRs serve purposes similar to their transmission counterparts, including the following.

- risk-averse market participants can hedge against nodal price volatility.
- Storage owners can recover upfront costs.
- The system operator can redistribute budget surpluses.

Further to the last point, passive storage cannot arbitrage over time in the same sense that transmission lines cannot arbitrage over space. This deprives it of a significant profit channel and leaves the system operator with *budget* or *merchandising surpluses*, which the rights redistribute.

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A multiperiod tracing scheme is also constructed for decomposing power and energy that has passed through storage and transmission into nodal contributions. The new scheme straightforwardly extends conventional *proportional sharing* approaches [17] using a new *mixing assumption* about energy stored between time periods, and can be similarly used for alternative pricing schemes or allocating fixed costs.

In Section V, FSRs and multiperiod tracing are implemented on a simple analytical example. The effect of FSRs on gaming is also examined when a collocated generator and load strategically report their cost and demand curves over two time periods. The analysis suggests that the rights actually mitigate gaming, particularly when evenly allocated between the load and generator.

II. Motivating Example

Consider a single generator with convex, differentiable real power cost $f(p)$. It sells p_1 in the first period and then p_2 in the following to a load with demands $0 < d_1 < d_2$. With no storage, power and demand must balance within each period, so that the cost is simply $f(d_1) + f(d_2)$. Suppose that a lossless, passive storage exists with energy capacity $\overline{c}$ and no charge rate limits, which enables power generated in the first period to be used in the second. Assume that the storage is empty in the first time period. The total system cost is then

$$\begin{cases} f(d_1+\overline{c}) + f(d_2-\overline{c}), & \text{if } 2\overline{c} < d_2 - d_1 \\ 2f\left(\frac{d_1+d_2}{2}\right), & \text{if } 2\overline{c} \geq d_2 - d_1. \end{cases}$$

If the storage is congested, $p_1 = d_1 + \overline{c}$ and $p_2 = d_2 - \overline{c}$. Under marginal cost pricing, the generator's net profits are

$$\sum_t \left(p_t \left. \frac{df(p)}{dp} \right|_{p=p_t} - f(p_t) \right)$$

and the load pays

$$(p_1 - \overline{c}) \left. \frac{df(p)}{dp} \right|_{p=p_1} + (p_2 + \overline{c}) \left. \frac{df(p)}{dp} \right|_{p=p_2}$$

over both time periods. Here, the power quantity $\overline{c}$ was bought from the generator at the first period price and sold to the load at the potentially far higher second period price. Consequently, the system operator profits by the amount

$$\sum_t (d_t - p_t) \left. \frac{df(p)}{dp} \right|_{p=p_t} \geq 0.$$

Observe that this exact quantity would be paid by the system operator to non-passive storage, i.e. if it bought and sold power at nodal prices. Therefore, reiterating, it is only for passive storage that this paper's approach is relevant.

This discrepancy between the price at which the stored quantity is bought and sold and the load's resulting cost increase may pose a considerable risk to the load; for instance, the larger second period price could be a spike, which the load would want to hedge against. Analogously, the generator may want to claim the system operator's resulting budget surplus as additional profits.

III. General Formulation

A sequence of power markets linked by storage is modeled with the multiperiod, linearized optimal power flow coupled by energy storage constraints over time periods $t = 1, \ldots, T$. For notational concision, it is assumed that each node has one generator and periods have unit spacing, i.e., the time between periods is one. Variables are first subscripted by node, storage, or line index and then by time. For instance, the real power produced or consumed at node i in period t is $p_{i,t}$. Hard generation and load constraints are encoded in the upper and lower power limits $\overline{p}_{i,t}$ and $\underline{p}_{i,t}$. Transmission lines have constant susceptances and capacities, b_{ij} and $\overline{s}_{ij}$.

A standard inventory model of energy storage is used [18], with the set indices of the storages at node i denoted by $\mathcal{S}_i$. Each storage is characterized by charge and discharge rate limits, $\underline{r}_{i,t}$ and $\overline{r}_{i,t}$, an energy capacity constraint $\overline{c}_{i,t}$, charge and discharge loss coefficients, $0 \leq \eta^+_{i,t} \leq 1$ and $\eta^-_{i,t} \geq 1$, and an energy leakage coefficient, $0 \leq \alpha_{i,t} \leq 1$. All energy storage parameters are permitted to vary with time to accommodate virtual energy storages comprised of flexible load aggregations [15], [16]. Storage i's state of charge at time t is denoted $e_{i,t}$, and its charge and discharge rates by $u^+_{i,t}$ and $u^-_{i,t}$, respectively. Note that these parameters are taken to refer to the portion of storage dedicated to load shifting, and do not pertain to portions committed to other services like frequency regulation and power balancing.

To accommodate general intertemporal costs, the objective is allowed to be any convex function of p over all nodes and time periods, $\mathcal{F}(p)$. For example, an objective of the form

$$\mathcal{F}(p) = \sum_{i,t} f_{i,t}(p_{i,t}) + g_{i,t}(p_{i,t}, p_{i,t+1})$$

can capture the cost of generation in each period plus period-to-period ramping costs.

