Measurment Model: Two possible constraints are Zero lateral and vertical velocities

The lateral velocity v^{lat} and vertical velocity v^{up} are obtained after expressing the velocity v in the body frame B.

$$v^B = C^T v = \begin{bmatrix} v^{for} \\ v^{lat} \\ v^{up} \end{bmatrix}$$

rotation matrix

$\to$ C is a shew-symmetric, orthogonal matrix

$$z = C^T v + n^{\text{noise}} \; : \quad \text{Assume } z \approx \overset{\text{Expected value}}{\bar z} + \delta z \; , \; C \approx (I - \delta\phi^\times)\bar C \; , \; v \approx \bar v + \delta v$$

$$\Rightarrow z = \bar z + \delta z = -(I - \delta\phi^\times)\bar C(\bar v + \delta v) + n = \bar C \bar v - \bar C \delta v + \delta\phi^\times \bar C \bar v + \delta\phi^\times \bar C \delta v + n$$

$$\xrightarrow{\bar z = \bar C^T \bar v + n} \delta z = -\bar C \delta v + \delta\phi^\times \bar C \bar v + \underbrace{\delta\phi^\times \bar C \delta v}_{\text{High-order term (neglected)}}$$

$$\xrightarrow{a^\times b = -b^\times a} \delta z = -\bar C \delta v - (\bar C \bar v)^\times \delta\phi$$

$$\Rightarrow H = \begin{bmatrix} 0 & -\bar C & -(\bar C \bar v)^\times & 0 & 0 \end{bmatrix} \quad \bigstar$$

$$C \approx (I - \delta\phi^\times)\bar C \Rightarrow C^T = \bar C^T (I + \delta\phi^\times) \Rightarrow$$

$$z = \bar z + \delta z = \bar C^T (I + \delta\phi^\times)(\bar v + \delta v) + n = \bar C^T \bar v + \bar C^T \delta v + \bar C^T \delta\phi^\times \delta v + \bar C^T \delta\phi^\times \bar v + n$$

$$\xrightarrow{\bar z = \bar C^T \bar v + n} \delta z = \bar C \delta v + \cancel{\bar C^T \delta\phi^\times \delta v}^{\to 0} + \bar C^T \delta\phi^\times \bar v$$

Compare it with $\bigstar$

$$\xrightarrow{a^\times b = -b^\times a} \delta z = \bar C^T \delta v - \bar C^T \bar v^\times \delta\phi \Rightarrow H = \begin{bmatrix} 0 & \bar C^T & -\bar C^T \bar v^\times & 0 & 0 \end{bmatrix}$$

$$\xrightarrow[\text{with velocity \& quaterion measurments}]{\text{The final H}} \quad H = \begin{bmatrix} 0 & \bar C^T & -\bar C^T \bar v^\times & 0 & 0 \\ 0 & 0 & I & 0 & 0 \end{bmatrix}$$