Ricardo Barbosa
April 08, 2021

CST 329: Reasoning with Logic
Dr. Shahidul Islam (Based on the materials from Dr. Bruns)

Work on the practice problems must be typed and submitted in pdf format only. Any other formats will not be accepted. Make your own copy (File > Make a copy) of the document. Edit your copy of the documents with your answers. Then, download a copy of your word doc as a PDF (File > Download > PDF) and submit it through the Canvas submission page.

Note: When asked to write proof, create the problems on proof-checker.org and paste the screenshots of your successful proofs in this document for your submission.

Consider working with others when needed on the following problems:

Lab: Examples, first-order logic

1. Use the following translation key:

   Fx: x is a female
   Gx: x is male
   Hx: x is human
   Tx: x is from Texas
   Sx: x is a computer scientist
   Lx: x is a mammal
   Mx: x is mortal

   Translate the sentences below to logic:

   - Nothing is a male computer scientist from Texas.

     ~Ex(Hx ^ Sx ^ Tx)

   - All male humans are mortal mammals.

     Ax(Hx -> (Mx ^ Lx))

   - Some female humans are computer scientists who live in Texas.

     Ex(Fx ^ Hx ^ Tx)

   - No male human is a computer scientist who lives in Texas.

     ~Ex((Mx^Hx) ^ (Sx ^ Tx))

2. This is from Problem 2 at the end of Chapter 12 in our textbook. Provide a key and translate the following expressions into first-order logic. Assume the domain of discourse if terrestrial organisms. For example $\forall x\, Fx$ would mean 'all terrestrial organisms are F'. Don't be concerned that some of these sentences are obviously false.

   Fx: x are horses

   Gx: x are mammals

   Sx: x lay eggs

   - Some egg-laying mammals are not horses.

     Ex((Sx ^ Gx) ^ ~Fx)

   - There are no horses.

     ~Ex(Fx)

   - There are some mammals.

     Ex(Gx)

   - Only horses are mammals.

     Ax(Gx->Fx)

   - All and only horses are mammals.

     Ax(Gx <-> Fx)

Lab: Universal instantiation

1. Prove the following argument with the proof checker

$$\frac{\forall x\,(Hxc \rightarrow Fx),\, Hbc}{Fb}$$

Check Your Proof:

Proof:

Construct a proof for the argument: *Ax*(*Hxc*->*Fx*), *Hbc* ∴ *Fb*

1	$\forall x(Hxc \rightarrow Fx)$	
2	Hbc	
3	$Hbc \rightarrow Fb$	1 Universal instantiation
4	Fb	2, 3 Modus Ponens

new line | new subproof

Congratulations! This proof is correct.

check proof | start over

Clear & Start a new Proof

2. Prove the following argument with the proof checker

$$\frac{\forall x\,(Fx \rightarrow Gx),\, Gc \rightarrow \neg Hc,\; Hc}{\neg Fc}$$

Check Your Proof:

Proof:

Construct a proof for the argument: *Ax*(*Fx*->*Gx*), *Gc* ->~*Hc*, *Hc* ∴ ¬*Fc*

1	$\forall x(Fx \rightarrow Gx)$	
2	$Gc \rightarrow \neg Hc$	
3	Hc	
4	$Fc \rightarrow Gc$	1 Universal instantiation
5	$\neg\neg Hc$	3 Double Negation
6	$\neg Gc$	2, 5 Modus Tollens
7	$\neg Fc$	4, 6 Modus Tollens

new line | new subproof

Congratulations! This proof is correct.

