CH 4

3)

Proposition: if a is an odd integer, then a^2+3a+5 is odd.

Proof:

Suppose a is odd

1) $\rightarrow a = 2x+1, \quad x \in \mathbb{Z}$

2) $\rightarrow a^2+3a+5 = (2x+1)^2+3(2x+1)+5$

$\rightarrow = 4x^2+4x+1+6x+3+5$

$\rightarrow = 4x^2+10x+9 = 4x^2+10x+8+1$

3) $\rightarrow 2(2x^2+5x+4)+1$

4) $\rightarrow a^2+3a+5 = 2(x)+1, \quad x = 2x^2+5x+4$

5) $\therefore$ a^2+3a+5 is odd

∎

5) Proposition: Suppose $x, y \in \mathbb{Z}$. if x is even, then xy is even.

Proof:

Suppose $x, y \in \mathbb{Z}$

1) $\rightarrow x = 2(a), \quad a \in \mathbb{Z}$

2) $\rightarrow xy = (2a)(y) = 2(a \cdot y)$

3) $\rightarrow xy = 2z, \quad z = a \cdot y$

4) $\therefore$ xy is even

∎

9) Proposition: Suppose a is an integer. if $7 | 4a$ then $7|a$

Proof: Suppose $7|4a$

1) $4a = 7x$, $x \in \mathbb{Z}$
2) $\rightarrow 2(2a) = 7x$, x is even
3) $\rightarrow 4a = 7(2n) \Rightarrow 4a = 14n \Rightarrow, 2a = 7n$
4) $\rightarrow$ n is even, $n = 2e$
5) $\rightarrow 2a = 14e \Rightarrow a = 7e$
6) $\therefore 7|a$

$\blacksquare$

11) Proposition: Suppose $a,b,c,d \in \mathbb{Z}$. if $a|b$ and $c|d$ then $ac|bd$

Proof: Suppose $a|b$ and $c|b$

1) $b = ax$, $x \in \mathbb{Z}$
2) $d = cn$, $n \in \mathbb{Z}$
3) $bd = (ax)(cn)$
4) $\rightarrow bd = (ac)(xn)$, $xn = e$
5) $\rightarrow bd = (ac)(e)$
6) $\therefore ac|bd$

$\blacksquare$

15) Proposition: if $n \in \mathbb{Z}$, then n^2+3n+4 is even

Proof: Suppose $n \in \mathbb{Z}$

Case 1: 1) Suppose n is even

2) $\Rightarrow n = 2a \quad a \in \mathbb{Z}$

3) $\Rightarrow (2a)^2 + 3(2a) + 4 = 4a^2 + 6a + 4$

4) $\Rightarrow 2(2a^2 + 3a + 2)$

5) $\Rightarrow n^2+3n+4 = 2b$, $b = 2a^2+3a+2$

Case 2: 1) Suppose n is odd

2) $\Rightarrow n = 2a+1$

3) $\Rightarrow (2a+1)^2 + 3(2a+1) + 4 = 4a^2+4a+1+6a+3+4$

4) $\Rightarrow 4a^2+10a+8 = 2(2a^2+5a+4)$

5) $n^2+3n+4 = 2b$, $b = 2a^2+5a+4$

6) $\therefore n^2+3n+4$ is even

$\blacksquare$

CH 5

1) Proposition: Suppose $n \in \mathbb{Z}$ if n^2 is even then n is even

Proof: Suppose n is odd

1) $n = 2x+1$, $x \in \mathbb{Z}$

2) $n^2 = (2x+1)^2 = 4x^2+4x+1 = 2(2x^2+2x)+1$

3) $\Rightarrow n^2 = 2b+1$, $b = 2x^2+2x$

4) $\therefore n^2$ is odd

$\blacksquare$

3) Proposition: $a, b \in \mathbb{Z}$, if $a^2(b^2-2b)$ is odd then a and b are odd — either case makes statement true

Proof: Suppose a $\boxed{\text{or}}$ b is even.

