Investor Sentiment in the Financial Market with Small World Networks

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Abstract—**This paper argues the dynamic evolution issue of investor sentiment based on the small world network theory, by reviewing the micro-mechanism of financial network. On the stochastic multi-agent platform of Nagurney's financial market, it takes the relative wealth and habit formation utility functions as the motive mechanism of this network structure evolution. With the degree distributions, mean geodetic distance between vertex pairs, and clustering coefficient as the evolving criterion of this network structure, it describes the dynamic evolution attributes of investor sentiment. Results of simulation indicate that investor sentiment evolution process has significant robustness with the set motive mechanism.**

Keywords-behavioral finance; investor sentiment; financial network; small world network

I. INTRODUCTION

With the development of internet and electric transaction, it is strengthened for quantity and complex degree of international financial markets. Therefore, scholars are focusing on the field of financial networks [1]. Networks have been used to formulate and study numerous problems arising in diverse application settings, including a variety of economic problems. Particularly, financial systems have provided an attractive and rich setting.

The origin of the use of networks for the representation of financial systems with many interacting decision-makers lies in the work of Quesnay [2], who depicted the circular flow of funds in an economy as a network. In the context of credit networks and made use of linear programming, Thore [3] had earlier introduced networks for the study of systems of linked portfolios, along with the mathematics. It is forming the financial network theory.

Although these works of financial networks are important and invaluable, it is limited within the standard finance paradigm, which ignores effects of investors' bounded rationality on financial system. Fama [4] argue that noise traders are unimportant in the financial asset price formation process. However, continuing evidence of market anomalies, challenge efficient markets hypothesis. Underlying noise trader models in finance is the premise that subsets of agents trade in response to extraneous variables.

This paper is structured as follows: In Section 2, we present the micro-mechanism of investor sentiment in the financial network. In Section 3, we form the motive mechanism of investor sentiment dynamic evolution, and simulate our model. Section 4 is a summary of results.

II. THE MICRO-MECHANISM ANALYSIS

In the paradigm of behavioral finance, De Long, Shleifer, Summers, and Waldmann propose the model of noise trading on equilibrium prices, named by DSSW model [5]. Noise traders acting in concert on non-fundamental signals can introduce a systematic risk that is priced. In their model, the deviations in price from fundamental value created by changes in investor sentiment are unpredictable.

Besides the DSSW model, there are two additional works on noise trading and its implications on asset prices. Barberis, Shleifer and Vishny [6] present a model of investor sentiment that explains the underreaction and overreaction of stock prices.

In contrast to DSSW that investor sentiment becomes more extreme and prices move even further away from fundamental values, Bhushan, Brown and Mello [7] show that myopia is not a sufficient or a necessary condition for noisy asset prices. The DSSW model has motivated empirical studies that noise trader risks influence price formation.

Because investor sentiment affects security prices in equilibrium, arbitrage fails to completely eliminate mispricing. Lee, Shleifer and Thaler [8] find that changes in closed-end fund discounts are highly correlated with the returns on small capitalization stocks. Bodurtha, Kim and Lee [9] report that changes in country fund discounts is related to the sentiment of investors.

Behavioral finance is the paradigm where financial markets are studied using models that are less narrow than those based on Von Neumann–Morgenstern expected utility theory and arbitrage assumptions.

Specifically, behavioral finance has two building blocks: cognitive psychology and the limits to arbitrage, which describe the influential effects among the structure, conduct and performance paradigm of behavioral finance [10]. Cognitive refers to how people think.

978-1-4244-2108-4/08/$25.00

There is a huge psychology literature documenting that people make systematic errors in the way that they think: They are overconfident, they put too much weight on recent experience. Their preferences may also create distortions. Behavioral finance uses this body of knowledge rather than taking the arrogant approach that it should be ignored.

People are overconfident that affects financial markets. With the development of financial network theory, several recent papers have argued that investor overconfidence or shifts in confidence offer a possible explanation for a range of anomalous empirical patterns in securities markets.

Palomino [11] shows that small market size can further enhance the survivorship of noise traders. An informed trader who knows he is trading against an overconfident informed opponent chokes back on his trades, to the benefit of the overconfident trader.

Hirshleifer and Luo [12] present a dynamic model in which overconfident traders survive because they more aggressively exploit information and therefore earn higher profits. Gigerenzer and Todd [13] specify an array of decision-making environments in which making mistakes relative to the Bayesian benchmark can improve fitness.

Bernardo and Welch [14] model herd-behavior and the socially constructive role overconfident individuals can play in long trends. Overconfident types benefit society by making private information public.

