ex 1

$$\dot{x}_1 = x_1 x_2^2 - x_1$$
$$\dot{x}_2 = -x_1^2 x_2 + u$$

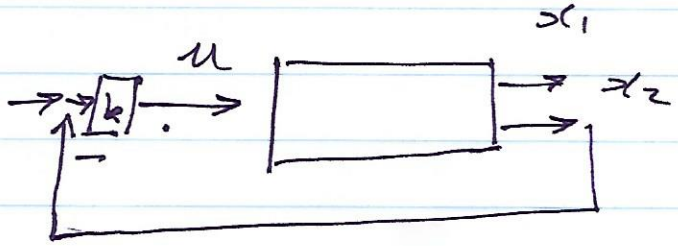
$$V = \frac{1}{2}(x_1^2 + x_2^2)$$
$$\dot{V} = x_1 \dot{x}_1 + x_2 \dot{x}_2$$
$$= x_1 (x_1 x_2^2 - x_1) + x_2 (-x_1^2 x_2 + u)$$
$$= \cancel{x_1^2 x_2^2} - x_1^2 - \cancel{x_1^2 x_2^2} + u x_2$$

$$\dot{V} = -x_1^2 + u x_2$$

$\dot{V} \leq 0$ if $u = 0 \Rightarrow$ SISL

select $u = -k x_2$

then $\dot{V} = -x_1^2 - k x_2^2 < 0 \Rightarrow$ AS



proportional feedback Regulator, $x \to 0$

Ex 3 $\dot{x}_1 = x_1 x_2^2 + u$

$\dot{x}_2 = -x_1^2 x_2 - x_2$

$V = \frac{1}{2}(x_1^2 + x_2^2)$

$\dot{V} = x_1 \dot{x}_1 + x_2 \dot{x}_2$

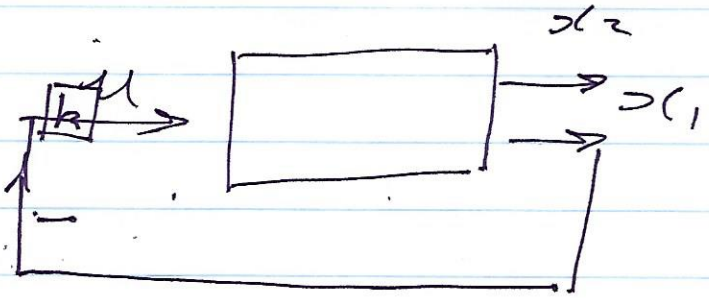
$= x_1 (x_1 x_2^2 + u) + x_2 (-x_1^2 x_2 - x_2)$

$= x_1 u - x_2^2$

$\dot{V} \leq 0$ if $u = 0 \Rightarrow$ SISL

if $u = -k x_1$

$\dot{V} = -k x_1^2 - x_2^2 < 0 \Rightarrow AS$



Feedback loop

Regulator, $x \to 0$

#2

$$\dot{x}_1 = x_1 x_2^2 - x_1$$
$$\dot{x}_2 = u$$

LFC $\quad V = \frac{1}{2}(x_1^2 + x_2^2)$

$$\dot{V} = x_1\dot{x}_1 + x_2\dot{x}_2$$
$$= x_1(x_1 x_2^2 - x_1) + x_2 u$$
$$\dot{V} = x_1^2 x_2^2 - x_1^2 + x_2 u$$

select $u = -x_1^2 x_2 - kx_2$

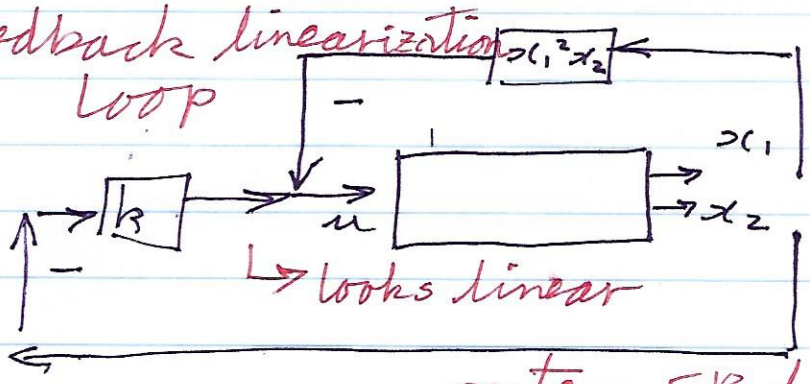
GROBS $\quad$ PIGS

$$\dot{V} = -x_1^2 - kx_2^2 \quad < 0 \Rightarrow AS$$

CALCULATE NONLINEARITIES

Feedback linearization loop

inner FB loop.