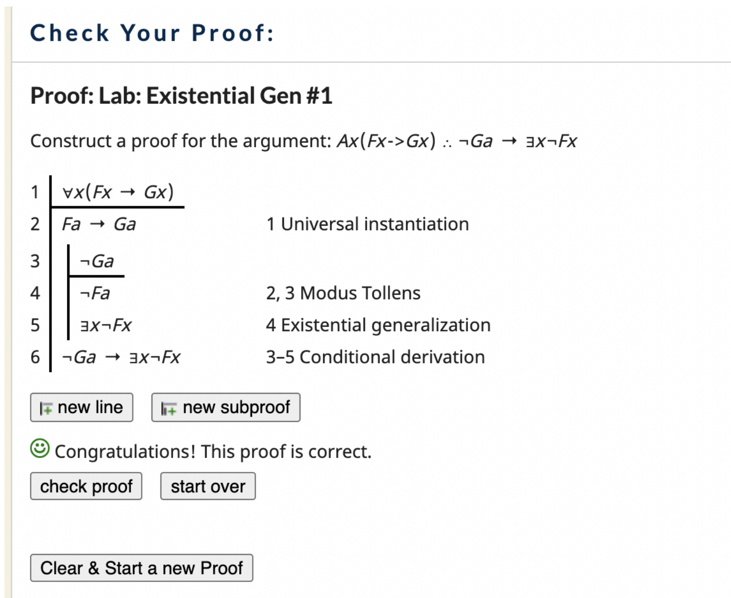
check proof | start over

Clear & Start a new Proof

Lab: Existential generalization

1. Prove the following argument with the proof checker

$$\frac{\forall x\,(Fx \rightarrow Gx)}{\neg Ga \rightarrow \exists x\,(\neg Fx)}$$



Check Your Proof:

Proof: Lab: Existential Gen #1

Construct a proof for the argument: *Ax(Fx->Gx)* ∴ ¬*Ga* → ∃*x*¬*Fx*

1	$\forall x(Fx \rightarrow Gx)$	
2	$Fa \rightarrow Ga$	1 Universal instantiation
3	$\quad \neg Ga$	
4	$\quad \neg Fa$	2, 3 Modus Tollens
5	$\quad \exists x \neg Fx$	4 Existential generalization
6	$\neg Ga \rightarrow \exists x \neg Fx$	3–5 Conditional derivation

new line | new subproof

Congratulations! This proof is correct.

check proof | start over

Clear & Start a new Proof

2. Is it true that if "Everything is F" then "Something is F"? Is this intuitively reasonable? Try to prove the following argument with the proof checker:

Yes. This is reasonable. If x is an element of a larger X, then there should exist a smaller y within x.

$$\frac{\forall x\, Fx}{\exists x\, Fx}$$

Check Your Proof:

Proof: EI Lab #2

Construct a proof for the argument: *Ax Fx* ∴ ∃*xFx*

1	$\forall xFx$	
2	Fa	1 Universal instantiation
3	$\exists xFx$	2 Existential generalization

new line   new subproof

☺ Congratulations! This proof is correct.

check proof   start over

Clear & Start a new Proof

Lab: Existential instantiation lab

1. Prove the following argument with the proof checker

$$\frac{\forall x\,(Wx \rightarrow Mx),\ \exists x\,(Wx \wedge Cx),\ \forall x\,(Cx \rightarrow Fx)}{\exists x\,(Mx \wedge Fx)}$$

Check Your Proof:

Proof: Existential Inst #1

Construct a proof for the argument: *Ax*(*Wx*->*Mx*), *Ex*(*Wx*^*Cx*), *Ax*(*Cx*->*Fx*) ∴ ∃*x*(*Mx* ∧ *Fx*)

1	$\forall x(Wx \rightarrow Mx)$	
2	$\exists x(Wx \wedge Cx)$	
3	$\forall x(Cx \rightarrow Fx)$	
4	$Wa \wedge Ca$	
5	$Wa \rightarrow Ma$	1 Universal instantiation
6	$Ca \rightarrow Fa$	3 Universal instantiation
7	Wa	4 Simplification
8	Ca	4 Simplification
9	Ma	5, 7 Modus Ponens
10	Fa	6, 8 Modus Ponens
11	$Ma \wedge Fa$	9, 10 Adjunction
12	$\exists x(Mx \wedge Fx)$	11 Existential generalization
13	$\exists x(Mx \wedge Fx)$	2, 4–12 Existential instantiation

