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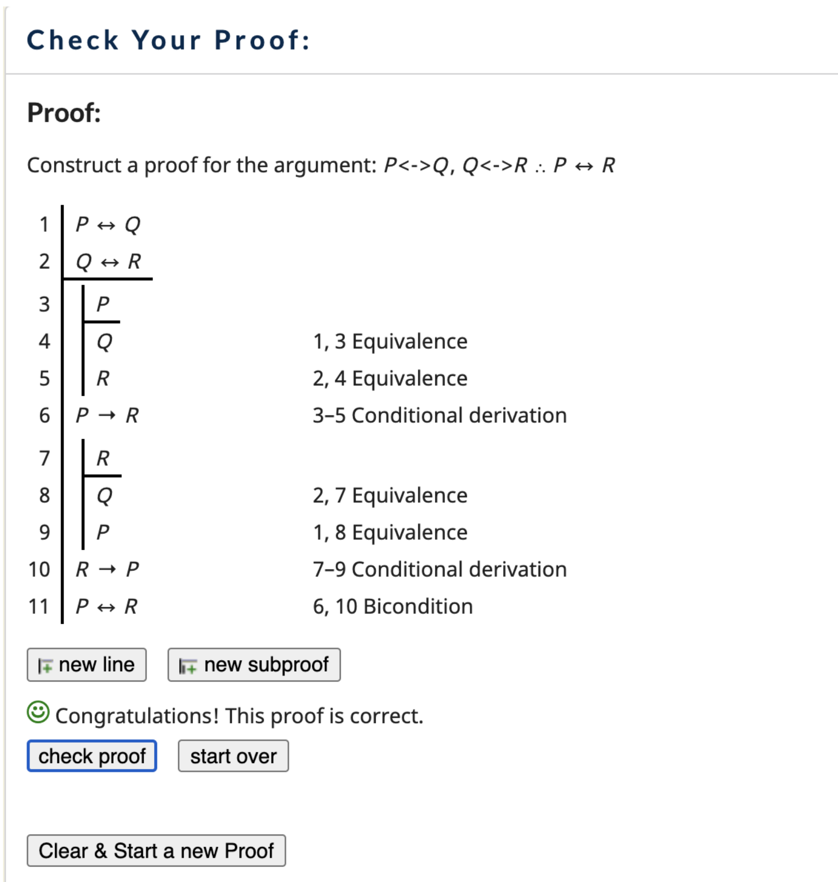
CST 329: Reasoning with Logic

Lab: Iff, Contingent sentences, and Theorems

1. Write a proof for this argument, and check your proof with the proof checker:

$$\frac{P \leftrightarrow Q, Q \leftrightarrow R}{P \leftrightarrow R}$$



Check Your Proof:

Proof:

Construct a proof for the argument: P<->Q, Q<->R ∴ $P \leftrightarrow R$

1	$P \leftrightarrow Q$	
2	$Q \leftrightarrow R$	
3	P	
4	Q	1, 3 Equivalence
5	R	2, 4 Equivalence
6	$P \to R$	3–5 Conditional derivation
7	R	
8	Q	2, 7 Equivalence
9	P	1, 8 Equivalence
10	$R \to P$	7–9 Conditional derivation
11	$P \leftrightarrow R$	6, 10 Bicondition

new line   new subproof

Congratulations! This proof is correct.

check proof   start over

Clear & Start a new Proof

2. **If $\Phi$is a theorem, then what is $\neg\Phi$?**

   No that's a contradiction

3. **If $\Phi$is a contingent sentence, then can $\neg\Phi$ be a theorem? Support your answer.**

No. A contingent sentence will always have a case where it is false, and cases where it is true. A theorem must always be true.

4. Give 2 examples each of:
   a. theorems

      $a^2 + b^2 = c^2$

      $e^{i\pi} + 1 = 0$

   b. contingent sentences

      $x^2 + y = 7$

      $2x + 1 = 3$

   c. contradictory sentences

      $5 = 3$

      $0 = 1$

   (This may sound like a silly question but it will help your learning a lot.)

