HW 02 - Nonlinear Systems Simulation

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Document Information:

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Duffing's Equation

Duffing’s equation is interesting in that it exhibits bifurcation, or dependence of stability properties and number of equilibrium points on a parameter. The undamped Duffing equation is

$$\ddot{x} + \alpha x + x^3 = 0$$

1. Find the equilibrium points. Show that for \alpha \greater 0 there is only one e.p. .

2. Simulate the Duffing oscillator and make time plot and phase plane plot.

3. Simulate the following cases,

- $\alpha = -1$.
- $\alpha = -0.1$.
- $\alpha = 1$.

For each case, take ICs spaced in a uniform mesh in a suitable box to show the behavior. Pick the box size. Make one phase plane plot for each case showing all trajectories for that case.

Duffing's Equation Answer

Define state variable equations:

$\dot{x}_1 = x_2$

$\dot{x}_2 = -\alpha x_1 x_1^3$

To find the equalibrium we set $\dot{x}_1 = 0$ and plug it the state variable equations.

$x_2 = 0$.

$-\alpha x_1 - x_1^3 = 0 \Rightarrow$

$\dot{x}_2 = -\alpha x_1 x_1^3 \quad \Rightarrow \quad -x_1(\alpha + x_1^2) = 0 \quad \Rightarrow \quad x_1 = 0, \quad x_1^2 = -\alpha.$

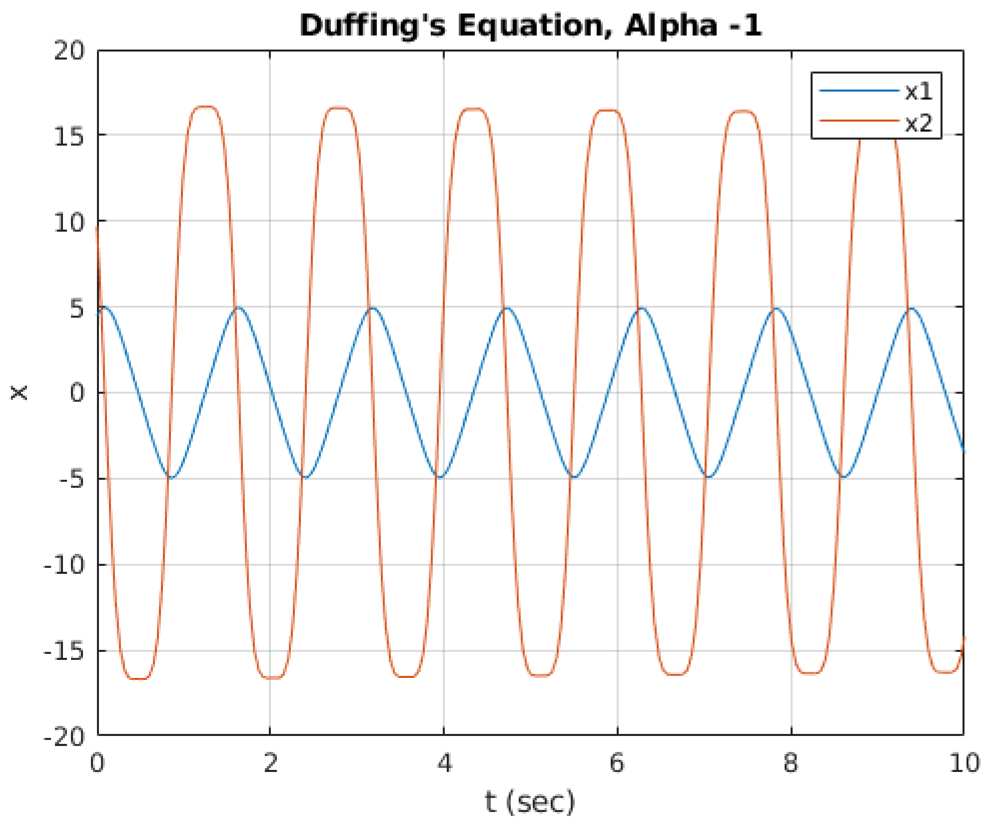
Equilibrium points must be real. If α is greater than or equal to zero, there is only one equalibrium point at (0,0). If α is smaller than zero, two e.p. at,

