Optimization of Advertising Budget Allocation Over Time Based on LS-SVMR and DE

Dapeng Niu, Ying Sun, and Fuli Wang

Abstract—**The advertising budget allocation problem for financial service is dealt with based on statistical learning and evolutionary computation in this paper. Taking the carry-over effects of the advertising into account, the least squares support vector machine regression (LS-SVMR) is used to construct the response model. A comparison between the proposed response model and traditional regression method based market response models is implemented. The results show the effectiveness and validity of the former model. Taking the budgets allocated to every month in the planning horizon as decision variables, the budget allocation optimization model is built and an improved differential evolution algorithm is used to find the optimal solutions. Finally, the proposed budget allocation method is illustrated by a practical problem.**

Note to Practitioners—**In modern society, advertising is an important part in brand marketing strategy. The planning and design of an advertising campaign involve several decisions, among which is advertising budget allocation. It aims to determine the optimal advertising expenditure among individual brands over a predetermined planning horizon, competing for a limited resource or geographic market segments. However, little work deals with the allocation of a given advertising budget for propagating services over time. In this study, a method for the optimal advertising budget allocation over the planning horizon of the advertising campaign for a financial service is provided. A response model is constructed using the LS-SVMR. Taking the budgets allocated to every month in the planning horizon as decision variables, the budget allocation optimization problem is proposed and an improved differential evolution algorithm is then applied to find the optimal solutions. The effectiveness of the proposed advertising budget allocation method is validated by a numerical example.**

Index Terms—**Advertising budget allocation, differential evolution algorithm, least squares support vector machine regression (LS-SVMR), optimization.**

I. Introduction

THE PLANNING and design of an advertising campaign involve several decisions, such as the advertising budget, the budget allocation over time or among brands, and the scheduling across media. These decisions are interrelated with each other in a complex form. Commonly, the planning process is decomposed to several stages and each one of the decisions is treated individually [1]. Advertising budget allocation is the process of determining the optimal advertising expenditure among individual brands over a predetermined planning horizon, to compete for a limited resource or geographic market segments.

Although several previous studies have considered the advertising budget allocation problem, most of them just solve the allocation among brand or over geographic market segments [2]–[4]. Some other researchers consider the budget allocation and media selection simultaneously, or it is more proper to say that the budget allocation is the byproduct of the media selection [5]–[7]. Literatures [8] and [9] have concerned the problem of allocating the budget over time. However, almost all of the previous work for advertising budget allocation focused on propagating the product or increasing sales. Little work has been done on allocation of a given advertising budget over time for propagating services. Hence, the purpose of this paper is to provide a method for the optimal allocation of the given advertising budget over the planning horizon of the advertising campaign for a financial service.

Unlike the budget allocation of advertising for product, the evaluation of advertising effect for financial service is unobvious. This is because the profit that service applicants bring cannot be calculated immediately. Therefore, the number of customers who will apply the propagated service is selected as the evaluation criterion of advertising effect. Considering the complex characteristics of customers' actions to advertising, the least squares support vector machine regression (LS-SVMR) method which is a learning technique based on the structural risk minimization principle is used to model the response versus advertising budget.

In this paper, we have built a response model for advertising based on theLS-SVMR method using the history data. We have also developed an optimization model for advertising budget allocation over time based on the response model. Establishment of the response model and the optimization model are both contributions of the paper. Furthermore, we have proposed an improved differential evolution algorithm to solve the optimization problem and improved the advertising effect, which is another major contribution of this paper.

The remainder of this paper is organized as follows. In Section II, the theory of LS-SVMR is introduced and a response model is built based on it. In Section III, the optimization problem of advertising budget allocation over planning horizon based on the response model is proposed, and the

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problem is then solved with an improved differential evolution algorithm. In Section IV, the application of proposed advertising budget allocation method is illustrated through a numerical example. Section V concludes this paper.

II. Response Model

Response model describes the relationship between advertising response and the expenditure [10]. It is the premise of getting the optimal budget allocation. Only when the response model has been properly constructed, does it become assessable to how well the budget is allocated over planning horizon.

Numerous previous studies have explored the models of market response to advertising [11]–[13]. However, most of the models aimed to predict the sales of product or the market share that might be acquired. To the best of our knowledge, there exist no response models with regard to advertising campaign for services.

