2.1

2 2 2

3 5 7 9

1) a) 2, 5, 10, 17, 26

$a_n = n^2+1$ $a_1 = (1)^2+1 = 2$ $a_2 = (2)^2+1 = 5$ $a_3 = (3)^2+1 = 10$

1 1 1 1

2 3 4 5 6

b) 0, 2, 5, 9, 14, 20

$a_n = \frac{n(n+1)}{2} - 1$ $a_1 = \frac{1(1+1)}{2} - 1 = 0$ $a_2 = \frac{2(2+1)}{2} - 1 = 2$

1 1 1

4 5 6 7

c) 8, 12, 17, 23, 30

$a_n = \frac{(n+2)(n+3)}{2} + 2$ $a_1 = \frac{(1+2)(1+3)}{2} + 2 = 8$ $a_2 = \frac{(2+2)(2+3)}{2} + 2 = 12$

64 426

14 78 504

4 18 96 600

d) 1, 5, 23, 119, 719

$a_n = (n+1)! - 1$ $a_1 = (1+1)! - 1 = 1$ $a_2 = (2+1)! - 1 = 5$

$a_3 = (3+1)! - 1 = 24 - 1 = 23$

3) a) $a_n = \frac{1}{2}(n^2+n)$ $a_0 = \frac{1}{2}(0^2+0) = 0$ $a_1 = \frac{1}{2}(1^2+1) = 1$ $a_2 = 3$

$a_3 = 6$ $a_4 = 10$ $a_n = a_{n-1} + n$ $a_2 = a_1 + 2$ $a_1 = 0 + 1 = 1$

$a_0 = 0$ $a_1 = 0 + 1 = 1$ $= 3$

b) $a_n = 2a_{n-1} - a_{n-2}$ $a_0 = 0$ $a_1 = 1$

$a_2 = 2a_1 - a_0 = 2(1) - 0 = 2$ $a_3 = 2a_2 - a_1 = 2(2) - 1 = 3$

$a_4 = 2a_3 - a_2 = 2(3) - 2 = 4$ $a_5 = 2a_4 - a_3 = 2(4) - 3 = 5$

$a_n = n$ $a_1 = 0$ $a_1 = 1$ $a_2 = 2 \ldots$

c) $a_n = n a_{n-1}$ $a_0 = 1$ $a_1 = 1 a_0 = 1$

$a_2 = 2a_1 = 2$ $a_3 = 3a_2 = 6$ $a_4 = 4a_3 = 24$

$a_5 = 5a_4 = 120$ $a_n = n!$ $a_0 = 1$ $a_1 = 1$ $a_2 = 2$ $a_3 = 6$

$a_4 = 24$ $a_5 = 120$

5) 0, 1, 1, 2, 3, 5, 8, 13 $F_0 = 0$

0, 0+1, 0+1+1 . . .

0, 1, 2, 0+1+1+2, 0+1+1+2+3, 0+1+1+2+3+5,

0+1+1+2+3+5+8

0, 1, 2, 4, 7, 12, 20 ...

b) $F_0 + F_1 \ldots F_n = ?$

1 1 1 1 2 3

1 1 2 3 5 8

0, 1, 2, 4, 7, 12, 20 $F_{n-2} - 1$

2.2

1) $\underbrace{5, 9}_{4}, \underbrace{13}_{4}, \underbrace{17}_{4}, \underbrace{21}_{4}$ .. $a_1 = 5$

$a_n = a_{n-1} + 4$ $a_0 = 1$ $a_1 = a_0 + 4 = 5$

$a_2 = a_1 + 4 = 9$ $a_3 = a_2 + 4 = 13$

b) $\overset{4}{\frown}$ $\overset{4}{\frown}$ $\overset{4}{\frown}$ $\overset{4}{\frown}$ $5, 9, 13, 17, 21$ $a_n = 5+4(n-1)$

