CH 12.1

1) $A = \{0,1,2,3,4\}$, $B = \{2,3,4,5\}$

$f = \{(0,3),(1,3),(2,4),(3,2),(4,2)\}$

Domain is $\{0,1,2,3,4\} = A$

Range is $\{2,3,4\}$

$f(2) = 4$ $\quad f(1) = 3$

3) $f: \{a,b\} \to \{0,1\}$

$f_1 = \{(a,0),(b,0)\}$ $\quad f_2 = \{(a,1),(b,0)\}$

$f_3 = \{(a,0),(b,1)\}$ $\quad f_4 \{(a,1),(b,1)\}$

5) $\{a,b,c,d\}$ to $\{d,e\}$

$f = \{(a,d),(b,d),(c,d),(d,d),(a,e),(b,e),(c,e),(d,e)\}$

$R = \{(a,d),(b,d),(c,d),(d,e)\}$

7) $f = \{(x,y) \in Z \times \cancel{Z} : 3x+y=4\}$ is this a function

Z to Z ? $\quad f: Z \to Z$

$3x+y=4$ if and only if $y = 4-3x$ then

$f: Z \to Z$

9) $f = \{(x^2, x) : x \in R\}$

No, since x^2 will make all integer postive

$\therefore$ if ~~x~~ is -2 or 2 4 will pop up

more than once $(4,2)$ $(4,-2)$

12.2 1-7 odd

1) $A = \{1, 2, 3, 4\}$ $B = \{a, b, c\}$

$f: A \to B$ not injective or surjective

$f\{(1,b), (2,b), (3,b), (4,b)\}$

3) $\cos: R \to R$

not injective $\cos 0 = \cos 2\pi$

not surjective no R for x where $\cos(x) = 5$

$\cos: R \to [-1, 1]$

surjective $\cos(0) = 1$ $\cos(\pi) = -1$

not injective

5) $f: Z \to Z$ $f(n) = 2n+1$

not surjective codomain contains 2 but

$\forall n \in Z$ for $f(n) \neq 2$

injective. $f(x) = f(n)$

$2x+1 = 2n+1 \Rightarrow 2x = 2n \Rightarrow x = n$

7) $f: Z \times Z \to Z$ $f((m,n)) = 2n - 4m$

not injective $(0,2) \neq (-1,0)$, $f(0,2) = f(-1,0)$

$f(0,2) = 2(2) - 4(0) = 4$

$f(-1,0) = 2(0) - 4(-1) = 4$

not surjective

$f((m,n)) = 2n - 4m = 2(n-2m)$

$f((m,n))$ = always even if b is odd

then $f((m,n)) \neq b$

12.3

1) Proposition if six integers are chosen at random, then at least two of them will have the same remainder when divided by 5

1) $f: A \to B$ A: {six integers} B: {5}

2) $f(6) = 5x + 1$

3) $\therefore$ Pigeonhole principle there will be two sets with same remainder.

$\boxed{E}$

3) Proposition: Given any six positive integers there are two for which their sum or difference is divisible by 9.

12.4

1) $A = \{5,6,8\}$, $B = \{0,1\}$, $C = \{1,2,3\}$

$f: A \to B$ $\quad f = \{(5,1),(6,0),(8,1)\}$

$g: B \to C$ $\quad g = \{(0,1),(1,1)\}$

$g \circ f = \{(5,1),(6,1),(8,1)\}$

3) $A = \{1,2,3\}$ $\quad f: A \to A$ $\quad f = \{(1,2),(2,2),(3,1)\}$

$g: A \to A$ $\quad g = \{(1,3),(2,1),(3,2)\}$

$g \circ f = \{(1,1),(2,1),(3,3)\}$

$f \circ g = \{(1,1),(2,2),(3,2)\}$

5) $f,g: R \to R$ $\quad f(x) = \sqrt[3]{x+1}$ $\quad g(x) = x^3$

$g \circ f$: $(\sqrt[3]{x+1})^3 = x+1$

$f \circ g$: $\sqrt[3]{x^3+1}$

12.5

1) $f: \mathbb{Z} \to \mathbb{Z}$ $f(n) = 6 - n$ bijective
compute f^{-1}
Injective $f(m) = f(n) \Rightarrow 6 - m = 6 - n \Rightarrow m = n$
surjective $f(6-b) = 6 - 6 + b = b$
$f^{-1}(n) = 6 - n$

2) $f: \mathbb{R} - \{2\} \to \mathbb{R} - \{5\}$ $f(x) = \frac{5x+1}{x-2}$ bijective

$y = \frac{5x+1}{x-2} \Rightarrow$ $yx - 2y = 5x + 1 \Rightarrow yx - 5x = 2y + 1$

$x(y-5) = 2y + 1 \Rightarrow x = \frac{2y+1}{y-5}$

$f^{-1}(y) = \frac{2y+1}{y-5}$

3) $B = \{2^n : n \in \mathbb{Z}\} = \{\ldots, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\}$ $f: \mathbb{Z} \to B$
$f(n) = 2^n$ bijective? f^{-1}?
injective $f(x) = f(n) \Rightarrow 2^x = 2^n \Rightarrow \log_2 2^x = \log_2 2^n$
$\Rightarrow x \log_2 2 = n \log_2 2 \Rightarrow x = n$

surjective $b \in B$ $b = 2^n$ $f(n) = 2^n = b$

$x = 2^n \Rightarrow \log x = \log 2^n \Rightarrow \log x = n \log 2$
$n = \frac{\log x}{\log 2} \Rightarrow n = \log_2 x$

$f^{-1}(n) = \log_2 x$

12.6

1) $f: \mathbb{R} \to \mathbb{R}$ $f(x) = x^3 + 3$ $f([3, 5])$
and $f^{-1}([12, 19])$

$f([3,5]) = ([12, 28])$ $f^{-1}([12,19]) = ([f(-3), f(4)])$
$= ([12, 19])$

3) $f: \{1,2,3,4,5,6,7\} \to \{0,1,2,3,4\}$
$|f^{-1}(\{3\})| = 3$
$f^{-1}(3) = 3 \Rightarrow f(3) = 3$
$(A^c)^{(B^c)} = (5-1)^{(7-1)} = 4^6$

7.2

1) one to one and onto

a) $f \subseteq A \times B$, $f = \{(1,a), (2,b) (3,c), (4,d)\}$
injective, subjective

b) $g \subseteq A \times B$, $g = \{(1,a), (2,a), (3,b) (4,d)\}$
none

c) $h \subseteq A \times B$, $h = \{(1,a), (2,b), (3,c)\}$
insective

d) $K \subseteq A \times B$, $K = \{(1,a), (2,b), (2,c), (3,a), (4,a)\}$
subjective

e) $L \subseteq A \times A$, $L = \{(1,1), (2,1), (3,1), (4,1)\}$
none

(inj) (sub)

3/ one to one or onto or both

a) $f_1 : R \to R$ $f_1(x) = x^3 - x$

subjective

b) $f_2 : Z \to Z$ by $f_2(x) = -x+2$

ingective and subjective

c) $f_3 : N \times N$ $f_3(j,k) = 2^J 3^K$

ingective

d) $f_4 : P \to P$ $f_4(n) = n/2$ $[x]$ is cieling of x the smallest integer greater than or equal to x

subjective

e) $f_5 : N \to N$ $f_5(n) = n^2 + n$

ingective

f) $f_6 : N \to N \times N$ $f_6(n) = (2n, 2n+1)$

injective.

5) $X = \{ socks \}$ $Y = \{ pairs \}$

$f : X \to Y$ $f(x) =$ pairs of sock's x

Ex : $X = Y$