The prediction of the response is a complicated problem. There are many factors that affect the response. Fortunately, if the history data can be effectively dealt with by the data mining techniques which are used to extract knowledge from information, the response may be predictable. In this paper, a response model based on LS-SVMR is constructed to predict the number of applicants attracted by advertising.

A. Least Squares Support Vector Machine Regression

Support vector machine (SVM) is a learning algorithm based on statistical learning theory, which is appropriate for small sample learning problem. Having many advantages compared with other approaches, SVM has been successfully used in many application fields and extended to regression problems during the past few years [14], [15]. The original SVM methods are time consuming and huge space demanding [16], however. So, the LS-SVMR, which only requires solving a set of linear equations and thus is much easier and computationally simpler, is used in this paper to model the response versus advertising budget. The concept of LS-SVMR will be briefly introduced in the coming paragraphs.

Given the training dataset $\{(\mathbf{x}_i, y_i)\}_{i=1,2,\ldots N}$, where the input variable $\mathbf{x}_i \in \mathbf{R}^d$ and the response variable $y_i \in \mathbf{R}$. The goal is to construct a regressor of the form

$$y = f(\mathbf{x}) = \mathbf{w}^T \varphi(\mathbf{x}) + b \tag{1}$$

where $\varphi(\cdot)$ maps a point in space $\mathbf{R}^d$ into the space $\mathbf{R}$.

LS-SVMR introduces a least squares version to SVM regression by formulating the regression problem as an optimization problem with the form as follows:

$$\min \frac{1}{2}\mathbf{w}^T\mathbf{w} + \frac{1}{2}\gamma \sum_{i=1}^{N} e_i^2 \tag{2}$$

$$\text{Subject to } y_i = \mathbf{w}^T \varphi(\mathbf{x}_i) + b + e_i \tag{3}$$

To derive the dual problem of (2) and (3), the Lagrange multipliers are introduced as follows:

$$J(\mathbf{w}, b, \mathbf{e}, \boldsymbol{\alpha}) = \frac{1}{2}\mathbf{w}^T\mathbf{w} + \frac{1}{2}\gamma \sum_{i=1}^{N} e_i^2 + \sum_{i=1}^{N} \alpha_i (y_i - \mathbf{w}^T \varphi(\mathbf{x}_i) - b - e_i) \tag{4}$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_N)$ is the Lagrange multiplier vector, which can be positive or negative in case of LS-SVMR formulation. Differentiate (4) with respect to $\mathbf{w}, b, e_i$ and α_i, respectively, and we will obtain the conditions for optimality [17], [18]

$$\frac{\partial J}{\partial \mathbf{w}} = 0 \rightarrow \mathbf{w} = \sum_{i=1}^{N} \alpha_i \varphi(\mathbf{x}_i) \tag{5}$$

$$\frac{\partial J}{\partial b} = 0 \rightarrow \sum_{i=1}^{N} \alpha_i = 0 \tag{6}$$

$$\frac{\partial J}{\partial e_i} = 0 \rightarrow \alpha_i = \gamma e_i \tag{7}$$

$$\frac{\partial J}{\partial \alpha_i} = 0 \rightarrow \mathbf{w}^T \varphi(\mathbf{x}_i) + b + e_i - y_i = 0. \tag{8}$$

Putting (5) into (1), the desired regression function is obtained as

$$y = \sum_{i=1}^{N} \alpha_i \varphi(\mathbf{x}_i)^T \varphi(\mathbf{x}) + b. \tag{9}$$

If we define a positive definite kernel as follows:

$$K(\mathbf{x}_i, \mathbf{x}_j) = \varphi(\mathbf{x}_i)^T \varphi(\mathbf{x}_j) \tag{10}$$

a point y_j can be evaluated as

$$y_j = \sum_{i=1}^{N} \alpha_i \varphi(\mathbf{x}_i)^T \varphi(\mathbf{x}_j) + b = \sum_{i=1}^{N} \alpha_i K(\mathbf{x}_i, \mathbf{x}_j) + b. \tag{11}$$

The selection of kernel will be introduced in the section on case study (Section IV).

