3.1 Applied Structures

1) d = "I like discrete structures", c = "I will pass this course" s = "I will do my assignments"

a) $d \wedge c$ b) $s \vee {-c}$ c) $-d \wedge -s$

d) $^{-}s \wedge {^{-}c}$

3) p = "2 ≤ 5", q = "8 is an even integer", r = "11 is a prime #"

a) 2 > 5 and 8 is even integer, false

b) if 2 ≤ 5 then 8 is even integer, True

c) if 2 ≤ 5 and 8 is an even integer then 11 is a prime #, True

d) if 2 ≤ 5 then 8 is an even integer or 11 is not a prime #, True

e) if 2 ≤ 5 then 8 is not... or 11 is not..., False

f) if 8 is not even integer then 2 > 5, True

5) a) if an integer is even then it is a multiple of 4, False

b) the fact that a rectangle is a polygon it is suffIcent condition that it is a square., false

c) if $x^2 = 25$ then $x = 5$ no false

d) if $x = 2$ or $x = 3$ then $x^2 - 5x + 6 = 0$, true

e) $x = y$ is a neccasary condition for $x^2 = y^2$, false

3.2

1) a) $\rho \vee \rho$

ρ	$\rho \vee \rho$
0	0
1	1

b) $\rho \wedge (\neg \rho)$

ρ	$\neg \rho$	$\rho \wedge \neg \rho$
0	1	0
1	0	0

c) $\rho \vee (\neg \rho)$

ρ	$\neg \rho$	$\rho \vee (\neg \rho)$
0	1	1
1	0	1

d) $\rho \wedge \rho$

ρ	$\rho \wedge \rho$
0	0
1	1

3) a) $(\neg((\rho) \wedge (r))) \vee (s) = \neg(\rho \wedge s) \vee s$

b) $((\rho) \vee (q)) \wedge ((r) \vee (q)) = (\rho \vee q) \wedge (r \vee q)$

5) four variables $= 2^4 = 16$

3.3

1) $a \Longleftrightarrow e$ - $(\rho \wedge r) \vee q \Longleftrightarrow (\rho \vee q) \wedge (r \vee q)$

$d \Longleftrightarrow f$ - $\neg r \vee \rho \Longleftrightarrow r \to \rho$

$g \Longleftrightarrow h$ $\quad r \vee \neg \rho \Longleftrightarrow \rho \to r$

3) $r \Rightarrow s$ converse $s \Rightarrow r$

s	r	$s \to r$	$r \to s$	$(s \to r) \leftrightarrow (r \to s)$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

$r \Rightarrow s$ is not equivalant to $s \Rightarrow r$

5) $p \quad q$ $\quad 4 = \text{rows} \quad 2$ choices

p	q
0	0
0	1
1	0
1	1

$2^4 = 16$ possible propositions

7) $0 \to p \quad p \to 1$ tautologies

contradiction

tautologies

0	p	$0 \to p$
0	1	1
0	0	1

p	1	$p \to 1$
0	1	1
1	1	1

3.4

1) p = "I study" $\quad q$ = "I will learn"

$((p \to q) \wedge \neg q) \leftrightarrow \neg p = z$

p	q	$p \to q$	$(p \to q) \wedge \neg q$	z
0	0	1	1	1
0	1	1	0	1
1	0	0	0	1
1	1	1	0	1

z is tautology

$\therefore$ True

3) $(p \Leftarrow q) \Leftrightarrow (q \Rightarrow p)$

in any true statement change $\wedge$ with $\vee$, $\vee$ with $\wedge$
0 with 1, 1 with 0, $\Leftarrow$ with $\Rightarrow$, $\Rightarrow$ with $\Leftarrow$

2.1

1) Statement, false

3) Statemet, True

5) Statement, True

7) Statement, false

9) not a statement don't know what x is open sentence.

11) not a statement don't know x open sentence

13) statement is always true no matter what x is

15) statement, it isn't true nor false. yet to be proven.

2.2

1) P = "The number 8 is even"
Q = "The number 8 is a powr of 2"
$P \wedge Q$

3) P = "$x = y$" open sentence
$\sim P = \sim(x = y) = \sim x = \sim y$

5) P = "$y < x$" $\sim P = \sim(y < x)$ open sentence

7) P = "number $x = 0$" Q = "number $y \neq 0$"
$P \wedge Q$ open sentence

9) P = "$x \in A$" Q = "$\sim(x \in B)$"
$P \wedge Q$ open sentence

11) P = "$A \subseteq N$" Q = "$|\bar{A}| < \infty$"
$P \wedge Q$

13) P = "human beings want to be good"
Q = "human being don't want to be too good"
R = "humany being don't want to be good all the time"
$P \wedge Q \wedge R$

2.3

1) if a matrix determinint is not zero then it is invertible.

