1. Judgment under uncertainty: Heuristics and biases

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Many decisions are based on beliefs concerning the likelihood of uncertain events such as the outcome of an election, the guilt of a defendant, or the future value of the dollar. These beliefs are usually expressed in statements such as "I think that . . . ," "chances are . . . ," "it is unlikely that . . . ," and so forth. Occasionally, beliefs concerning uncertain events are expressed in numerical form as odds or subjective probabilities. What determines such beliefs? How do people assess the probability of an uncertain event or the value of an uncertain quantity? This article shows that people rely on a limited number of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. In general, these heuristics are quite useful, but sometimes they lead to severe and systematic errors.

The subjective assessment of probability resembles the subjective assessment of physical quantities such as distance or size. These judgments are all based on data of limited validity, which are processed according to heuristic rules. For example, the apparent distance of an object is determined in part by its clarity. The more sharply the object is seen, the closer it appears to be. This rule has some validity, because in any given scene the more distant objects are seen less sharply than nearer objects. However, the reliance on this rule leads to systematic errors in the estimation of distance. Specifically, distances are often overestimated when visibility is poor because the contours of objects are blurred. On the other hand, distances are often underestimated when visibility is good because the objects are seen sharply. Thus, the reliance on clarity as an indication of distance leads to common biases. Such biases are also found in the intuitive judgment of probability. This article describes three heuristics

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that are employed to assess probabilities and to predict values. Biases to which these heuristics lead are enumerated, and the applied and theoretical implications of these observations are discussed.

Representativeness

Many of the probabilistic questions with which people are concerned belong to one of the following types: What is the probability that object A belongs to class B? What is the probability that event A originates from process B? What is the probability that process B will generate event A? In answering such questions, people typically rely on the representativeness heuristic, in which probabilities are evaluated by the degree to which A is representative of B, that is, by the degree to which A resembles B. For example, when A is highly representative of B, the probability that A originates from B is judged to be high. On the other hand, if A is not similar to B, the probability that A originates from B is judged to be low.

For an illustration of judgment by representativeness, consider an individual who has been described by a former neighbor as follows: "Steve is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail." How do people assess the probability that Steve is engaged in a particular occupation from a list of possibilities (for example, farmer, salesman, airline pilot, librarian, or physician)? How do people order these occupations from most to least likely? In the representativeness heuristic, the probability that Steve is a librarian, for example, is assessed by the degree to which he is representative of, or similar to, the stereotype of a librarian. Indeed, research with problems of this type has shown that people order the occupations by probability and by similarity in exactly the same way (Kahneman & Tversky, 1973, 4). This approach to the judgment of probability leads to serious errors, because similarity, or representativeness, is not influenced by several factors that should affect judgments of probability.

Insensitivity to prior probability of outcomes

One of the factors that have no effect on representativeness but should have a major effect on probability is the prior probability, or base-rate frequency, of the outcomes. In the case of Steve, for example, the fact that there are many more farmers than librarians in the population should enter into any reasonable estimate of the probability that Steve is a librarian rather than a farmer. Considerations of base-rate frequency, however, do not affect the similarity of Steve to the stereotypes of librarians and farmers. If people evaluate probability by representativeness, therefore, prior probabilities will be neglected. This hypothesis was tested in an experiment where prior probabilities were manipulated

(Kahneman & Tversky, 1973, 4). Subjects were shown brief personality descriptions of several individuals, allegedly sampled at random from a group of 100 professionals – engineers and lawyers. The subjects were asked to assess, for each description, the probability that it belonged to an engineer rather than to a lawyer. In one experimental condition, subjects were told that the group from which the descriptions had been drawn consisted of 70 engineers and 30 lawyers. In another condition, subjects were told that the group consisted of 30 engineers and 70 lawyers. The odds that any particular description belongs to an engineer rather than to a lawyer should be higher in the first condition, where there is a majority of engineers, than in the second condition, where there is a majority of lawyers. Specifically, it can be shown by applying Bayes' rule that the ratio of these odds should be $(.7/.3)^2$, or 5.44, for each description. In a sharp violation of Bayes' rule, the subjects in the two conditions produced essentially the same probability judgments. Apparently, subjects evaluated the likelihood that a particular description belonged to an engineer rather than to a lawyer by the degree to which this description was representative of the two stereotypes, with little or no regard for the prior probabilities of the categories.

The subjects used prior probabilities correctly when they had no other information. In the absence of a personality sketch, they judged the probability that an unknown individual is an engineer to be .7 and .3, respectively, in the two base-rate conditions. However, prior probabilities were effectively ignored when a description was introduced, even when this description was totally uninformative. The responses to the following description illustrate this phenomenon:

> Dick is a 30 year old man. He is married with no children. A man of high ability and high motivation, he promises to be quite successful in his field. He is well liked by his colleagues.