The multiperiod, linearized optimal power flow is given below. Each constraint's dual multipliers are listed to their left, with complementarity relationships indicated by the $\perp$ symbol. Unless otherwise specified, each constraint holds over all nodes $i = 1, \ldots, N$ and periods $t = 1, \ldots, T$ as follows:

$$\min_{p,\theta,e,u^+,u^-} \quad \mathcal{F}(p) \quad \text{subject to}$$

$$\lambda_{i,t}: \quad p_{i,t} = \sum_{j \in \mathcal{S}_i} \left(u^+_{j,t} + u^-_{j,t}\right) + \sum_j b_{ij}(\theta_{i,t} - \theta_{j,t}) \tag{1}$$

$$\xi^l_{i,t}, \xi^u_{i,t} \geq 0 \perp \underline{p}_{i,t} \leq p_{i,t} \leq \overline{p}_{i,t} \tag{2}$$

$$\mu_{ij,t} \geq 0 \perp b_{ij}(\theta_{i,t} - \theta_{j,t}) \leq \overline{s}_{ij} \tag{3}$$

$$\gamma^{+,l}_{i,t}, \gamma^+_{i,t} \geq 0 \perp 0 \leq u^+_{i,t} \leq \overline{r}_{i,t} \tag{4}$$

$$\gamma^-_{i,t}, \gamma^{-,u}_{i,t} \geq 0 \perp \underline{r}_{i,t} \leq u^-_{i,t} \leq 0 \tag{5}$$

$$\chi^l_{i,t}, \chi_{i,t} \geq 0 \perp 0 \leq e_{i,t} \leq \overline{c}_{i,t} \tag{6}$$

$$\sigma_{i,t}: \quad e_{i,t+1} = \alpha_{i,t} e_{i,t} + \eta^+_{i,t} u^+_{i,t} + \eta^-_{i,t} u^-_{i,t} \tag{7}$$

$$\sigma_{i,0}: \quad e_{i,1} = 0. \tag{8}$$

$\lambda_{i,t}$ and $\mu_{ij,t}$ are, respectively, the standard nodal and line shadow prices [19], [20]. Note that (3) is enforced over all lines in both directions, e.g., both $b_{12}(\theta_{1,t} - \theta_{2,t}) \leq \overline{s}_{12}$ and

$b_{21}(\theta_{2,t} - \theta_{1,t}) \leq \overline{s}_{21}$ are present if nodes one and two are connected. The initial states of charge are arbitrarily set to zero because the beginning of a multiperiod market is the end of the preceding and because it is not relevant to the subsequent developments.

Remark 1 (Simultaneous Charging and Discharging): Simultaneous charging and discharging of storage is permitted in the above model. This may be optimal in some scenarios, e.g., when a nodal price is negative, but may also diminish storage lifespans or be unphysical for some technologies or timescales. Solutions with simultaneous charging and discharging may be proscribed using integer variables, but at the expense of computational tractability and the existence of optimal dual multipliers. Such a formulation may be amenable to heuristic approaches for pricing integer programs [21], [22]; this is not pursued here. Note that, if $\eta^+_{i,t} = \eta^-_{i,t} = 1$, there is no possible benefit in simultaneous charging and discharging, and $u^+_{i,t}$ and $u^-_{i,t}$ may be combined into one variable, $u_{i,t}$.

An optimal solution must satisfy the below stationarity conditions, which are obtained by differentiating the Lagrangian by the primal variables as follows:

$$\frac{d\mathcal{F}(p)}{dp_{i,t}} - \lambda_{i,t} + \xi^u_{i,t} - \xi^l_{i,t} = 0 \quad (9)$$

$$\sum_j b_{ij}(\lambda_{i,t} - \lambda_{j,t} + \mu_{ij,t} - \mu_{ji,t}) = 0 \quad (10)$$

$$\lambda_{j:i\in\mathcal{S}_j,t} - \gamma^{+,l}_{i,t} + \gamma^+_{i,t} + \eta^+_{i,t}\sigma_{i,t} = 0 \quad (11)$$

$$\lambda_{j:i\in\mathcal{S}_j,t} - \gamma^-_{i,t} + \gamma^{-,u}_{i,t} + \eta^-_{i,t}\sigma_{i,t} = 0 \quad (12)$$

$$\chi_{i,t} - \chi^l_{i,t} + \alpha_{i,t}\sigma_{i,t} - \sigma_{i,t-1} = 0 \quad (13)$$

$$\chi_{i,T+1} - \chi^l_{i,T+1} - \sigma_{i,T} = 0 \quad (14)$$

The following budget balance condition relates the system operator's budget surplus to the subsequent financial right definitions:

$$\sum_t \sum_i \lambda_{i,t} p_{i,t} + \sum_j \mu_{ij,t}\overline{s}_{ij} + \sum_{j\in\mathcal{S}_i} \gamma^+_{j,t}\overline{r}_{j,t} - \gamma^-_{j,t}\underline{r}_{j,t} + \chi_{j,t}\overline{c}_{j,t} = 0. \quad (15)$$

This condition can be derived through arithmetic manipulation of the Karush–Kuhn–Tucker conditions and therefore is satisfied by any optimal pair of primal and dual solutions. Specifically, it is obtained by applying the following steps.

Step 1) Multiple (1) by $\lambda_{i,t}$ and sum the product over all i and t.

Step 2) Using (10) and complementary slackness, replace the resulting term $\sum_{ij,t} \lambda_{i,t} b_{ij}(\theta_{i,t} - \theta_{j,t})$ with $-\sum_{ij,t} \mu_{ij,t}\overline{s}_{ij}$.

Step 3) Using (7), (11)–(13), and complementary slackness, replace the term $\sum_{i,t} \lambda_{i,t} \sum_{j\in\mathcal{S}_i}(u^+_{j,t} + u^-_{j,t})$ with $\sum_{i,t}\sum_{j\in\mathcal{S}_i} -\gamma^+_{j,t}\overline{r}_{j,t} + \gamma^-_{j,t}\underline{r}_{j,t} - \chi_{j,t}\overline{c}_{j,t}$.

The first term in (15) is the system operator's budget surplus resulting from its nodal payments. Observe that, if any of (3)–(6) are active at their nonzero limits, the associated dual multipliers and the budget surplus are positive. The remaining terms in (15) identify financial storage and transmission rights that redistribute this surplus. Specifically, the second term corresponds to a flowgate transmission right [6], [20], defined as an entitlement to a payment of $\mu_{ij,t}$ times a contracted quantity of power flow. The last three terms represent storage congestion charges and motivate the following financial rights.

