<u>EE 5323 Nonlinear Control Systems</u>

Homework Pledge of Honor

On all homeworks in this class - YOU MUST WORK ALONE.

Any cheating or collusion will be severely punished.

It is very easy to compare your software code and determine if you worked together It does not matter if you change the variable names.

Please sign this form and include it as the first page of all of your submitted homeworks.

....

Typed Name: ____VRUSHABH SURESH DONGE_____________

Pledge of honor:

"On my honor I have neither given nor received aid on this homework."

Signature: V.S. Donge.

EE 5323 Homework 2

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1 Duffing's equation

The undamped Duffing equation is

$$\ddot{x} + \alpha x + x^3 = 0 \tag{1.1}$$

The second order differential equation in 1.1 can be written in state space variables with states $x_1 = x$ and $x_2 = \dot{x}$.

$$\dot{x}_1 = x_2 \tag{1.2}$$

$$\dot{x}_2 = -\alpha x_1 - x_1^3 \tag{1.3}$$

1. Equilibrium points,

$$x_2 = 0 \tag{1.4}$$

$$-\alpha x_1 - x_1^3 = 0 \longrightarrow x_1 = 0, x_1^2 = -\alpha. \tag{1.5}$$

   Therefore, for $\alpha \geq 0$, there is only one equilibrium point (0,0). For $\alpha < 0$, equilibrium points are $(+\sqrt{-\alpha}, 0)$ and $(-\sqrt{-\alpha}, 0)$.

2.

MATLAB Code

```
function xdot=duffingeq(t, x)
%  alpha=-1;
%  alpha=-0.1;
 alpha=1;
 xdot(1, 1) = x(2);
 xdot(2, 1) = -alpha*x(1)-x(1)^3;
end
```

Listing 1: Matlab function file

```
close all;
clc;
clear all;

tspan=[0 20];

for i=-2:0.5:2
    for j=-2:0.5:2

        x0=[i;j];
        [t,x]=ode23('duffingeq',tspan,x0);

        plot(x(:,1),x(:,2))
        grid on;hold on;

    end
end

xlabel('x1');
ylabel('x2');
axis([-5 5 -5 5])
figure
        plot(t,x(:,1))
        hold on;
        plot(t,x(:,2))
        xlabel('t');
ylabel('x');
```