B. Response Model Based on LS-SVMR

Responses to advertising campaigns do not often take place instantly. Carryover effect is the general term used to describe the influence of a current advertising expenditure on the response in future periods. Current advertising response is decided not only by current advertising budget but also by previous advertising budgets. Therefore, the advertising response of current period can be expressed as function of current advertising budget and previous advertising budgets.

Taking the carryover effect into account, the structure of the response model can be illustrated as Fig. 1. In the figure, c_t is the advertising budget of current period; r_t is the advertising

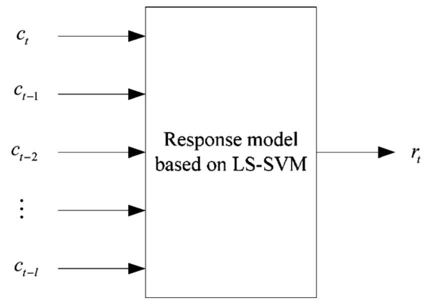


Fig. 1. Structure of the response model.

response of current period; c_{t-l} is the carryover effect of advertising expenditure in past lth period.

The lagging number l can be determined by comparing the Akaike information criterion (AIC) values of models with different lagging numbers, which can be obtained through the following expression:

$$\mathrm{AIC} = 2k + N \cdot \ln\left(\frac{1}{N}\sum_{i=1}^{N}(y_i - f(\mathbf{x}_i))^2\right) \tag{12}$$

where k is the number of model parameters to be estimated. The lagging number l, which corresponds to the model with minimum AIC value, is determined as optimal.

The history data are composed of the advertising budget of current period and the carryover effects of advertising expenditure in each of the past l periods, respectively. We separate the data into training data, validation data, and test data. The following steps then proceed to calculate the optimized l:

1) Let $l = 0, 1, \ldots, T-1$, respectively, and use the training data to build a response model corresponding to each value of l, based on LS-SVMR.
2) Calculate the AIC value of each model, and choose the lagging number, which corresponds to the minimum value of AIC as the optimized l.

When the lagging number is determined, we have also obtained the best regression model in fact. Then, we can use the test data for model testing.

Advertising response is not only affected by the advertising expenditure but also other factors. One of the important factors is the seasonality. Raw history data of the number of service applicants must be adjusted before modeling. To eliminate the seasonal components, we use the ratio-to-moving-average method to deal with the original data.

III. Advertising Budget Allocation Optimization

A. Mathematical Model

Unlike the existing methods of advertising budget allocation, which generally focuses on propagating the product or increasing sales, we propose an advertising budget allocation problem based on the response model, which aims to maximize the number of customers who will apply the propagated service. Advertising budget allocations are made periodically in practice. Considering a planning horizon consisting of discrete periods $t = 1, 2, \ldots, T$, the advertising budget allocation optimization model is formulated as follows:

$$\text{Objective}: \quad \max \sum_{t=1}^{T} r_t(c_{t-l}, \ldots, c_{t-1}, c_t) \tag{13}$$

$$\text{Subject to}: \quad \sum_{t=1}^{T} c_t \le C \tag{14}$$

$$\frac{c_t}{r_t(c_{t-l}, \ldots, c_{t-1}, c_t)} \le C_u \tag{15}$$

$$c_t > 0 \tag{16}$$

where $r_t(\cdot)$ is the advertising response–the number of customers applying the propagated service in period t, which is calculated by the response model introduced in Subsection II-B; C is the total budget of the advertising campaign; C_u is the unit cost of response determined by decision maker.

Through the optimization model, we can see that the response model is the premise for the optimization problem, because the response model describes the relationship between the cost and the number of customers who would apply the propagated service. The form of $r_t(\cdot)$ implies that the current advertising response is decided by previous advertising budgets before current period and current advertising budget. If there is a sequence of advertising budget over time, it will decide a unique objective through expression (13). And using a certain optimization algorithm, we can find the optimal budget allocation, which corresponds to the maximum value of the total response under the given constraints.

Inequation (14), which means that the total advertising budget should be no more than the previously determined amount, is a constraint on the sum of advertising expenditure allocated to each period. The number of the service applicants may increase as the spent advertising expenditure becomes more. However, when the response gets close to the saturation level, it becomes too costly to attract one more customer. This situation will lower the return on investment (ROI) of the advertising campaign. Inequation (15) reflects the constraint on the advertising expenditure for each service applicant, which will avoid the decrease of the ROI. Inequation (16) is a constraint meaning that the advertising expenditure allocated to each period must be a positive number.