new line | new subproof

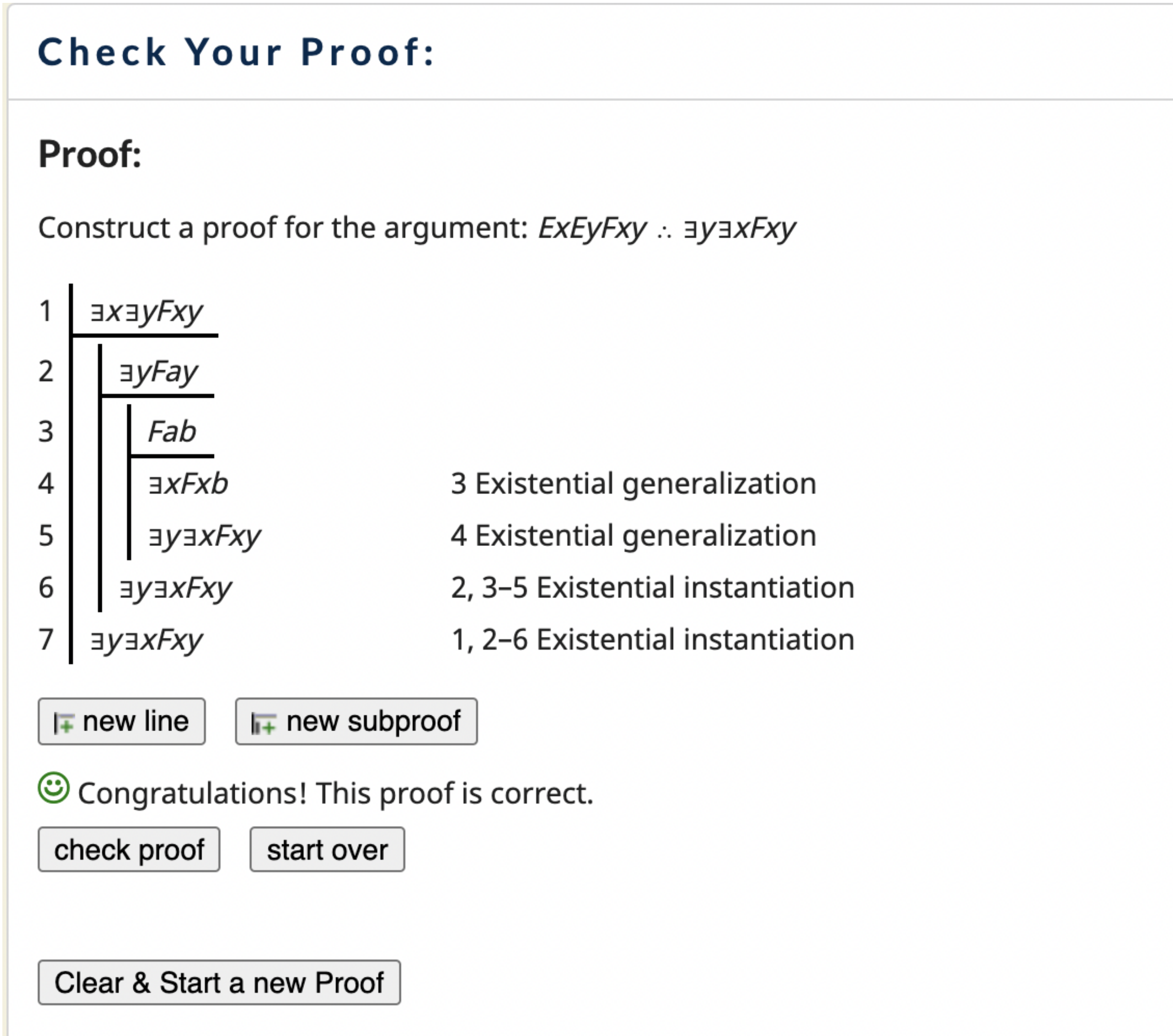
Congratulations! This proof is correct.

