Bardia Mayra

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1000766739

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ee5323

HW07

1

$$\dot{x}_1 = x_2 \sin(x_1)$$

$$\dot{x}_2 = x_1 x_2 + u$$

$$V(x) = \frac{1}{2}(x_1^2 + x_2^2) > 0$$

$$\dot{V} = \dot{x}_1 x_1 + \dot{x}_2 x_2 = 0 \Rightarrow$$

$$\dot{V} = (x_2 \sin(x_1))\, x_1 + (x_1 x_2 + u) x_2 = 0$$

$$\Rightarrow \underbrace{x_1 x_2 \sin(x_1)}_{\text{keep}} + \underbrace{x_1 x_2^2 + u x_2}_{\text{strike}} = 0$$

set $u = -x_1 x_2$

$$x_1 x_2 \sin(x_1) + \cancel{x_1 x_2^2} - \cancel{(x_1 x_2) x_2} = 0$$

$$x_1 x_2 \sin(x_1) = 0$$

SISL

2.

$\dot{x}_1 = x_1 x_2^2 + u_1$

$\dot{x}_2 = x_1^3 x_2^7 + u_2$

$V = \frac{1}{2}(x_1^2 + x_2^2)$; $V > 0$

$\dot{V} = \dot{x}_1 x_1 + \dot{x}_2 x_2 = 0$

$\dot{V} = x_1^2 x_2^2 + x_1 u_1 + x_1^3 x_2^8 + x_2 u_2 = 0$

SISL: keep $x_1^3 x_2^8$; toss $x_1^2 x_2^2$

$\boxed{\text{set } u_1 = -x_2^2 x_1 \quad \& \quad u_2 = 0}$

$\Rightarrow \dot{V} = \cancel{x_1^2 x_2^2} + x_1(-x_2^2 x_1) + x_1^3 x_2^8 + \cancel{x_2(0)} = 0$

$\cancel{-x_1^2 x_2^2}$

$\Rightarrow \dot{V} = x_1^3 x_2^8 = 0$ ~~LISL~~ SISL

AS: ~~toss~~ keep $x_1^2 x_2^2$; toss $x_1^3 x_2^8$

~~$\Rightarrow \dot{V}$~~ $\boxed{\text{set } u_1 = -x_1^2 x_2^8 \quad ; \quad u_2 = 0}$

$\dot{V} = x_1^2 x_2^2 + \cancel{x_1 u_1} + \cancel{x_1^3 x_2^8} + \cancel{x_2(0)} = 0 \Rightarrow \dot{V} = x_1^2 x_2^2 = 0$

$\cancel{-x_1^3 x_2^8}$

$\dot{V} \geq 0$ AS

3

3

$$\begin{cases} \dot{x}_1 = \sin x_2 \\ \dot{x}_2 = x_1^4 \cos x_2 + u \end{cases}$$

Design a tracker for $x_d(t)$; assume $x_d(t)$, $\dot{x}_d(t)$, $\ddot{x}_d(t)$ are known

$y = x_1$

$\dot{y} = \sin x_2$

$$\ddot{y} = \cos x_2 . \dot{x}_2 = \underbrace{x_1^4 \cos^2 x_2}_{f(x)} + \underbrace{u \cos x_2}_{u.g(x)}$$

$\ddot{e} = -k_v \dot{e} - k_p e$

need $\ddot{y} = -k_v \dot{y} - k_p y$ for stability

$$\text{select} \quad u = \underbrace{\frac{1}{g(x)} \left(-f(x) + \ddot{y}_d \underbrace{- k_v \dot{y} - k_p y}_{\text{fass}}\right)}_{\text{GROBS}}$$

Keep

closed loop system

$\hookrightarrow \ddot{y} = -k_v \dot{y} - k_p y$

$\Rightarrow \ddot{y} + k_v \dot{y} + k_p \dot{y} = 0$

pick $u = \frac{1}{g_1(x)}\left(-f_1(x) + \ddot{y}_d - k_v \dot{y} - k_p y\right)$; $\ddot{y}_d = 0$ default

$$u = \frac{1}{\cos x_2}\left(-x_1^4 \cos^2 x_2 + \ddot{y}_d - k_v \dot{y} - k_p y\right)$$

Internal Dynamics. $\dot{x}_2$

The plant is ~~first~~ second order; ~~the controller is also first order~~. The ~~or~~ over all system is second order (robust control) because the desired output is second order; thus we have a relative degree of ~~one~~ two which means the internal dynamics is also first order, because it is stable w/ a second order control system. There are no problems because we used Robust Control. $(\ddot{e} \to 0)$ $\therefore$ I.D. is stable.

4.a

$$\begin{cases} \dot{x}_1 = x_2 \sin x_1 - x_1 + u \\ \dot{x}_2 = -x_1 + x_2^2 \end{cases}$$

a. Pick $y = x_1$

$$\dot{y} = \dot{x}_1 = x_2 \sin x_1 - x_1 + \underline{u}$$

→ u showed up!
→ relative degree = 1

$e = y_d - y$; $\dot{e} = \dot{y}_d - \dot{y}$

$$\Rightarrow \dot{e} = \dot{y}_d + \dot{y} - x_2 \sin x_1 + x_1 - u$$

Pick $\boxed{u = \dot{y}_d - x_2 \sin x_1 + x_1 + k_p e}$

internal dynamics: $\dot{x}_2 = -x_1 + x_2^2$

is it stable? No, it compounds as a quadratic!

→ No need to check zero dynamics;
we need to select a different y.

4. b

$$\begin{cases} x_1' = x_2 \sin x_1 - x_1 + u \\ x_2' = -x_1 + x_2^2 \end{cases}$$

b. $y = x_2$; Find $u(t)$; does it work? I.D.?

$$y' = x_2' = -x_1 + x_2^2$$

$$y'' = -x_1' + 2x_2 x_2' \Rightarrow$$

$$y'' = -x_2 \sin x_1 + x_1 \underline{- u} + 2x_2(-x_1 + x_2^2)$$

↳ u showed up!

↳ Relative degree = $\underline{2}$

$$e = y_d - y \;;\; e' = y_d' - y' \;;\; e'' = y_d'' - y''$$

$$e'' = \underline{y_d''} + \underbrace{x_2 \sin x_1 - x_1} + u + \underbrace{2x_1 x_2 + 2x_2^3}$$

Pick

$$\boxed{u = y_d'' - x_2 \sin x_1 + x_1 \cancel{- 2x_1x_2 - 2x_2^3} + k_p e + k_v \dot{e}}$$

Internal dynamics: $\underline{\dot{x}_1} = \cancel{x_2 \sin x_1} \cancel{- x_1} + \underline{u}$

$$+ y_d'' - \cancel{x_2 \sin x_1} + \cancel{x_1} \cancel{- 2x_1x_2 - 2x_2^3}$$

$$+ k_p e + \cancel{k_v} \dot{e}$$

$$\cancel{k_v(y_d' - y')}$$

$$\cancel{k_v(y_d' + x_1 - x_2^2)}$$

Interal Dynam

$$\boxed{\Rightarrow \dot{x}_1 = y_d'' + ke + k_v \dot{e}}$$

$y_d, y_d', y_d'' \Rightarrow 0$; $e, \dot{e} \to 0$; $\boxed{\text{stable? yes}}$