B. Model Solution

The selection of optimization method for the proposed optimization problem depends on the characteristics of the optimization objective and the constraints, shown by (13)–(16). The specific form of the response model, which appears in the objective function and the constraints, has a great influence on the complexity of the optimization problem. If the lagging number l in the response model is 1, the advertising response of current period will be only influenced by the advertising budget of current period and the carryover effect of advertising budget in last period. Hence, the classical dynamic programming can efficiently find the optimal solutions to this optimization problem. However, the carryover effect usually lasts for a longer time, which means that the effects of advertising budgets in several previous periods also have influences on the response of current period. Moreover, inequation (15) is a nonlinear function of decision variable c_t. Therefore, the optimization problem is essentially a nonlinear optimization problem.

The situations discussed above make the procedure of searching for the optimal solutions quite complicate. Due to the complexity of the optimization problem, a stochastic search approach with high efficiency-differential evolution (DE) is adopted. The differential evolution algorithm, developed by Storn and Price [19], has proven to be a promising candidate to solve real valued optimization problems [20], [21]. It is very simple and easy to implement, with only a few parameters required to be set by the user [22].

When using the DE algorithm to solve the optimization problem, we should encode the candidate solutions, i.e., $\mathbf{Z}_i^G = \{z_{1,i}^G, z_{2,i}^G, \ldots, z_{D,i}^G\}, i = 1, 2 \ldots, NP$, where $\mathbf{Z}_i^G$ represents the ith D-dimensional individual in the population of Gth generation, $z_{j,i}^G, j = 1, 2, \ldots, D$ is the jth component of the individual vector, and NP is the total number of individuals in the population. Here, in this paper, according to the specific optimization problem, $z_{j,i}^G$ corresponds to each of the decision variables—the monthly advertising budget allocation c_t. The initial population is produced by uniformly randomizing individuals within the constrained search space, which is the first step of the procedures for the DE algorithm. Following initialization of the evolution population are the main three operations of DE, i.e., mutation operation, crossover operation, and selection operation.

Mutation Operation: Mutation operation is employed to produce mutant individuals. A difference vector with respect to each individual in the current population, also called the parent individual, is first formed by two randomly chosen individuals $(\mathbf{Z}_{r1}^G, \mathbf{Z}_{r2}^G)$ from the current population, shown as

$$\mathbf{DV} = \mathbf{Z}_{r1}^G - \mathbf{Z}_{r2}^G. \tag{17}$$

According to the different forms of integrating the difference vector with the parent individual, there have been many mutation strategies for differential evolution algorithm [23]. We select the most frequently used mutation strategy, i.e., "DE/rand/1". A mutant individual is generated as

$$\hat{\mathbf{Z}}_i^{G+1} = \mathbf{Z}_i^G + F\left(\mathbf{Z}_{r1}^G - \mathbf{Z}_{r2}^G\right) \tag{18}$$

where $\hat{\mathbf{Z}}_i^{G+1}$ and $\mathbf{Z}_i^G$ are the produced mutant individual and parent individual, respectively. F is the mutation factor, which scales the difference vector. As F becomes larger, DE trends to search more stochastically; if F turns smaller, the population diversity of DE will be reduced.

Crossover Operation: After the mutation operation, crossover operation is employed to produce the trial individual $\mathbf{V}_i^G$, which is performed between the parent individual $\mathbf{Z}_i^G$ in the current population and its corresponding mutant individual $\hat{\mathbf{Z}}_i^{G+1}$. A user-specified crossover probability factor is used to control the fraction of parameter values copied from the mutant individual. The crossover operation equation is

$$\mathbf{V}_{j,i}^G = \begin{cases} \hat{\mathbf{Z}}_{j,i}^{G+1}, & \mathrm{rand}(0,1) \le CR \\ \mathbf{Z}_{j,i}^G, & \mathrm{otherwise} \end{cases} \tag{19}$$

where $\mathrm{rand}(0,1)$ is a randomly produced number within the range [0, 1], $j = 1, 2, \ldots, D$ represents the jth gene of the individual, and D is the total number of genes in each individual, i.e., the dimension of the candidate solution. CR is the crossover probability factor.