Textbook page 108:

5. You can instantiate a sentence $\Phi$by replacing atomic sentences in Φ with sentences. But these substitutions have to be done consistently and "in parallel". Which of the following are legal instantiations of this sentence: $P \rightarrow (Q \ -> \ P)$?
   a. $Q \rightarrow (Q \rightarrow Q)$
   b. $R \rightarrow (Q \rightarrow \neg R)$
   c. $(P \wedge Q) \rightarrow (Q \rightarrow (P \wedge Q))$
   d. $\neg P \rightarrow (Q \rightarrow \neg P)$

Textbook Page 109:

6. Refer to the lecture slides or the textbook for the theorems T1-T10. Is this sentence a legal instantiation of one of the theorems T1-T10, and if so, which theorem?
   a. $(P \rightarrow (R \rightarrow P)) \rightarrow (\neg(R \rightarrow P) \rightarrow \neg P)$

      T10

Textbook page 109:

7. Why is the instantiation of a theorem also a theorem? For example, can you explain why, if $P \rightarrow (R \rightarrow P)$ is a theorem, then also $(P \wedge R) \rightarrow (\neg Q \rightarrow (P \wedge R))$ is a theorem?

   We have defined a theorem to be a tautology. These theorems can be treated as true sentences in the form of phi or psi in metalanguage, which allow substitution of any of any atomic sentence in the theorem with any other sentence if and only if we replace each initial instance of that atomic sentence in the theorem with the same sentence.

Lab: Limitations of Propositional Logic and Quantifiers

1. Write a translation key for the text below showing the proper names and predicates.

   Bob is a poriferan.

   b Bob

   Fx b is a poriferan

2. Write a translation key for the text below showing the proper names and predicates.

Tal wants to return to the CSUMB campus.

a **tal**

b **CSUMB campus**

Fx **wants to**

Gx **return to**

3. Using the translation key below:

   Hx x is human

   Mx x is mortal

   a Ana

   b Bubba

   Translate each of the following sentence to logic:

   a. Ana is human

      Ha

   b. Bubba is mortal

      Mb

   c. if Bubba is mortal then Ana is mortal

      Mb -> Ha

   d. Some humans are mortal

      ∃x (Hx ^ Mx)

4. Translate these two formulas to English. Carefully explain the difference in their meaning.

   a. $\exists x\,(Hx \wedge Mx)$

      Some humans are mortal

   b. $\exists x\,(Hx \rightarrow Mx)$

      There are some humans who, if human, are mortal.

5. Do 4 again, but use these sentences:

   a. Ax (Hx ^ Mx)

Everyone is human and mortal

b. Ax (Hx -> Mx)
All humans are mortal

Lab: Syntax of first-order logic

1. Which of the following are well-formed formulas?
    a) Fax
    b) Ga ^ Fab
    c) forall x Fa
    d) ~(exists x)
    e) Ga ^ b
2. For which of the following formulas, x is free?
    a) ~Fax
    b) exists x ~Fax
    c) (exists x Fx) ^ (forall y Gx ^ Gy)
3. For which of the following formulas, x is free?
    a) Fa
    b) forall x (Fx ^ Gy)

c) exists y (Fx ^ Gy)

4. Which of these sentences are of first-order logic?
   a) forall y (Fx ^ Gy)
   b) ~(exists x (exists y Fxy))
   c) exists x (forall y (x v y))
   d) forall x Gxy

Lab: Semantics of first-order logic

Please work with your teammates on the following problems, which come from Section 12.4 of our textbook.

Provide a key and translate the following expressions into first-order logic. Assume the domain of discourse is terrestrial organisms. For example, $\forall x\, Fx$ would mean 'all terrestrial organisms are F'. Don't be concerned that some of these sentences are obviously false.

Write your answers down on paper or a document!

Fx: $x\ is\ a\ horse$

Gx: $x\ is\ a\ mammal$

Tx: $x\ is\ chestnut\ horse$

Sx: $x\ is\ an\ egg - laying\ species$

1. All horses are mammals.

$\forall x(Fx \land Gx)$

2. Some horses are mammals.

   $\exists x(Fx \land Gx)$

3. No horses are mammals.

   $\neg\exists x(Fx \land Gx)$

4. Some horses are not mammals.

   $\exists x(Fx \land \neg Gx)$

5. Some mammals lay eggs, and some mammals do not.

   $\exists x(Gx \rightarrow Sx) \land \exists x(Gx \land \neg Sx)$

6. Some chestnut horses are mammals that don't lay eggs.

   $\exists x(Tx \rightarrow (Gx \land \neg S(x))$

7. No chestnut horses are mammals that lay eggs.

   $\neg\exists x(Tx \land Gx) \land Sx$