CST 329: Reasoning with Logic
Dr. Shahidul Islam (Based on the materials from Dr. Bruns)

Work on the practice problems must be typed and submitted in pdf format only. Any other formats will not be accepted. Make your own copy (File > Make a copy) of the document. Edit your copy of the documents with your answers. Then, download a copy of your word doc as a PDF (File > Download > PDF) and submit it through the Canvas submission page.

Note: When asked to write proof, create the problems on proof-checker.org and paste the screenshots of your successful proofs in this document for your submission.

Consider working with others when needed on the following problems:

Lab: disjunction semantics

1. From memory, write the truth table for disjunction.

P	Q	P v Q
T	T	T
T	F	T
F	T	T
F	F	F

2. Show that $Q \vee \neg Q$ is a tautology.

Q	~Q	Q v ~Q
T	F	T
F	T	T

3. Write a translation key for the following sentence, then translate it to our logical language:

   I'm going to finish this chapter or lie down and take a nap.

   P: I'm going to finish this chapter

   Q: I am going to lie down

   R: I am going to take a nap

   P v (Q ^ R)

4. Write a translation key for the following sentence, then translate it to our logical language:

   If I don't finish this chapter I'm going to lie down and take a nap.

   P: I finish this chapter

   Q: I'm going to lie down

   R: I am going to take a nap

   ~P -> (Q^R)

5. Do you think the English sentences in problems 3 and 4 mean the same thing? Do you think the logical sentences you produced mean the same thing?

   Yes

6. Write a translation key for the following sentence, then translate it to our logical language:

   I don't want mushrooms and I don't want pineapple.

   P: I want mushrooms

   Q: I want pineapple

   ~P ^ ~Q

7. In the previous problem, did you use disjunction in your logical sentence? Could you have used disjunction in your logical sentence?

   ~(PvQ)

Lab: Disjunction inference

1. From memory, what are the four inference rules for disjunction?

   Modus tollendo pollens (2 rules), addition (2 rules)

   Solve problem 2 from Section 7.6 of our textbook. It is a problem about Wittgenstein and Meinong. Don't forget the part about proving the argument valid.

   Either Wittgenstein or Meinong stole the diamonds. If Meinong stole the diamonds, then he was in the billiards room. But if Meinong was in the library, then he was not in the billiards room. Therefore, if Meinong was in the library, Wittgenstein stole the diamonds.

   P: Wittgenstein stole the diamonds

   Q: Meinong stole the diamonds

   R: Meinong was in the billiards room

   S: Meinong was in the library

   1. **P v Q**
   2. **Q -> R**
   3. **S -> ~R**

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   4. **S -> P**

2. Solve problem 1 from Section 7.6 of our textbook.

   Either Dr. Kronecker or Bishop Berkeley killed Colonel Cardinality. If Dr. Kronecker killed Colonel Cardinality, then Dr. Kronecker was in the kitchen. If Bishop Berkeley killed Colonel Cardinality, then he was in the drawing room. If Bishop Berkeley was in the drawing room, then he was wearing boots. But Bishop Berkeley was not wearing boots. So, Dr. Kronecker killed the Colonel.

   P: Dr. Kronecker killed Colonel Cardinality

   Q: Bishop Berkeley killed Colonel Cardinality

   R: Dr. Kronecker was in the kitchen

   S: Dr Kronecker was in the drawing room

   T: Bishop Berekely was in the drawing room

   U: Bishop Berekely was wearing boots

   P v Q

   P -> R

   Q -> S

   T -> U

   ~U

   P

3. If you have time, try proving that P v ~P is a theorem using the inference rules we've covered so far.

Lab: Reductio Ad Absurdum

1. Write a proof for this argument, and check your proof with the proof checker:

$$\frac{\neg(P \wedge Q), Q}{\neg P}$$

Check Your Proof:

Proof:

Construct a proof for the argument: ~(P^Q), Q ∴ ¬P

1	¬(P ∧ Q)	
2	Q	
3	¬¬P	
4	P	3 Double Negation
5	P ∧ Q	2, 4 Adjunction
6	¬(P ∧ Q)	1 Repeat
7	¬P	3–6 Reductio Ad Absurdum

new line new subproof

Congratulations! This proof is correct.

check proof start over

Clear & Start a new Proof

2. Write a proof for this argument, and check your proof with the proof checker:

$$\frac{P}{\neg P \rightarrow Q}$$

Check Your Proof:

Proof:

Construct a proof for the argument: $P \therefore \neg P \rightarrow Q$

1	P		
2	$\quad \neg P$		
3	$\quad P \vee Q$		1 Addition
4	$\quad Q$		2, 3 Modus Tollendo Ponens
5	$\neg P \rightarrow Q$		2–4 Conditional derivation

new line new subproof

Congratulations! This proof is correct.

check proof start over

Clear & Start a new Proof

Lab: Proof by Contradiction, 2

1. Is $Q \ \wedge \ \neg Q$ a contradiction?

   Yes

2. Write a proof for this argument, and check your proof with the proof checker:

$$\frac{\neg(P \vee Q)}{\neg P \wedge \neg Q}$$

Check Your Proof:

Proof:

Construct a proof for the argument: ~(*PvQ*) ∴ ¬*P* ∧ ¬*Q*

1	¬(*P* ∨ *Q*)	
2	¬¬*P*	
3	*P*	2 Double Negation
4	*P* ∨ *Q*	3 Addition
5	¬(*P* ∨ *Q*)	1 Repeat
6	¬*P*	2–5 Reductio Ad Absurdum
7	¬¬*Q*	
8	*Q*	7 Double Negation
9	*P* ∨ *Q*	8 Addition
10	¬(*P* ∨ *Q*)	1 Repeat
11	¬*Q*	7–10 Reductio Ad Absurdum
12	¬*P* ∧ ¬*Q*	6, 11 Adjunction

new line new subproof

Congratulations! This proof is correct.

check proof start over

Clear & Start a new Proof

3. Write a proof for this argument, and check your proof with the proof checker:

$$\frac{P \vee Q}{Q \vee P}$$

Anaconda-Navigator

Check Your Proof:

Proof:

Construct a proof for the argument: *PvQ* ∴ $Q \vee P$

1	$P \vee Q$	
2	$\neg(Q \vee P)$	
3	$\neg Q$	
4	P	1, 3 Modus Tollendo Ponens
5	$Q \vee P$	4 Addition
6	$\neg(Q \vee P)$	2 Repeat
7	Q	3–6 Reductio Ad Absurdum
8	$Q \vee P$	7 Addition
9	$\neg(Q \vee P)$	2 Repeat
10	$Q \vee P$	2–9 Reductio Ad Absurdum

new line | new subproof

Congratulations! This proof is correct.

check proof | start over

Clear & Start a new Proof

Lab: Proof strategies

1. Solve (or re-solve) this argument. However, think carefully about how you decided to do the proof. Write down the steps in your thought process. Don't rush this -- it's a very helpful exercise.

$$\frac{\neg Q \rightarrow P}{Q \vee P}$$

   You can work as a team, or compare your strategy afterward.

Check Your Proof:

Proof:

Construct a proof for the argument: $\sim Q \rightarrow P \therefore Q \vee P$

1	$\neg Q \rightarrow P$	
2	$\neg(Q \vee P)$	
3	$\neg\neg Q$	
4	Q	3 Double Negation
5	$Q \vee P$	4 Addition
6	$\neg(Q \vee P)$	2 Repeat
7	$\neg Q$	3–6 Reductio Ad Absurdum
8	P	1, 7 Modus Ponens
9	$Q \vee P$	8 Addition
10	$\neg(Q \vee P)$	2 Repeat
11	$Q \vee P$	2–10 Reductio Ad Absurdum

new line new subproof

Congratulations! This proof is correct.

check proof start over

Clear & Start a new Proof

2. Repeat the previous problem, but use this argument:

$$\frac{}{P \vee \neg P}$$

Got stuck still working with team

3. Repeat the previous problem, but use this argument:

$$\overline{(P \wedge \neg Q) \rightarrow \neg(P \rightarrow Q)}$$

Got stuck still working with team

Check Your Proof:

Proof:

Construct a proof for the argument: $\therefore (P \wedge Q) \rightarrow \neg(P \rightarrow Q)$

1	$P \wedge \neg Q$	
2	$\neg\neg(P \rightarrow Q)$	
3	$P \rightarrow Q$	2 Double Negation
4	P	1 Simplification
5	$\neg Q$	1 Simplification
6	$\neg P$	3, 5 Modus Tollens
7	P	1 Simplification
8	$\neg(P \rightarrow Q)$	2–7 Reductio Ad Absurdum
9	$(P \wedge Q) \rightarrow \neg(P \rightarrow Q)$	1–8 Conditional derivation

new line new subproof

☹ Sorry there were errors.
Line 9: Is not a proper application of the rule →I (for the line(s) cited).

check proof start over

Clear & Start a new Proof