This description was intended to convey no information relevant to the question of whether Dick is an engineer or a lawyer. Consequently, the probability that Dick is an engineer should equal the proportion of engineers in the group, as if no description had been given. The subjects, however, judged the probability of Dick being an engineer to be .5 regardless of whether the stated proportion of engineers in the group was .7 or .3. Evidently, people respond differently when given no evidence and when given worthless evidence. When no specific evidence is given, prior probabilities are properly utilized; when worthless evidence is given, prior probabilities are ignored (Kahneman & Tversky, 1973, 4).

Insensitivity to sample size

To evaluate the probability of obtaining a particular result in a sample drawn from a specified population, people typically apply the representa-

tiveness heuristic. That is, they assess the likelihood of a sample result, for example, that the average height in a random sample of ten men will be 6 feet (180 centimeters), by the similarity of this result to the corresponding parameter (that is, to the average height in the population of men). The similarity of a sample statistic to a population parameter does not depend on the size of the sample. Consequently, if probabilities are assessed by representativeness, then the judged probability of a sample statistic will be essentially independent of sample size. Indeed, when subjects assessed the distributions of average height for samples of various sizes, they produced identical distributions. For example, the probability of obtaining an average height greater than 6 feet was assigned the same value for samples of 1000, 100, and 10 men (Kahneman & Tversky, 1972b, 3). Moreover, subjects failed to appreciate the role of sample size even when it was emphasized in the formulation of the problem. Consider the following question:

A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50 percent of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50 percent, sometimes lower.

For a period of 1 year, each hospital recorded the days on which more than 60 percent of the babies born were boys. Which hospital do you think recorded more such days?

The larger hospital (21)
The smaller hospital (21)
About the same (that is, within 5 percent of each other) (53)

The values in parentheses are the number of undergraduate students who chose each answer.

Most subjects judged the probability of obtaining more than 60 percent boys to be the same in the small and in the large hospital, presumably because these events are described by the same statistic and are therefore equally representative of the general population. In contrast, sampling theory entails that the expected number of days on which more than 60 percent of the babies are boys is much greater in the small hospital than in the large one, because a large sample is less likely to stray from 50 percent. This fundamental notion of statistics is evidently not part of people's repertoire of intuitions.

A similar insensitivity to sample size has been reported in judgments of posterior probability, that is, of the probability that a sample has been drawn from one population rather than from another. Consider the following example:

Imagine an urn filled with balls, of which $\frac{2}{3}$ are of one color and $\frac{1}{3}$ of another. One individual has drawn 5 balls from the urn, and found that 4 were red and 1 was white. Another individual has drawn 20 balls and found that 12 were red and 8 were white. Which of the two individuals should feel more confident that the urn

contains ⅔ red balls and ⅓ white balls, rather than the opposite? What odds should each individual give?

In this problem, the correct posterior odds are 8 to 1 for the 4:1 sample and 16 to 1 for the 12:8 sample, assuming equal prior probabilities. However, most people feel that the first sample provides much stronger evidence for the hypothesis that the urn is predominantly red, because the proportion of red balls is larger in the first than in the second sample. Here again, intuitive judgments are dominated by the sample proportion and are essentially unaffected by the size of the sample, which plays a crucial role in the determination of the actual posterior odds (Kahneman & Tversky, 1972b). In addition, intuitive estimates of posterior odds are far less extreme than the correct values. The underestimation of the impact of evidence has been observed repeatedly in problems of this type (W. Edwards, 1968, **25;** Slovic & Lichtenstein, 1971). It has been labeled "conservatism."

Misconceptions of chance

People expect that a sequence of events generated by a random process will represent the essential characteristics of that process even when the sequence is short. In considering tosses of a coin for heads or tails, for example, people regard the sequence H-T-H-T-T-H to be more likely than the sequence H-H-H-T-T-T, which does not appear random, and also more likely than the sequence H-H-H-H-T-H, which does not represent the fairness of the coin (Kahneman & Tversky, 1972b, **3**). Thus, people expect that the essential characteristics of the process will be represented, not only globally in the entire sequence, but also locally in each of its parts. A locally representative sequence, however, deviates systematically from chance expectation: it contains too many alternations and too few runs. Another consequence of the belief in local representativeness is the well-known gambler's fallacy. After observing a long run of red on the roulette wheel, for example, most people erroneously believe that black is now due, presumably because the occurence of black will result in a more representative sequence than the occurrence of an additional red. Chance is commonly viewed as a self-correcting process in which a deviation in one direction induces a deviation in the opposite direction to restore the equilibrium. In fact, deviations are not "corrected" as a chance process unfolds, they are merely diluted.

