Toward Real-Time State Estimation and Tracking of Elastic Rods

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***Abstract*—This is abstract**

I. Introduction

This is the intro

A. Related Work

This is related

B. Objectives

The objective of this article is to present a concise, intuitive and transferable the solution to a classical optimal control problem. Our objective is to model the plant using Lagrange's equations of motion, derive state dynamics, obtain a generalized cost function, and to optimally control the double pendulum by reaching a stable configuration in the upright position while given minim-energy control input to the system.
Also — testing
Also —

C. Article Structure

II. Estimation Formulation

There are many benefits to using the *state space* framework, notably that is well generalized and conveniently implemented on modern computers.

A. System Setup

Consider a resting cart on a flat rigid surface along the x-axis, on top of which are stacked two inverted pendulums, figure 01. The pendulums resemble a robotic arm but rather without a motor or external torque input. At the end of each pendulum is assumed to be a point-mass, m_1, m_2. Each pendulum moves freely and independently and is connected by lengths l_1 and l_2 from their joints, q_1 and q_2 to their point-masses, m_1, m_2. The mass of the cart is considered point-mass as well and is denoted by m_c with its position q_c along the x-axis. θ_1 and θ_2 denote angular deviations from upright position for q_1 and q_2, respectively.

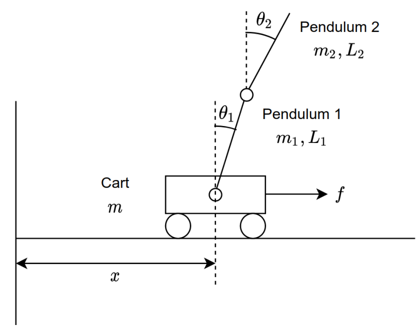

Fig. 1. Physical System

B. State Definition

Suppose we have a continuous-time, nonlinear system presented in state space form.

$$\dot{x} = f(x, u), \tag{1}$$

$$y = h(x). \tag{2}$$

whats f - prediction model whats h - obs model omit obs noise, it requires EKF??

Our goal is to stabilize the described highly nonlinear system in the upright configuration

C. Noise and Friction

D. Global Coordinate Frame

We need to form a homogenous configuration space to present state variables x, θ_1, θ_2 by forming a global coordinate frame with $x(t = 0)$ as the origin.

$$q_c = \begin{bmatrix} q \\ 0 \end{bmatrix}, \;\; q_1 = \begin{bmatrix} q + l_1 \sin\theta_1 \\ l_1 \cos\theta_1 \end{bmatrix}, \;\; q_2 = \begin{bmatrix} q + l_1 \sin\theta_1 + l_2 \sin\theta_2 \\ l_1 \cos\theta_1 + l_2 \cos\theta_2 \end{bmatrix},$$

We denote position of point-masses for the cart, m_1, and m_2 by $q_c, q_1, q_2 \in R^2$ on xy coordinates.

E. Other Considerations

Noise and Friction modeling, process noise versus observation noise.

III. Model Derivation

A. Equations of Motion

As it is layed out in Lagrangian mechanics; first we must obtain the Lagrangian of the physical model, $\mathcal{L} = K - P$. K and P represent the *kinetic* and *potential* energies of the system. K and P are obtained by,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m_c\|\dot{q_c}\|^2 + m_1\|\dot{q_1}\|^2 + m_2\|\dot{q_2}\|^2 \quad (3)$$

B. Continues-Time, Nonlinear Model

C. Optimal Control

D. Discrete, Nonlinear Model

E. Discrete, Linear Model

F. General Time-Invariant Cost Function

G. Linear Quadratic Regulator Solution

IV. Simulations and Results

V. Conclusion

The conclusion goes here.

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The authors would like to thank...

References

[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.