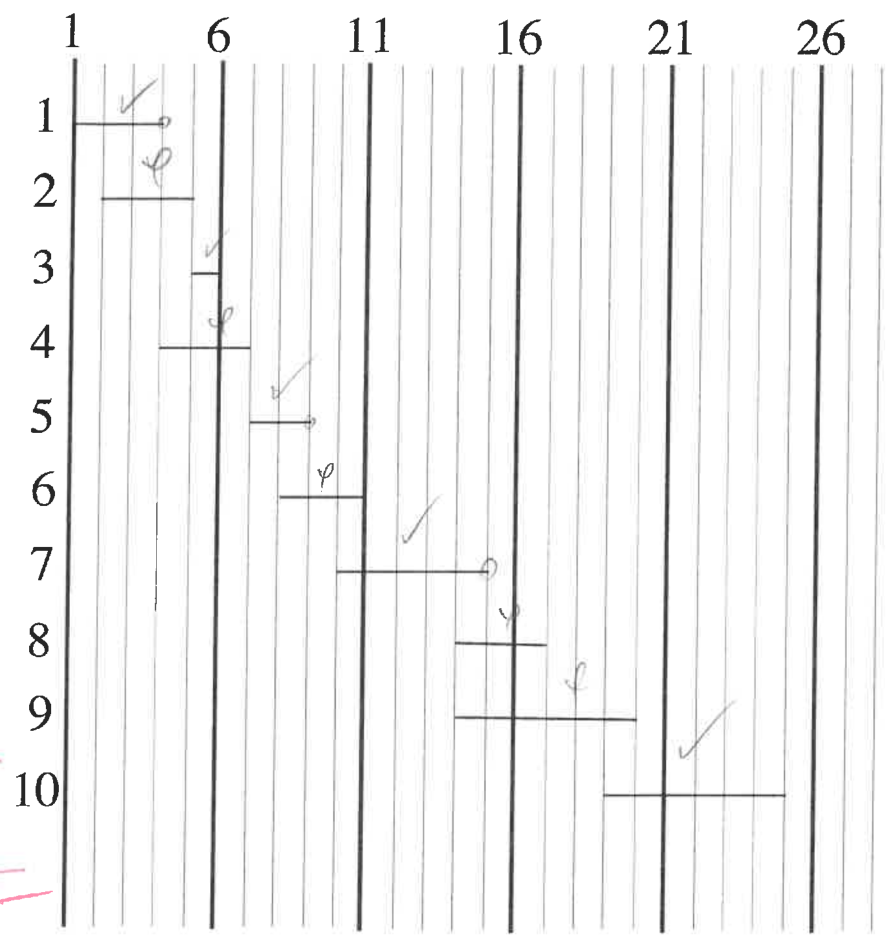
Name Bardia Majra

CSE 5311
Test 1 - Open Book
Spring 2021

1. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 15 points



i	v_i	p_i	$m(i)$
1	3	0	3
2	3	0	3
3	1	2	4
4	3	1	6
5	2	4	~~6~~ 8
6	3	4	9
7	5	5	~~11~~ 13
8	3	6	13
9	6	6	15
10	6	8	~~17~~ 19

$P_1 + P_3 + P_5 + P_7 + P_{10}$

$3 + 1 + 2 + 5 + 6 = 17$

6
6
7
12
6
4
4

45

55

6

2. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{2}\right) + n^2$ is in $O(n^2)$. (15 points)

