EE 5323 Homework 5

Lyapunov Stability Analysis

1. Consider the system

$$\dot{x}_1 = x_1 x_2^2 - x_1$$
$$\dot{x}_2 = -x_1^2 x_2$$

Use Lyapunov to study the stability. SISL? AS?

2. Consider the system

$$\dot{x}_1 = x_1 x_2^2 - x_1$$
$$\dot{x}_2 = -x_1^2 x_2 - x_2$$

Use Lyapunov to study the stability. SISL? AS?

3. AS.
   a. Use Lyapunov to show that the system

$$\dot{x}_1 = x_1 x_2{}^2 + x_1 (x_1{}^2 + x_2{}^2 - 3)$$
$$\dot{x}_2 = -x_1{}^2 x_2 + x_2 (x_1{}^2 + x_2{}^2 - 3)$$

is locally asymptotically stable. Find the Region of Asymptotic Stability.

   b. Simulate the system from many uniformly spaced ICs.

4. SISL

   a. Use quadratic Lyapunov Function to show this system is locally SISL

$$\dot{x}_1 = x_2 + x_1 (x_1^2 - 2)$$
$$\dot{x}_2 = -x_1$$

Find a region within which $\dot{V} \le 0$.

   b. Simulate the system from many uniformly spaced ICs.