CSE 5301 - HW02

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Exercise 1

We have a random variable Z which belongs to the binomial distribution. Suppose the probability of the success is 0.2 and number of experiments is 12, what is the probability for each one of the below statements: (5 points each)
Binomial distribution:

$$P_z = P(Z = z) = \binom{n}{z} p^z (1-p)^{n-z}\ ; \quad z \in \mathbb{Z}\ (or\ z \in \{\mathbb{N}, 0\})$$

$$P_z = \frac{n!}{z!(n-z)!}\ .\ p^z\ .\ (1-p)^{n-z}$$

Where:

$$p = 0.2,\ \ n = 12$$

a) P(Z=5)

$$P(Z = 5) = \frac{n!}{z!(n-z)!}\ .\ p^z\ .\ (1-p)^{n-z} \Longrightarrow \frac{12!}{5!(12-5)!}\ .\ (.2)^5\ .\ (.8)^7\ \ =\ \ 0.0532$$

b) P(Z<3)

$$P(Z < 3) = P(0) + P(1) + P(2) = \frac{12!\ (.2)^0\ (.8)^{12}}{0!(12)!} + \frac{12!\ (.2)^1\ (.8)^{11}}{1!(11)!} + \frac{12!\ (.2)^2\ (.8)^{10}}{2!(10)!};\ \ 0! \equiv 1$$

$$\Longrightarrow .0687 + 0.2061 + .2835 = .5583$$

c) P (Z>=7)

$$P(Z \geqslant 7) = P(7) + P(8) + P(9) + P(10) + P(12) =$$

$$\frac{12!\ (.2)^7(.8)^5}{7!(5)!} + \frac{12!\ (.2)^8(.8)^4}{8!(4)!} + \frac{12!\ (.2)^9(.8)^3}{9!(3)!} + \frac{12!\ (.2)^{10}(.8)^2}{10!(2)!} + \frac{12!\ (.2)^{11}(.8)^1}{11!(1)!} + \frac{12!\ (.2)^{12}(.8)^0}{12!(0)!}$$

$$.0033 + .0005 + .000057672 + .000004325 + .000000197 + .000000004\ \ =\ \ .003862198$$

Exercise 2

Suppose that we investigated a fast food chain and saw that on average 6 out of 10 customers order drinks along with their meals. We choose 20 customers randomly, find the probability that : (5 points each)

$$P_z = \frac{n! \,.\, p^z \,.\, (1-p)^{n-z}}{z!(n-z)!}; \; n = 20, \; p = .6$$

a) Exactly 10 customer order drinks with their meals.

$$P_{10} = \frac{20! \,.\, (.6)^{10} \,.\, (.4)^{10}}{10!(10)!} = .117141551$$

b) More than 15 customer order drinks with their meals.

$$P_{>15} = \frac{20! \,.\, (.6)^{16} \,.\, (.4)^{4}}{16!(4)!} + \frac{20! \,.\, (.6)^{17} \,.\, (.4)^{3}}{17!(3)!} + \frac{20! \,.\, (.6)^{18} \,.\, (.4)^{2}}{18!(2)!} + \frac{20! \,.\, (.6)^{19} \,.\, (.4)^{1}}{19!(1)!} + \frac{20! \,.\, (.6)^{20} \,.\, (.4)^{0}}{20!(0)!}$$

$$\Longrightarrow \; .0349 + .01234 + .00309 + .00049 + .000036 = .050856$$

c) What is the variance of the number of drinks?

$$Var_{binomial} = n.p.(1-p) \Longrightarrow 20 \cdot .6 \cdot .4 = 4.8$$

d) Name the distribution
Binomial distribution.

Exercise 3

We have a deck of cards (52 cards) and we draw 6 cards randomly without replacement. calculate the probability that: (5 points each)

a) We have 4 red cards.

$$\frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} \cdot \frac{23}{49} \;=\; .0552 = 5.52\%$$

b) We have 3 face cards.

$$\frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} \;=\; .00995 \;=\; .995\%$$

c) Name the distribution.
Normal distribution.

Exercise 4

Suppose that we saw the power outage on average rate of 4 per month. What is the probability that in the next 2 months we see : (5 points each)

$$P(K = k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}\ ;\ \ \lambda \ = \ 4\, per\ month \ = \ 8\ per\ 2\ months$$

a) Exactly 8 power outage?

$$P(k = 8) = e^{-8} \cdot \frac{8^8}{8!} = .13958$$

b) At most 4 power outage?

$$P(k \leqslant 4) = P(0) + P(1) + P(2) + P(3) + P(4)$$

$$\Longrightarrow e^{-8} \cdot \frac{8^0}{0!} + e^{-8} \cdot \frac{8^1}{1!} + e^{-8} \cdot \frac{8^2}{2!} + e^{-8} \cdot \frac{8^3}{3!} + e^{-8} \cdot \frac{8^4}{4!} \ = \ .00034 + .00268 + .01073 + .02863 + .05725 \ = \ .09963$$

c) What is the expected value of the number of power outage?

$$MLE_{Poisson} = \mu\ [mean] = \lambda = 4\ \ per\ \ month\ or\ 8\ for\ the\ next\ 2\ months$$

d) Name the distribution
Poisson distribution.

Exercise 5

We are examining a product which was made by a company and we noticed that the product is defective with the probability of 20%. Calculate the below statements: (5 points each)
Geometric distribution:

$$PDF_{Geometric} = P(Z = z) = q^{z-1} \cdot p; \quad z \in \mathbb{N}$$

$$CDF_{Geometric} = P(Z \leqslant z) = 1 - q^{z}; \quad z \in \mathbb{N}$$

a) The probability that the first defective product is the 4th one than we examine.

$$P(z = 4) = (.8)^3 \cdot .2 = .1024$$

b) Calculate the probability that we find the first defective product in the first 5 inspections?

$$P(.) = P(1) + P(2) + P(3) + P(4) + P(5) = 1 - (.8)^5 = .67232$$

c) Name the distribution.
Geometric distribution.

Exercise 6

We have a fair die which has 4 faces (1, 2, 3, 4) and our random variable X shows the number when the die is rolled. (5 points each)
a) What is the probability mass function for the random variable x?

$$P(X = x) = \frac{1}{4}$$

b) What is the expected value of X?

$$\mu = \frac{1 + 2 + 3 + 4}{4} = 2.5$$

c) Name the distribution
Uniform distribution.