Review for final part 2

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(Based on the materials from Dr. Bruns)
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Review of concepts

To help make the ideas of syntax and semantics stronger, we'll organize the review into these 5 parts:

	prop logic	FOL
syntax	part 1	part 3
semantics	part 2	part 4
connection	part 5	

Prop. logic: syntax-related concepts

concept	page in text
sentence, atomic sentence	9
top-level operator	-
argument	30
premise, conclusion	30
inference rule	46
direct proof	48
logical equivalence	101
instantiating a theorem	109

Sentence of prop. logic

A sentence Φ is either:

- an atomic sentence (P, Q, …)
- $\Phi_1 \wedge \Phi_2$ (where Φ_1, Φ_2 are sentences)
- $\Phi_1 \vee \Phi_2$
- $\Phi_1 \rightarrow \Phi_2$
- Φ_1 ↔️ Φ_2
- $\neg\Phi$

parentheses are not included in the definition

Top-level operator

What's the top-level operator of

$$\neg(P \to Q) \to \neg(Q \to R)$$

?

The implication in the middle.

Argument

What is the definition of argument?

A list of one or more sentences.

The last is the conclusion.

The others are the premises.

Argument

Is is possible to have an argument with no premises?

Yes, we'd seen this a lot.

Direct proof

What is the definition of direct proof?

A direct proof has no sub-proofs.

It's the simplest kind of proof.

Logical equivalence

How to prove two sentences Φ and Ψ are logically equivalence in a syntactic way?

Prove the theorem $\Phi \leftrightarrow \Psi$

Prop. logic: semantic concepts

concept	page in text
truth table	17
valid argument	31
sound argument	33
logical equivalence	64
tautology	65
contradictory sentence	93
contingent sentence	106

Valid argument

What is a valid argument?

An argument in which, if the premises are true, then the conclusion is true.

Valid argument

What is the semantic method for showing that an argument is valid?

Create a truth table with the premises and the conclusion as columns.

The argument is valid if, on every row the premises are all true, then the conclusion is true.

The argument in logic:

$P \rightarrow Q$
$\neg Q$

$\neg P$

		premise	premise	conclusion
P	**Q**	**$P \rightarrow Q$**	**$\neg Q$**	**$\neg P$**
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Valid argument

(T/F) In a valid argument, the conclusion is true

False. If the premises are not true, there is no guarantee the conclusion is true

example:

$P \rightarrow Q$
$\neg Q$

$\neg P$

Tautology

(T/F) Every sentence is either a contingent sentence or a tautology.

False. The sentence could be a contradictory sentence.

FOL: syntax-related concepts

concept	page in text
name	120
predicate	123
quantifier	129
term	130
free variable	130
well-formed formula (WFF)	130
sentence	130

(Symbolic) term

What is a term?

Either a name (like a,b,c,…) or a variable (like x,y,z)

Terms

Does the following WFF have a syntax error?

$$x \lor \exists y\, Fxy$$

Yes, a term (x) is not a WFF.

A term represents an object in the domain, not a truth value.

Syntax of first-order logic

A term is either: (also called "symbolic term")

- a name $(a, b, c, \ldots)$
- a variable (u, v, w, x, y, z)

A well-formed formula Φ is either:

- an n-ary predicate followed by $\beta_1\,\beta_2 \cdots \beta_n$
- $\Phi_1 \wedge \Phi_2$
- $\Phi_1 \vee \Phi_2$
- $\Phi_1 \rightarrow \Phi_2$
- $\Phi_1 \leftrightarrow \Phi_2$
- $\neg\Phi$
- $\forall\alpha\ \Phi$
- $\exists\alpha\ \Phi$

α is a variable
β is a term

FOL sentence

Define "FOL sentence"

A WFF with no free variables.

Free variables

Does the following WFF have any free variables?

$$(Ax\ Fx) \wedge (\exists y\ Gxy)$$

Yes. x is free in the second part of the WFF.

FOL: semantic concepts

concept	page in text
logically true FOL sentence	152
relations (transitive, symmetric, reflexive)	160
functions	161

Interpretations for first-order logic

$$\forall x \, \exists y \, (Fx \rightarrow \neg Gb)$$

What do we need to interpret this sentence?

An interpretation for first-order logic specifies:

- a set of objects (the domain)
- for each proper name, the object it points to
- the meaning of each predicate

A FOL sentence is a logically true if its true in every interpretation.