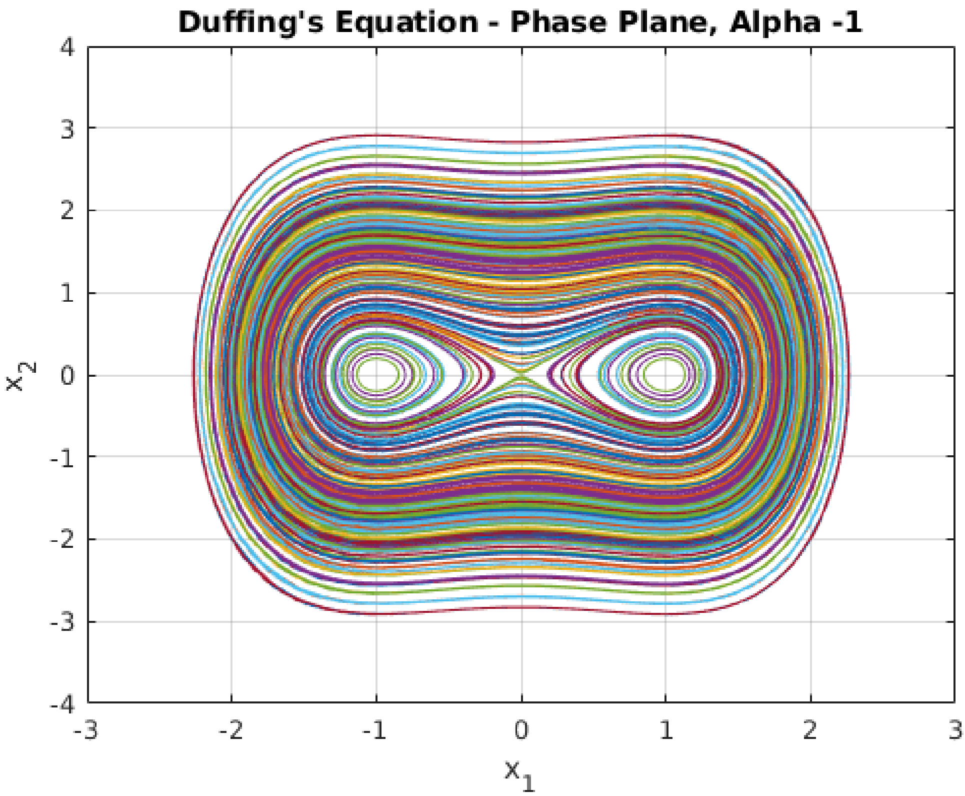
$(\sqrt{-\alpha}, 0)$ and $(-\sqrt{-\alpha}, 0)$

```
clc
close all
warning('off','all')
warning

x0_set = -2:.2:2;
t_intv= [0 10];
x_0= [4.5, 9.7]'; % initial conditions for x(t)

% part a, alpha = -1;
figure
[t,x]= ode23('duffinga', t_intv, x_0);
plot(t,x)
hold on;
grid on;
title("Duffing's Equation, Alpha -1");
ylabel('x');
xlabel('t (sec)');
legend('x1', 'x2');

t_intv= [0 10];
figure
for i=x0_set
  for j=x0_set
    x0 = [i; j];
    [t,x]= ode45('duffinga', t_intv, x0);
    plot(x(:,1),x(:,2))
    hold on;
  end
end
```

```
title("Duffing's Equation - Phase Plane, Alpha -1");
ylabel('x_2');
xlabel('x_1');
axis([-3 3 -4 4]);
grid on;
```