Case 1
1) a is even
2) $\to a = 2x,\ x \in \mathbb{Z}$
3) $\to (2x)^2(b^2-2b) = 4x^2b^2 - 4x^2b$
4) $\to\ = 2(2x^2b^2 - 2x^2b),\ z = (2x^2b^2 - 2x^2b)$

Case 2
1) b is even
2) $\to b = 2x,\ x \in \mathbb{Z}$
3) $\to a^2((2x)^2 - 2(2x)) = 4x^2a^2 - 4a^2x$
4) $\to 2(2x^2a^2 - 2a^2x),\ z = (2x^2a^2 - 2a^2x)$
5) $\therefore a^2(b^2-2b)$ is even

■

9) Proposition: Suppose $n \in \mathbb{Z}$ if $3 \nmid n^2$, then $3 \nmid n$

Proof: Suppose $3 \mid n$
1) $n = 3a,\ a \in \mathbb{Z}$
2) $n^2 = (3a)^2 = 9a^2 = 3(3a^2),\ x = 3a^2$
3) $\to n^2 = 3x$
4) $\therefore 3 \mid n^2$ is true

■

15) Proposition: Suppose $x \in Z$ if x^3-1 is even, then x is odd

Proof: Suppose x is even

1) $x = 2a \quad , a \in Z$
2) $\rightarrow (2a)^3 - 1 = 8a^3 - 1 = (2a)^3 - 1^3$
   $\rightarrow 2a^2 + 2a + 1 = 2(a^2+a) + 1 \ , \ b = a^2+a$
3) $\rightarrow \cancel{x}^3 - 1 = 2b+1$
4) $\therefore n^3-1$ is odd

$\blacksquare$

17) Proposition: If n is odd, then $8 \mid (n^2-1)$

Proof: Suppose n is odd

1) $n = 2x+1 \ , x \in Z$
2) $n^2-1 = 8a \ , x \in Z$
3) $\rightarrow (2x+1)^2 - 1 = 8a$
4) $\rightarrow 4x^2 + 4x + 1 - 1 = 8a$
5) $\rightarrow 2(2x^2+2x) = 8a$
6) $\rightarrow 4x(x+1) = 8a$
7) $\rightarrow x(x+1) = 2a$
8) $\rightarrow 4(2a) = 8a$
9) $\therefore n^2-1 = 8a$
10) $\therefore 8 \mid (n^2-1)$

$\blacksquare$

CH6

5) Proposition: Prove that $\sqrt{3}$ is irrational.

Proof: $\sqrt{3}$ is a rational #

1) $\sqrt{3} = a/b$, $a, b \in \mathbb{Z}$ and a/b is reduced
2) $3 = a^2/b^2$
3) $\rightarrow 3b^2 = a^2 \Rightarrow 3 \mid a^2 \Rightarrow 3 \mid a$
4) $\rightarrow 3 \nmid a$ remainder of 1
5) $\rightarrow a = 3x+1 \Rightarrow a^2 = (3x+1)^2 = 9x^2+6x+1$
6) $\rightarrow 3(3x^2+2x)+1 \Rightarrow 3 \nmid a^2$
7) $\rightarrow \therefore 3 \mid a$ and $3 \mid a^2$
8) $\rightarrow 3 \mid a \Rightarrow a = 3x \Rightarrow 3b^2 = (3x)^2$
9) $\rightarrow 3b^2 = 9x^2 \Rightarrow b^2 = 3x^2$
10) $\therefore 3 \mid b$ and $3 \mid b^2$
11) $\therefore 3 \mid a$ and $3 \mid b$
12) This is a contradiction since the fraction shouldn't be reducable.
13) $\therefore \sqrt{3}$ is irrational

$\blacksquare$

7) Proposition: if $a, b \in \mathbb{Z}$, then $a^2 - 4b - 3 \neq 0$

Proof: Suppose $a^2 - 4b - 3 = 0$, $a, b \in \mathbb{Z}$

1) $a^2 = 4b + 3 = (4b+2) + 1 = 2(2b+1) + 1$
2) $\rightarrow a^2$ is odd $\rightarrow a = 2x + 1$, $x = (2b+1)$
3) $\rightarrow (2x+1)^2 - 4b - 3 = 0$
4) $\rightarrow 4x^2 + 4x + 1 - 4b - 3 = 0$
5) $\rightarrow 4x^2 + 4x - 4b = 2$
6) $\rightarrow 2(2x^2 + 2x - 2b) = 2$
7) $\rightarrow 2(x^2 + x - b) = 1$
8) 1 is even which contradicts that a^2 is odd
9) $\therefore a^2 - 4b - 3 \neq 0$

$\blacksquare$

11) Proposition: there exist no integers a and b for which $18a + 6b = 1$

Proof: $a, b \in \mathbb{Z}$, $18a + 6b = 1$

1) $2(9a + 3b) = 1$
2) $\rightarrow 18a + 6b = 1$ is even
3) This is a contradiction 1 is not even
4) $\therefore$ There exist no integers a and b for which $18a + 6b = 1$

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