Syntactic and semantic definitions

Concept	Semantic definition	Syntactic definition
tautology	truth table has only T under top-level operator	a theorem -- a sentence that can be derived with no premises
contradictory sentence	truth table has only F under top-level operator	negation of sentence can be derived with no premises
contingent sentence	truth table has both T and F under top-level operator	a sentence that's not a theorem or contradiction
equivalent sentences	columns under the top-level operators are identical	$\Phi \leftrightarrow \Psi$ is a theorem
valid argument	on all lines where premises are true, the conclusion is true	an argument where the conclusion can be derived from the premises

this table is based on a table in "forall x"

Showing a property is present/absent

Logical property	To prove it is present	To prove it is absent
being a theorem	derive the sentence	find a false line in the truth table for the sentence
being a contradictory sentence	derive the negation of the sentence	find a true line in the truth table for the sentence
contingency	find a false line and true line in the truth table for the sentence	prove the sentence or its negation
equivalence	prove $\Phi \leftrightarrow \Psi$ for the sentences	find a line in the truth table where the sentences have different values
validity	derive the conclusion from the premises	find a line in the truth table where the premises are true and the conclusion is false

this table is based on a table in "forall x"

Proof strategies

Working backwards: summary

What are you trying to prove?

Conjunction:

Φ

Ψ

$\Phi \land \Psi$

Implication:

Φ

Ψ

$\Phi \to \Psi$

Negation:

$\neg\neg\Phi$

Ψ

$\neg\Psi$

$\neg\Phi$

Alternatively: work forwards

What are you starting from?

Conjunction:

$\Phi \land \Psi$

Φ ^E

Ψ ^E

Disjunction:

$\Phi \lor \Psi$

$\vdots$

$\neg\Phi$

Ψ MTP

Implication:

$\Phi \to \Psi$

$\vdots$

Φ

Ψ MP

Negation:

$\neg\Psi$

$\neg\Phi$

$\vdots$

Φ

Ψ

Using all information

Don't forgot to think about how to use every premise in your proof.

It is unlikely that some of the premises will be unused.

First-order logic proof rules

The 2 easy rules for FOL

Universal instantiation

$$\frac{\forall \alpha\ \Phi(\alpha)}{\Phi(\beta)}$$

Every occurrence of α in Φ must be replaced with β

example:

m | $\forall x\ (Fx \vee \neg Gx)$
| $Fb \vee \neg Gb$ — m, univ. instant.

"Universal elimination"

Existential generalization

$$\frac{\Phi(\beta)}{\exists \alpha\ \Phi(\alpha)}$$

Not every occurrence of β in Φ must be replaced with α

α must not appear in $\Phi(\beta)$

example:

m | $Fxc \wedge Gc$
| $\exists y\ (Fxc \wedge Gy)$ — m, exist. gen.

"Existential introduction"

Existential instantiation

$$
\begin{array}{l|l}
m & \exists\alpha\,\Phi(\alpha) \\
i & \quad \Phi(c) \\
 & \quad \vdots \\
j & \quad \Psi \\
 & \Psi \qquad \text{m, i-j existential instantiation}
\end{array}
$$

- α is a variable $(x, y, z, \ldots)$
- c is a name

This is the only FOL rule with a subproof

"existential elimination"

- c must not occur in $\exists\alpha\,\Phi(\alpha)$
- c must not occur in Ψ
- c must not appear in any "open" (unfinished) subproof or main proof

example

$$
\begin{array}{l|l}
m & \exists x\, Fx \\
i & \quad Fc \\
 & \quad \vdots \\
j & \quad \exists x\, Gx \\
 & \exists x\, Gx \qquad \text{m, i-j exist. inst.}
\end{array}
$$

Universal derivation

m	$\Phi(c)$	
	$\forall x\, \Phi(x)$	m, universal derivation

- α is a variable $(x, y, z, \ldots)$
- c is a name

- x must not appear in $\Phi(c)$
- c must not appear in any open subproof or main proof
- every occurrence of c in $\Phi(c)$ must be replaced by x

example:

m	$F(c)$	
	$\forall x\, F(x)$	m, univ. derivation

Identity rules

Identity Introduction

$$\frac{}{\beta = \beta}$$

β is a term (name or variable)

No line number needed.

Substitution of identicals

$$\frac{\begin{array}{c}\alpha = \beta \\ \Phi(\alpha)\end{array}}{\Phi(\beta)}$$

β is a term (name or variable)

This says: if two things are identical, then anything true of one thing will be true of the other.