$T(n) \leq ck^2$; $k < n$

$T(n/2) \leq c\frac{n^2}{4}$

$T(n) = 2T(n/2) + n^2$

$T(n) \leq \frac{\cancel{2}cn^2}{\cancel{4}\,2} + n^2$

$T(n) \leq \frac{c}{2}n^2 + n^2$ → extra

$\leq cn^2$

got what we started with

$\Rightarrow O(n^2)$

6

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (15 points)

```
n=6
q[0]=0.000000
key[1]=1
p[1]=0.030000
q[1]=0.090000
key[2]=2
p[2]=0.100000
q[2]=0.040000
key[3]=3
p[3]=0.120000
q[3]=0.020000
key[4]=4
p[4]=0.010000
q[4]=0.120000
key[5]=5
p[5]=0.200000
q[5]=0.030000
key[6]=6
p[6]=0.200000
q[6]=0.040000
w[0][0]=0.000000
w[0][1]=0.120000
w[0][2]=0.260000
w[0][3]=0.400000
w[0][4]=0.530000
w[0][5]=0.760000
w[0][6]=1.000000
w[1][1]=0.090000
w[1][2]=0.230000
w[1][3]=0.370000
w[1][4]=0.500000
w[1][5]=0.730000
w[1][6]=0.970000
```

```
w[2][2]=0.040000
w[2][3]=0.180000
w[2][4]=0.310000
w[2][5]=0.540000
w[2][6]=0.780000
w[3][3]=0.020000
w[3][4]=0.150000
w[3][5]=0.380000
w[3][6]=0.620000
w[4][4]=0.120000
w[4][5]=0.350000
w[4][6]=0.590000
w[5][5]=0.030000
w[5][6]=0.270000
w[6][6]=0.040000
Building c(0,2) using roots 1 thru 2
Building c(1,3) using roots 2 thru 3
Building c(2,4) using roots 3 thru 4
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 2 thru 2
Building c(1,4) using roots 2 thru 3
Building c(2,5) using roots 3 thru 5
Building c(3,6) using roots 5 thru 5
Building c(0,4) using roots 2 thru 3
Building c(1,5) using roots 3 thru 5
Building c(2,6) using roots 5 thru 5
Building c(0,5) using roots 3 thru 3
Building c(1,6) using roots 3 thru 5
Building c(0,6) using roots ? thru ?
Counts - root trick 30 without root
    trick 50
Average probe length is ????
```

```
trees in parenthesized prefix
c(0,0) cost 0.000000
c(1,1) cost 0.000000
c(2,2) cost 0.000000
c(3,3) cost 0.000000
c(4,4) cost 0.000000
c(5,5) cost 0.000000
c(6,6) cost 0.000000
c(0,1) cost 0.120000 1
c(1,2) cost 0.230000 2
c(2,3) cost 0.180000 3
c(3,4) cost 0.150000 4
c(4,5) cost 0.350000 5
c(5,6) cost 0.270000 6
c(0,2) cost 0.380000 2(1,)
c(1,3) cost 0.550000 2(,3)
c(2,4) cost 0.460000 3(,4)
c(3,5) cost 0.530000 5(4,)
c(4,6) cost 0.860000 5(,6)
c(0,3) cost 0.700000 2(1,3)
c(1,4) cost 0.880000 3(2,4)
c(2,5) cost 1.000000 5(3(,4),)
c(3,6) cost 1.040000 5(4,6)
c(0,4) cost 1.060000 3(2(1,),4)
c(1,5) cost 1.490000 3(2,5(4,))
c(2,6) cost 1.510000 5(3(,4),6)
c(0,5) cost 1.670000 3(2(1,),5(4,))
c(1,6) cost 2.120000 5(3(2,4),6)
c(0,6) cost ???????? ??????????????
```

Not a BST

3: C(0,2) + C(3,6) . W[0][6]

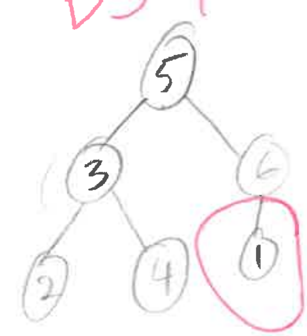
.38 1.04 1 = 1.42

4: C(0,3) + c(4,6) . W[0][6] = ?

.2 .86 1 = 1.56

5: C(0,4) + C(5,6) W[0][6] = $\boxed{1.33}$

1.06 + .27 1



7

4. Suppose all $2^k - 1$ nodes ($k \geq 3$), along with the sentinel, in a red-black tree are colored black. Explain what will happen if any key is deleted. (15 points)

① Delete the node

② fix the child to its parent

or

if has multiple

make a rotation ?

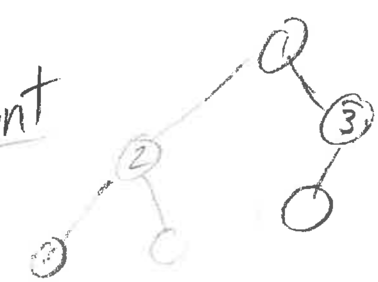
③ Connect the child to the parent

④ Do some adjustment & rotation to balance the height of tree.

⑤ Check for Colors

↳ here we Dont need to Convert node 3 from black to red

↳ Because it has left



12

CSE 5311
Test 1 - Closed Book
Spring 2021

Name Bardia Mojra

Multiple Choice:

1. Write the letter or value of your answer on the line (______) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. Which of the following is not true regarding the amortized analysis of binary tree traversals?

A A. INIT had an amortized cost of 1.
B. SUCC had an actual cost determined by the number of edges followed.
C. SUCC had an amortized cost of 2.
D. The potential was defined with regard to the type of traversal being performed.

