Ricardo Barbosa
CST 329
April 14, 2021

CST 329: Reasoning with Logic
Dr. Shahidul Islam (Based on the materials from Dr. Bruns)

Work on the practice problems must be typed and submitted in pdf format only. Any other formats will not be accepted. Make your own copy (File > Make a copy) of the document. Edit your copy of the documents with your answers. Then, download a copy of your word doc as a PDF (File > Download > PDF) and submit it through the Canvas submission page.

Note: When asked to write proof, create the problems on proof-checker.org and paste the screenshots of your successful proofs in this document for your submission.

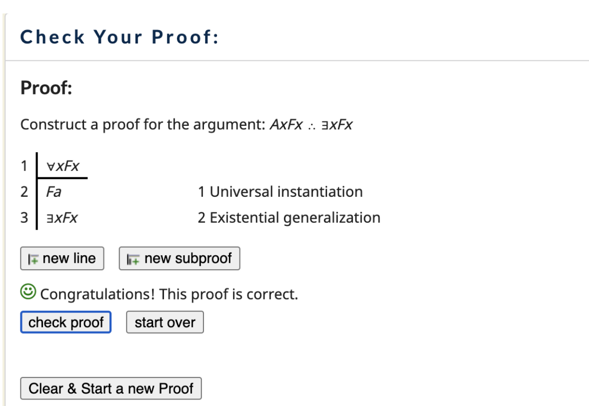
Consider working with others when needed on the following problems:

Lab: Logically true FOL sentences and invalid arguments

1. Is the following sentence of FOL logically true? Prove it.

   $(\forall x\, Fx) \;\rightarrow\; (\exists x\, Fx)$

   Hint: Can you prove that the sentence is a theorem? If it is a theorem, then is it logically true?



2. Show that the following argument is invalid using the method described in lecture:

$$\frac{\forall x\, Fxx}{\forall x\, \forall y\, Fxy}$$

Hint: start by picking a numeric domain. Then decide on the meaning of F and G. Then decide on the meaning of a and b.

Domain: {0,1,2…}

Fxy: x is equal to y

a: 3

b: 4

3. Show that the following argument is invalid using the method described in lecture:

$$\frac{\forall x\, (Fx \to Gx), \neg Ga}{\neg Fb}$$

Domain: {0,1,2,...}

Fx: x > 100

Gx: x > 50

a: 10

b: 1000

4. If you have time, show that the following argument is invalid using the method described in lecture:

$$\frac{\forall x\, (Fx \lor Gx), \neg Fa}{Gb}$$

Domain: {0,1,2,...}

Fx: x is 2
Gx: x is 7
a: 3
b: 4

Lab: Relations, functions, identity, and multiple quantifiers

1. Write a FOL formula to express that predicate F (with arity 2) is reflexive.

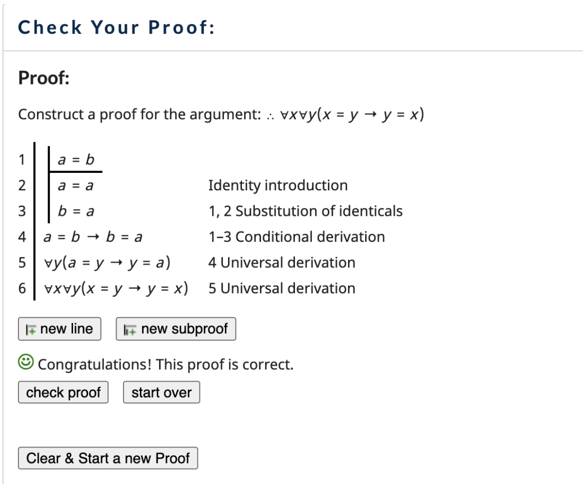
   Fxx

2. Write a FOL formula to express that predicate F (with arity 2) is transitive.

   Fxy and Fyz then Fxz

3. Prove the theorem we proved in class on your own.

$\forall x\, \forall y\, (x = y \rightarrow y = x)$



Check Your Proof:

Proof:

Construct a proof for the argument: $\therefore \forall x \forall y(x = y \rightarrow y = x)$

1	$\quad a = b$	
2	$\quad a = a$	Identity introduction
3	$\quad b = a$	1, 2 Substitution of identicals
4	$a = b \rightarrow b = a$	1–3 Conditional derivation
5	$\forall y(a = y \rightarrow y = a)$	4 Universal derivation
6	$\forall x \forall y(x = y \rightarrow y = x)$	5 Universal derivation

new line new subproof

Congratulations! This proof is correct.

check proof start over

Clear & Start a new Proof

4. Prove this theorem (no premises) using the proof checker:

   $\forall x\, \forall y\, \forall z\, ((x = y \wedge y = z) \rightarrow x = z)$

   The proof is not too different from the previous problem.

Check Your Proof:

Proof:

Construct a proof for the argument: $\therefore \forall x \forall y \forall z[(x = y \wedge y = z) \rightarrow x = z]$

1	$a = b \wedge b = c$	
2	$a = b$	1 Simplification
3	$a = a$	Identity introduction
4	$b = a$	2, 3 Substitution of identicals
5	$b = c$	1 Simplification
6	$b = b$	Identity introduction
7	$c = b$	5, 6 Substitution of identicals
8	$c = c$	Identity introduction
9	$b = c$	7, 8 Substitution of identicals
10	$c = a$	4, 9 Substitution of identicals
11	$a = c$	8, 10 Substitution of identicals
12	$[(a = b \wedge b = c) \rightarrow a = c]$	1–11 Conditional derivation
13	$\forall z[(a = b \wedge b = z) \rightarrow a = z]$	12 Universal derivation
14	$\forall y \forall z([a = y \wedge y = z) \rightarrow a = z]$	13 Universal derivation
15	$\forall x \forall y \forall z([x = y \wedge y = z) \rightarrow x = z]$	14 Universal derivation

new line | new subproof

Congratulations! This proof is correct.

check proof | start over

Clear & Start a new Proof

5. Translate "there are no more than two apples" to logic. Use the translation key "Fx x is an apple".

   Ex(Fx ^ Fx ^ ~(x=x))

6. Translate "there are exactly two apples" to logic. Use the translation key "Fx x is an apple".

   Ex(Fx ^ Ax(Fx -> (x=x))