check proof | start over

Clear & Start a new Proof

2. Prove the following argument with the proof checker

$$\frac{\exists x\, \exists y\, Fxy}{\exists y\, \exists x\, Fxy}$$



Lab: Universal derivation lab

1. Prove the following argument with the proof checker.

$$\frac{\forall x\,(Fx \rightarrow Gx), \forall x\,(Gx \rightarrow Hx), \forall x\,(Hx \rightarrow Mx)}{\forall x\,(Fx \rightarrow Mx)}$$

Check Your Proof:

Proof: Universal Derivation 1

Construct a proof for the argument: *Ax*(*Fx*->*Gx*), *Ax*(*Gx*->*Hx*), *Ax*(*Hx*->*Mx*) ∴ $\forall x(Fx \rightarrow Mx)$

1	$\forall x(Fx \rightarrow Gx)$	
2	$\forall x(Gx \rightarrow Hx)$	
3	$\forall x(Hx \rightarrow Mx)$	
4	$Fa \rightarrow Ga$	1 Universal instantiation
5	$Ga \rightarrow Ha$	2 Universal instantiation
6	$Ha \rightarrow Ma$	3 Universal instantiation
7	$\quad Fa$	
8	$\quad Ga$	4, 7 Modus Ponens
9	$\quad Ha$	5, 8 Modus Ponens
10	$\quad Ma$	6, 9 Modus Ponens
11	$Fa \rightarrow Ma$	7–10 Conditional derivation
12	$\forall x(Fx \rightarrow Mx)$	11 Universal derivation

new line | new subproof

☺ Congratulations! This proof is correct.

check proof | start over

Clear & Start a new Proof

2. I want you to translate backwards from a logical argument to English. Here is a logical argument:

$$\frac{\forall x\,(Fx \leftrightarrow Gx), Gb}{\exists x\,(Gx \land Fx)}$$

Write a translation key, and using the translation key, translate the logic to English. Your English argument should be a paragraph, not an ordered list of sentences.

Fx: x is a Fish

Gx: x has gills

All and only fish have gills and we know that fish have gills. So there must exist a fish with gills.

3. Explain why the following "proof" is incorrect:

Construct a proof for the argument: *Ax Fxx* ∴ ∀*x*∀*yFxy*

1	∀*xFxx*	
2	*Faa*	1 Universal instantiation
3	∀y Fay	2 Universal derivation
4	∀*x* ∀*y Fxy*	3 Universal derivation

Note: Think about the restrictions on the use of the rules. Once you think you have an answer, use the proof checker and see what it says about it.

The proof-checker says that this is not a proper application of the rule Universal Derivation. I believe this is wrong because there must be a subproof after Faa that assumed AyFay

4. If you have time, finish the following proof by adding justification to every line that needs it.

1	$\forall x(J(x) \to K(x))$	
2	$\exists x \forall y\, L(x,y)$	
3	$\forall x\, J(x)$	
4	$\quad \forall y\, L(a,y)$	
5	$\quad L(a,a)$	
6	$\quad J(a)$	
7	$\quad J(a) \to K(a)$	
8	$\quad K(a)$	
9	$\quad K(a) \land L(a,a)$	
10	$\quad \exists x(K(x) \land L(x,x))$	
11	$\exists x(K(x) \land L(x,x))$	

Check Your Proof:

Proof:

Construct a proof for the argument: *Ax(Jx->Kx)*, *ExAy(Lxy)*, *AxJx* ∴ ∃x(*Kx* ∧ *Lxx*)

1	∀x(*Jx* → *Kx*)	
2	∃x∀y*Lxy*	
3	∀x*Jx*	
4	∀y*Lay*	
5	*Laa*	4 Universal instantiation
6	*Ja*	3 Universal instantiation
7	*Ja* → *Ka*	1 Universal instantiation
8	*Ka*	6, 7 Modus Ponens
9	*Ka* ∧ *Laa*	5, 8 Adjunction
10	∃x(*Kx* ∧ *Lxx*)	9 Existential generalization
11	∃x(*Kx* ∧ *Lxx*)	2, 4–10 Existential instantiation

new line | new subproof

☺ Congratulations! This proof is correct.

check proof | start over

Clear & Start a new Proof