Misconceptions of chance are not limited to naive subjects. A study of the statistical intuitions of experienced research psychologists (Tversky & Kahneman, 1971, **2**) revealed a lingering belief in what may be called the "law of small numbers," according to which even small samples are highly representative of the populations from which they are drawn. The responses of these investigators reflected the expectation that a valid

hypothesis about a population will be represented by a statistically significant result in a sample – with little regard for its size. As a consequence, the researchers put too much faith in the results of small samples and grossly overestimated the replicability of such results. In the actual conduct of research, this bias leads to the selection of samples of inadequate size and to overinterpretation of findings.

Insensitivity to predictability

People are sometimes called upon to make such numerical predictions as the future value of a stock, the demand for a commodity, or the outcome of a football game. Such predictions are often made by representativeness. For example, suppose one is given a description of a company and is asked to predict its future profit. If the description of the company is very favorable, a very high profit will appear most representative of that description; if the description is mediocre, a mediocre performance will appear most representative. The degree to which the description is favorable is unaffected by the reliability of that description or by the degree to which it permits accurate prediction. Hence, if people predict solely in terms of the favorableness of the description, their predictions will be insensitive to the reliability of the evidence and to the expected accuracy of the prediction.

This mode of judgment violates the normative statistical theory in which the extremeness and the range of predictions are controlled by considerations of predictability. When predictability is nil, the same prediction should be made in all cases. For example, if the descriptions of companies provide no information relevant to profit, then the same value (such as average profit) should be predicted for all companies. If predictability is perfect, of course, the values predicted will match the actual values and the range of predictions will equal the range of outcomes. In general, the higher the predictability, the wider the range of predicted values.

Several studies of numerical prediction have demonstrated that intuitive predictions violate this rule, and that subjects show little or no regard for considerations of predictability (Kahneman & Tversky, 1973, 4). In one of these studies, subjects were presented with several paragraphs, each describing the performance of a student teacher during a particular practice lesson. Some subjects were asked to *evaluate* the quality of the lesson described in the paragraph in percentile scores, relative to a specified population. Other subjects were asked to *predict*, also in percentile scores, the standing of each student teacher 5 years after the practice lesson. The judgments made under the two conditions were identical. That is, the prediction of a remote criterion (success of a teacher after 5 years) was identical to the evaluation of the information on which the prediction was based (the quality of the practice lesson). The students who made

these predictions were undoubtedly aware of the limited predictability of teaching competence on the basis of a single trial lesson 5 years earlier; nevertheless, their predictions were as extreme as their evaluations.

The illusion of validity

As we have seen, people often predict by selecting the outcome (for example, an occupation) that is most representative of the input (for example, the description of a person). The confidence they have in their prediction depends primarily on the degree of representativeness (that is, on the quality of the match between the selected outcome and the input) with little or no regard for the factors that limit predictive accuracy. Thus, people express great confidence in the prediction that a person is a librarian when given a description of his personality which matches the stereotype of librarians, even if the description is scanty, unreliable, or outdated. The unwarranted confidence which is produced by a good fit between the predicted outcome and the input information may be called the illusion of validity. This illusion persists even when the judge is aware of the factors that limit the accuracy of his predictions. It is a common observation that psychologists who conduct selection interviews often experience considerable confidence in their predictions, even when they know of the vast literature that shows selection interviews to be highly fallible. The continued reliance on the clinical interview for selection, despite repeated demonstrations of its inadequacy, amply attests to the strength of this effect.

The internal consistency of a pattern of inputs is a major determinant of one's confidence in predictions based on these inputs. For example, people express more confidence in predicting the final grade-point average of a student whose first-year record consists entirely of B's than in predicting the grade-point average of a student whose first-year record includes many A's and C's. Highly consistent patterns are most often observed when the input variables are highly redundant or correlated. Hence, people tend to have great confidence in predictions based on redundant input variables. However, an elementary result in the statistics of correlation asserts that, given input variables of stated validity, a prediction based on several such inputs can achieve higher accuracy when they are independent of each other than when they are redundant or correlated. Thus, redundancy among inputs decreases accuracy even as it increases confidence, and people are often confident in predictions that are quite likely to be off the mark (Kahneman & Tversky, 1973, 4).