$a_1 = 5 = 4+1 \quad = 5$ a, $a_1 = 5+4(1-1) = 5$

$a_2 = 9 = 4+3+2 \quad = 4+5$ $a_2 = 5+4(2-1) = 9$

$a_3 = 13 = 4+3+3+2+1 = 4+5+3$ $a_3 = 5+4(3-1) = 13$

$a_4 = 17 = 4+4+3+3+3 = 4+5+8$

c) 2013 a term $4\overline{)2012}^{\,503}$

$a_{503} = 5+4(503-1) = 2013$

d) $5, 9, 13, 17, 21 \ldots 533$ $4\overline{)532}^{\,133}$

133

e) $5+9+13+17+21+\ldots 533$ 133 terms

$533+5$ $\rightarrow \frac{538 \cdot 133}{2} = 35777$

f) $b_n = ?$ $\overset{4\;\;4\;\;4}{\overset{5\;\;9\;\;13\;\;17}{1, 6, 15, 28, 45, \ldots}}$ $b_0 = 1$

$b_n = 1 + \left(\frac{4n+6}{2}\right)^n$

3) $\overset{7}{\frown}$ $4 + 11 + 18 + 25 + \ldots + 244$ $a_0 = 1$

$36 \cdot 7 = 252 - 3$

$a_1 = 4 = 4(1)$ $a_1 = 4 + 7(n-1)$

$a_2 = 11 = 4 + 7$

$a_3 = 18 = 4 + 7 + 7$

$a_4 : 25 = 4 + 7 + 7 + 7$

b) $\dfrac{253 \cdot 36}{2} = 4554$

5) $\underbrace{5 + 7}_{2} + \underbrace{9}_{2} + \underbrace{11}_{2} + \ldots + 521$ $5 + 2(n-1) = 521$

$a_n = 5 + 2(n-1)$ $n = 259$

$S = 5 + 7 + 9 + 11 + \ldots + 521$ $2(n-1)$

$+ \; S = 521 + 519 + 517 + 515 \ldots + 5$

$2S = 526 + 526 + 526 + 526 \ldots + 526$

$\dfrac{259 \cdot 526}{2} = 68117$

2.4

3) $a_n = a_{n-1} + 2^n$ $a_0 : 5$

$a_1 = a_0 + 2^1 = 7$

$a_2 = a_1 + 2^2 = 11$

$a_3 = a_2 + 2^3 = 19$

$a_4 = a_3 + 2^4 = 35$

$2 = 2(1)$
$3 = 2 + 1$
$8 = 2 + 6$
$16 = 2 + 14$

5, 7, 11, 19, 35

$a_1 - a_0 = 2 = 2^1$

$a_2 - a_1 = 4 = 2^2$ -1

$a_3 - a_2 = 8 = 2^3$ $a_n - a_{n-1} = 2^n$

$a_4 - a_3 = 16 = 2^4$ $a_n - a_0 = 2^{n+1} - 2$ $a_n = 2^{n+1} + 3$

2.5

2) Proof: $\forall n \in N \quad \sum_{k=0}^{n} 2^k = 2^{n+1} - 1$

1) $\overset{a_0}{2^0} + \overset{a_1}{2^1} + \overset{a_2}{2^2} + 2^3 \ldots 2^n = 2^{n+1} - 1, \forall_n \in N$

2) $1 = 2^{0+1} - 1 = 1$

3) $2k + 2^{k+1} = 2^{k+2} - 1 = 2^{k+1} - 1 + 2^{k+1} = 2 \cdot 2^{k+1} - 1$

$= 2^{k+2} - 1$

$\therefore \forall_n \in N \ \sum_{k=0}^{n} 2^k = 2^{n+1} - 1$

▨

3) Proof: $7^n - 1$ is multiple of 6, $\forall_n \in N$

1) $0 = 7^0 - 1 = 0$

2) $7^k - 1 = 6n \rightarrow 7^{k+1} - 1$

3) $7^{k+1} - 1 = 7^{k+1} (-7 \cdot \overset{-1}{+} 6)$

$= 7(7^{k+1} - 1) + 6$

$= 7(6n) + 6$

$= 6(7n+1) \Rightarrow n = 7n + 1 \Rightarrow 6n$

$\therefore$ $7^n - 1$ is multiple of 6 $\forall_n \in N$

▨

4) Proof $1+3+5+\ldots+(2n-1) = n^2 \quad \forall n \geq 1$

1) $1 = (2(1)-1) = 1^2 = 1$

2) $2k-1 = k^2 \Rightarrow (2k-1)-(2k+1) = (k+1)^2$

$= k^2 + (2k+1)) = (k+1)^2$

$\rightarrow \therefore (k+1))^2 = k^2$

$\therefore (2n-1) = n^2 \quad \forall n \geq 1$

▨