Let hatted variables denote the quantity of power or energy associated with each right. For example, $\hat{e}_{i,t}$ would be a prespecified amount of kilowatt-hours, which must be less than $\overline{c}_{i,t}$. Define the following financial rights:

- **Power capacity right (PCR)**: The owner of a PCR collects $\gamma^+_{i,t}\hat{u}^+_{i,t} - \gamma^-_{i,t}\hat{u}^-_{i,t}$.
- **Energy capacity right (ECR)**: The owner of an ECR collects $\chi_{i,t}\hat{e}_{i,t}$.
- **Financial storage right (FSR)**: The owner of an FSR collects $\gamma^+_{i,t}\hat{u}^+_{i,t} - \gamma^-_{i,t}\hat{u}^-_{i,t} + \chi_{i,t}\hat{e}_{i,t}$.

Only one PCR has been defined for both charging and discharging for simplicity and for reasons discussed in Remark 1. Similarly, although an FSR is just the combination of a PCR and ECR, it is pragmatic for its simplicity. An ECR may however be preferable to an FSR for a market participant that contributes to a binding energy but not power constraint, e.g., a generator that stores power over many time periods but does not charge or discharge during periods of high ramping. Likewise, a PCR may be preferable to a market participant that does not significantly contribute to stored quantities but which produces or consumes large amounts over a few time periods.

Physical storage rights entitling the owner to control over usage could defined as well; however, only financial rights are considered here because they cannot contradict real-time system operations, among other analogous reasons from the transmission case [7], [23].

The following are salient properties of the storage rights.

- Like flowgate transmission rights, a storage right cannot take on negative values and thus cannot leave its holder with financial obligations.
- When power traverses both transmission lines and storage on its paths from producers to consumers, it may be reasonable to jointly assign transmission and storage rights in the form of spatiotemporal paths or networks.
- If storage is perfectly efficient and has no power limits, an ECR $\chi_{i,t}\hat{e}_{i,t}$ is equivalent to the nodal price difference, $\lambda_{j:i\in\mathcal{S}_j,t} - \lambda_{j:i\in\mathcal{S}_j,t-1}$, times $\hat{e}_{i,t}$. However, whereas such relations may be useful for radial transmission lines with small losses, realistic storage losses can be more substantial, and power constraints may bind before energy constraints. Inclusion of these details into such relationships is possible but leads to more complicated expressions. For these reasons, such reformulations are discouraged.

Remark 2 (Nodal Price Volatility and Hedging): The mathematical outcome of multiperiod optimal power flow and hence the resulting nodal prices do not depend on whether storage is passive or buys its energy at nodal prices. Whereas active storage profits directly from nodal price differences via intertemporal arbitrage, passive storage would profit through sales of rights, e.g. an auction. Ideally, the right purchase price equals the total rent collected over the right's lifespan. By purchasing rights, risk-averse market participants exchange potentially volatile nodal price transactions for one upfront payment. Active storage does not offer this flexibility.

Remark 3 (Transmission Versus Physical and Virtual Storage): An important distinction between storage and transmission arises from the one-way nature of the energy constraints (6) and (7). Were the lower limit in (6) negative rather than zero, storage would be able to move power backward in time just as a transmission line can move power in two directions. Mathematically, this manifests in ECRs only having value forward in time; more precisely, although the dual multiplier of the lower limit in (6), $\chi^{l}_{i,t}$, may be positive, the associated "backward" ECR would be worthless because the lower limit is zero. However, deferrable load aggregations that shift demand forward in time virtually move energy backward in time. This could be simply modeled by making the lower limit of (6) negative, in which case a new term would appear in (15) and ECRs could be defined in two directions. In this regard, idealized flexible load aggregations are more versatile resources than physical energy storage.

A. Imperfect Information and Ancillary Services

The multiperiod linearized power flow and consequently FSRs are restrictive because:

- they rely on accurate predictions of future power requirements, and
- they only describe load shifting.

Indeed, the envisioned benefits of some storage services are primarily realized under uncertainty, for example buffering renewable intermittency by absorbing random power imbalances and ancillary services like frequency regulation; neither of these are described by the multiperiod optimal power flow in this paper. A consequence of this limitation is that dedicating a storage's full capacity and finances to this framework may result in its dramatic underutilization. Some potential approaches to incorporating these considerations into FSRs are now discussed.

First, storage can bolster stability and power quality through reactive power support. This ancillary service can be accommodated on slow time scales by using a more detailed multiperiod optimal power flow. In particular, second-order cone and semidefinite power flow relaxations capture reactive power and voltage magnitudes [13], [14], and also admit useful nodal prices due to strong duality. Developing storage rights within such a model is a straightforward extension of the current work.

Load shifting, absorbing power imbalances, and frequency regulation all primarily involve real power on different time scales, each with different physics. Rigorously modeling the latter two requires descriptions of uncertainty and fast power system dynamics like generator inertias. Unfortunately, most formal approaches like stochastic and dynamic programming do not straightforwardly offer duality-based tools; an exception is the linear quadratic regulator, which can model frequency regulation and has a simple pricing interpretation [24].

A generic, *ad-hoc* approach to this issue is to replicate multiperiod optimal power flow on multiple timescales. For instance, FSRs could be computed for hourly and five-minute power flow routines. In this case, care must be taken to model the dependencies between the routines, e.g., by modeling the faster time scale as the second of a two-stage program of the hourly routine or by using robust optimization to model worst-case uncertainties. Another possibility is to use model predictive control and simply update the rights as new information becomes available.

Finally, the reader is referred to [25] for discussion on auctioning portions of storage capacity for different services. This approach would potentially circumvent these issues by enabling financial storage rights to be applied only to the portion of a storage's capacity allocated to load shifting. This approach, however, can also lead to underutilization due to fixed portions of storage capacity being allocated to different services prior to the realization of uncertainties.

IV. Tracing Through Space and Time

Tracing is a method of extracting the contributions of individual nodes from bulk power flows in transmission networks [17]. In this section, a multiperiod tracing technique is developed. The core mechanism in tracing is the *proportional sharing rule*, which assumes that the aggregate makeup of a node's incoming power flows is identical to the makeup of each outgoing power flow. Tracing is now extended to the identification of nodal contributions to power and energy that was stored in prior time stages; for instance, how much of the power in a transmission line in the current time period is attributable to generators that stored energy in previous time periods? The key mechanism is a new *mixing assumption*, which states that the makeup of power removed from storage is equal to that of its current stored energy. For concision, each node is now assumed to have at most one storage and one generator.