Selection Operation: The objective function value of the trial individual $\mathbf{V}_i^G$ is then compared with that of its corresponding parent individual $\mathbf{Z}_i^G$. The better one enters the next generation of population, while the other one is eliminated.

Adaptive Mutation Operator: In the basic DE, a constant mutation factor F is employed to scale the difference vector. An appropriate F is of crucial importance to the efficiency of the DE algorithm. The most appropriate F is very hard to decide, however. Therefore, we introduce an adaptive mutation operator into the basic DE, which will decide F adaptively according to the evolutionary course [24]

$$F = F_0 \times 2^{e^{\frac{G}{G - G_{\max}}}} \tag{20}$$

where F_0 is a mutation constant, $G_{\max}$ is the maximum evolutional generation, G is the current evolutional generation $(G = 0, 1, \ldots, G_{\max} - 1)$. The adaptive mutation operator has a maximum mutation factor $2F_0$ when the population evolution begins, thus it can maintain the population diversity and prevent prematurity during the early stages of the evolution course. As the population evolves gradually, the mutation factor becomes smaller and closer to F_0 in the later stage of evolution, so to keep the better individuals in the population and improve the algorithm's efficiency.

Since the budget allocation problems is a constrained problem, infeasible solutions may cover a big proportion of the solution space. In this paper, a multiplication of penalty function is adopted to give a much severe penalty to infeasible solutions. The mathematical form of the penalty function is as follows:

$$p(\mathbf{Z}) = 1 - \frac{1}{M} \sum_{m=1}^{M} \left(\frac{\Delta b_m(\mathbf{Z})}{\Delta b_m^{\max}} \right)^{\alpha} \tag{21}$$

where $\Delta b_m(\mathbf{Z}) = \max\{0, g_m(\mathbf{Z}) - b_m\}$; $\Delta b_m^{\max} = \max\{\varepsilon, \Delta b_m(\mathbf{Z}) - b_m \mid \mathbf{Z} \in \mathbf{P}^G\}$. $g_m(\mathbf{Z}) - b_m$ is the ith constraint; $\Delta b_m(\mathbf{Z})$ is violation value of chromosome $\mathbf{Z}$ for the ith constraint; $\Delta b_m^{\max}$ is the maximum violation value for the ith constraint in the current population $\mathbf{P}^G$; and ε is a small positive number used to avoid zero-division.

This penalty approach adjusts the ratio of penalties adaptively at each generation in order to achieve a balance between preventing infeasibility and avoiding over-penalty. The fitness function of the optimization algorithm in this paper is calculated as the multiplication of the objective function and the penalty function.

IV. Case Study

This section will illustrate the application of the proposed modeling and optimization methods for advertising budget allocation by a practical project for a financial institute. This institute spends a huge amount of expenditure on TV, newspaper, magazine, internet and traffic advertising each year for propagating the loan service it provides for customers. The data of TV advertising collected from January 2003 to December 2010 are used in this paper. Monthly advertising expenditure

TABLE I
SEASONAL ADJUSTMENT RATIOS FOR RAW DATA OF RESPONSE

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Oct.	Nov.	Sep.	Dec.
Ratios	0.94	0.88	1.03	1.05	1.23	1.03	0.95	1.04	1.05	1.02	0.99	0.79

and monthly number of service applicants exposed to TV advertising are available. The goal of this decision procedure is to optimally allocate the total budget set for TV advertising monthly, thus to attract the maximum total number of customers, who may be exposed to the TV advertising, to apply the service which the financial institute provides.

A. Building of the Response Model

We have eight years of history data in total. When building the response model, we choose randomly one year of advertising data as the test data and the remaining seven years of data are used as the training and validation data. As discussed in Section II, the monthly response data should be preprocessed first. Average monthly response for 2003 is set to the reference value 1. Then, ratio-to-moving average method is adopted to seasonally adjust the raw data. Calculated seasonal adjustment ratios are listed in Table I.

