Quaternion EKF

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***Abstract*— This electro**

I. INTRODUCTION

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A. Notation

For clarity, we decided to dedicate this section to

In this paper, scalar values are represented by <u>underlined</u> variables, i.e. $\underline{a}, \underline{x}, \underline{y}, \underline{z}$. Vectors are represented by normal font variables, i.e. a, x, y, z. Matrices are represented by UPPERCASE letters, i.e. $\mathbf{A}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}$. Approximated vectors are represented as normal vectors with a *tilde* on top to signify its *approximation* designation, i.e. $\widetilde{\mathbf{a}}, \widetilde{\mathbf{x}}, \widetilde{\mathbf{y}}, \widetilde{\mathbf{z}}$. The same notation is applied to matrices and scalars. Estimated vectors are represented as normal vectors with a *hat* on top to signify its *estimation* designation, i.e. $\widehat{\mathbf{a}}, \widehat{\mathbf{x}}, \widehat{\mathbf{y}}, \widehat{\mathbf{z}}$. The same notation is applied to matrices and scalars.

II. OBSERVATION AND ESTIMATION MODELS

A. Inertial Model

We start with a Newtonian dynamic model, where the system is described by forces acting onto robot's rigid body.

First, we calculate the *proper acceleration*, a_{proper}, which presents forces acting onto robot's rigid body. We subtract gravity from the absolute acceleration or *coordinate acceleration*, a_{abs}

and its *Body Frame* acceleration, a. We remove Earth's gravity from system dynamics.

$$a_{proper} = C_B^I (a_{abs} - g), \tag{1}$$

Where matrix C_B^I represents the orthonormal rotation from *inertial frame*, I, to robots *body frame*, B.

Linear are force

$$\widetilde{a} \ or \ f = a + \omega_a \tag{2}$$

$$\widetilde{\omega} = \omega + \omega_\omega \tag{3}$$

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B. Observation State Definition

The measurement state definition is defined by linear position, r, linear velocity, v, angular velocity, ω, and angular orientation in the quaternion space, q. The observation state vector, z, is defined as the following:

$$z^T := [r \ v \ \omega \ q], \tag{4}$$

Where the observed quaternion state is used to compute the corresponding state rotation matrix, C_B^I. It is important to note that the observation data is treated as the groundtruth.

C. EKF Model

To deploy a modified Kalman filter, we start with the assumption of continuous-time nonlinear system described by the following:

$$\dot{x} = f(x, u, \omega_f), \tag{5}$$

$$y = h(x, u, \omega_h). \tag{6}$$

Where $f()$ represents the *process* model and $h()$ represent the *observation* model. Variables ω_f and ω_h represent the *process* and *observation* noise, respectively. Vector u represent input to the system.

D. Estimation State Definition

The estimation state is defined by the robot's linear position, r, and velocity, v, and the body frame orientation in quaternion space.

$$x^T := [r \ v \ q] \tag{7}$$

$$P := Cov(\delta x), \tag{8}$$

Estimation residual is denoted by δx

$$\delta x^T = [\delta r \ \delta v \ \delta \phi] \tag{9}$$

E. Estimation Model

$$\delta x^T = [\delta r \ \delta v \ \delta \phi] \tag{10}$$

III. QUATERNION ALGEBRA

Quaternion intro paragraph Quaternion space is a non-minimal representation belonging to *SO(3)* Lie group.

A. Unit Quaternion

Moreover, the quaternion term from the dataset has *four terms* with *xyzw* format. Hamilton's quaternion defined by 3 perpendicular imaginary axes i, j, k with real scalars x, y, z and a real term w which constraints other 3 dimension to a *unit magnitude*. Thus, the fourth term normalizes the vector's magnitude conveniently and preserves the 3D rotation (3 DOF). We define **Unit Hamiltonian** or **Unit Quaternion** as,

$$\begin{aligned} \mathbb{H}^1 \quad := \quad & \{q_{wxyz} = w + xi + yj + zk \ \in \mathbb{H} \mid \\ & w^2 + x^2 + y^2 + z^2 = 1\} \end{aligned} \tag{11}$$

Where superscript 1 in $\mathbb{H}^1$ denotes a unit quaternion space with 4 terms. There are two equal representations for $\mathbb{H}^1$ subgroup; thus, we provide a concise definition and notation for both to avoid confusion. The the first representation is shown in **??** where the four terms of the quaternion are arranged in *wxyz* order and it is represented by q_{wxyz}. The second quaternion is arranged in *xyzw* format and is represented by q_{xyzw}. It is important to note the difference as both are used in our derivation and implementation.

$$q_{wxyz} = q_{xyzw} \ ; \quad q_{wxyz},\ q_{xyzw} \in \mathbb{H}^1 \tag{12}$$

B. Pure Quaternion

As previously mention, the three imaginary terms of the quaternion represent the angles of interest in 3D and the fourth dimension constraints the magnitude. Thus to avoid computational errors, we use quaternion only with its three imaginary terms, *xyz*. This quaternion space representation is defined by $\mathbb{H}^0$ and denoted by q_xyz variables.

$$\begin{aligned} \mathbb{H}^0 \quad := \quad & \{q_{xyz} = xi + yj + zk \ \in \mathbb{H} \mid \\ & x, y, z \in \mathbb{R}\} \backsimeq \mathbb{R}^3 \end{aligned} \tag{13}$$

C. Exponential Map

For calculating incremental rotation in

Incremental rotation estimation using the skew-symmetric matrix obtained form the rotational rate vector and matrix exponential mapping function, [QEKF01]. Gamma, Γ, represents incremental

$$\Gamma_0 := \sum_{i=0}^{\infty} \frac{(\Delta t^{i+n})}{(i+n)} \omega^{\times i}, \tag{14}$$

Where $(.)^\times$

D. Updating Quaternion State

$$q_{i+1} = \delta q_i \otimes \widehat{q}_i \tag{15}$$

E. Capturing Quaternion Error

We use the mapping function $\zeta(.)$ to calculate the quaternion state error from the error rotation vector, [QEKF01].

$$\delta q = \zeta(\delta\phi), \tag{16}$$

$$\zeta : v \rightarrow \zeta(v) = \begin{bmatrix} \sin(\frac{1}{2}\|v\|)\frac{v}{\|v\|} \\ \cos(\frac{1}{2}\|v\|) \end{bmatrix} \tag{17}$$

IV. MATH

Before you

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Finally, comp

A. Abbreviations and Acronyms

Defib

B. Units

C. Equations

The equations

Note

D. Some Common Mistakes

V. USING THE TEMPLATE

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A. Headings, etc

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B. Figures and Tables

Positioning Figure

TABLE I

AN EXAMPLE OF A TABLE

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Three	Four

We suggest

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VI. CONCLUSIONS

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APPENDIX

Appendixes should appear before the acknowledgment.

ACKNOWLEDGMENT

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References are

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