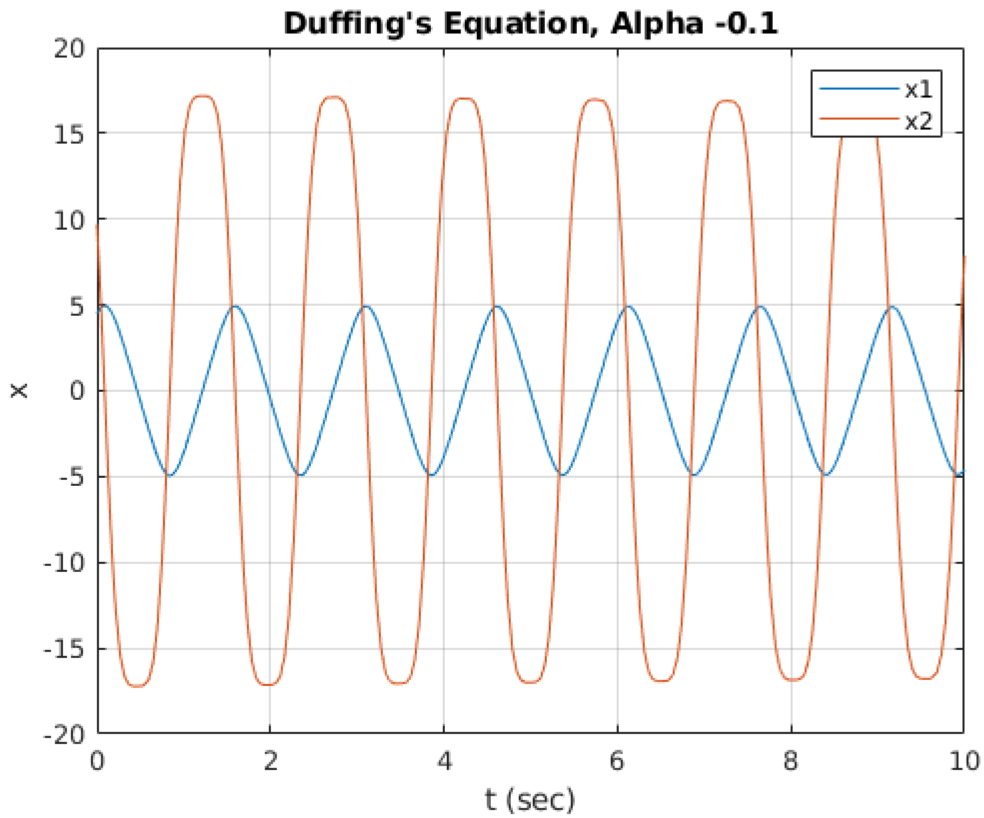
All warnings have the state 'off'.

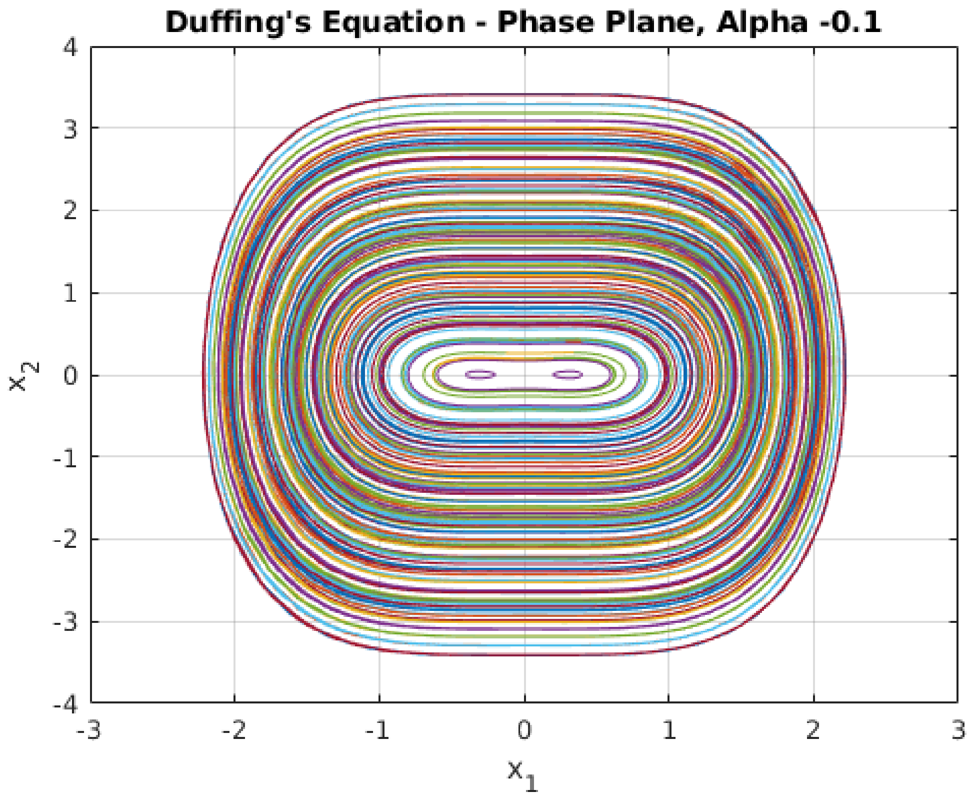





```
function xdot = duffinga(t,x)
  alpha = -1;
  xdot = [x(2); -alpha*x(1) - x(1)^3];
end

close all
% part b, alpha = -0.1
figure
[t,x]= ode23('duffingb', t_intv, x_0);
plot(t,x)
hold on;
grid on;
title("Duffing's Equation, Alpha -0.1");
ylabel('x');
xlabel('t (sec)');
legend('x1', 'x2');

t_intv= [0 10];
figure
for i=x0_set
  for j=x0_set
    x0 = [i; j];
    [t,x]= ode45('duffingb', t_intv, x0);
    plot(x(:,1),x(:,2))
    hold on;
  end
end
```