Divided by the corresponding seasonal adjustment ratios, the raw data of response can be seasonally adjusted. Then, the raw data of advertising expenditure and seasonally adjusted response data are used to build the response model. We determine the lagging number l by comparing the AIC values of models with different lagging numbers, as mentioned in Section II-B. Finally, we get an optimal lagging number $l = 3$. This just accords with the fact that the TV advertising schedule is usually updated quarterly.

For LS-SVMR, there are many kernel functions. These kernels are linear, polynomial, radial basis function (RBF), spline, bspline, sigmoid, etc. The most widely used RBF kernel function is selected when building the response models, which is defined as (22)

$$K(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{\sigma^2}\right) \tag{22}$$

where σ^2 is the squared variance of the Gaussian function.

Based on the grid search, the optimum parameter σ and regularization parameter γ are tuned by cross validation procedure, during which one year of the data is selected as validation data and all the remaining six years of data are the training data for modeling.

A comparison between output of the finally obtained response model based on least squares vector machine regression and the test data is made to evaluate the model's prediction performance. And the results are shown in Fig. 2, from which we can see that the proposed model can depict the advertising response with acceptable accuracy.

Moreover, the comparison between the response model based on LS-SVMR and the general market response models based on traditional regression method were also made. The coefficient of determination is used to evaluate the effectiveness of regression models, which is defined as

$$D = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \bar{y})^2} \tag{23}$$

where y_i is the observed value, f_i is the model predicted value, and $\bar{y}$ is the average value of all model predicted values. The coefficients of determination range from 0 to 1. The bigger the coefficients are, the better the model reflects the relationship between the independent variables and the dependent variable. If each of the model prediction number of service applicants equals to the observed one, then the coefficient of determination has a maximum value of 1. The coefficients for usually used traditional response models are given in Table II, in comparison with that of LS-SVMR regression based model.

From Table II, it can be seen that the proposed response model based on LS-SVMR method has the maximum coefficient of determination, which means that it predicts the advertising response with higher accuracy than general market response models.

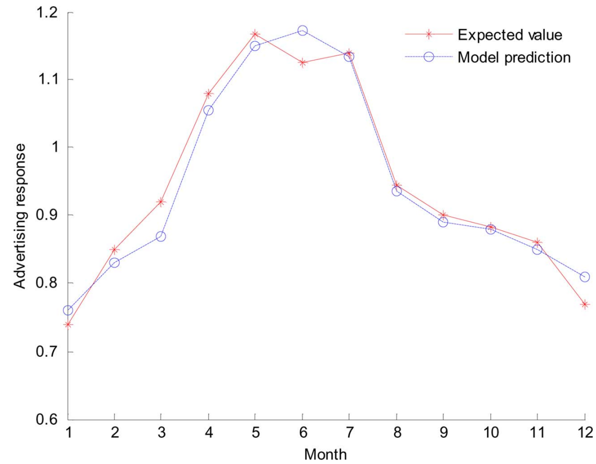


Fig. 2. Comparison between model prediction and test data.

TABLE II
COMPARISON BETWEEN LS-SVMR BASED MODEL AND GENERAL MARKET RESPONSE MODELS BASED ON TRADITIONAL REGRESSION METHOD

MODEL	COEFFICIENT OF DETERMINATION
LS-SVMR REGRESSION BASED MODEL	0.980
LINEAR MODEL	0.931
POWER SERIES MODEL	0.967
FRACTIONAL ROOT MODEL	0.932
SEMILOG MODEL	0.933
EXPONENTIAL MODEL	0.936
LOGISTIC MODEL	0.951
ADBUDG MODEL	0.953

TABLE III
BUDGET ALLOCATION RESULTS OF PROPORTIONAL TO SALES METHOD AND PROPOSED METHOD

MONTH	PROPORTIONAL TO SALES METHOD	PROPOSED METHOD
JAN.	0.683	0.457
FEB.	0.658	0.468
MAR	0.746	0.461
APR.	0.808	1.460
MAY	0.971	0.492
JUNE	0.779	0.488
JULY	0.738	1.323
AUG.	0.779	1.265
SEP.	0.785	0.498
OCT.	0.760	1.147
NOV.	0.723	0.482
DEC.	0.590	0.479

B. Optimization

After the response model is built, we can obtain the objective function of the advertising budget allocation optimization problem described in Section III-A as follows

Objective:

$$\max \sum_{t=1}^{12} r_t(c_t, c_{t-1}, c_{t-2}, c_{t-3}). \tag{24}$$

The improved DE with adaptive mutation operator, as proposed in Section III-B is used to find the optimal advertising budget allocation for each month in a year. Therefore, the advertising budget allocated to each month is the decision variable, which is to be encoded as a gene of each individual in the evolution population when using DE to solve the optimization problem.

