Some Results on Almost Distributive Lattices*

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Abstract

The concept of an almost distributive lattice (or shortly an ADL) was first introduced by U.M. Swamy and G. C. Rao [2] in 1980 as a common abstraction to most of the existing ring theoretic and lattice theoretic generalization of Boolean algebras. An ADL is an algebra with two binary operations $\vee$ and $\wedge$ which satisfies almost all the properties of a distributive lattice with smallest element 0 except possibly the commutativity of $\vee$, the commutativity of $\wedge$ and the right distributivity of $\vee$ over $\wedge$. It was also observed that any one of these three properties converts an ADL into a distributive lattice.

In this paper, we delve deeper into the study of ADLs and explore some structural properties. We first provide a plenty of examples (finite as well as infinite) supported by Hasse diagrams, exhibiting a variety of ADL properties. In general, it is not known so far whether the $\vee$ operation in ADLs is associative or not. We present a counter example showing that not every ADL is $\vee$-associative. Moreover, we obtain a set of necessary and sufficient conditions for an ADL to be $\vee$-associative. Motivated by this particular example, we obtain a number of subdirectly irreducible finite ADLs other than those given in [2]. But it is still an open problem to prove whether or not there are no more subdirectly irreducible ADLs.

Moreover, we present a number of congruence properties that the class of distributive lattices satisfy but the class of ADLs fails to satisfy. We further state some open problems on finding the sub varieties of the class of ADLs having these congruence properties.

Continuing our investigation on finite ADLs, we obtain an algorithm (or a formula) to determine the cardinality of finite ADLs. Our algorithm is inductive that helps to describe the cardinality of an ADL in terms of the cardinality of a finite distributive lattice (the lattice of its principal ideal); where the cardinality of distributive lattices is given in [1].

Contents

References

[1] J. Berman and P. Köhler. The cardinality of finite distributive lattices.

[2] U. M. Swamy and G. C. Rao. Almost distributive lattices, 1981–2011.

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