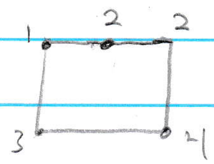
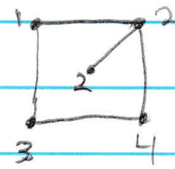
Emmanuel Cardenas

4.1

3) Yes it is possible to have two different non-isomorphic graphs to have the same number of verticies and edges.

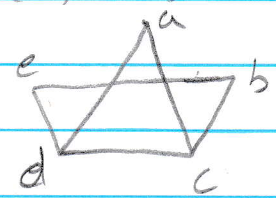




4) Are the graph equal?

G1: $V = \{a, b, c, d, e\}$, $E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}\}$

G2:



Not equal G1 has $\{a, b\}$ G2 does not.

Are they Isomorphic?

Yes, if $f: G_1 \to G_2$, $f(a) = d$ $f(b) = c$ $f(c) = e$ $f(d) = b$ $f(e) = a$

6) What is largest amount of edges in a graph with 10 verticies?

$$K_{10} = \frac{10(10-1)}{2} = 45 \text{ edges}$$

largest amount of edges in a bipartite graph with 10 verticies?

V_1	V_2	
1	9	$K_{19} = 9$ edges
2	8	$K_{28} = 16$ edges
3	7	$K_{37} = 21$ edges
5 | 5 → 4	6	$K_{46} = 24$ edges

$K_{55} = 25$ edges

what is the largest number of edges in a tree with 10 verticies?

$(n-1) = 10-1 = 9$ edges

15) Prove any graph with at least two verticies must have two verticie of the same degree?

Pigeon's hole

2 n, n

n

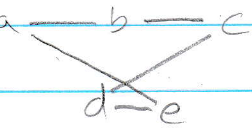
$\therefore$ two verticies in the same set will have same degrees

$\square$

4.2

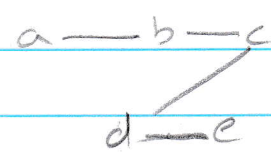
1) which are trees?

a) $G=(V,E)$, $V=\{a,b,c,d,e\}$ & $E=\{\{a,b\},\{a,e\},\{b,c\},\{c,d\},\{d,e\}\}$



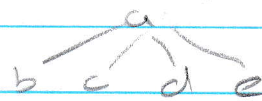
not a tree to many edges $5 \neq (5-1)$

b) $G=(V,E)$, $V=\{a,b,c,d,e\}$, $E=\{\{a,b\},\{b,c\},\{c,d\},\{d,e\}\}$



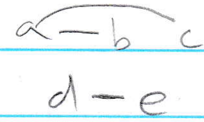
yes, connected no cycle and edges $4=(5-1)$

c) $G=(V,E)$, $V=\{a,b,c,d,e\}$; $E=\{\{a,b\},\{a,c\},\{a,d\},\{a,e\}\}$



yes, all connected no cycle and edges $4=(5-1)$

d) $G=(V,E)$, $V=\{a,b,c,d,e\}$, $E=\{\{a,b\},\{a,c\},\{d,e\}\}$



no, not connected and edge $3 \neq (5-1)$

2) for the degree sequence's are they, must always? must never? or could possibly be a sequence for a tree.

a) $(4,1,1,1,1)$

verticies 5 sum of degrees 8

edges $= \frac{8}{2} = 4$ $(4+1)=5$ $\therefore$ is a tree.

b) $(3,3,2,1,1)$

verticies $= 5$ sum of degrees $= 10$

edges $= \frac{10}{2} = 5$ $(5+1) \neq 5$ $\therefore$ not a tree

c) $(2,2,2,1,1)$

verticies $= 5$ sum of degree $= 8$ edges $= \frac{8}{2} = 4$

$(4+1)=5$ $\therefore$ is a tree.

d) $(4,4,3,3,3,2,2,1,1,1,1,1,1,1)$

verticies $= 14$ sum of degrees $= 28$ edges $= \frac{28}{2} = 14$

$(14+1) \neq 14$ $\therefore$ not a tree

4.3

1) is it possible for a planar graph to have 6 verticies, 10 edges, 5 faces.

$v-e+f=2 \qquad v=6 \quad e=10 \quad f=5$

$=6-10+5 \neq 2 \qquad \therefore$ Not possible

3) is it possible for a connected graph with 7 verticies and 10 edges to be drawn so no edges cross and create 4 faces?

