Quaternion EKF

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***Abstract*— This electro**

I. INTRODUCTION

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A. Notation

For clarity, we decided to dedicate this section to

Vectors are represented by normal font variables, i.e. a, x, y, z. Matrices are represented by UPPERCASE letters, i.e. $\mathbf{A}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}$. Approximated vectors are denoted with a *tilde* on top to signify its *approximation* designation, i.e. $\widetilde{a}, \widetilde{x}, \widetilde{y}, \widetilde{z}$. For clarity, we define approximate value representation in **??** as the original function with added bias and uniform distribution noise. This form representation is used to represent high-resolution measurements that are used as approximated input to the system. This is an essential part the EKF implementation which we describe in detail in later sections. Estimated vectors are represented with a *hat* on top to signify its *estimation* designation, i.e. $\widehat{a}, \widehat{x}, \widehat{Y}, \widehat{Z}$.

II. OBSERVATION AND ESTIMATION MODELS

A. Inertial Model

We start with a Newtonian dynamic model, where the system is described by forces acting on a rigid body.

For this paper, we decided to use linear and angular velocities as known input to the system, where we define u as the following.

$$u^T := [v^T w^T], \tag{1}$$

$$u_B = C_B^I u, \tag{2}$$

Where matrix C_B^I represents the orthanormal rotation from *inertial frame*, I, to robots *body frame*, B.

$$\widetilde{v} = v + n_v + b_v, \tag{3}$$

$$\widetilde{\omega} = \omega + n_\omega + b_\omega. \tag{4}$$

where n and b represent a normal distribution noise and bias added to the

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B. Observation State Definition

The measurement state definition is defined by linear position, r, linear velocity, v, angular velocity, ω, and angular orientation in the quaternion space, q. The observation state vector, z, is defined as the following:

$$z^T := \left[r^T \;\; v^T \;\; \omega^T \;\; q_{xyzw}^T \right], \tag{5}$$

Where the observed the quaternion state is used to compute the corresponding state rotation matrix, C_B^I. It is important to note that the observation data is treated as the groundtruth (Keep?????).

C. EKF Model

To deploy a modified Kalman filter which uses *Quaternions* for representing rotation. We explain these modifications in detail in the *Quaternion Algebra*. We start with the LaGrangian system representation, a continuous-time nonlinear system described by the following:

$$\dot{x} = f(x, u), \tag{6}$$

$$y = h(x, u). \tag{7}$$

Where $f()$ represents the *process* model and $h()$ represent the *observation* model. Vector u represents input to the system. Vector y represents the system output.

D. Estimation State Definition

The estimation state is defined by the robot's linear position, r, and velocity, v, and the body frame orientation in the quaternion space.

$$x^T := \left[r^T \;\; v^T \;\; q_{xyz}^T \right] \tag{8}$$

$$P := Cov(\delta x), \tag{9}$$

Estimation residual is denoted by δx

$$\delta x^T = [\delta r^T \;\; \delta v^T \;\; \delta \phi^T] \tag{10}$$

How was $\delta \phi^T$ obtained?

E. Estimation Model

Calculate the incremental rotation matrix R_k from q_{xyzw} from observation vector z, 5.

How is this done? scipy spatial transform module, Rotation object.

This spatial transform is carried out using an *extension of Euler's formula*

$$R_k = Rot_{from_q}\left(\; q_{xyzw}^T \right) \tag{11}$$

III. QUATERNION ALGEBRA

Quaternion intro paragraph Quaternion space is a non-minimal representation belonging to *SO(3)* Lie group.

