Q3.

$$D = \{ ((3,4), -1), ((2,3), -1), ((2,1), 1), ((1,2), 1), ((1,3), 1), ((4,4), -1) \}$$

→ assuming inputs as feature: we setup the kernel

$$\phi(x) = x \quad \& \quad K(x_1, x_2) = x_1^T x_2$$

↳ assume normalization of Distance w/ decision boundry

$$t_n((w^T x_n) + b) = 1$$

$$\Rightarrow t_n(w^T(x_n) + b) \geq 1$$

Set up the lagrangian

$$\mathcal{L}(w,b,a) = \frac{1}{2}\|w\|^2 - \sum_{n=1}^{N}\left(t_n(w^T x+b)-1)\right)$$

$$\rightarrow \frac{dL}{dw} = w - \frac{d}{dw}\sum_{n=1}^{N} t_n(w^T x+b))-1)$$

$$\Rightarrow \frac{dL}{dw} = w - \frac{d}{dw}\left(\sum_{n=1}^{N} a_n t_n w^T x_n + \sum_{n=1}^{N} a_n t_n b - \sum_{n=1}^{N} a_n\right) = 0$$

$$w = \cancel{\frac{d}{dw}}\left(\sum_{n=1}^{N} a_n t_n x_n\right) \Rightarrow w = \sum_{n=1}^{N} a_n t_n x_n$$

<u>For b</u>

$$\frac{d}{db}L = \frac{d}{db}\sum_{n=1}^{N} a_n t_n w^T x_n + \sum_{n=1}^{N} a_n t_n b - \sum_{n=1}^{N} a_n$$

$$\rightarrow \sum_{n=1}^{N} a_n t_n = 0$$

Now back substitute ~~for~~ w & b into $\mathcal{L}(w,b,a)$

$$L = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_i t_i a_j t_j k(x_i; x_j) + \sum_{i=1}^{N} a_n - \sum_{i=1}^{N}\sum_{j=1}^{N} a_i t_i a_j t_j k(x_i; x_j)$$

$$= \boxed{\sum_{i=1}^{N} a_n - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_i t_i \cdot a_j t_j k(x_i; x_j)}$$

3.b

$$XY = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & y \end{array} \\ \begin{bmatrix} 3, & 4 & \cancel{-1} \\ \cancel{3}2, & 3, & \cancel{+1} \\ 2, & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 1 \\ 4 & 4 & \cancel{-1} \end{bmatrix} \end{array} \begin{matrix} -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \end{matrix}$$

$X_1 = [3, 4]$, $Y_1 = -1$

$X_2 = [2, 3]$, $Y_2 = -1$

$\Rightarrow \eta = k(x_1, x_1) + k(x_2, x_2) - 2k(x_1, x_2) =$

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} [3 \ \ 4] + \begin{bmatrix} 2 \\ 3 \end{bmatrix} [2 \ \ 3] - 2\left(\begin{bmatrix} 3 \\ 4 \end{bmatrix} [2 \ \ 3] \ \ \cancel{[3, 4]} \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right)$$

$$18 \ \cancel{9 + 16} \quad + \quad 4 + \cancel{9} \quad - 2(\cancel{6 + 12}) \ 18$$

$$38 \qquad - 36 \quad = \quad 2$$

$$\begin{cases} L = \max(0, a_2 - a_1), \ H = \min(C, C + a_2 - a_1); & ; y_1 \neq y_2 \\ L = \max(0, a_2 + a_1 - C), \ H = \min(C, a_2 + a_1) & ; y_1 = y_2 \end{cases}$$

So that $y_1 = y_2$ can used as

$$L = \max(0, -1) = 0, \ H = \min(1, 0) = 1$$

now that; $a_2^{new} = a_2 + \dfrac{y_2(E_1 - E_2)}{\eta}$

$$\Rightarrow a_2^{new} = \overset{0}{a_2} + \frac{y_2\left((\overset{0}{\hat{y}_1^{-1}} - y_1) - (\overset{0}{\hat{y}_2^{-1}} - y_2)\right)}{\eta}$$