2. The expected number of rolls to get a 3 with a fair conventional six-sided (1 .. 6) die is:

6 6

3. Suppose you already have 16 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 16 you already have?

5 $\frac{20}{4}$

4. When is path compression used?

~~B~~ C A. With a UNION operation.
B. After an insertion into a splay tree.
C. With a FIND operation.
D. After an insertion into any type of balanced binary search tree.

5. What is the worst-case number of rotations when performing deletion on an AVL tree?

B ~~A~~ A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. No rotations are ever needed

6. To reduce the probability of having any collisions to < 0.5 when hashing n keys, the table should have at least this number of elements.

C A. n B. $n \ln n$ C. n^2 D. n^3

7. How many inversions are there for the lists ~~5~~, 1, 2, 3, 4 and 1, 2, ~~5~~, 4, 3?

3

5 > 2
3 > 1

8. In the worst case, the number of rotations for inserting a key in a treap with n keys is:

C ~~B~~ A. $\theta(1)$ B. $\theta(\log n)$ C. $\theta(n)$ D. $\theta(n \log n)$

6

9. Which property does not hold for binomial heaps?

~~A~~ B ___ A. Performing n INSERT operations into an empty heap will take $O(n)$ time.
B. MINIMUM takes $O(1)$ time.
C. The number of trees is based on the binary representation of the number of stored items.
D. DECREASE-KEY takes $O(\log n)$ time.

10. What is the main contribution of leftist heaps?

B ___ A. The height of the tree is $O(\log n)$.
B. The UNION is computed in $O(\log n)$ time.
C. The MINIMUM is found in $O(1)$ time.
D. The amortized complexity of DECREASE-KEY is $O(1)$.

11. What is the purpose of the X pointer in the four cases for red-black tree deletion?

~~C~~ A ___ A. To designate a node with an "extra" black color.
B. To designate a node whose key must eventually be removed.
C. To designate the node that is the successor of the node whose key is to be removed.
D. To designate a node that will be the target of a rotation.

12. The main difference between MTF and OPT for self-organizing linear lists is:

D ___ A. MTF can do transpositions
B. MTF is given the entire request sequence in advance, while OPT receives the requests one-at-a-time
C. OPT counts inversions
D. OPT is given the entire request sequence in advance, while MTF receives the requests one-at-a-time

13. Assuming a random n-permutation is provided, the expected number of hires for the hiring problem is:

A ___ A. H_n B. 2 C. n^2 D. $\ln \ln n$

14. Which priority queue is defined using the notion of null path length?

A ___ A. Leftist heap B. Binomial heap C. Pairing heap D. Binary heap

15. When performing selection in worst-cast linear time for n numbers, roughly how many column medians are computed in the first round?

A ___ A. $n/5$ B. m, the median-of-medians C. $0.7n$ D. $W(n/5)$

16. During which operation on a leftist heap may subtree swaps be needed?

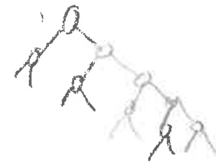
D ___ A. DECREASE-KEY B. EXTRACT-MIN
C. UNION D. All of A., B., and C.

-4

17. If a Fibonacci tree appears as a subtree of an AVL tree, which nodes would be assigned a balance factor of 0?

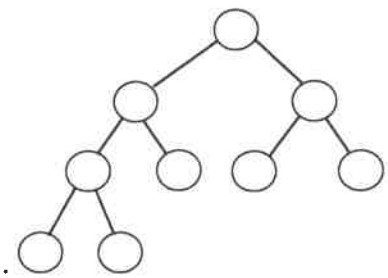
~~D~~ B

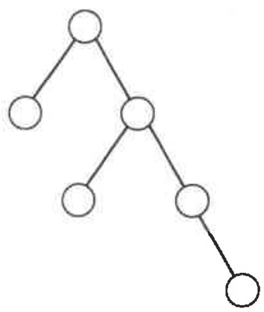
A. none of them B. only the leaves
C. only the root D. the leaves and the root

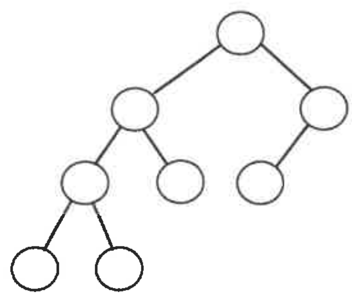


18. Which of the following binary trees has *multiple* legal colorings as a red-black tree?

B A. 

B. 

C. 

D. 

19. Brent's rehash improves the retrieval performance of:

C A. linear probing B. perfect hashing C. double hashing D. Bloom filters

20. Pairing heaps are a practical alternative to:

~~D~~ A. binary heaps B. binomial heaps C. Fibonacci heaps D. leftist heaps

C

4