EE 5323 Homework 7

Lyapunov Controls Design, Feedback Linearization

1. **Controls Design.** A system is given by

$$\dot{x}_1 = x_2 \,\text{sgn}(x_1)$$
$$\dot{x}_2 = x_1 x_2 + u$$

Select Lyapunov function candidate

$$V(x) = \tfrac{1}{2}(x_1^2 + x_2^2)$$

Use Lyapunov to design a controller *u(x)* to make system SISL.

2. **Multi-input Control.** Use Lyapunov to design controls u_1, u_2 to make this system

$$\dot{x}_1 = x_1 x_2^2 + u_1$$
$$\dot{x}_2 = x_1^3 x_2^7 + u_2$$

a. SISL, and then
b. AS

3. (Slotine and Li problem 6.3) A system is given by

$$\dot{x}_1 = \sin x_2$$
$$\dot{x}_2 = x_1^4 \cos x_2 + u$$

with output $y(t) = x_1(\text{t})$

a. Design a FB linearization controller to make the output follow a desired trajectory $y_d(t)$ That is, find $u(t)$
b. Discuss the internal dynamics. Are they a problem?

4. Effect of Output Choice in i/o FB Linearization

It is desired to stabilize a system given by

$$\dot{x}_1 = x_2 \sin x_1 - x_1 + u$$
$$\dot{x}_2 = -x_1 + x_2^2$$

a. Select the output as $y = x_1$ and use FB lin. design to select the control *u(t)* to follow the desired trajectory $y_d(t)$. Check the internal dynamics. Set *y=0* to get the zero dynamics. Is the system minimum phase?

b. Select the new output $y = x_2$. Find the FB lin. controller u(t). Does this work? What about the internal dynamics?