Listing 2: Matlab main file

Simulation results

1. For initial condition $\alpha = -1$

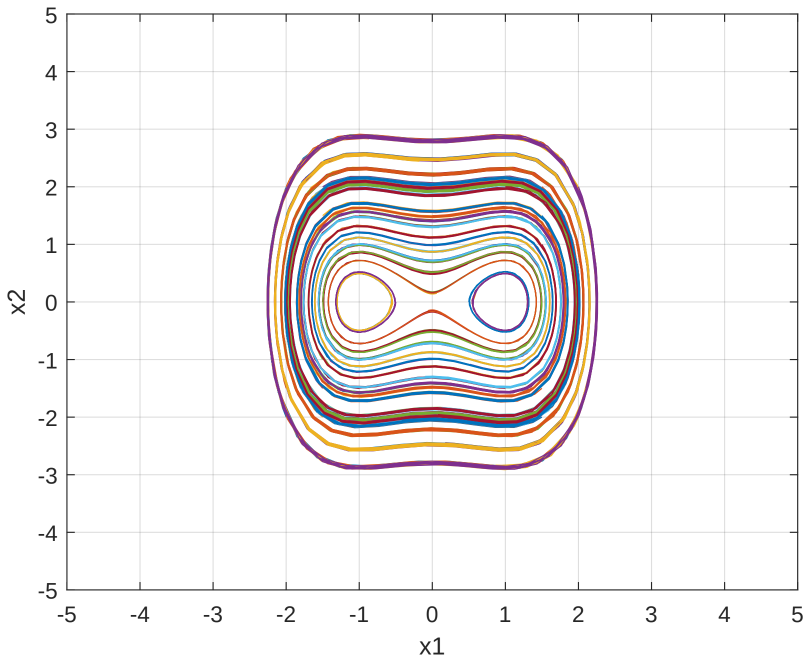

Figure 1: Phase plane plot

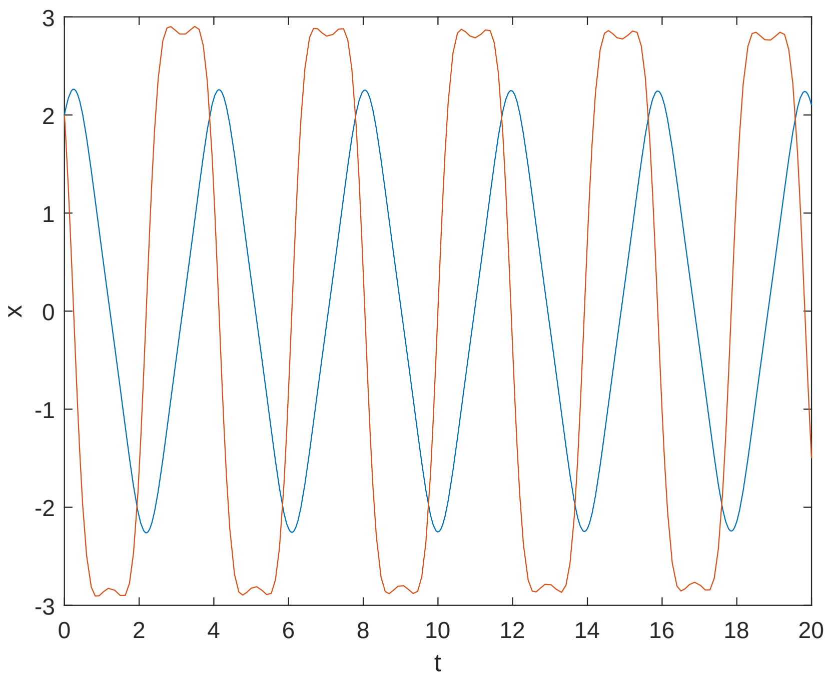

Figure 2: Time plot

2. For initial condition $\alpha = -0.1$

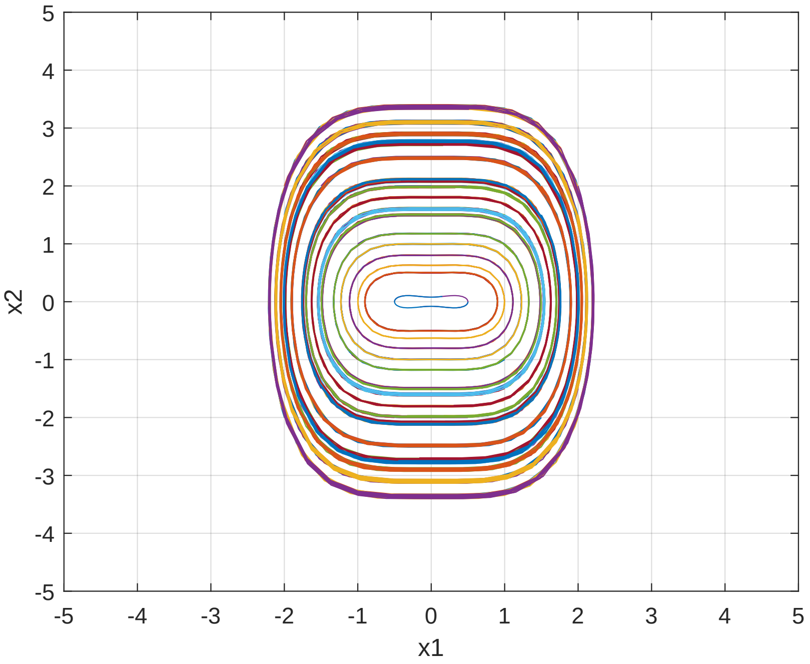

Figure 3: Phase plane plot

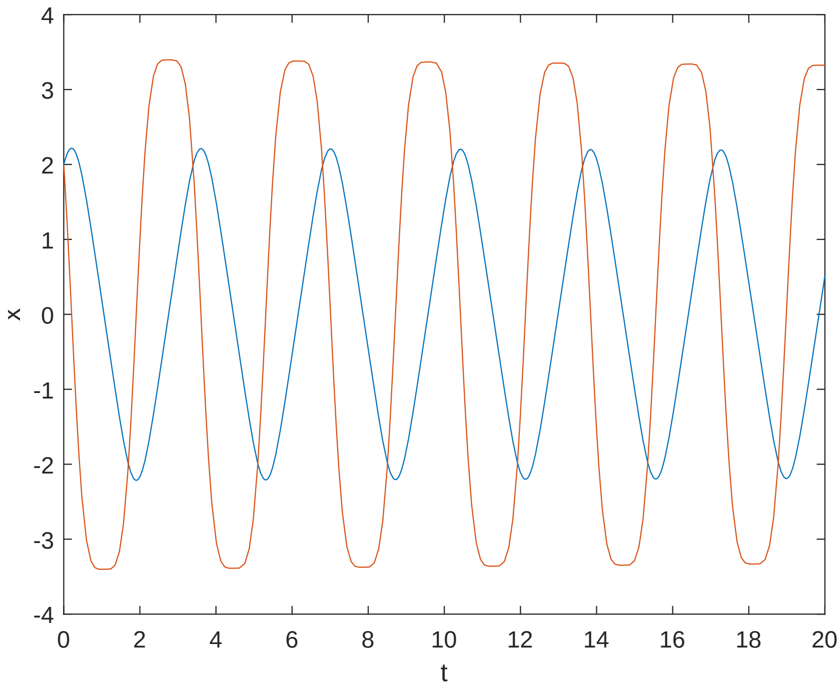

Figure 4: Time plot