Let $p_{ij,t}$ denote the flow in line ij at time t. Let $\mathcal{I}_{i,t}$ and $\mathcal{O}_{i,t}$ be the sets of lines with power arriving to and departing from node i at time t, respectively, and let

$$\mathbb{I}_{ik} = \begin{cases} 1, & i = k \\ 0, & i \neq k. \end{cases}$$

The multiperiod tracing decomposition is defined as

$$p^{k}_{il,t} = p_{il,t} \frac{\mathbb{I}_{ik} p^{+}_{i,t} - u^{+,k}_{i,t} - u^{-,k}_{i,t} - \sum_{j \in \mathcal{I}^{p}_{i}} p^{k}_{ij,t}}{p^{+}_{i,t} - u^{+}_{i,t} - u^{-}_{i,t} - \sum_{j \in \mathcal{I}^{p}_{i}} p_{ij,t}}, \quad \text{for each} \quad l \in \mathcal{O}_{i,t} \tag{16}$$

$$u^{+,k}_{i,t} = u^{+}_{i,t} \frac{\mathbb{I}_{ik} p^{+}_{i,t} - u^{-,k}_{i,t} - \sum_{j} p^{k}_{ij,t}}{p^{+}_{i,t} - u^{-}_{i,t} - \sum_{j} p_{ij,t}} \tag{17}$$

$$u^{-,k}_{i,t} = u^{-}_{i,t} \frac{e^{k}_{i,t}}{e_{i,t}} \tag{18}$$

$$e^{k}_{i,t+1} = \alpha_{i,t} e^{k}_{i,t} + \eta^{+}_{i,t} u^{+,k}_{i,t} + \eta^{-}_{i,t} u^{-,k}_{i,t}. \tag{19}$$

Equation (16) is essentially the original proportional sharing rule, augmented to include each node's contribution from storage. (17) says that the fraction of power put into storage i at time t from generator k is equal to that entering the node from generator k through transmission, generation, and, if it is simultaneously charging and discharging, storage. By using $p^{+}_{i,t}$ in (16) and (17), it is assumed that if a node's demand exceeds its supply, any power generated there does not enter the transmission system or storage.

The mixing assumption is applied in (18), which says that the fraction of the power removed from storage originating from

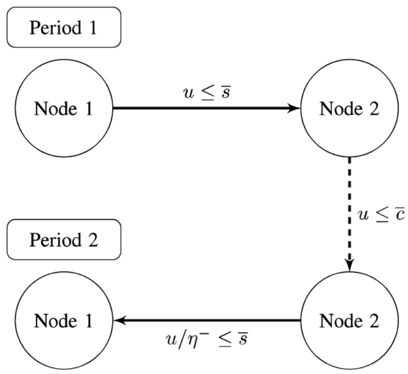

Fig. 1. Power transfers in the two-node, two-period example.

generator k is equal to the total fraction of stored energy originating from generator k. (19) is simply the state of charge evolution, imposed individually on each k.

Summing both sides of each line over k results in primal variables equaling themselves, establishing their consistency. Note that this algorithm traces generated power forward in time, and a similar algorithm could be defined that traces consumed power backward in time.

V. Examples

Here, simple examples are dissected to highlight the properties of storage rights and tracing. First, their implementation is demonstrated in Sections V-A and V-B, and then the effect of storage rights on gaming is examined in Section V-C.

A. Financial Rights

Consider the parallel between flowgate and storage rights for hedging against transmission and storage congestion, respectively. Two nodes over two time periods are connected by a transmission line. Suppose that node two has a storage which is empty in the first period and loses no energy between periods or during charging, but discharging is inefficient ($\eta^- > 1$). Node one has a generator that produces power at cost $f(p) = ap^2 + bp$, and demands in the first and second time periods that satisfy $2a(d_2 - \eta^- d_1) - b(\eta^- - 1) \ge 0$; this ensures that it is optimal to store a nonnegative amount in the first period. Node two has no generation or demand.

Generation costs are reduced by shifting load from the first to second period. The quantity u is moved from node one into the storage at node two in the first period, and then sent back to node one in the second. Load shifting is limited by the capacity of the transmissionline, $\overline{s}$ and the energy capacity of the storage $\overline{c}$. Fig. 1 illustrates the power transfer and the transmission and storage constraints it must satisfy. The shadow prices associated with the transmission capacity and storage energy capacity constraints are denoted by μ and χ, respectively, and the price of power at node one and time t by λ_t.

To facilitate the exposition, numerical values are included alongside analytical expressions, parameter values for which are given by

$$a = 1 \quad b = 1 \quad d_1 = 1 \quad d_2 = 4 \quad \eta^{-1} = 1.1$$

1) No Congestion: First assume that there is unlimited storage and transmission capacity ($\overline{s} = \overline{c} = \infty$), so that congestion cannot occur. The optimal solution perfectly balances load shifting and discharging losses between periods. The optimal amount to store in the first period is

$$u^* = \frac{2a(d_2 - \eta^- d_1) - b(\eta^- - 1)}{2a(\eta^- + 1/\eta^-)} \approx 1.4$$

which is nonnegative by the initial assumption. With perfectly efficient storage ($\eta^- = 1$), this reduces to $(d_1 - d_2)/2 = 1.5$, half the difference of the demands. Because no capacity constraints are active, $\lambda_1 = \lambda_2/\eta^{-1} \approx 5.8$, both storage and transmission rights have zero value, and the system operator's budget balances. Observe that even without congestion, the prices differ slightly due to storage losses.