A. Unit Quaternion

Moreover, the quaternion term from the dataset has *four terms* with *xyzw* format. Hamilton's quaternion defined by 3 perpendicular imaginary axes i, j, k with real scalars x, y, z and a real term w which constraints other 3 dimension to a *unit magnitude*. Thus, the fourth term normalizes the vector's magnitude conveniently and preserves the 3D rotation (3 DOF). We define **Unit Hamiltonian** or **Unit Quaternion** as,

$$\begin{aligned} \mathbb{H}^1 \quad := \quad & \{q_{wxyz} = w + xi + yj + zk \ \in \mathbb{H} \mid \\ & w^2 + x^2 + y^2 + z^2 = 1\} \end{aligned} \tag{12}$$

Where superscript 1 in $\mathbb{H}^1$ denotes a unit quaternion space with 4 terms. There are two equal representations for $\mathbb{H}^1$ subgroup; thus, we provide a concise definition and notation for both to avoid confusion. The the first representation is shown in 5 where the four terms of the quaternion are arranged in *wxyz* order and it is represented by q_{wxyz}. The second quaternion is arranged in *xyzw* format and is represented by q_{xyzw}. It is important to note the difference as both are used in our derivation and implementation.

$$q_{wxyz} = q_{xyzw} \ ; \quad q_{wxyz},\ q_{xyzw} \in \mathbb{H}^1 \tag{13}$$

B. Pure Quaternion

As previously mentioned, the three imaginary terms of the quaternion represent the 3D angles of interest in radians and the fourth dimension constraints the vector magnitude. Thus to avoid computational errors, in the prediction step, we use the unit quaternion where it only has its three imaginary terms, *xyz*. This quaternion space representation is defined by $\mathbb{H}^0$ and denoted by q_xyz variables.

$$\begin{aligned} \mathbb{H}^0 \quad := \quad & \{q_{xyz} = xi + yj + zk \ \in \mathbb{H} \mid \\ & x, y, z \in \mathbb{R}\} \simeq \mathbb{R}^3 \end{aligned} \tag{14}$$

C. Exponential Map

For calculating incremental rotation in

Incremental rotation estimation using the skew-symmetric matrix obtained form the rotational rate vector and matrix exponential mapping function, [QEKF01]. Gamma, Γ, represents incremental

$$\Gamma_0 := \sum_{i=0}^{\infty} \frac{(\Delta t^{i+n})}{(i+n)} \omega^{\times i}, \tag{15}$$

Where $(.)^\times$ represents skew-symmetry matrix of a vector

D. Updating Quaternion State

$$q_{i+1} = \delta q_i \otimes \widehat{q}_i \tag{16}$$

E. Capturing Quaternion Error

We use the mapping function $\zeta(.)$ to calculate the quaternion state error from the error rotation vector, [QEKF01].

$$\delta q = \zeta(\delta\phi), \tag{17}$$

$$\zeta : v \rightarrow \zeta(v) = \begin{bmatrix} \sin(\frac{1}{2}\|v\|)\frac{v}{\|v\|} \\ \cos(\frac{1}{2}\|v\|) \end{bmatrix} \tag{18}$$

IV. MATH

Before you

Diagonal matrix Q is in $dim_x \times dim_x$ dimensions and represent the process noise tolerance or innovation (if I recall correctly———).

$$Q_c = [Q_t] \tag{19}$$

H is the *observation model*. H is dimz by dimx which 12 by 9 where q_xyz is used.

$$H = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix} \tag{20}$$

L is dimx by dimx matrix and here it is 9x9.

$$L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -C^T & 0 \\ 0 & 0 & -I \end{bmatrix} \tag{21}$$

F is dimx by dimx matrix and here it is 9x9.

$$F = \begin{bmatrix} I & \Delta t I & 0 \\ 0 & I & 0 \\ 0 & 0 & I - \Delta t \omega^\times \end{bmatrix} \tag{22}$$

Finally, comp

A. Abbreviations and Acronyms

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B. Units

C. Equations

The equations
Note

D. Some Common Mistakes

V. USING THE TEMPLATE

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A. Headings, etc

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B. Figures and Tables

Positioning Figure

VI. CONCLUSIONS

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TABLE I

AN EXAMPLE OF A TABLE

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Three	Four

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APPENDIX

Appendixes should appear before the acknowledgment.

ACKNOWLEDGMENT

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References are

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