3. For initial condition $\alpha = 1$

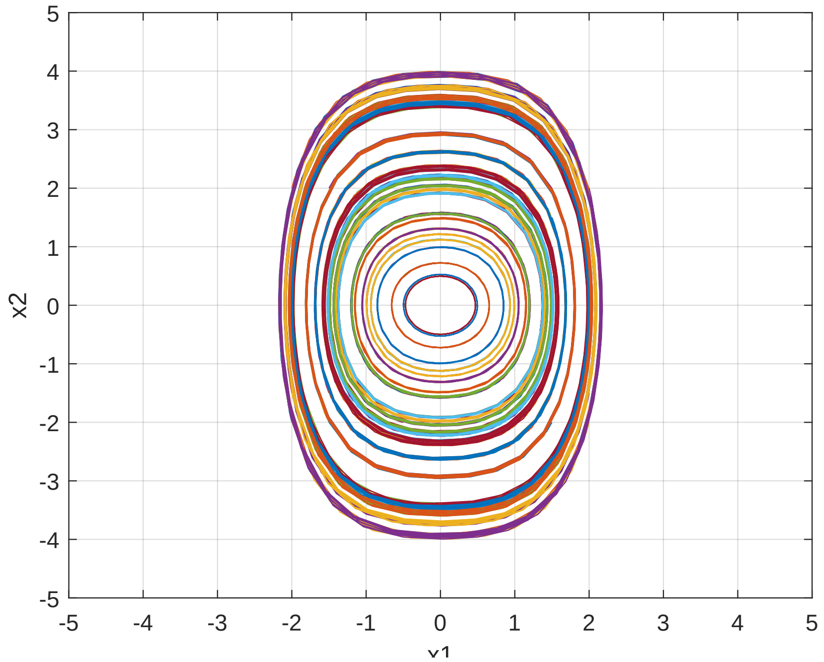

Figure 5: Phase plane plot

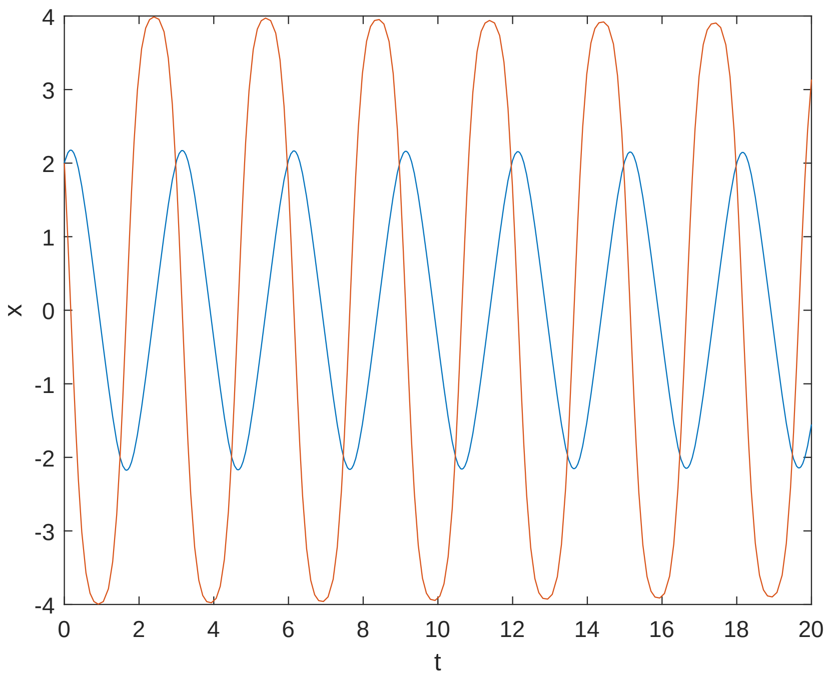

Figure 6: Time plot

2

Consider the system,

$$\dot{x}_1 = x_2(1 + x_1 - x_2^2) \quad [2.1]$$

$$\dot{x}_2 = x_1(1 + x_2 - x_1^2) \quad [2.2]$$

MATLAB Code

```
function xdot=seceq(t, x)

 xdot(1, 1) = x(2)*(1+x(1)-x(2)^2);
 xdot(2, 1) = x(1)*(1+x(2)-x(1)^2);
end
```

Listing 3: Matlab function file

```
close all;
clc;
clear all;

tspan=[0 10];

for i=-10:0.5:10
    for j=-10:0.5:10
%          for i=-3:0.5:3
%             for j=-3:0.5:3
        x0=[i;j];
        [t,x]=ode23('seceq',tspan,x0);

        plot(x(:,1),x(:,2))
        grid on;hold on;
%             end
%           end
    end
end

xlabel('x1');
ylabel('x2');
% axis([-5 5 -5 5])
axis([-15 15 -15 15])
```