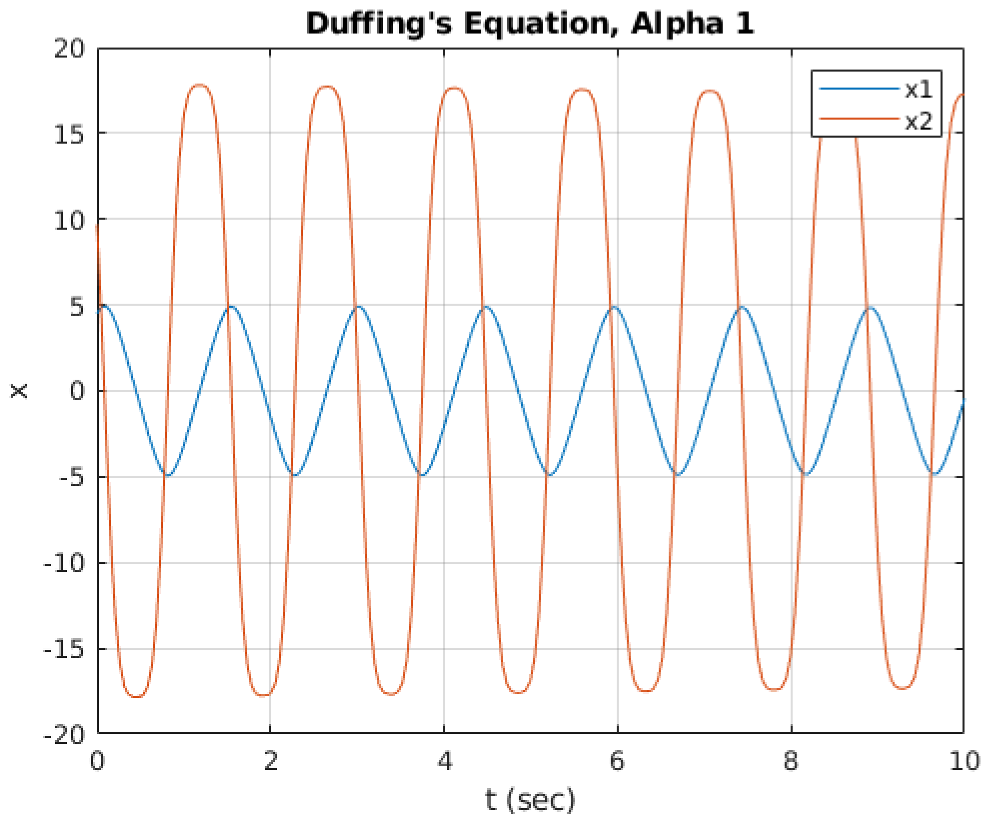
```
title("Duffing's Equation - Phase Plane, Alpha -0.1");
ylabel('x_2');
xlabel('x_1');
axis([-3 3 -4 4]);
grid on;
```