Based on above research, the investor sentiment in the financial market involves not only a single decision-maker but many interacting in some fashion on what may be viewed as multitiered networks. Storoy, Thore and Boyer [15] presented a network model of the interconnection of capital markets and demonstrated how decomposition theory of mathematical programming could be exploited.

Nagurney, Dong and Hughes [16] presented a model and recognized the network structure underlying the sub-problems encountered in their proposed decomposition scheme. Nagurney and Ke [17] focused on modeling the behavior of not only the sources of funds but also on modeling the behavior of the intermediaries. The introduction of small world connection topology [18] has allowed the possibility of having different kinds of structures for investor sentiment.

III. EVOLUTION OF INVESTOR SENTIMENT

A. Platform of Financial Network

There is limitation of structural evolution on complex social networks. Including Liven-Nowell and Kleinberg's work [19], the attributes of individual Selection are ignored in complex networks. Existing network model can not describe the real conditions of the financial network.

For the financial network composition, the site denotes an individual or social entity. The border represents the mutual effect between entities, which depends on the individual Selection rule. Jackson and Rogers [20] point out that it should focus on both "how" and "why" issues.

From the viewpoint of individual investor Selection, it is adjusted for Nagurney and Dong's financial network platform [21]. There is a financial network including n investors and m kinds of financial tools. Let x_{ij} denote the capital quantity of the j financial tool held by the i investor.

Let y_{ij} denote the debt quantity of the j financial tool held by the i investor. They are measured with the value form. Let r_j denote the j financial tool price. $(x_{ij} - y_{ij})$ defines the net capital of the j financial tool held by the i investor.

The financial network is formed as the work of Nagurney and Dong. It is hypothesized with the complete competition and rational investors. The investment behavior of investors is taken as independent in the short term.

Furthermore, it does not require the equilibrium of capital and debt. Hence, the optimization for the various financial tools held by investors is transferred as the extremum issue for the utility function of the m financial tools.

B. Motive Mechanism of Evolution

Yang, Chen and Duan [22] propose the evolution motive mechanism of the small world network, from views of individual Selection, with the economic network model based on game theory. Let $v_i(t)$ represents the value of network site i at the t time.

Without the static state, the action of network site i depends on the rule of $v_i(t+1) > v_i(t)$. Considering itself state and relation condition with neighbor other network sites, the action of network site i will make decisions among holding, cutting and strengthening the borders.

However, the existing works on individual Selection do no considering the bounded rational conditions of individual decision-makers. This paper solves this problem with the utility function based on the relative wealth and habit formation.

Let c_t denote the consumer rate of investor at the t time. a and b are the positive parameters respectively. The bigger a represents the smaller weight of past consumption.

The bigger b indicate the smaller weight among the consumption, wealth and habit. "$a = b = 0$" means that habit can not form. The habit formation H_t is partially stochastic, derived from the cognitive theory.

Let W_t and S_t represent the absolute and relative wealth. For the index of social wealth V_t,

$$dV_t / V_t = \mu_{V,t} dt + \sigma_{V,t} d\omega_{V,t} \tag{1}$$

$\mu_{V,t}$ and $\sigma_{V,t}$ are the conditional mean and variance of the growth rate respectively. $\omega_{V,t}$ is the standard Brown movement process. The relative wealth S_t is described as the function $S_t = f(W_t, V_t)$.

If $f_V < 0$, the higher index of social wealth corresponds the lower relative wealth by contraries.

$U(c_t, S_t, H_t, t)$ represents the utility function of investor at the t time, which depends on the current consumption rate c_t, the t time, the relative wealth S_t and the habit H_t.

And, $U(c_t, S_t, H_t, t)$ has the attribute of twice continuum differential coefficient, when $U_c > 0$, $U_S > 0$, and $U_H < 0$. The utility function of investor [23] is defined as,

$$U(c_t, W_t, V_t, H_t, t) = U(c_t, f(W_t, V_t), H_t, t) \tag{2}$$

By defining the utility function of investor based on the relative wealth and habit formation, the core motive mechanism of evolution is described.

Influenced by the factors of c_t, W_t, V_t, H_t, different investors have different value standard of risks, with diverse utility functions, which are adjusted by prompting strategies.

C. Dynamic Attributes of Sentiment Evolution

Bounded rational investors make the decision of individual Selection on the adjusted Nagurney and Dong's financial network platform, with the rule based on the relative wealth and habit formation utility function.

It is transferred into the structural evolution issue of the Watts-Strogatz classical model.

A small world network is an attempt to represent realistically the network of contacts between individuals, where local links predominate but social and geographical mobilities imply a fraction of random connections through which long-range transmission may occur.