The total budget allocated to TV advertising has been predefined before this study. Unit cost of response is set as the average unit cost of last years. After scaling the value of total budget and unit cost, the constraints can be expressed as follows:

$$\sum_{t=1}^{12} c_t \leq 9.020 \tag{25}$$

$$\frac{c_t}{r_t(c_t, c_{t-1}, c_{t-2}, c_{t-3})} \leq 1.482 \tag{26}$$

$$c_t > 0. \tag{27}$$

DE parameters used in this study are the following: population size $\mathrm{NP} = 50$, generation number $G_{\max} = 100$, crossover probability factor $\mathrm{CR} = 0.8$, mutation constant $F_0 = 0.5$.

The obtained optimal monthly advertising budget allocation based on the response model and the optimization method proposed in this paper is presented in Table III, and the budget allocation each month based on proportional to sales method is also listed in the table for comparison.

TABLE IV
PREDICTED RESPONSE OF BUDGET ALLOCATION DECIDED BY PROPORTIONAL TO SALES METHOD AND PROPOSED METHOD

MONTH	PROPORTIONAL TO SALES METHOD	PROPOSED METHOD
JAN.	0.732	0.875
FEB.	0.729	0.841
MAR	0.839	0.946
APR.	0.847	0.980
MAY	1.028	1.177
JUNE	0.894	0.980
JULY	0.826	0.922
AUG.	0.869	0.989
SEP.	0.852	1.041
OCT.	0.809	0.944
NOV.	0.776	0.939
DEC.	0.614	0.747
TOTAL	9.815	11.381

Strictly speaking, whether the budget allocation strategy obtained through the proposed method is optimal or not should be tested by a company which would use other methods simultaneously. However, this is unavailable. In this study, the test is implemented by comparing the predicted response of budget allocation based on the proposed method and proportional to sales method. The results are shown in Table IV.

The results show that budget allocation based on the proposed method can attract more service applicants than the one based on proportional to sales method, which is the company pursues. It should be declared that acquired results are all the scaled values. Multiplied by corresponding ratios, the values will be changed to meaningful variables which can be used in practical projects.

V. CONCLUSION

In this paper, we deal with the monthly advertising budget allocation problem for financial service. Considering both the complexity of the advertising response and the characteristics of available history data, we construct a response model based on the LS-SVMR. Different from existing response models generally concerned with the sales of product or the market share, the proposed response model is special in predicting the number of service applicants. Results of comparison between the model and the market response models based on traditional regression methods show that our proposed response model can predict the number of applicants more accurately. Thereafter, we propose the monthly budget allocation optimization problem on the basis of the response model and use an improved differential evolution algorithm to solve the problem. The obtained monthly advertising budget allocation is then compared with that generated by proportional to sales allocation method. From the results, we can see that under the given budget constraints, the budget allocation obtained through the proposed optimization approach can attract more applicants. That is to say our approach outperforms the proportional to sales allocation method.

REFERENCES

[1] D. Pekelman and S. Sethi, "Advertising budgeting, wearout and copy replacement," *J. Oper. Res. Soc.*, vol. 29, no. 7, pp. 651–659, 1978.

[2] D. H. Gensch and U. P. Welam, "An optimum budget allocation model for dynamic, interacting market segments," *Manage. Sci.*, vol. 20, no. 2, pp. 179–190, 1973.

[3] G. L. Urban, "Allocating ad budgets geographically," *J. Advertising Res.*, vol. 15, no. 6, pp. 7–16, 1975.

[4] P. D. Berger and N. N. Bechwati, "The allocation of promotion budget to maximize customer equity," *Omega*, vol. 29, no. 1, pp. 998–1013, 2001.