2) Congested Transmission: Now assume that the transmission capacity is less than the storage capacity and optimal stored quantity in the uncongested case, with numerical values $\overline{s} = 0.5 \le u^*$ and $\overline{s} \le \overline{c}$. It is now optimal to store $\overline{s} = 0.5$ in the first time period and withdraw $\overline{s}/\eta^- \approx 0.45$ in the second. Since transmission is congested and not storage, the line's flowgate right takes on positive value in the first period, and the storage right has zero value. Due to discharging losses, the line cannot be congested in the second period, at which time the flowgate right also has zero value.

The transmission capacity shadow price in the first time period is

$$\mu = 2a(d_1 + \overline{s}) + b - \frac{2a\left(\eta^- d_2 - \overline{s}\right) + \eta^- b}{\eta^{-2}} \approx 3.4.$$

The value of the flowgate right is hence $\mu\hat{s}$, where $\hat{s}$ is the portion of transmission capacity associated with the right.

The congestion causes the price at node one to be substantially higher in period two than one, i.e. $\lambda_2 \approx 8.1 > \lambda_1 = 4$. Consequently, the load buys the power quantity $\overline{s}/\eta^{-1}$ at a higher price than it is generated at, and the generator sells $\overline{s}$ at a lower price than it is bought at. Either party could purchase a flowgate right to offset this fluctuation.

3) Congested Storage: Lastly, let $\overline{c} = 1 \le u^*$ and $\overline{c} \le \overline{s}$, so that it is optimal to store $\overline{c}$ in the first period and to withdraw $\overline{c}/\eta^-$ in the second. Mathematically, this case is almost identical to the preceding, but with transmission capacity replaced by the storage's energy capacity.

The storage's energy capacity shadow price in the first time period is

$$\chi = 2a(d_1 + \overline{c}) + b - \frac{2a(\eta^- d_2 - \overline{c}) + \eta^- b}{\eta^{-2}} \approx 1.5$$

and the value of the ECR is $\chi\hat{c}$, where $\hat{c}$ is the associated portion of the storage's energy capacity. Since the energy capacity is larger than the previous part's transmission capacity, the shadow price is smaller ($\chi < \mu$) and the nodal prices are now closer together, $\lambda_2 \approx 7.2 > \lambda_1 = 5$. As in the preceding case, the generator or load can acquire an ECR to capture the system operator's revenue surplus and offset the nodal price fluctuation.

B. Tracing

Here, the tracing method of Section IV is demonstrated on an augmented version of the two-node system. Suppose now that nodes one and two respectively produce $p_{1,t}$ and $p_{2,t}$ to serve the demands $d_{1,t}$ and $d_{2,t}$ for $t = 1, 2$. Assume that $p_{1,1} > d_{1,1}$ and $p_{2,1} > d_{2,1}$ and the excess is stored in the first period and then withdrawn in the second. Also assume that $p_{2,2} \geq d_{2,2}$ so that node one receives power from node two in the second period. Recall that the superscript refers to the index of the contributing node.

From (16), $p^1_{12,1} = p_{1,1} - d_{1,1}$, and from (17), node k's contribution to the power added to storage in the first period is

$$u_1^{+,k} = u_1^+ \frac{p_{k,1} - d_{k,1}}{p_{1,1} - d_{1,1} + p_{2,1} - d_{2,1}},$$

which is trivially equal to $p_{k,1} - d_{k,1}$. From (19), the stored energy from node k is $e^k_t = p_{k,1} - d_{k,1}$. By (18), the portion withdrawn attributable to node k is

$$u_2^{-,k} = u_2^- \frac{p_{k,1} - d_{k,1}}{p_{1,1} - d_{1,1} + p_{2,1} - d_{2,1}},$$

where $u_2^- = p_{1,2} - d_{1,2} + p_{2,2} - d_{2,2} < 0$. Applying (16) at the second period yields

$$\begin{aligned} p^1_{21,2} &= p_{21,2} \frac{-u_2^{-,1}}{p_{2,2} - d_{2,2} - u_2^-} \\ p^2_{21,2} &= p_{21,2} \frac{p_{2,2} - d_{2,2} - u_2^{-,2}}{p_{2,2} - d_{2,2} - u_2^-} \end{aligned}$$

where $p_{21,2} = d_{1,2} - p_{1,2}$. The tracing rule has thus identified how much of the power in the transmission line in the second period is attributable to each generator.

C. Gaming

This final example assesses the influence of FSRs on gaming, and to what extent their allocation can be used as a mitigator. A single node is now considered, which facilitates game-theoretic analysis, but provides qualitative insights rather than accurate predictions. Note that this setup is reminiscent of that in [23] for financial transmission rights, except that storage can only move power in one direction (forward in time) and the strategic quantities are the players' bid parameters. As such, this example may be regarded as type of supply function competition [26].

Consider a single node with one storage, generator, and load over two periods. The storage is perfectly efficient ($\alpha = \eta^+ = \eta^- = 1$), has unit energy capacity ($\overline{c} = 1$), and no power constraints ($\overline{r} = -\underline{r} = \infty$); note that an additional power constraint would be mathematically redundant to the energy constraint because there are only two periods. Power can be procured from the generator in either time period at cost $f(p) = p^2 + ap$. A load consumes power only in the second period and derives the utility $g(d) = -(d - z)^2$. This is a simplistic representation of load utility, in which the parameter z represents the ideal level of consumption. The system operator accepts bids from both the generator and the load; see [27] for a more realistic discussion of load utility and bidding. The generator and load both strategically distort their cost and utility functions by reporting $\hat{f}(p) = p^2 + \hat{a}p$ and $\hat{g}(d) = -(d - \hat{z})^2$. Define the actual and reported social welfares to be

$$\begin{aligned} \mathcal{W} &= g(d) - f(p_1) - f(p_2) \\ \hat{\mathcal{W}} &= \hat{g}(d) - \hat{f}(p_1) - \hat{f}(p_2). \end{aligned}$$

Using the bids, the system operator maximizes social welfare by solving

$$\begin{aligned} \max_{d,p_1,p_2,u} \quad & \hat{\mathcal{W}} \\ \text{subject to} \quad & p_1 - u = 0 \\ & p_2 + u = d \\ & 0 \leq u \leq \overline{c} \\ & p_1 \geq 0,\ p_2 \geq 0. \end{aligned}$$