The spread of infectious disease through a structured population was indeed one of the applications envisioned by the creators of the small world network model. Although for typical diseases, other types of contact networks [24] have been shown to be more realistic.

However, for most transmission mechanisms including investor sentiment, a small world network is the simplest model compatible with the available data.

We study the investor sentiment in the financial market of a small-world network on the phase diagram and critical behavior of the majority-vote model with noise [25]. Long-range connections are introduced in the system.

We consider the Watts–Strogatz algorithm, in which each nearest-neighbor bond connecting site i on a two-dimensional square lattice is rewired with probability p, that is, we move one end of the bond to a new randomly chosen site.

Thus, the summation does not consider nearest neighbors only, as in the case of a regular lattice ($p = 0$), but it runs over all those sites connected to site i.

The majority-vote model is a simple nonequilibrium two-state spin system to study the evolution of opinion of individuals in a social community.

Corresponding the noise trader and the rational trader, in this model each individual has to decide by one of the two possible options, namely "yes or no", and its Selection is influenced by its neighborhood.

In terms of the noise parameter q, an individual changes its opinion at time t with probability,

$$w(\sigma_i) = \frac{1}{2}\left[1 - (1 - 2q)\sigma_i S\left(\sum_{\delta=1}^{k} \sigma_{i+\delta}\right)\right] \tag{3}$$

Where a spin variable $\sigma_i = \pm 1$, denoting the opposite states yes and no, is ascribed to each site on the lattice, $S(x) = sign(x)$ for $x \neq 0$, $S(0) = 0$, and the summation is carried out over the k spins connected to the spin at site i.

Numerical simulations on regular lattices of the model defined by the above flipping probability showed that it presents a phase transition from an ordered to a disordered state at a critical value of the noise parameter, $q = q_c$, which depends on the lattice topology [26].

The corresponding critical phenomenon is in the same class of universality as the equilibrium model, and so the critical exponents depend only on the lattice dimensionality.

Based on above analysis, with the degree distributions $p(k)$, mean geodetic distance between vertex pairs l, and clustering coefficient c as the evolving criterion of this network structure, it describes the dynamic evolution attributes of investor sentiment.

In the network with n sites, let d_{ij} denote the minimum distance from site i to j. The mean geodetic distance between vertex pairs l is defined as the following.

$$l = 2\sum_{i \geq j} d_{ij} \Big/ n(n+1) \tag{4}$$

The clustering coefficient c describes the group form, which is calculated as the mean arithmetical value of the site i partial clustering coefficient C_i, the ratio of the triangle quantity including the site i and the continuum three-dimensional quantity with the site i as the center.

D. Simulation Analysis

Supposing investor has two border upon sites s and r, which face the objects A and B. With the transform model of the utility function based on the relative wealth and habit formation, it's realized targets of dynamic stochastic adjustment for border upon sites weights δ_s and δ_r.

Then, this model is changed into a bi-level optimization issue, which is a non-protruding issue. Even for the simple linear bi-level issue, it's complex for a legible results in practice. The key of solving is transforming target function optimization issue of two borders upon sites into standard stochastic programming model.

For the latter kind of model, there are many mature algorithmic to solve. With a 0.1 space, it's indicated the simulation iterative process, which describes dynamic transform attributes of investor sentiment in the financial market with small world networks.

IV. CONCLUSIONS

This paper argues the evolution mechanism of investor sentiment in the financial market with small world network, which are situated for the financial system conditions. We developed a framework for the formulation, qualitative analysis, and computation of solutions to investor sentiment evolution problems in the financial network with small world networks. The financial network consisted of a multi-tiered network in which non-investment is also permitted.

We described the behavior of the decision-makers consisting of the noise trader and the rational trader in terms of source agents, the financial intermediaries, and the consumers in the financial markets. From the simulation results of investor sentiment in the financial market, it indicates that the financial network has the typical small world effect.

Our work has the same guiding value for other complex network problems in the social economic fields. Since Jackson and Rogers points out that the individual Selection is the powerful tool for solving problems of complex networks, particularly for the economic networks.

With the rapid development of behavioral finance theories, people reveal the individual Selection rule of complex networks, which forms a new research domain of complex network problems in the social and economic fields.

Just by the above study way, this paper demonstrated that investor sentiment problems in the financial network with different investors in the presence of risk attitudes associated with the small world networks can be formulated in a rigorous fashion accordingly.

Future research will include the extension of this framework to the general economic arena, the incorporation of other criteria, the introduction of dynamics, as well as some empirical applications.

ACKNOWLEDGEMENTS

This study was supported by the Social and Management Science Fund of Northwestern Polytechnical University under grant RW200709.

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