[5] A. Mihiotis and I. Tsakiris, "A mathematical programming study of advertising allocation problem," *Appl. Math. Comput.*, vol. 148, pp. 373–379, 2004.

[6] E. Cetin and S. T. Esen, "A weapon-target assignment approach to media allocation," *Appl. Math. Comput.*, vol. 175, pp. 1266–1275, 2006.

[7] A. Tektas and E. D. Alakavuk, "Allocation model: A tool to develop effective media plans for Turkey," *Int. J. Advertising*, vol. 22, pp. 333–348, 2003.

[8] F. S. Zufryden, "Optimal multi-period advertising budget allocation within a competitive environment," *J. Oper. Res. Soc.*, vol. 26, no. 4, pp. 743–754, 1975.

[9] H. I. Mesak and T. L. Means, "Modeling advertising budgeting and allocation decision using modified multinomial logit market share models," *J. Oper. Res. Soc.*, vol. 49, no. 12, pp. 1260–1269, 1998.

[10] K. Weaver and D. Merrick, "Budget allocation revisited," *Admap*, no. 452, Jul. 26–28, 2004.

[11] M. L. Vidale and H. B. Wolfe, "An operations-research study of sales response to advertising," *Oper. Res.*, vol. 5, no. 3, pp. 370–381, 1957.

[12] M. Nerlove and K. J. Arrow, "Optimal advertising policy under dynamic conditions," *Economica*, vol. 29, no. 7, pp. 129–142, 1962.

[13] D. M. Hanssens, "Market response, competitive behavior, and time series analysis," *J. Market. Res.*, vol. 17, no. 4, pp. 470–485, 1980.

[14] Y. H. Liu, H. P. Huang, and Y. S. Lin, "Dynamic scheduling of flexible manufacturing system using support vector machines," in *Proc. IEEE Int. Conf. Autom. Sci. Eng.*, 2005, pp. 387–392.

[15] X. J. Peng and Y. F. Wang, "A normal least squares support vector machine(NLS-SVM) and its learning algorithm," *Neurocomputing*, vol. 72, pp. 3734–3741, 2009.

[16] M. T. Gencoglu and M. Uyar, "Prediction of flashover voltage of insulators using least squares support vector machines," *Expert Syst. With Appl.*, vol. 36, pp. 10789–10798, 2009.

[17] J. A. Suykens and J. Vandewalle, "Least squares support vector machine classifiers," *Neural Proces. Lett.*, vol. 9, no. 3, pp. 293–300, 1999.

[18] D. Tsujinishi and S. Abe, "Fuzzy least squares support vector machines for multiclass problems," *Neural Netw.*, vol. 16, pp. 785–792, 2003.

[19] S. Rainer and K. Price, "Differential evolution—A simple and efficient heuristic for global optimization over continuous spaces," *J. Global Opt.*, vol. 11, no. 4, pp. 341–359, 1997.

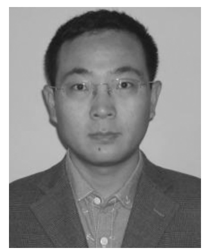
[20] A. Noktehdan, B. Karimi, and A. H. Kashan, "A differential evolution algorithm for the manufacturing cell formation problem using group based operators," *Expert Syst. With Appl.*, vol. 37, no. 7, pp. 4822–4829, 2010.

[21] L. W. H. Vincent and S. G. Ponnambalam, "A differential evolution-based algorithm to schedule flexible assembly lines," *IEEE Trans. Autom. Sci. Eng.*, vol. PP, no. 99, pp. 1–5, Nov. 2012.

[22] H. R. Cai, C. Y. Chung, and K. P. Wong, "Application of differential evolution algorithm for transient stability constrained optimal power flow," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 719–728, May 2008.

[23] A. K. Qin, V. L. Huang, and P. N. Suganthan, "Differential evolution algorithm with strategy adaptation for global numerical optimization," *IEEE Trans. Evol. Comput.*, vol. 13, no. 2, pp. 398–417, Apr. 2009.

[24] X. F. Yan, J. Yu, and F. Qian, "Adaptive mutation differential evolution algorithm and its application to estimate soft sensor parameters," *Control Theory Appl.*, vol. 23, no. 5, pp. 744–748, 2008.



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