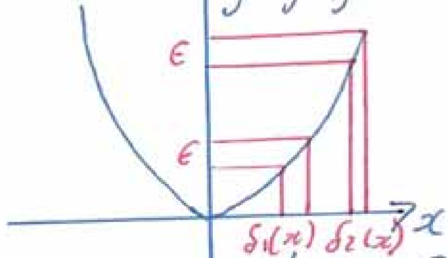
Continuity, Diff., Lipschitz

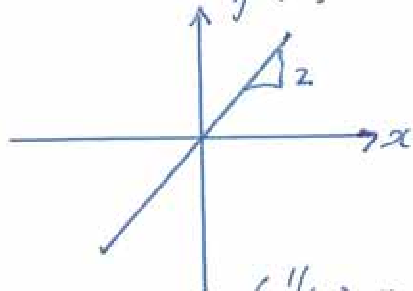
Example 1

$y = f(x) = x^2$



contin.
not unif. contin.

$f'(x) = 2x$

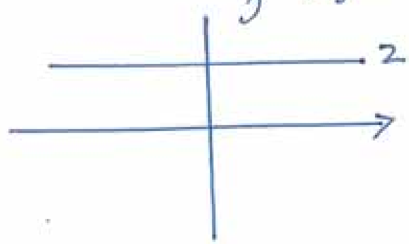


locally Lips.
not globally Lips.

diff. $f(x) \in C^1$

$f''(x) = 2$

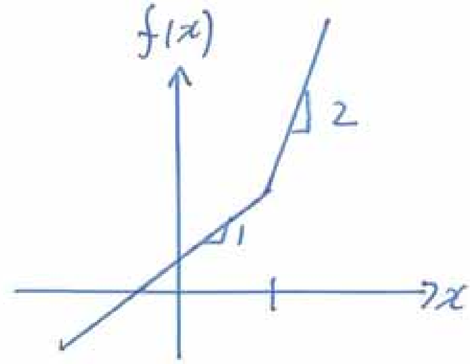
cont. diff.



$f(x) \in C^2$

$f(x) \in C^\infty$

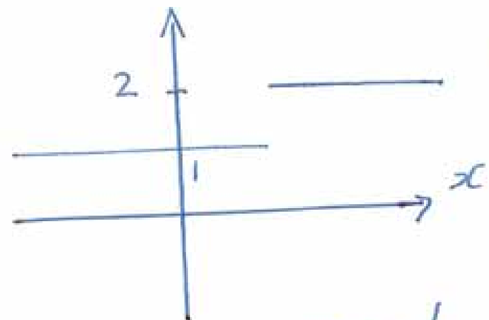
Example 2



contin

unif. cont.

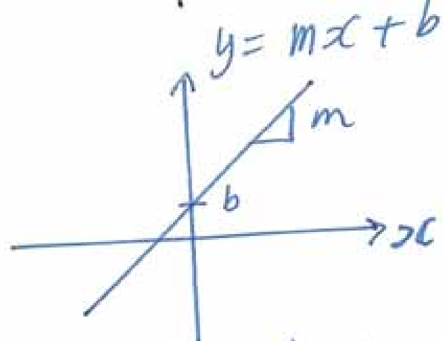
globally Lips.



diff. $f(x) \in C^1$

not cont. diff.

Example 3



a contraction map if slope $m < 1$

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1$$

$$\int \frac{1}{x} \, dx = \log|x|$$

$$\frac{d}{dx} \log(f(x)) = \frac{f'(x)}{f(x)}$$

$$\int_a^b u \, dv = uv\Big|_a^b - \int_a^b v \, du$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}$$

p.7 ex 1.2

$$\frac{dx}{dt} = \dot{x} = -x + x^2$$

$$\int_{x_0}^{x} \frac{1}{x^2 - x}\, dx = \int_{t_0}^{t} 1\, dt \qquad \frac{d}{dx}\frac{U}{V} = \frac{U'V - V'U}{V^2}$$

note

$$\frac{d}{dx} \log \frac{x-1}{x} = \frac{x}{x-1} \frac{d}{dx} \frac{x-1}{x}$$

$$= \frac{x}{x-1} \frac{1(x) - (x-1)}{x^2} = \frac{x}{x-1} \frac{1}{x^2} = \frac{1}{x(x-1)}$$

$$= \frac{1}{x^2 - x}.$$

$$\int_{x_0}^{x} \frac{1}{x^2 - x} = \int d \log\left(\frac{x-1}{x}\right) = \log \frac{x-1}{x}\Big|_{x_0}^{x}$$

$$\log \frac{x-1}{x} - \log \frac{x_0 - 1}{x_0} = t - t_0$$

$$\log\left(\frac{x-1}{x} \Big/ \frac{x_0 - 1}{x_0}\right) = t - t_0$$

$$\frac{x-1}{x} \Big/ \frac{x_0-1}{x_0} = e^{t - t_0}$$

$$\frac{x-1}{x} = \cancel{\frac{x_0}{x_0 - 1}} \frac{x_0 - 1}{x_0} e^{t - t_0}$$

$$\frac{1-x}{x} = \frac{1-x_0}{x_0} e^{t - t_0}$$

$$1 - x = x\left(\frac{1-x_0}{x_0}\right) e^{t - t_0} \qquad \rightarrow \text{next page}$$

$$x = \frac{x_0 e^{-(t - t_0)}}{(1 - x_0) + x_0 e^{-(t - t_0)}}$$

$$1-x = x\left(\frac{1-x_0}{x_0}\right)e^{t-t_0}$$

$$1 = x\left(1+\frac{1-x_0}{x_0}e^{t-t_0}\right)$$

$$x = \frac{1}{1+\frac{1-x_0}{x_0}e^{t-t_0}}$$

$$= \frac{x_0 e^{-(t-t_0)}}{(1-x_0)+x_0 e^{-(t-t_0)}}$$