Incidence Matrix and Wiener Index of Zero Divisor Graphs

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Abstract

Consider R be a commutative ring with nonzero identity, and let $Z(R)$ be the set of zero divisors of R. The zero divisor graph of ring R, denoted by $\Gamma(R)$, is a (undirected) graph whose vertex set is $Z^*(R) = Z(R) \setminus \{0\}$, the nonzero zero-divisors of R and two distinct vertices u and v are adjacent in the zero divisor graph $\Gamma(R)$ if and only if $uv = 0$ [1]. In this article, we study the incidence matrix [3], p-ary linear code derived from the incidence matrix [2], and the Wiener index of the zero divisor graph [4] for the ring of integers modulo n, the ring of Gaussian integers modulo n, and the Cartesian product of finite fields $Z_p \times Z_q$. We prove that the Wiener index of $\Gamma(Z_p \times Z_q)$ is $p^2+q^2+pq-4p-4q+5$, and $W(\Gamma(Z_p[i])) = 3p^2 - 8p + 5$. Additionally, a Python code for computing the Wiener index of the zero divisor graph has been provided.

References

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