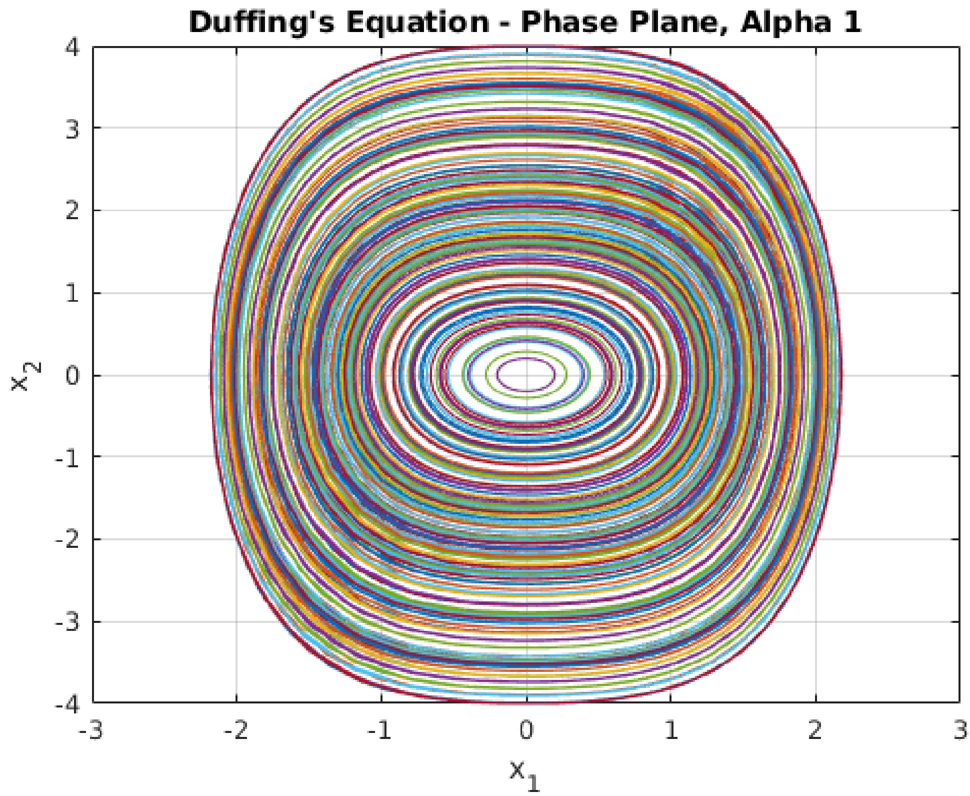





```
function xdot = duffingb(t,x)
  alpha = -0.1;
  xdot = [x(2); -alpha*x(1) - x(1)^3];
end

close all
% part c, alpha = 1
figure
[t,x]= ode23('duffingc', t_intv, x_0);
plot(t,x)
hold on;
grid on;
title("Duffing's Equation, Alpha 1");
ylabel('x');
xlabel('t (sec)');
legend('x1', 'x2');

t_intv= [0 10];
figure
for i=x0_set
  for j=x0_set
    x0 = [i; j];
    [t,x]= ode45('duffingc', t_intv, x0);
    plot(x(:,1),x(:,2))
    hold on;
  end
end
```

```
title("Duffing's Equation - Phase Plane, Alpha 1");
ylabel('x_2');
xlabel('x_1');
axis([-3 3 -4 4]);
grid on;
```