Misconceptions of regression

Suppose a large group of children has been examined on two equivalent versions of an aptitude test. If one selects ten children from among those

who did best on one of the two versions, he will usually find their performance on the second version to be somewhat disappointing. Conversely, if one selects ten children from among those who did worst on one version, they will be found, on the average, to do somewhat better on the other version. More generally, consider two variables X and Y which have the same distribution. If one selects individuals whose average X score deviates from the mean of X by k units, then the average of their Y scores will usually deviate from the mean of Y by less than k units. These observations illustrate a general phenomenon known as regression toward the mean, which was first documented by Galton more than 100 years ago.

In the normal course of life, one encounters many instances of regression toward the mean, in the comparison of the height of fathers and sons, of the intelligence of husbands and wives, or of the performance of individuals on consecutive examinations. Nevertheless, people do not develop correct intuitions about this phenomenon. First, they do not expect regression in many contexts where it is bound to occur. Second, when they recognize the occurrence of regression, they often invent spurious causal explanations for it (Kahneman & Tversky, 1973, **4**). We suggest that the phenomenon of regression remains elusive because it is incompatible with the belief that the predicted outcome should be maximally representative of the input, and, hence, that the value of the outcome variable should be as extreme as the value of the input variable.

The failure to recognize the import of regression can have pernicious consequences, as illustrated by the following observation (Kahneman & Tversky, 1973, **4**). In a discussion of flight training, experienced instructors noted that praise for an exceptionally smooth landing is typically followed by a poorer landing on the next try, while harsh criticism after a rough landing is usually followed by an improvement on the next try. The instructors concluded that verbal rewards are detrimental to learning, while verbal punishments are beneficial, contrary to accepted psychological doctrine. This conclusion is unwarranted because of the presence of regression toward the mean. As in other cases of repeated examination, an improvement will usually follow a poor performance and a deterioration will usually follow an outstanding performance, even if the instructor does not respond to the trainee's achievement on the first attempt. Because the instructors had praised their trainees after good landings and admonished them after poor ones, they reached the erroneous and potentially harmful conclusion that punishment is more effective than reward.

Thus, the failure to understand the effect of regression leads one to overestimate the effectiveness of punishment and to underestimate the effectiveness of reward. In social interaction, as well as in training, rewards are typically administered when performance is good, and punishments are typically administered when performance is poor. By

regression alone, therefore, behavior is most likely to improve after punishment and most likely to deteriorate after reward. Consequently, the human condition is such that, by chance alone, one is most often rewarded for punishing others and most often punished for rewarding them. People are generally not aware of this contingency. In fact, the elusive role of regression in determining the apparent consequences of reward and punishment seems to have escaped the notice of students of this area.

Availability

There are situations in which people assess the frequency of a class or the probability of an event by the ease with which instances or occurrences can be brought to mind. For example, one may assess the risk of heart attack among middle-aged people by recalling such occurrences among one's acquaintances. Similarly, one may evaluate the probability that a given business venture will fail by imagining various difficulties it could encounter. This judgmental heuristic is called availability. Availability is a useful clue for assessing frequency or probability, because instances of large classes are usually reached better and faster than instances of less frequent classes. However, availability is affected by factors other than frequency and probability. Consequently, the reliance on availability leads to predictable biases, some of which are illustrated below.

Biases due to the retrievability of instances

When the size of a class is judged by the availability of its instances, a class whose instances are easily retrieved will appear more numerous than a class of equal frequency whose instances are less retrievable. In an elementary demonstration of this effect, subjects heard a list of well-known personalities of both sexes and were subsequently asked to judge whether the list contained more names of men than of women. Different lists were presented to different groups of subjects. In some of the lists the men were relatively more famous than the women, and in others the women were relatively more famous than the men. In each of the lists, the subjects erroneously judged that the class (sex) that had the more famous personalities was the more numerous (Tversky & Kahneman, 1973, 11).

In addition to familiarity, there are other factors, such as salience, which affect the retrievability of instances. For example, the impact of seeing a house burning on the subjective probability of such accidents is probably greater than the impact of reading about a fire in the local paper. Furthermore, recent occurrences are likely to be relatively more available than earlier occurrences. It is a common experience that the subjective probability of traffic accidents rises temporarily when one sees a car overturned by the side of the road.

Biases due to the effectiveness of a search set

Suppose one samples a word (of three letters or more) at random from an English text. Is it more likely that the word starts with *r* or that *r* is the third letter? People approach this problem by recalling words that begin with *r* (road) and words that have *r* in the third position (car) and assess the relative frequency by the ease with which words of the two types come to mind. Because it is much easier to search for words by their first letter than by their third letter, most people judge words that begin with a given consonant to be more numerous than words in which the same consonant appears in the third position. They do so even for consonants, such as *r* or *k*, that are more frequent in the third position than in the first (Tversky & Kahneman, 1973, **11**).