outer FB loop

outer P regulation loop

Feedback Linearization

Regulator $x \to 0$

ex4 ex3.23 S+L p-94

$$\dddot{x} - \dot{x}^3 + x^2 = u \quad , \quad \dddot{x} = \dot{x}^3 - x^2 + u \quad (1)$$

$$V = \frac{1}{2}(x^2 + \dot{x}^2)$$

$$\dot{V} = x\dot{x} + \dot{x}\ddot{x}$$

$$= x\dot{x} + \dot{x}(\dot{x}^3 - x^2 + u)$$

$$= \dot{x}(x + \dot{x}^3 - x^2 + u)$$

$(\dot{x} b(\dot{x}) > 0 \quad x c(x) > 0)$

cf $\ddot{x} + b(\dot{x}) + c(x) = 0$

set $u = -x + x^2 - 2\dot{x}^3 = \underbrace{-x + x^2 - \dot{x}^3}_{\text{GROBS}} \underbrace{- \dot{x}^3}_{\text{PICS}}$

$$\dot{V} = \dot{x}(-\dot{x}^3) = -\dot{x}^4 \leq 0 \quad \text{SISL}$$

LaSalle ext.

$\dot{V} \to 0 \Rightarrow \dot{x} \to 0$

put into (1) 1) closed-loop system

$$\Rightarrow \dddot{x} = \dot{x}^3 - x^2 + u$$

$$= \dot{x}^3 - x^2 - x + x^2 - 2\dot{x}^3 = -\dot{x}^3 - x$$

$$\ddot{x} + \dot{x}^3 + x = 0 \quad , \quad \ddot{x} + b(\dot{x}) + c(x) = 0$$

2) LaSalle

$\dot{V} \to 0, \dot{x} \to 0$

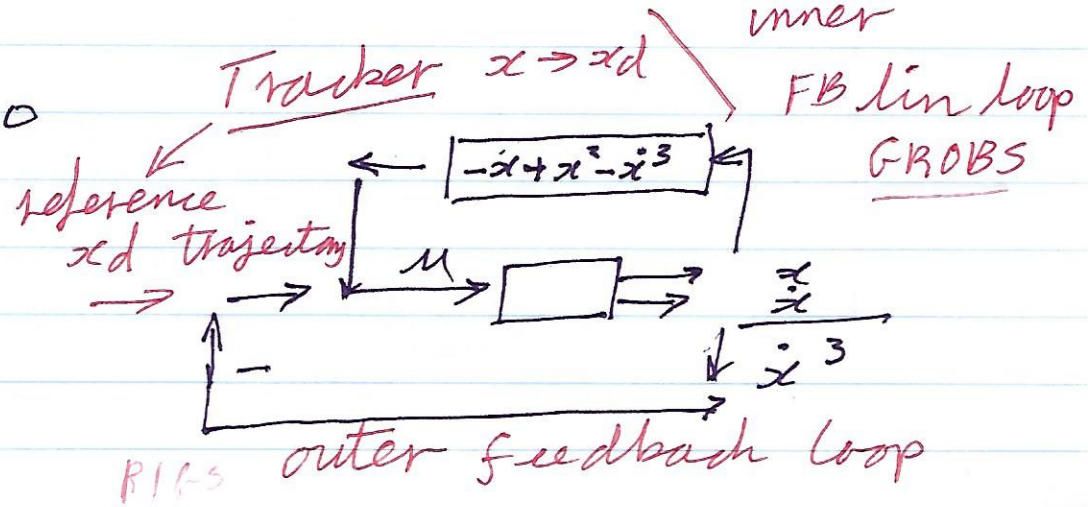
$\ddot{x} = -x$

$\Rightarrow$ AS

Regulator

$x \to 0$

$\dot{x} \to 0$



Tracker (vs. Regulator)

Ex 5 Ex 3.23.1 S&L p. 94

$\dddot{x} - \ddot{x}^3 + x^2 = u$, $\dddot{x} = +\ddot{x}^3 - x^2 + u$

Desired trajectory $x_d(t)$

Tracking error $e = x_d - x$

Error Dynamics $\dot{e} = \dot{x}_d - \dot{x}$

$\ddot{e} = \ddot{x}_d - \ddot{x} = \ddot{x}_d - (\ddot{x}^3 - x^2 + u)$ (1)

