Take Home Exam 2 | Name ____________________

CSCI 006 | April 16, 2020

Make sure to show all your work. You will not recieve credit for magic answers. You will only earn full points possible by showing all steps required in an organized way.

1. **(10 points)** Find me three examples of Theorems from the book and in each, state the hypothesis and the premise.

   (a) Theorem:

      i. Premise:

      ii. Conclusion:

   (b) Theorem:

      i. Premise:

      ii. Conclusion:

   (c) Theorem:

      i. Premise:

      ii. Conclusion:

2. **(5 points)** Name 4 types of proofs that we have done in class.

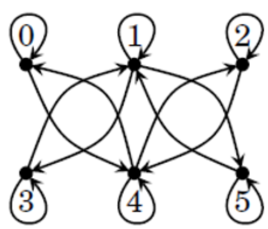
3. **(8 points)** Prove the following only using the definitions of even and odd integers.
Let $x, y \in \mathbb{Z}$. Prove that $(x+1)y^2$ is even if and only if x is odd or y is even.

4. **(6 points)**

   (a) Write an equivalent statement to the following statement:
   If $x, y \in \mathbb{Q}$ and $x < y$, then $\exists\, z \in \mathbb{R}$ such that $x < z < y$.

   (b) What is the name for this kind of equivalent statement?

5. **(6 points)** Here is a diagram for a relation R on a set A.



   (a) Write the set A and the set R.

   (b) Is R reflexive?

   (c) Is R symmetric?

   (d) Is R transitive?

6. **(14 points)**

   (a) **(4 points)** Create an equivalence relation, R, on a set, A, with 6 elements that has 3 equivalent classes.

   (b) **(1 point)** Why is this an equivalent class?

   (c) **(3 points)** State the equivalence classes and how you determined the equivalence classes.

   (d) **(1 point)** Create a new and different relation, S, on A with 3 equivalence classes.

   (e) **(1 point)** State the equivalence classes and how you determined the equivalence classes.

(f) **(4 points)** Create an isomorphism from R to S. If you cannot, create another equivalence relation that is isomorphic to R. Explain why this is an isomorphism.

7. **(10 points)** Let a function $f : \mathbb{Z} \to \mathbb{Z}$ be defined as $f(n) = 3n + 5$.

   (a) Is f injective?

   (b) Is f surjective?

   (c) Let $A = \{3n : n \in \mathbb{Z}\}$, find a function, $g : A \to \mathbb{Z}$, such that $f \circ g : A \to A$ and $f \circ g : \mathbb{Z} \to \mathbb{Z}$ is bijective.

8. **(8 points)** Let there be a 5×6 rectangle with 16 points randomly placed inside of it. Prove that two of the points must have a distance between them of $\sqrt{10}$ units or less.

9. **(8 points)** Prove that $4 \mid (5^n - 1)$ for every nonnegative integer n using Induction.

10. **(4 points)** Consider the following geometric sequence. Find the recurrence definition of this sequence.

$$16, 8, 4, 2, 1, \frac{1}{2}, \ldots$$

11. **(4 points)** Consider the following arithmetic sequence. Find the closed formula if $a_1 = 2$ and find the closed forumula if $a_{11} = 2$.

$$2, 5, 8, 11, 14, \ldots$$