Different tasks elicit different search sets. For example, suppose you are asked to rate the frequency with which abstract words (*thought, love*) and concrete words (*door, water*) appear in written English. A natural way to answer this question is to search for contexts in which the word could appear. It seems easier to think of contexts in which an abstract concept is mentioned (*love* in love stories) than to think of contexts in which a concrete word (such as *door*) is mentioned. If the frequency of words is judged by the availability of the contexts in which they appear, abstract words will be judged as relatively more numerous than concrete words. This bias has been observed in a recent study (Galbraith & Underwood, 1973) which showed that the judged frequency of occurrence of abstract words was much higher than that of concrete words, equated in objective frequency. Abstract words were also judged to appear in a much greater variety of contexts than concrete words.

Biases of imaginability

Sometimes one has to assess the frequency of a class whose instances are not stored in memory but can be generated according to a given rule. In such situations, one typically generates several instances and evaluates frequency or probability by the ease with which the relevant instances can be constructed. However, the ease of constructing instances does not always reflect their actual frequency, and this mode of evaluation is prone to biases. To illustrate, consider a group of 10 people who form committees of k members, $2 \leq k \leq 8$. How many different committees of k members can be formed? The correct answer to this problem is given by the binomial coefficient $\binom{10}{k}$ which reaches a maximum of 252 for $k = 5$. Clearly, the number of committees of k members equals the number of committees of $(10 - k)$ members, because any committee of k members defines a unique group of $(10 - k)$ nonmembers.

One way to answer this question without computation is to mentally construct committees of k members and to evaluate their number by the

ease with which they come to mind. Committees of few members, say 2, are more available than committees of many members, say 8. The simplest scheme for the construction of committees is a partition of the group into disjoint sets. One readily sees that it is easy to construct five disjoint committees of 2 members, while it is impossible to generate even two disjoint committees of 8 members. Consequently, if frequency is assessed by imaginability, or by availability for construction, the small committees will appear more numerous than larger committees, in contrast to the correct bell-shaped function. Indeed, when naive subjects were asked to estimate the number of distinct committees of various sizes, their estimates were a decreasing monotonic function of committee size (Tversky & Kahneman, 1973, **11**). For example, the median estimate of the number of committees of 2 members was 70, while the estimate for committees of 8 members was 20 (the correct answer is 45 in both cases).

Imaginability plays an important role in the evaluation of probabilities in real-life situations. The risk involved in an adventurous expedition, for example, is evaluated by imagining contingencies with which the expedition is not equipped to cope. If many such difficulties are vividly portrayed, the expedition can be made to appear exceedingly dangerous, although the ease with which disasters are imagined need not reflect their actual likelihood. Conversely, the risk involved in an undertaking may be grossly underestimated if some possible dangers are either difficult to conceive of, or simply do not come to mind.

Illusory correlation

Chapman and Chapman (1969) have described an interesting bias in the judgment of the frequency with which two events co-occur. They presented naive judges with information concerning several hypothetical mental patients. The data for each patient consisted of a clinical diagnosis and a drawing of a person made by the patient. Later the judges estimated the frequency with which each diagnosis (such as paranoia or suspiciousness) had been accompanied by various features of the drawing (such as peculiar eyes). The subjects markedly overestimated the frequency of co-occurrence of natural associates, such as suspiciousness and peculiar eyes. This effect was labeled illusory correlation. In their erroneous judgments of the data to which they had been exposed, naive subjects "rediscovered" much of the common, but unfounded, clinical lore concerning the interpretation of the draw-a-person test. The illusory correlation effect was extremely resistant to contradictory data. It persisted even when the correlation between symptom and diagnosis was actually negative, and it prevented the judges from detecting relationships that were in fact present.

Availability provides a natural account for the illusory-correlation effect. The judgment of how frequently two events co-occur could be

based on the strength of the associative bond between them. When the association is strong, one is likely to conclude that the events have been frequently paired. Consequently, strong associates will be judged to have occurred together frequently. According to this view, the illusory correlation between suspiciousness and peculiar drawing of the eyes, for example, is due to the fact that suspiciousness is more readily associated with the eyes than with any other part of the body.

Lifelong experience has taught us that, in general, instances of large classes are recalled better and faster than instances of less frequent classes; that likely occurrences are easier to imagine than unlikely ones; and that the associative connections between events are strengthened when the events frequently co-occur. As a result, man has at his disposal a procedure (the availability heuristic) for estimating the numerosity of a class, the likelihood of an event, or the frequency of co-occurrences, by the ease with which the relevant mental operations of retrieval, construction, or association can be performed. However, as the preceding examples have demonstrated, this valuable estimation procedure results in systematic errors.