$v-e+f=2 \qquad 7-10+4 \neq 2$

$\therefore$ no it is not possible

8) Prove Eulers formula using induction on the number of edges.

$v-e+f=2, \; e=0$

$v, f = 1$

$v-(k+1)-f=2$

$\Rightarrow v-k-1-f=2$, disconnecting an edge

$\Rightarrow v-k+f-1=2$ will decrease the faces

or

$v-1-k+f=2$, disconnecting an edge will degrees vertex

$\therefore v-e+f=2 \Rightarrow v-(k+1)+f=2$

9) Prove eulers formula using induction on the number of vertices

$v - e + f = 2$, $v = 1$

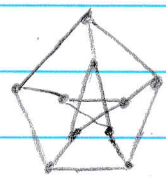
$\Rightarrow f = e + 1$ $\therefore$ $v - e + f = 2$

G' $(v-1)$, $(e-1)$, f

$\Rightarrow v - 1 - (e-1) + f = 2$

$\therefore$ $v - e + f = 2$

11) Prove Petersen graph is not planar



$v - e + f = 2$, $v = 10$ $e = 15$

$10 - 15 + f = 2 \Rightarrow f = 7$

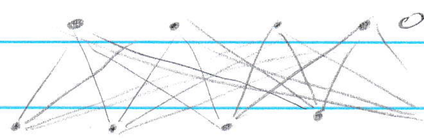
if 7 faces exist then 35 edges exist

divide by 2 the 17 edges which mean

$17 > 15$ $\therefore$ Petersen graph is not planar

4.4

1) smallest number of colors for $K_{4,5}$, chromatic #?

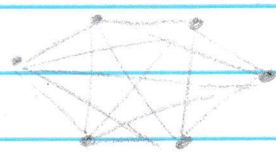


$\therefore$ chromatic # = 2

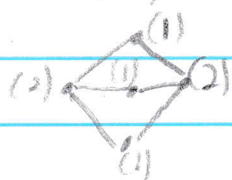
bipartite

2) Draw a graph with chromatic number 6, Could it be planar?

based on 4-color theorem all planar graph can be colored with 4 or fewer colors.



3) find the chromatic color od each of the following graphs



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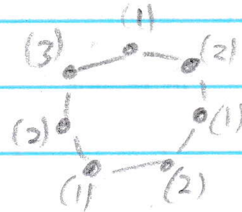

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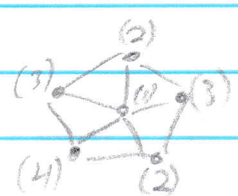

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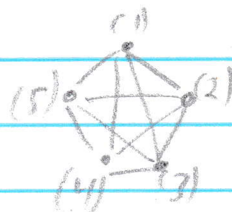

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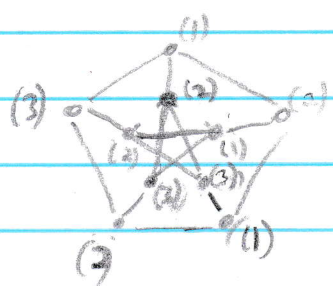
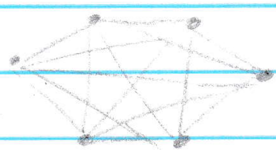


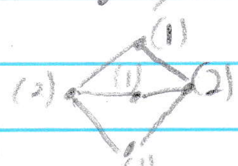
3

2) Draw a graph with chromatic number 6, Could it be planar?

based on 4-color theorem all planar graph can be colored with 4 or fewer colors.



3) find the chromatic color of each of the following graphs



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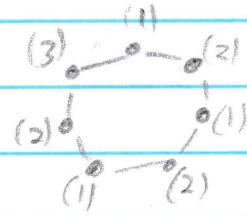

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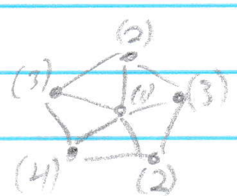

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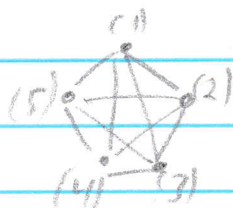

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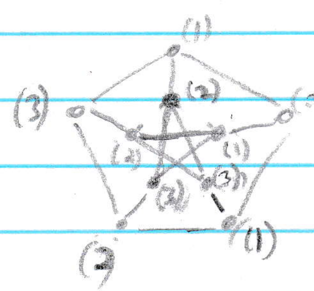

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