For concision, the energy capacity constraint has been applied directly to u instead of introducing a state of charge variable. Let $\mathbb{T} = 2\hat{z} - \hat{a} - 6$. When $\mathbb{T} = 0$, the dispatched load level is exactly twice the storage capacity, so that any larger load cannot be shared evenly over both time periods. The optimal solution is given by

$$d^* = \begin{cases} \frac{1}{4}(2 + 2\hat{z} - \hat{a}), & \text{if } \mathbb{T} \geq 0 \\ \frac{1}{3}(2\hat{z} - \hat{a}), & \text{if } \mathbb{T} < 0 \end{cases}$$

$$p_1^* = \begin{cases} 1, & \text{if } \mathbb{T} \geq 0 \\ \frac{1}{6}(2\hat{z} - \hat{a}), & \text{if } \mathbb{T} < 0 \end{cases}$$

The resulting power prices in the first and second periods are then $\lambda_1 = 2p_1^* + \hat{a}$ and $\lambda_2 = 2p_2^* + \hat{a} = -2(d^* - \hat{z})$. From (11)–(13), the shadow price associated with the storage's energy capacity is

$$\chi = \lambda_2 - \lambda_1 = 2\left(p_2^* - p_1^*\right).$$

The generator and load own ECRs that respectively entitle them to collect $\beta_1\chi$ and $\beta_2\chi$, where $\beta_1 \geq 0, \beta_2 \geq 0$, and $\beta_1 + \beta_2 \leq 1$.

The generator and load utilities are given by

$$\begin{aligned} \mathcal{G}(\hat{a}, \hat{z}) &= \lambda_1 p_1^* - f\left(p_1^*\right) + \lambda_2 p_2^* - f\left(p_2^*\right) + \beta_1\chi \\ \mathcal{L}(\hat{a}, \hat{z}) &= g(d^*) - \lambda_2 d^* + \beta_2\chi. \end{aligned}$$

The generator and load respectively solve

$$\max_{\hat{a}} \ \mathcal{G}(\hat{a}, \hat{z}) \quad \text{and} \quad \max_{\hat{z}} \ \mathcal{L}(\hat{a}, \hat{z}).$$

This defines a two-player game. The Nash equilibrium is characterized in the Appendix. Since both utilities are quasiconcave, a pure strategy Nash equilibrium exists [28]. Gaming is assessed by comparing pure strategy Nash equilibria under different ECR allocations and the non-strategic, socially optimal outcome. Since there are only two players, some strategic effects appear more intensely than what would be expected in markets with more players.

First, examine the effect of the actual ideal load level, z, on strategic outcomes by interpreting the explicit expressions for the Nash equilibria in the Appendix. Regardless of whether the storage is congested, increasing z leads to larger reported generation costs $\hat{a}$ and lower reported load levels $\hat{z}$, corresponding to more gaming by both the generator and load.

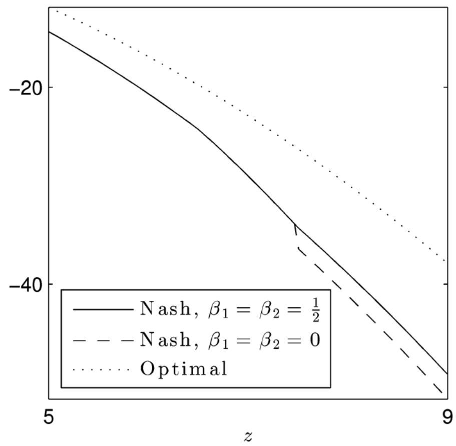

Fig. 2. Social welfare $\mathcal{W}$ as a function of the actual ideal load level z. When storage is congested, ECRs improve social welfare in strategic case.

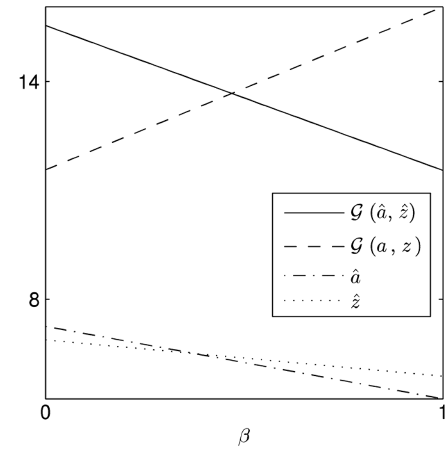

Fig. 3. Generator utilities and reported cost and load level parameters as a function of ECR allocation, β.

The ECRs come into effect when the storage is congested. Allocating the ECRs to the generator ($\beta_1 \gg \beta_2$) results in less gaming by the generator but more by the load, and vice versa when ECRs are allocated to the load ($\beta_1 \ll \beta_2$). When they are allocated evenly ($\beta_1 \approx \beta_2 \approx 1/2$), $\hat{a}_1$ and $\hat{z}_1$ respectively increase and decrease toward their true values, a and z, corresponding to less gaming by both players. The result is that the ECRs improve social welfare, $\mathcal{W}$, during congestion, as shown in Fig. 2.

Now suppose that $\beta_1 = \beta$, $\beta_2 = 1 - \beta$, and $z = 8$ so that the storage is congested at the Nash equilibrium. Fig. 3 shows the generator's utility as β is increased from zero to one, which corresponds to shifting the ECR allocation from the load to the generator. As expected, the generator's utility increases in the non-strategic, optimal case. However, in the strategic case the counterintuitive, opposite result is observed. This is explained by looking at the corresponding values of $\hat{a}$ and $\hat{z}$ in Fig. 3. The shifting ECR allocation revokes a profit channel from the load and gives it to the generator. This induces the load to report lower demand, $\hat{z}$, to cut costs, and in turn induces the generator to lower its reported costs, $\hat{a}$, to restore demand. Consequently, the generator charges a lower price for a lower demand, decreasing its profits more than the additional ECR revenue. The load utility $\mathcal{L}(\hat{a}, \hat{z})$, exhibits analogous, flipped dependence on β.