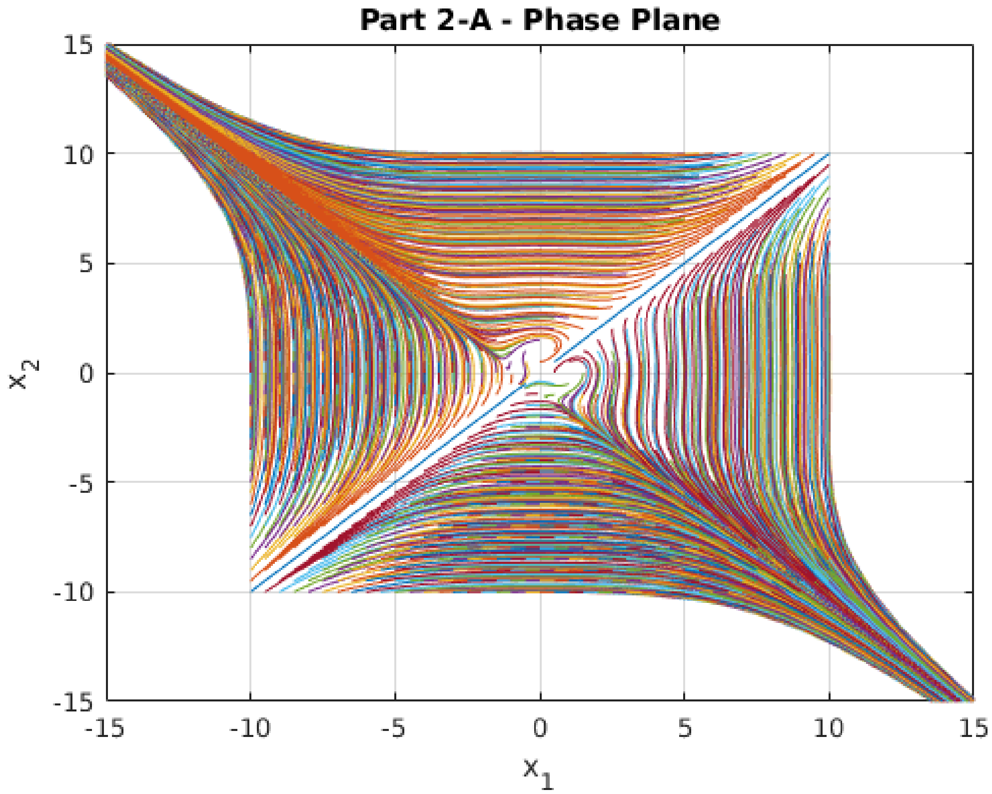



```
function xdot = duffingc(t,x)
  alpha = 1;
  xdot = [x(2); -alpha*x(1) - x(1)^3];
end
```

Part 2-A

Define state variable equations:

```
clc
close all
tspan = [0 5];
for k=-10:0.5:10
  for j=-10:0.5:10
    x0 = [k;j];
    [t,x] = ode23('part2', tspan, x0);
    plot(x(:,1),x(:,2))
    hold on
  end
end
title("Part 2-A - Phase Plane");
ylabel('x_2');
xlabel('x_1');
axis([-15 15 -15 15]);
grid on;
```

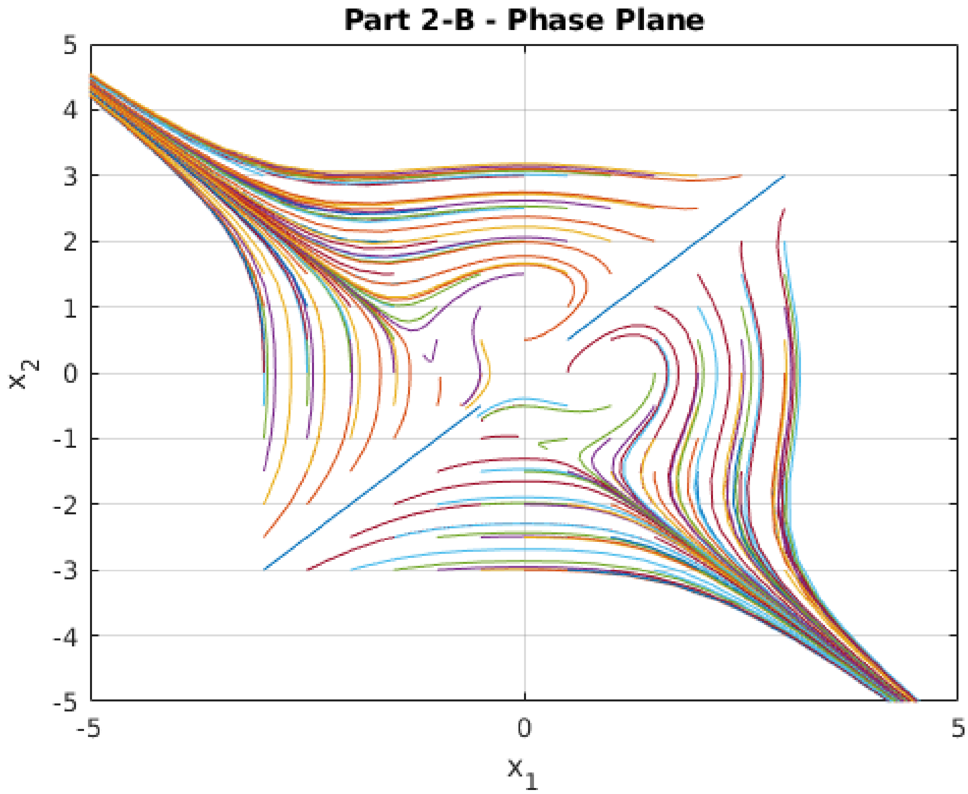



```
function xdot = part2(t,x)
  xdot = [x(2)*(1+x(1)-x(2)^2); x(1)*(1+x(2)-x(1)^2)];
end
```

