EE 5323 - HW03

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HW03 – Nonlinear System Simulations
EE 5323 – Nonlinear Systems
Dr. Lewis

Exercise 1

Voltera Predator-Prey System
Consider the Voltera predator-prey system,

$$\dot{x}_1 = -x_1 + x_1 x_2$$

$$\dot{x}_2 = x_2 - x_1 x_2$$

Find the equilibrium points and their nature.

Answer
State variable is given as:

$$\dot{x}_1 = -x_1 + x_1 x_2 \; \dot{x}_2 = x_2 - x_1 x_2$$

The Voltera predator-prey system has limit cycles therefore the system is at equilibrium when the population of both predator and prey remain constant; thus, the derivative should be zero. To find the equilibrium, I set $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$. Solve the system for its roots.

$$\dot{x}_1 = 0 \implies 0 = -x_1 + x_1 x_2 \; \dot{x}_2 = 0 \implies 0 = x_2 - x_1 x_2$$

$$0 = x_1(\beta x_2 - \alpha) \implies x_1 = 0; \; x_2 = \alpha/\beta$$

$$0 = x_2(\gamma - \sigma x_1) \implies x_1 = \gamma/\sigma; \; x_2 = 0$$

There are two equilibrium points at $(x_1, \; x_2)$,

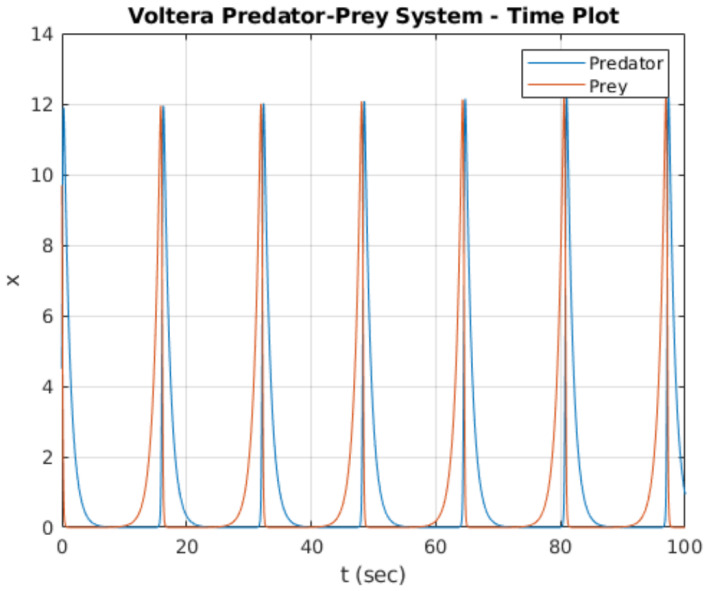
- At zero, $(0, \; 0)$,
- Any positive pair of integers $(\alpha/\beta, \; \gamma/\sigma)$

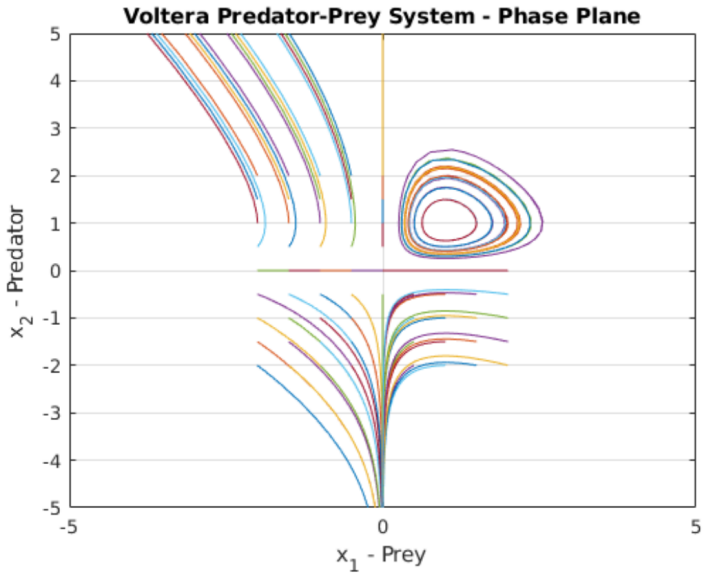
The equilibrium point nature of the zero is a stable center point that is a limit cycle. The other e.p. has a saddle point nature because it is stable in one dimension (goes to zero) and unstable in the other (goes to infinity).

Matlab Code

```
%% HW 03 - Nonlinear Systems Simulation
% Author: Bardia Mojra
% Date: 09/28/2021
% Title: HW 03 - Nonlinear Systems Simulation

clc
close all
warning('off','all')
warning

x0_set = -2:.5:2;
t_intv= [0 100];
x_0= [4.5, 9.7]'; %'initial conditions for x(t)

figure
[t,x]= ode23('Voltera', t_intv, x_0);
plot(t,x)
hold on;
grid on;
title('Voltera Predator-Prey System - Time Plot');
ylabel('x');
xlabel('t (sec)');
legend('Predator', 'Prey');
t_intv= [0 10];
figure
for i=x0_set
  for j=x0_set
    x0 = [i; j];
    [t,x]= ode45('Voltera', t_intv, x0);
    plot(x(:,1),x(:,2))
    hold on;
  end
end
title('Voltera Predator-Prey System - Phase Plane');
ylabel('x_2 - Predator');
xlabel('x_1 - Prey');
axis([-5 5 -5 5]);
grid on;

function xdot = Voltera(t,x)
  xdot = [-x(1)+x(1)*x(2); x(2)-x(1)*x(2)];
end
```

Figures


Voltera Predator-Prey System - Time Plot
Predator
Prey
x
t (sec)
0
2
4
6
8
10
12
14
0
20
40
60
80
100


Voltera Predator-Prey System - Phase Plane
x2 - Predator
x1 - Prey
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
0
5

Exercise 2

Equilibrium points and linearization
Consider the following system,

$$\dot{x}_1 = x_2(-x_1 + x_2 - 1)$$
$$\dot{x}_2 = x_1(x_1 + x_2 + 1)$$

(a) Find all equilibrium points

(b) Find Jacobian

(c) Find the nature of all e.p.s

Answer
State