3) if a function is continuous then it is integrable.

5) if an integer is divisible by 8 then it is divisible by 4.

7) if a series converges absolutly then it convergers

9) if a function is continuous, then it is integrable

11) if you fail then you have stopped coritly

13) if people agree with me then I feel I am wrong.

2.4

1) matrix a is invertible if and only if $det(A) \neq 0$

3) $xy = 0$ if and only if y or $x = 0$

5) an occurence is an adventure if and only if one can recount it.

2.5

1) $P \vee (Q \Rightarrow R)$

P	Q	R	$Q \Rightarrow R$	$P \vee (Q \Rightarrow R)$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	0
1	0	0	1	1
0	1	1	1	1
1	0	1	1	1
1	1	0	0	1
1	1	1	1	1

3) $\sim(P \Rightarrow Q)$

P	Q	$P \Rightarrow Q$	$\sim(P \Rightarrow Q)$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

5) $(P \wedge \sim P) \vee Q$

P	Q	$\sim P$	$(P \wedge \sim P)$	$(P \wedge \sim P) \wedge Q$
0	0	1	0	0
0	1	1	0	1
1	0	0	0	0
1	1	0	0	1

7) $(P \wedge \sim P) \Rightarrow Q$

P	Q	$\sim P$	$P \wedge \sim P$	$(P \wedge \sim P) \Rightarrow Q$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	1

9) $\sim(\sim P \vee \sim Q)$

P	Q	$\sim P$	$\sim Q$	$(\sim P \vee \sim Q)$	$\sim(\sim P \vee \sim Q)$
0	0	1	1	1	0
0	1	1	0	1	0
1	0	0	1	1	0
1	1	0	0	0	1

11) if P is false then $(P \wedge Q)$ is false so for $(R \Rightarrow S) \Leftrightarrow (P \wedge Q)$ to be true then $(R \Rightarrow S)$ needs to be false therefore R is true and S is false

2.6)

1) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$

P	Q	R	$Q \vee R$	$P \wedge Q$	$P \wedge R$	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
1	0	1	1	0	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

3) $P \Rightarrow Q = \sim P \vee Q$

P	Q	$\sim P$	$P \Rightarrow Q$	$\sim P \vee Q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

5) $\sim(P \vee Q \vee R) = \sim P \wedge \sim Q \wedge \sim R$

P	Q	R	$\sim P$	$\sim Q$	$\sim R$	$P \vee Q \vee R$	$\sim(P \vee Q \vee R)$	$\sim P \wedge \sim Q \wedge \sim R$
0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	1	0	0
0	1	0	1	0	1	1	0	0
1	0	0	0	1	1	1	0	0
0	1	1	1	0	0	1	0	0
1	0	1	0	1	0	1	0	0
1	1	0	0	0	1	1	0	0
1	1	1	0	0	0	1	0	0

7) $P \Rightarrow Q = (P \wedge \sim Q) \Rightarrow (Q \wedge \sim Q)$

P	Q	$\sim Q$	$(P \wedge \sim Q)$	$(Q \wedge \sim Q)$	$(P \wedge \sim Q) \Rightarrow (Q \wedge \sim Q)$	$(P \Rightarrow Q)$
0	0	1	0	0	1	1
0	1	0	0	0	1	1
1	0	1	1	0	0	0
1	1	0	0	0	1	1

9) $P \wedge Q$ and $\sim(\sim P \vee \sim Q)$

the pair are logically equivalent Demorgan's law

$\sim\sim P \vee \sim\sim Q = P \wedge Q$

11) $(\sim P) \wedge (P \Rightarrow Q)$ and $\sim(Q \Rightarrow P)$

P	Q	$\sim P$	$\sim Q$	$P \Rightarrow Q$	$Q \Rightarrow P$	$\sim(Q \Rightarrow P)$	$(\sim P) \wedge (P \Rightarrow Q)$
0	0	1	1	1	1	0	1
0	1	1	0	1	0	1	1
1	0	0	1	0	1	0	0
1	1	0	0	1	1	0	0

not logically equivalent

13) $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge R$

P	Q	R	$Q \wedge R$	$P \vee Q$	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge R$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	1	0	0
1	0	0	0	1	1	0
0	1	1	1	1	1	1
1	0	1	0	1	1	1
1	1	0	0	1	1	0
1	1	1	1	1	1	1

not logically equivalent