Part 2-B

Define state variable equations:

```
clc
close all
tspan = [0 5];
for k=-3:0.5:3
  for j=-3:0.5:3
    x0 = [k;j];
    [t,x] = ode23('part2', tspan, x0);
    plot(x(:,1),x(:,2))
    hold on
  end
end
title("Part 2-B - Phase Plane");
ylabel('x_2');
xlabel('x_1');
axis([-5 5 -5 5]);
grid on;
```

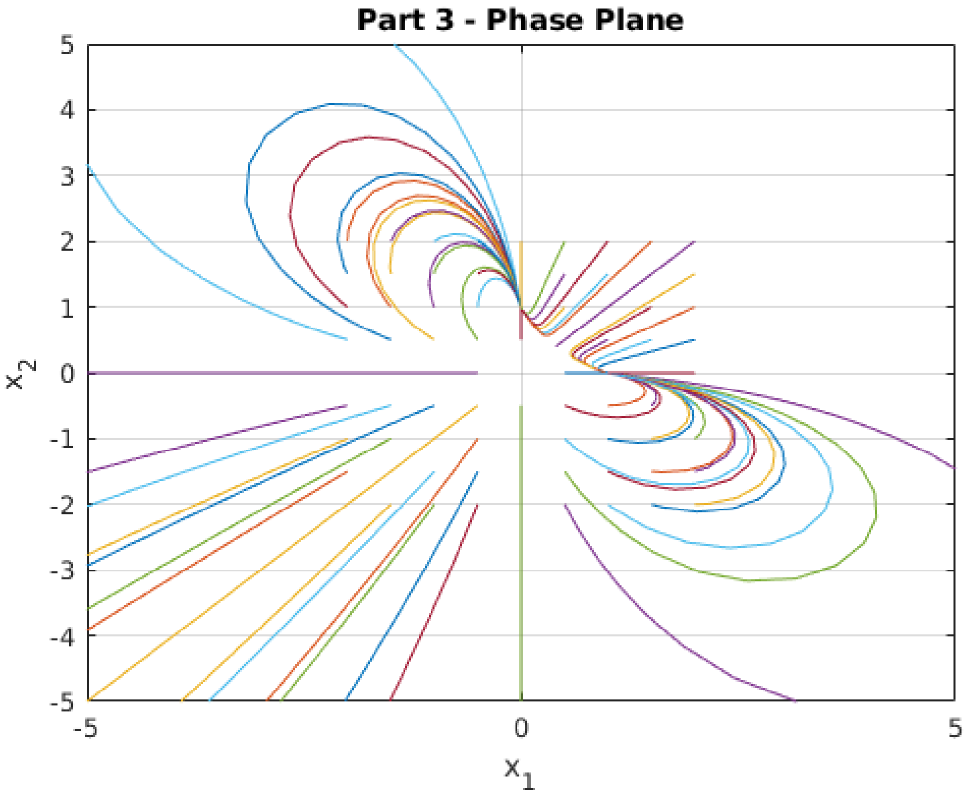



```
function xdot = part2(t,x)
  xdot = [x(2)*(1+x(1)-x(2)^2); x(1)*(1+x(2)-x(1)^2)];
end
```

Part 3

Define state variable equations:

```
clc
close all
tspan = [0 5];
for k=-2:0.5:2
  for j=-2:0.5:2
    x0 = [k;j];
    [t,x] = ode23('part3', tspan, x0);
    plot(x(:,1),x(:,2))
    hold on
  end
end
title("Part 3 - Phase Plane");
ylabel('x_2');
xlabel('x_1');
axis([-5 5 -5 5]);
grid on;
```




```
function xdot = part3(t,x)
  a=2; c=2; d=2; f=2;
  b=3; e=3;
  xdot = [a*x(1)-b*x(1)*x(2)-c*x(1)^2; d*x(2)-e*x(1)*x(2)-f*x(2)^2];
end
```

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