

EE 5322- Intelligent Control Systems

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Simulation of Stochastic Discrete Time Systems

The discrete time stochastic system

$$x_{k+1} = Ax_k + Bu_k + Gw_k, \quad x_0$$

with w_k the process noise, is simulated in MATLAB using the M file

```
function [ki,x]=dtsystem(a,b,g,x0,u);
N=size(u); N=N(1); ki(1)= 1;
n=size(x0); n=n(1);
x=zeros(N,n); x(1,:)=x0';
for k=1:N-1
    ki(k+1)=k+1;
    x(k+1,:)= (a*x(k,:) + b*u(k,:) + g*rand)';
    % x(k+1)=a*x(k) + b*u(k) + g*w(k);
end
ki=ki';
```

Note that the state vector is stored, e.g., as

```
x =
      0      0
0.1790  1.1790
1.3675  3.1514
3.2184  5.5225
5.6099  8.1635
8.2577 10.7957
10.8255 13.1127
13.1399 14.9953
15.1018 16.4036
```

which is a 2-D state vector for $k=1,2,3,4,5,6,7,8,9$. That is, one has

$$\begin{matrix} x_1^T \\ x_2^T \\ \vdots \end{matrix}$$

over the time interval $k=[1,2,\dots,N]$ stored as an $N \times n$ matrix, with $x_k \in R^n$.

Prior to using `dtsystem`, one must generate the initial condition x_0 and the control input u_k , stored like the state above. i.e. the m -vector control input over the k interval of interest $[1,N]$ is stored as an $N \times m$ matrix. The script code needed is, e.g.,

```
>> a=[0 1
      -.89 1.8];
```

```

>> b=[0 1]' ;

>> g=[1 1]' ;

>> x0=[0 0]' ;

>> k=[1:1:200]' ;

>> u=ones(size(k)) ;

>> [ki,x]=dtsystem(a,b,0.2*g,x0,u);
>> plot(ki,x)

```

Generating Random Noise in MATLAB

The two main MATLAB random noise generators are:

1. RAND Uniformly distributed random numbers.

RAND(N) is an N-by-N matrix with random entries, chosen from a uniform distribution on the interval (0.0,1.0).
 RAND(M,N) and RAND([M,N]) are M-by-N matrices with random entries.
 RAND(M,N,P,...) or RAND([M,N,P,...]) generate random arrays.
 RAND with no arguments is a scalar whose value changes each time it is referenced. RAND(SIZE(A)) is the same size as A.

RAND produces pseudo-random numbers. The sequence of numbers generated is determined by the state of the generator. Since MATLAB resets the state at start-up, the sequence of numbers generated will be the same unless the state is changed.

2. RANDN Normally distributed random numbers.

RANDN(N) is an N-by-N matrix with random entries, chosen from a normal distribution with mean zero, variance one and standard deviation one.
 RANDN(M,N) and RANDN([M,N]) are M-by-N matrices with random entries.
 RANDN(M,N,P,...) or RANDN([M,N,P...]) generate random arrays.
 RANDN with no arguments is a scalar whose value changes each time it is referenced. RANDN(SIZE(A)) is the same size as A.

RANDN produces pseudo-random numbers. The sequence of numbers generated is determined by the state of the generator. Since MATLAB resets the state at start-up, the sequence of numbers generated will be the same unless the state is changed.

Obtaining the System Output

The system output $z_k \in R^p$

$$z_k = Hx_k + v_k$$

can be obtained by adding lines to the m-file dtsystem, or by using dtsystem to generate x_k and then using

$$Y = X * H^T$$

where X is the $N \times n$ matrix above. This batch method has problems adding the measurement noise.