These results indicate that in scenarios with strategic loads and generators, FSRs can mitigate gaming and improve social welfare. Intuitively, this is because FSRs are more profitable when generator and load utilities reflect larger desired power transfers. This offsets the generator's incentive to strategically increase price and the load's incentive to strategically lower demand. However, it may be more realistic that only one participant, usually the generator, is strategic. In this case, transcribing the results of [23] for transmission rights to storage rights suggests that they may increase the market power of one monopolistic market participant.

VI. Discussion and Future Work

Financial rights and multiperiod tracing have been developed for electricity markets with passive storage. Multiperiod tracing is a straightforward extension of conventional tracing, and should be used similarly in multiperiod markets whether or not passive storage is present. The potential merits and faults of passive storage relative to storage that buys and sells power at wholesale prices are now discussed, and some venues for future work are identified.

Coupled with FSRs, passive storage elongates the process through which storage owners profit: rather than directly arbitraging temporal price variations, they profit through sales of financial rights, whose value depends on price variations. Risk-averse market participants like generators and loads can beneficially enter this process by purchasing FSRs as insurance or hedges against price volatility. This option does not exist with standard storage setups.

As with transmission lines, an additional benefit of passive storage is the decoupling of ownership and operation. This is advantageous for the following reasons.

- Potential investors who wish to avoid power system operations may find passive storage more appealing because they can profit through right sales without involvement in day-ahead and real-time electricity markets.
- Passive storage is more naturally operated by system operators because financially interested owners are not involved in nodal price transactions. This facilitates optimal utilization of automated components like batteries that system operators can remotely dispatch.

On the other hand, FSRs increase complexity by making right holders intermediaries between storage owners and electricity markets. As in the transmission case [8], [9], the rights' ability to redistribute the system operator's budget surpluses and direct profits to storage owners depends on effective allocation mechanisms like auctions, which add yet more complexity. Furthermore, smaller companies may have trouble sustaining the risk of backing FSRs, and thus face difficulty entering rights markets and bidding into planning processes. New research on these topics is necessary for practical implementations of storage rights. While the analysis in Section V-C suggests that FSRs decrease their holders' market power, further analysis is needed to determine if their added complexity creates other channels for gaming and their effect relative to existing approaches to market power mitigation. Indeed, most system operators currently employ stringent measures to mitigate generator market power, somewhat diminishing any similar benefits from FSRs.

As discussed in Section III-A, storage can provide different services at different timescales. It must therefore be determined which, if any, services FSRs are appropriate for. Presumably, such a distinction would occur at some minimum timescale threshold, e.g., between load shifting and power balancing, or between power balancing and frequency regulation. This issue is tied to the uncertainty at each timescale; for example, random disturbances and contingencies are the primary reason regulation is needed, whereas load shifting is influenced but not dominated by uncertainty. As discussed in Section III-A, addressing these issues would likely invoke problems in stochastic control and optimization, for which shadow-price based interpretations of dual multipliers may not be straightforwardly available. These are topics of future work.

Appendix

Define

$$\begin{aligned}
\mathcal{G}_1(\hat{a},\hat{z}) =& -\frac{3}{16}\hat{a}^2 + \frac{1}{4}(\hat{z} - 2\beta_1 + 4)\hat{a} \\
&+ (\hat{z}-3)\left(\beta_1 + \frac{1}{4}(\hat{z}-1)\right) \\
\mathcal{G}_2(\hat{a},\hat{z}) =& -\frac{5}{18}\hat{a}^2 + \frac{1}{9}(4\hat{z}+3)\hat{a} + \frac{\hat{z}}{9}(2\hat{z}-6) \\
\mathcal{L}_1(\hat{a},\hat{z}) =& -\frac{3}{4}\hat{z}^2 + \frac{1}{4}\left(\hat{a} + 4(z+\beta_2) - 2\right)\hat{z} - z^2 - 4\beta_2 \\
&- \frac{3}{16}(\hat{a}-2)^2 - \frac{1}{4}(\hat{a}-2)(2\beta_2 - \hat{a} + 2z + 2) \\
\mathcal{L}_2(\hat{a},\hat{z}) =& -\frac{8}{9}\hat{z}^2 + \frac{2}{9}(\hat{a}+6z)\hat{z} + \frac{\hat{a}^2}{9} - z^2 - \frac{2}{3}\hat{a}z.
\end{aligned}$$

The generator's profits are given by

$$\mathcal{G}(\hat{a},\hat{z}) = \begin{cases} \mathcal{G}_1(\hat{a},\hat{z}), & \text{if } \mathbb{T} \geq 0 \\ \mathcal{G}_2(\hat{a},\hat{z}), & \text{if } \mathbb{T} < 0 \end{cases}$$

and the load's net utility by

$$\mathcal{L}(\hat{a},\hat{z}) = \begin{cases} \mathcal{L}_1(\hat{a},\hat{z}), & \text{if } \mathbb{T} \geq 0 \\ \mathcal{L}_2(\hat{a},\hat{z}), & \text{if } \mathbb{T} < 0 \end{cases}$$

where $\mathbb{T} = 2\hat{z} - \hat{a} - 6$.

Lemma 1: $\mathcal{G}(\hat{a},\hat{z})$ and $\mathcal{L}(\hat{a},\hat{z})$ are quasiconcave in $\hat{a}$ and $\hat{z}$, respectively.