Adjustment and anchoring

In many situations, people make estimates by starting from an initial value that is adjusted to yield the final answer. The initial value, or starting point, may be suggested by the formulation of the problem, or it may be the result of a partial computation. In either case, adjustments are typically insufficient (Slovic & Lichtenstein, 1971). That is, different starting points yield different estimates, which are biased toward the initial values. We call this phenomenon anchoring.

Insufficient adjustment

In a demonstration of the anchoring effect, subjects were asked to estimate various quantities, stated in percentages (for example, the percentage of African countries in the United Nations). For each quantity, a number between 0 and 100 was determined by spinning a wheel of fortune in the subjects' presence. The subjects were instructed to indicate first whether that number was higher or lower than the value of the quantity, and then to estimate the value of the quantity by moving upward or downward from the given number. Different groups were given different numbers for each quantity, and these arbitrary numbers had a marked effect on estimates. For example, the median estimates of the percentage of African countries in the United Nations were 25 and 45 for groups that received 10 and 65, respectively, as starting points. Payoffs for accuracy did not reduce the anchoring effect.

Anchoring occurs not only when the starting point is given to the

subject, but also when the subject bases his estimate on the result of some incomplete computation. A study of intuitive numerical estimation illustrates this effect. Two groups of high school students estimated, within 5 seconds, a numerical expression that was written on the blackboard. One group estimated the product

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

while another group estimated the product

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$$

To rapidly answer such questions, people may perform a few steps of computation and estimate the product by extrapolation or adjustment. Because adjustments are typically insufficient, this procedure should lead to underestimation. Furthermore, because the result of the first few steps of multiplication (performed from left to right) is higher in the descending sequence than in the ascending sequence, the former expression should be judged larger than the latter. Both predictions were confirmed. The median estimate for the ascending sequence was 512, while the median estimate for the descending sequence was 2,250. The correct answer is 40,320.

Biases in the evaluation of conjunctive and disjunctive events

In a recent study by Bar-Hillel (1973) subjects were given the opportunity to bet on one of two events. Three types of events were used: (i) simple events, such as drawing a red marble from a bag containing 50 percent red marbles and 50 percent white marbles; (ii) conjunctive events, such as drawing a red marble seven times in succession, with replacement, from a bag containing 90 percent red marbles and 10 percent white marbles; and (iii) disjunctive events, such as drawing a red marble at least once in seven successive tries, with replacement, from a bag containing 10 percent red marbles and 90 percent white marbles. In this problem, a significant majority of subjects preferred to bet on the conjunctive event (the probability of which is .48) rather than on the simple event (the probability of which is .50). Subjects also preferred to bet on the simple event rather than on the disjunctive event, which has a probability of .52. Thus, most subjects bet on the less likely event in both comparisons. This pattern of choices illustrates a general finding. Studies of choice among gambles and of judgments of probability indicate that people tend to overestimate the probability of conjunctive events (Cohen, Chesnick, & Haran, 1972, **24**) and to underestimate the probability of disjunctive events. These biases are readily explained as effects of anchoring. The stated probability of the elementary event (success at any one stage) provides a natural starting point for the estimation of the probabilities of both conjunctive and disjunctive events. Since adjustment from the starting point is typically

insufficient, the final estimates remain too close to the probabilities of the elementary events in both cases. Note that the overall probability of a conjunctive event is lower than the probability of each elementary event, whereas the overall probability of a disjunctive event is higher than the probability of each elementary event. As a consequence of anchoring, the overall probability will be overestimated in conjunctive problems and underestimated in disjunctive problems.

Biases in the evaluation of compound events are particularly significant in the context of planning. The successful completion of an undertaking, such as the development of a new product, typically has a conjunctive character: for the undertaking to succeed, each of a series of events must occur. Even when each of these events is very likely, the overall probability of success can be quite low if the number of events is large. The general tendency to overestimate the probability of conjunctive events leads to unwarranted optimism in the evaluation of the likelihood that a plan will succeed or that a project will be completed on time. Conversely, disjunctive structures are typically encountered in the evaluation of risks. A complex system, such as a nuclear reactor or a human body, will malfunction if any of its essential components fails. Even when the likelihood of failure in each component is slight, the probability of an overall failure can be high if many components are involved. Because of anchoring, people will tend to underestimate the probabilities of failure in complex systems. Thus, the direction of the anchoring bias can sometimes be inferred from the structure of the event. The chain-like structure of conjunctions leads to overestimation, the funnel-like structure of disjunctions leads to underestimation.