Listing 4: Matlab main file

Simulation results

1. Initial condition: $x_1 = [-10, 10]$, $x_2 = [-10, 10]$

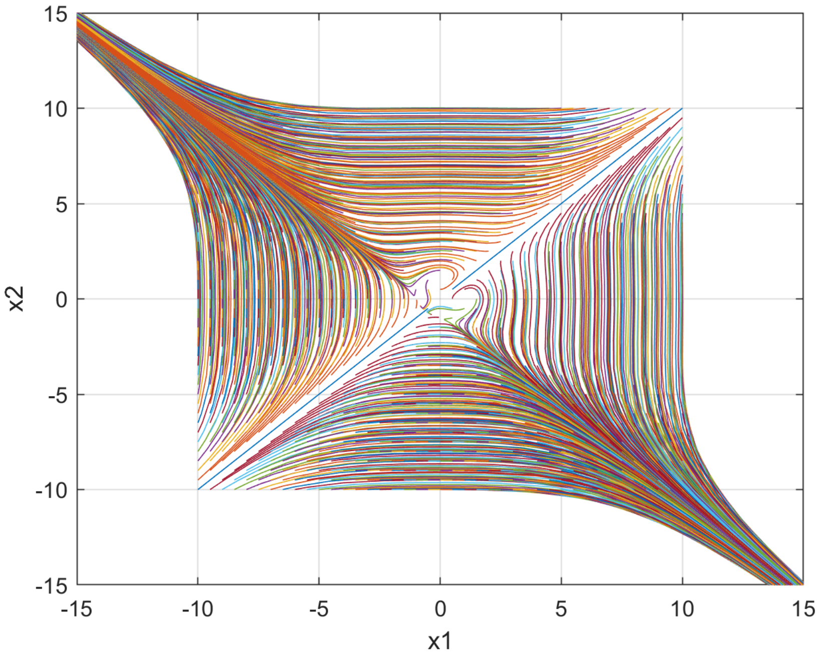

Figure 7: Phase plane plot on square $[-15, 15]x[-15, 15]$

2. Initial condition: $x_1 = [-3, 3]$, $x_2 = [-3, 3]$

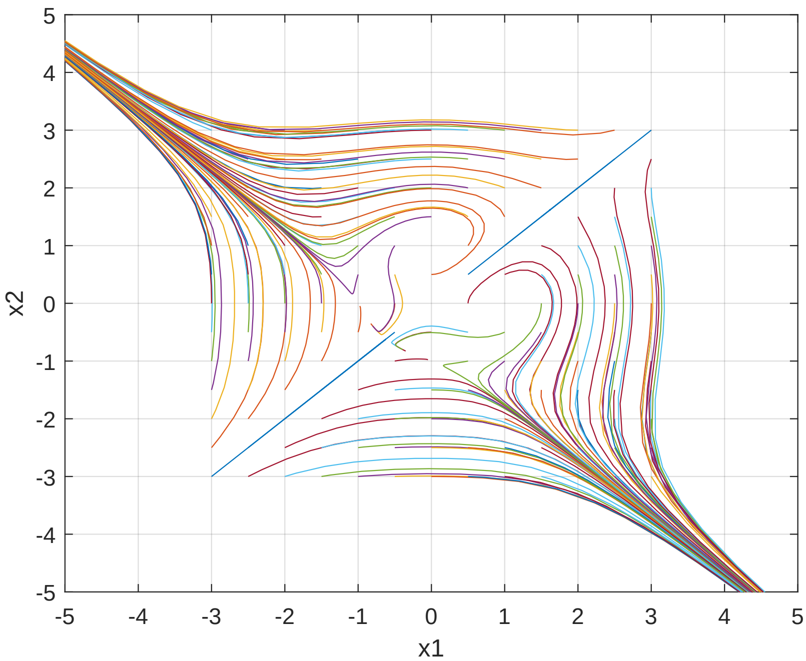

Figure 8: Phase plane plot on square $[-5, 5]x[-5, 5]$

3

Consider the system,

$$\dot{x}_1 = ax_1 - bx_1x_2 - cx_1^2 \quad [3.1]$$

$$\dot{x}_2 = dx_2 - ex_1x_2 - fx_2^2 \quad [3.2]$$

MATLAB Code

```
function xdot=thirdeq(t, x)
a=2;c=2;d=2;f=2;b=3;e=3;
 xdot(1, 1) = a*x(1)-b*x(1)*x(2)-c*x(1)^2;
 xdot(2, 1) = d*x(2)-e*x(1)*x(2)-f*x(2)^2;
end
```

Listing 5: Matlab function file

```
close all;
clc;
clear all;

tspan=[0 5];


        for i=-2:0.1:2
           for j=-2:0.1:2
        x0=[i;j];
        [t,x]=ode23('thirdeq',tspan,x0);

        plot(x(:,1),x(:,2))
        grid on;hold on;
           end
         end


xlabel('x1');
ylabel('x2');
axis([-5 5 -5 5])
```

Listing 6: Matlab main file

Simulation results Initial condition: $x_1 = [-2, 2]$, $x_2 = [-2, 2]$

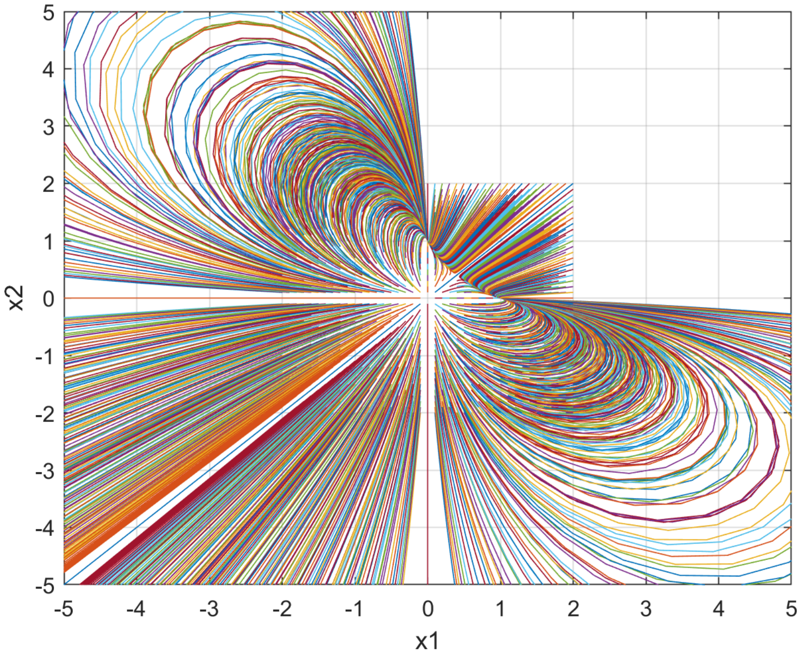

Figure 9: Phase plane plot on square $[-5, 5]x[-5, 5]$