LFC $V = \frac{1}{2}(e^2 + \dot{e}^2)$

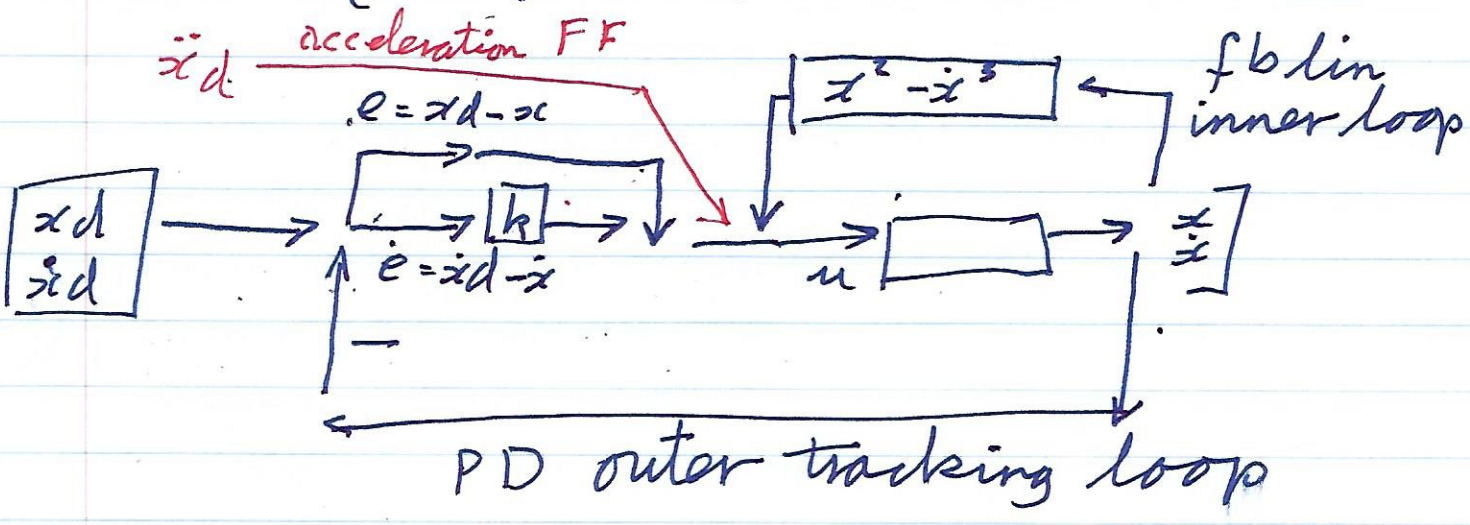
$\dot{V} = e\dot{e} + \dot{e}\ddot{e}$

$= \dot{e}(e + \ddot{x}_d - \ddot{x}^3 + x^2 - u)$

GROBS PIGS

$u = \ddot{x}_d + e + \ddot{x}^3 + x^2 + k\dot{e}$

$\dot{V} = \dot{e}(-k\dot{e}) = -k\dot{e}^2 \leq 0$ SISL



Feed Back Linearization Tracker

$\dot{x} = f(x) + g(x)u$

$e = x_d - x \quad , \quad \dot{e} = \dot{x}_d - \dot{x}$

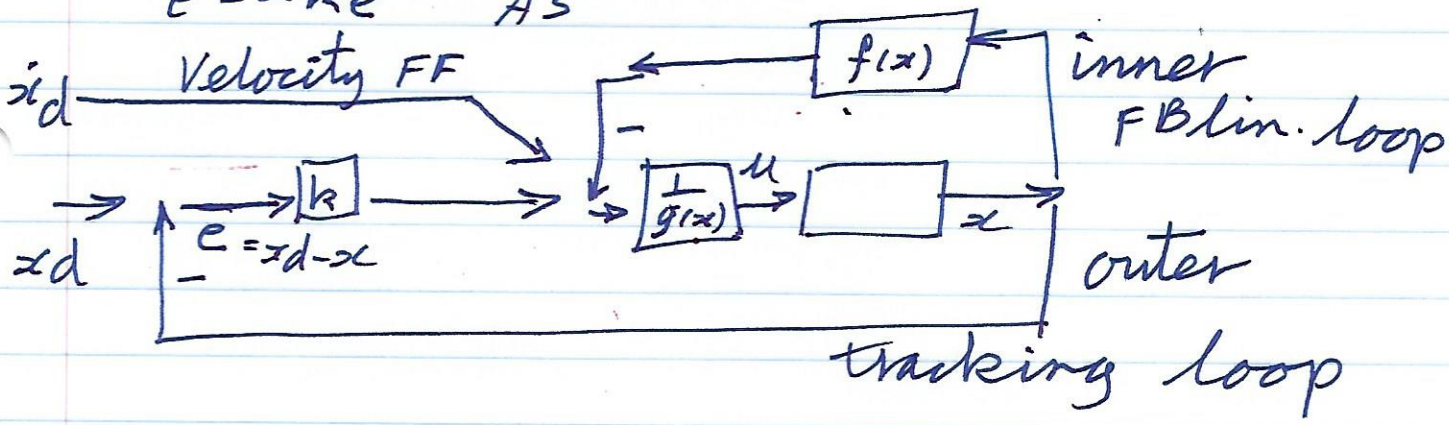
$\dot{e} = \dot{x}_d - f(x) - g(x)u$

select $u = \frac{1}{g(x)}(-f(x) + \dot{x}_d + ke)$

closed-loop system

$\dot{e} = \dot{x}_d - f - g\left(\frac{1}{g}(-f + \dot{x}_d + ke)\right)$

$\dot{e} = -ke \quad$ AS



practically

$$u = \begin{cases} \frac{1}{g(x)}(-f(x) + \dot{x}_d + ke) & , \|g(x)\| > \epsilon \\ \frac{1}{\epsilon} sgn(g)(-f + \dot{x}_d + ke) & , \|g(x)\| \leq \epsilon \end{cases}$$

$\Rightarrow$ UUB