Anchoring in the assessment of subjective probability distributions

In decision analysis, experts are often required to express their beliefs about a quantity, such as the value of the Dow-Jones average on a particular day, in the form of a probability distribution. Such a distribution is usually constructed by asking the person to select values of the quantity that correspond to specified percentiles of his subjective probability distribution. For example, the judge may be asked to select a number, X_{90}, such that his subjective probability that this number will be higher than the value of the Dow-Jones average is .90. That is, he should select the value X_{90} so that he is just willing to accept 9 to 1 odds that the Dow-Jones average will not exceed it. A subjective probability distribution for the value of the Dow-Jones average can be constructed from several such judgments corresponding to different percentiles.

By collecting subjective probability distributions for many different quantities, it is possible to test the judge for proper calibration. A judge is properly (or externally) calibrated in a set of problems if exactly II percent of the true values of the assessed quantities falls below his stated values of

X_{Π}. For example, the true values should fall below X_{01} for 1 percent of the quantities and above X_{99} for 1 percent of the quantities. Thus, the true values should fall in the confidence interval between X_{01} and X_{99} on 98 percent of the problems.

Several investigators (Alpert & Raiffa, 1969, **21;** Staël von Holstein, 1971b; Winkler, 1967) have obtained probability disruptions for many quantities from a large number of judges. These distributions indicated large and systematic departures from proper calibration. In most studies, the actual values of the assessed quantities are either smaller than X_{01} or greater than X_{99} for about 30 percent of the problems. That is, the subjects state overly narrow confidence intervals which reflect more certainty than is justified by their knowledge about the assessed quantities. This bias is common to naive and to sophisticated subjects, and it is not eliminated by introducing proper scoring rules, which provide incentives for external calibration. This effect is attributable, in part at least, to anchoring.

To select X_{90} for the value of the Dow-Jones average, for example, it is natural to begin by thinking about one's best estimate of the Dow-Jones and to adjust this value upward. If this adjustment – like most others – is insufficient, then X_{90} will not be sufficiently extreme. A similar anchoring effect will occur in the selection of X_{10}, which is presumably obtained by adjusting one's best estimate downward. Consequently, the confidence interval between X_{10} and X_{90} will be too narrow, and the assessed probability distribution will be too tight. In support of this interpretation it can be shown that subjective probabilities are systematically altered by a procedure in which one's best estimate does not serve as an anchor.

Subjective probability distributions for a given quantity (the Dow-Jones average) can be obtained in two different ways: (i) by asking the subject to select values of the Dow-Jones that correspond to specified percentiles of his probability distribution and (ii) by asking the subject to assess the probabilities that the true value of the Dow-Jones will exceed some specified values. The two procedures are formally equivalent and should yield identical distributions. However, they suggest different modes of adjustment from different anchors. In procedure (i), the natural starting point is one's best estimate of the quality. In procedure (ii), on the other hand, the subject may be anchored on the value stated in the question. Alternatively, he may be anchored on even odds, or 50–50 chances, which is a natural starting point in the estimation of likelihood. In either case, procedure (ii) should yield less extreme odds than procedure (i).

To contrast the two procedures, a set of 24 quantities (such as the air distance from New Delhi to Peking) was presented to a group of subjects who assessed either X_{10} or X_{90} for each problem. Another group of subjects received the median judgment of the first group for each of the 24 quantities. They were asked to assess the odds that each of the given values exceeded the true value of the relevant quantity. In the absence of any bias, the second group should retrieve the odds specified to the first group,

that is, 9:1. However, if even odds or the stated value serve as anchors, the odds of the second group should be less extreme, that is, closer to 1:1. Indeed, the median odds stated by this group, across all problems, were 3:1. When the judgments of the two groups were tested for external calibration, it was found that subjects in the first group were too extreme, in accord with earlier studies. The events that they defined as having a probability of .10 actually obtained in 24 percent of the cases. In contrast, subjects in the second group were too conservative. Events to which they assigned an average probability of .34 actually obtained in 26 percent of the cases. These results illustrate the manner in which the degree of calibration depends on the procedure of elicitation.

Discussion

This article has been concerned with cognitive biases that stem from the reliance on judgmental heuristics. These biases are not attributable to motivational effects such as wishful thinking or the distortion of judgments by payoffs and penalties. Indeed, several of the severe errors of judgment reported earlier occurred despite the fact that subjects were encouraged to be accurate and were rewarded for the correct answers (Kahneman & Tversky, 1972b, **3**; Tversky & Kahneman, 1973, **11**).

