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## HW3

Prob1:

ex. 2:  $\dot{x} = -ax$

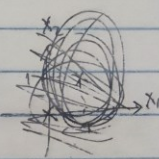
$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_1 x_2 \\ x_2 - x_1 x_2 \end{bmatrix} \quad \textcircled{1} \text{ linearize: } \dot{x} = Ax + Bu = \frac{\partial f}{\partial x} x + \frac{\partial f}{\partial u} u$$

$$\dot{x} = \frac{\partial f}{\partial x} x = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} \end{bmatrix} x = \begin{bmatrix} -1+x_2 & x_1 \\ -x_2 & 1-x_1 \end{bmatrix} x = Ax \quad \rightarrow \text{Jacob}$$

Prob1. ① find e.p.s, let  $\dot{x} = 0 \Rightarrow \begin{cases} -x_1 + x_1 x_2 = 0 \\ x_2 - x_1 x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 1 \\ x_2 = 1 \end{cases} \text{ or } \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases}$

1)  $J(0,0) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \lambda_1 = -1, \lambda_2 = 1$  center point saddle points

2)  $J(1,1) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \Rightarrow \lambda_1 = i, \lambda_2 = -i$  center point



Prob2:

prob 2. a. find e.p.s.  $\dot{x} = \begin{bmatrix} x_2(-x_1+x_2-1) \\ x_1(x_1+x_2+1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases} \text{ or } \begin{cases} x_1 = -1 \\ x_2 = 0 \end{cases}$

b. Jacobian  $J = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -x_2 & -x_1+2x_2-1 \\ 2x_1+x_2+1 & x_1 \end{bmatrix} \text{ or } \begin{cases} x_1 = 0 \\ x_2 = +1 \end{cases}$

c.  $J(0,0) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Rightarrow \lambda_1 = i, \lambda_2 = -i \Rightarrow$  center point

$J(-1,0) = \begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix} \Rightarrow \lambda_1 = 0, \lambda_2 = -1 > 0 \Rightarrow$  marginally stable ~~form~~

$J(0,1) = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix} \Rightarrow \lambda_1 = -2, \lambda_2 = 1 \Rightarrow$  saddle point

### Prob3:

```
function [t,x] = HW3_3()
clc
close all
tspan = [0 50];
for i = -3:0.3:3
    for j = -3:0.3:3
        x0 = [i,j];
        [t, x] = ode23(@HW33,tspan,x0);
        plot(x(:,1),x(:,2),'b'); hold on
    end
end
xlabel('x1')
ylabel('x2')
title('')
axis([-5 5 -5 5])

function dx = HW33(tspan,x)

dx = [x(2)*(-x(1)+x(2)-1); x(1)*(x(1)+x(2)+1)];
end
end
```