$\rightarrow$ all zero since at initial condition

$$a_2^{new} = 0 + \frac{(\cancel{+1}(\cancel{0}-0))}{2} = 0 + \frac{1}{2}, \quad [L, H] = [0, 1]$$

clipping

$$\alpha_1^{new} = a_1 + y_1 y_2 (a_2 - a_2^{new\ 1/2}) \Rightarrow$$

$+1$

$$\underline{a_1^{new} = \cancel{0}\ 1/2}$$

Now find b_1 & b_2

$b=0$ $\boxed{IC}$

$$b_1 = (\hat{y}_1 - y_1) + y_1(\cancel{a_1^{new\,0} - a_1})(k(x_1, x_1)) + y_2(a_2^{new} - a_2)k(x_1, x_2) + b$$

$$b_2 = (\hat{y}_2 - y_2) +$$

$$\Rightarrow b_1 = \cancel{(0-(-1))}^{+1} + \cancel{(-1)(1/2 - 0)}^{-1/2}\cdot \cancel{36}^{.18} + \cancel{(-1)}(-1/2)^{+1/2}.18 + 1$$

-9 $\quad 9$

$b_1 = 0$

$$W = W + (y_1)(a_1^{new} - a_1)x_1 + y_2(a_2^{new} - a_2)x_2$$

$$\underline{b_2 = 0}$$

$$W^{new} = [0\ 0]^T - (-1)(+1/2)[3\ 4] + (-1)(+1/2)[2\ 3]$$

$-1.5 \quad 2 \qquad 1 \quad 1.5$

$$W^{new} = [-2.5\ \ -3]$$

Second Iteration

$$a = [0, 0, 0 0 0 0], \quad b = -1; \quad w = [-2.5 \; -3]$$

$$x_1 = [1, 2] \quad x_2 = [1 \; 3] \quad y_1 = 1; \; y_2 = 1$$

$$\eta = k(x_1 x_1) + k(x_2 x_2) - 2k(x_1, x_2)$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} [1 \; 2] + \begin{bmatrix} 1 \\ 3 \end{bmatrix} [1 \; 3] - 2\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} [1 \; 3]\right.$$

$$(\cancel{1+4}) + (\cancel{1+9}) - 2(\overset{7}{\cancel{1+6}})$$

$$5 + 10 - 14 = \underline{1} \; ; \; \eta = 1$$

$$a_2^{new} = a_2 + \frac{\cancel{y_2((\overset{0}{y_1} - \overset{1}{y_1}) - (\overset{0}{y_2} - y_2))}}{\eta} = a_2 = 1/2$$

$$a_1^{new} = a_1 + y_1 y_2 (\underset{0}{\cancel{a_2}} - \underset{0}{\cancel{a_2^{new}}}) =$$

$$= 1/2 + (1)(1) \quad \Rightarrow a_1^{new} = 1/2$$

New b_1 & b_2

$$b_1 = (\hat{y}_1 - y_1) + y_1(\alpha_1^{new} - a_1)k(x_1 x_1) + y_2(\hat{a}_2 - a_2)k(x_1 x_2) + b$$

$$b_2 = (\hat{y}_2 - y_1) + \cdots \quad ''$$

$$b_1 = (\overset{-1}{\cancel{0 - 1}}) + (1)(\overset{0}{\cancel{1/2 - 1/2}}) \cdot (5) + (1)(\overset{0}{\cancel{0}}) \cdot (7) + 1$$

$b_1 = 0$?

$b_2 = 0$

$$w^{new} = \cancel{[0\ 0]}\ [-2.5\ \ -3] - (+1)(1/2)\overset{[-1/2\ \ -1]}{[1\ 2]}$$

$$+ (+1)(1/2)[1\ \ 3]$$

$$[+1/2\ \ 3/2]$$

$$w^{new} = \left[(-2.5 - 1/2 + 1/2)\ \ (\overset{-4}{\cancel{-3 - 1}} + 3/2)\right] \quad \overset{2.5}{\cancel{1.5}}$$

$\rightarrow w^{new} = [-2.5\ \ 2.5]$, $b = 0$