Proof: By inspection, $\mathcal{G}_1(\hat{a},\hat{z})$ and $\mathcal{G}_2(\hat{a},\hat{z})$ are strictly concave in $\hat{a}$ and $\mathcal{L}_1(\hat{a},\hat{z})$ and $\mathcal{L}_2(\hat{a},\hat{z})$ are strictly concave in $\hat{z}$. $\mathcal{G}(\hat{a},\hat{z})$ and $\mathcal{L}(\hat{a},\hat{z})$ are therefore quasiconcave if they are quasiconcave at the transition at $\mathbb{T} = 0$. This is true for $\mathcal{G}(\hat{a},\hat{z})$ if when

$$\left.\frac{d\mathcal{G}_1(\hat{a},\hat{z})}{d\hat{a}}\right|_{2\hat{z}=\hat{a}+6} \leq 0,$$

then

$$\left.\frac{d\mathcal{G}_2(\hat{a},\hat{z})}{d\hat{a}}\right|_{2\hat{z}=\hat{a}+6} \leq 0.$$

Suppose that

$$\begin{aligned}
\left.\frac{d\mathcal{G}_1(\hat{a},\hat{z})}{d\hat{a}}\right|_{2\hat{z}=\hat{a}+6} &= \frac{1}{4}(7 - \hat{a} - 2\beta_1) \\
&\leq 0.
\end{aligned}$$

Then, since $0 \leq \beta_1 \leq 1$,

$$\begin{aligned}
\left.\frac{d\mathcal{G}_2(\hat{a},\hat{z})}{d\hat{a}}\right|_{2\hat{z}=\hat{a}+6} &= \frac{1}{3}(5 - \hat{a}) \\
&\leq \frac{2}{3}(\beta_1 - 1) \\
&\leq 0.
\end{aligned}$$

$\mathcal{L}(\hat{a},\hat{z})$ is handled in the same manner. Suppose that

$$\begin{aligned}
\left.\frac{d\mathcal{L}_2(\hat{a},\hat{z})}{d\hat{z}}\right|_{2\hat{z}=\hat{a}+6} &= \frac{4}{3}(z - \hat{z} - 1) \\
&\leq 0.
\end{aligned}$$

Then, since $0 \leq \beta_2 \leq 1$,

$$\begin{aligned}
\left.\frac{d\mathcal{L}_1(\hat{a},\hat{z})}{d\hat{z}}\right|_{2\hat{z}=\hat{a}+6} &= \beta_2 + z - \hat{z} - 2 \\
&\leq \beta_2 - 1 \\
&\leq 0.
\end{aligned}$$

Therefore, both $\mathcal{G}(\hat{a},\hat{z})$ and $\mathcal{L}(\hat{a},\hat{z})$ are quasiconcave. ∎

Because both utilities are quasiconcave, a pure strategy Nash equilibrium must exist [28], which is characterized below. Define the following:

$$\begin{aligned}
\hat{a}_1 &= \max_{\hat{a}}\ \mathcal{G}_1(\hat{a},\hat{z}_1) \\
\hat{z}_1 &= \max_{\hat{z}}\ \mathcal{L}_1(\hat{a}_1,\hat{z}).
\end{aligned}$$

Simply differentiating and solving a linear system of equations yields

$$\begin{aligned}
\hat{a}_1 &= \frac{1}{4}(2\beta_2 - 6\beta_1 + 2z + 11) \\
\hat{z}_1 &= \frac{1}{8}(6\beta_2 - 2\beta_1 + 6z + 1).
\end{aligned}$$

Similarly,

$$\begin{aligned}
\hat{a}_2 &= \max_{\hat{a}}\ \mathcal{G}_2(\hat{a},\hat{z}_2) \\
&= \frac{2}{3}(z+1) \\
\hat{z}_2 &= \max_{\hat{z}}\ \mathcal{L}_2(\hat{a}_2,\hat{z}) \\
&= \frac{1}{12}(10z+1).
\end{aligned}$$

The Nash equilibrium is given by the following cases.

A. $2\hat{z}_1 > \hat{a}_1 + 6$

In this case, $(\hat{a}_1, \hat{z}_1)$ is a Nash equilibrium. The condition $2\hat{z}_1 \geq \hat{a}_1 + 6$ reduces to $z \geq 8.5 - \beta_1 - \beta_2$.

B. $2\hat{z}_2 < \hat{a}_2 + 6$

In this case, $(\hat{a}_2, \hat{z}_2)$ is a Nash Equilibrium. The condition $2\hat{z}_2 \leq \hat{a}_2 + 6$ reduces to $z \leq 6.5$.

C. $2\hat{z}_1 \leq \hat{a}_1 + 6$ *and* $2\hat{z}_2 \geq \hat{a}_2 + 6$

In this case, the Nash equilibria are not at the smooth maxima of $\mathcal{G}(\hat{a}, \hat{z})$ and $\mathcal{L}(\hat{a}, \hat{z})$. Since both are quasiconcave and strictly concave on either side of the kinks at $\mathbb{T} = 0$, equilibria occur when

$$\left.\frac{d\mathcal{G}_1(\hat{a}, \hat{z})}{d\hat{a}}\right|_{\mathbb{T}=0} \geq 0 \quad \text{and} \quad \left.\frac{d\mathcal{G}_2(\hat{a}, \hat{z})}{d\hat{a}}\right|_{\mathbb{T}=0} \leq 0$$
$$\left.\frac{d\mathcal{L}_1(\hat{a}, \hat{z})}{d\hat{z}}\right|_{\mathbb{T}=0} \leq 0 \quad \text{and} \quad \left.\frac{d\mathcal{L}_2(\hat{a}, \hat{z})}{d\hat{a}}\right|_{\mathbb{T}=0} \geq 0$$

which is explicitly written

$$\begin{aligned} &5 \leq \hat{a} \leq 7 - 2\beta_1 \\ &z + \beta_2 - 2 \leq \hat{z} \leq z - 1 \\ &\mathbb{T} = 0. \end{aligned}$$

Observe that this system is feasible when $6.5 \leq z \leq 8.5 - \beta_1 - \beta_2$, the complement of the other two cases. When $\beta_1 = 1$ and $\beta_2 = 0$, the equilibrium is (5,5.5), and when $\beta_1 = 0$ and $\beta_2 = 1$, the equilibrium is $(2z - 8, z - 1)$. For intermediary values like $\beta_1 = \beta_2 = 1/2$, a range of equilibria exists. It can be shown the social welfare is constant over this range.

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