The reliance on heuristics and the prevalence of biases are not restricted to laymen. Experienced researchers are also prone to the same biases – when they think intuitively. For example, the tendency to predict the outcome that best represents the data, with insufficient regard for prior probability, has been observed in the intuitive judgments of individuals who have had extensive training in statistics (Kahneman & Tversky, 1973, **4**; Tversky & Kahneman, 1971, **2**). Although the statistically sophisticated avoid elementary errors, such as the gambler's fallacy, their intuitive judgments are liable to similar fallacies in more intricate and less transparent problems.

It is not surprising that useful heuristics such as representativeness and availability are retained, even though they occasionally lead to errors in prediction or estimation. What is perhaps surprising is the failure of people to infer from lifelong experience such fundamental statistical rules as regression toward the mean, or the effect of sample size on sampling variability. Although everyone is exposed, in the normal course of life, to numerous examples from which these rules could have been induced, very few people discover the principles of sampling and regression on their own. Statistical principles are not learned from everyday experience because the relevant instances are not coded appropriately. For example, people do not discover that successive lines in a text differ more in average word length than do successive pages, because they simply do not attend to the average word length of individual lines or pages. Thus, people do

not learn the relation between sample size and sampling variability, although the data for such learning are abundant.

The lack of an appropriate code also explains why people usually do not detect the biases in their judgments of probability. A person could conceivably learn whether his judgments are externally calibrated by keeping a tally of the proportion of events that actually occur among those to which he assigns the same probability. However, it is not natural to group events by their judged probability. In the absence of such grouping it is impossible for an individual to discover, for example, that only 50 percent of the predictions to which he has assigned a probability of .9 or higher actually come true.

The empirical analysis of cognitive biases has implications for the theoretical and applied role of judged probabilities. Modern decision theory (de Finetti, 1968; Savage, 1954) regards subjective probability as the quantified opinion of an idealized person. Specifically, the subjective probability of a given event is defined by the set of bets about this event that such a person is willing to accept. An internally consistent, or coherent, subjective probability measure can be derived for an individual if his choices among bets satisfy certain principles, that is, the axioms of the theory. The derived probability is subjective in the sense that different individuals are allowed to have different probabilities for the same event. The major contribution of this approach is that it provides a rigorous subjective interpretation of probability that is applicable to unique events and is embedded in a general theory of rational decision.

It should perhaps be noted that, while subjective probabilities can sometimes be inferred from preferences among bets, they are normally not formed in this fashion. A person bets on team A rather than on team B because he believes that team A is more likely to win; he does not infer this belief from his betting preferences. Thus, in reality, subjective probabilities determine preferences among bets and are not derived from them, as in the axiomatic theory of rational decision (Savage, 1954).

The inherently subjective nature of probability has led many students to the belief that coherence, or internal consistency, is the only valid criterion by which judged probabilities should be evaluated. From the standpoint of the formal theory of subjective probability, any set of internally consistent probability judgments is as good as any other. This criterion is not entirely satisfactory, because an internally consistent set of subjective probabilities can be incompatible with other beliefs held by the individual. Consider a person whose subjective probabilities for all possible outcomes of a coin-tossing game reflect the gambler's fallacy. That is, his estimate of the probability of tails on a particular toss increases with the number of consecutive heads that preceded that toss. The judgments of such a person could be internally consistent and therefore acceptable as adequate subjective probabilities according to the criterion of the formal

theory. These probabilities, however, are incompatible with the generally held belief that a coin has no memory and is therefore incapable of generating sequential dependencies. For judged probabilities to be considered adequate, or rational, internal consistency is not enough. The judgments must be compatible with the entire web of beliefs held by the individual. Unfortunately, there can be no simple formal procedure for assessing the compatibility of a set of probability judgments with the judge's total system of beliefs. The rational judge will nevertheless strive for compatibility, even though internal consistency is more easily achieved and assessed. In particular, he will attempt to make his probability judgments compatible with his knowledge about the subject matter, the laws of probability, and his own judgmental heuristics and biases.

Summary

This article described three heuristics that are employed in making judgments under uncertainty: (i) representativeness, which is usually employed when people are asked to judge the probability that an object or event A belongs to class or process B; (ii) availability of instances or scenarios, which is often employed when people are asked to assess the frequency of a class or the plausibility of a particular development; and (iii) adjustment from an anchor, which is usually employed in numerical prediction when a relevant value is available. These heuristics are highly economical and usually effective, but they lead to systematic and predictable errors. A better understanding of these heuristics and of the biases to which they lead could improve judgments and decisions in situations of uncertainty.