EE 5323 Homeworks
Fall 2021

Updated: Saturday, November 06, 2021

DO NOT DO HOMEWORK UNTIL IT IS ASSIGNED.
THE ASSIGNMENTS MAY CHANGE UNTIL ANNOUNCED.

- Some homework assignments refer to the textbook: Slotine and Li, etc.
- For full credit, show all work.
- Some problems require hand calculations. In those cases, do not use MATLAB except to check your answers.

It is OK to talk about the homework beforehand.

BUT, once you start writing the answers, MAKE SURE YOU WORK ALONE.

The purpose of the Homework is to evaluate you individually, not to evaluate a team.

Cheating on the homework will be severely punished.

The next page must be signed and turned in at the front of ALL homeworks submitted in this course.

<u>EE 5323 Nonlinear Control Systems</u>

Homework Pledge of Honor

On all homeworks in this class - YOU MUST WORK ALONE.

Any cheating or collusion will be severely punished.

It is very easy to compare your software code and determine if you worked together It does not matter if you change the variable names.

Please sign this form and include it as the first page of all of your submitted homeworks.

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Typed Name: Gauri Jadhav

Pledge of honor:

"On my honor I have neither given nor received aid on this homework."

e-Signature: Gauri Jadhav 1001759704

EE 5323- Take Home Exam 2

Fall 2021

This exam has 6 pages in all. There are 4 problems.
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Almost all questions require numerical calculations to arrive at the answers. To obtain full credit, show all your work. No partial credit will be given without the supporting work.

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Name: Gauri Jadhav

Pledge of honor:

"On my honor I have neither given nor received aid on this examination."

Signature: Gauri Jadhav 1001759704

Q1] Lyapunov Function.

a] $\dot{x}_1 = x_2 \sin x_1 - x_1$

$\dot{x}_2 = -x_1 \sin x_1 - x_2$

Lyapunov ~~Eo~~ Function Candidate: $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$.

$\dot{V}(x) = x_1\dot{x}_1 + x_2\dot{x}_2$

Put the system dynamics:

$\dot{V}(x) = x_1[x_2 \sin x_1 - x_1] + x_2[-\cancel{8}x_1 \sin x_1 - x_2]$.

$= x_1^2(\sin x_2 - 1) + (-\sin x_1^2 x_2 - x_2^2)$.

$= x_1^2 \cancel{\sin} x_2 - x_1^2 - \cancel{\sin} x_1^2 x_2 - x_2^2$

$= -x_1^2 - x_2^2 < 0$.

$V < 0$ if $-x_1^2 - x_2^2 < 0$

~~If~~ System is Globally Asymptotic stable ~~of~~ as it is Negative Definite.

Q1 b] $\dot{x}_1 = x_2 \sin x_1 - x_1$

$\dot{x}_2 = -x_1 \sin x_1$.

Lyapunov candidate Function: $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$.

$\dot{V}(x) = x_1\dot{x} + x_2\dot{x}_2$

Put in system Dynamics:

$= x_1[x_2 \sin x_1 - x_1] + x_2[-x_1 \sin x_1]$.

$= x_1^2(\sin x_2 - 1) - x_2 \sin x_1^2$.

$\dot{V}(x) = x_1^2(\sin x_2 - 1 - \sin x_2)$.

$= -x_1^2 \le 0$.

It is Negative Semi Definite.

System is Globally SISL.

0

Q2] LaSalle's Extension

$\ddot{x} + k_1\dot{x} + k_2\dot{x}^3 + k_3 x^5 = 0$

Lyapunov candidate function:

$V = KE + PE$.

$V = \frac{1}{2}\dot{x}^2 + \int_0^x (k_3 y^5)\,dy$.

$\dot{V} = \dot{x}\ddot{x} + \dot{x}(k_3 x^5)$

$= \dot{x}(\ddot{x} + k_3 x^5)$. $= \dot{x}(-k_1\dot{x} - k_2\dot{x}^3 - k_3 x^5 + k_3 x^5)$.

$= \dot{x}(-k_1\dot{x} - k_2\dot{x}^3) = -\dot{x}^2(k_1 + k_2\dot{x}^2) \le 0$.

$\dot{V} \le 0 \Rightarrow -\dot{x}^2(k_1 + k_2\dot{x}^2) \le 0$,

It is negative semi definite.

∴ System is SISL.

Q2] b] $\dot{V} \to 0$ $\dot{x} \to$ $\dot{x} = 0$.

$\ddot{x} + k_1\dot{x} + k_2\dot{x}^3 + k_3 x^5 = 0$

$\ddot{x} + k_3 x^5 = 0$

$\ddot{x} = -k_3 x^5$

$x \to 0$, $\dot{x} = 0$.

∴ system is Asymptotically stable

Q3] a] $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ 0 & -6 \end{bmatrix} x$.

$A^T P + PA = -Q$.

~~So~~ Go for AS. $(Q = I)$.

$$\begin{bmatrix} 0 & 0 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} + \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & P_1 - 6P_2 \\ P_1 - 6P_2 & 2P_2 - 12P_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

$0 = -1$ $\longrightarrow$ cannot be considered

$P_1 - 6P_2 = 0$.

$2P_2 - 12P_3 = -1$

$P > 0$ doesn't exist for $Q = I$.

$\therefore$ Not AS.

Go for

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & P_1 - 6P_2 \\ P_1 - 6P_2 & 2P_2 - 12P_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}.$$

$\therefore P_1 - 6P_2 = 0 \quad P_1 = 6P_2 \quad P_2 = 1/6 P_1$.

$2P_2 - 12P_3 = -1$

$\therefore 2 \times \frac{1}{6} \times P_1 - 12P_3 = -1$

$\therefore P_1/3 - 12P_3 = -1$

$\therefore P_3 = 1/9 P_1$.

$$P = \begin{bmatrix} P_1 & 1/6 P_1 \\ 1/6 P_1 & 1/9 P_1 \end{bmatrix}$$

$P > 0$ $\therefore$ P is positive semi definite

$\therefore$ System is SISL.

Q3b]. $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x$

$A^T P + PA = -Q$.

Go for A.S. $Q = I$.

$$\begin{bmatrix} 0 & -3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} + \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

$$\begin{bmatrix} -3P_2 & -3P_3 \\ P_1 - 4P_2 & P_2 - 4P_3 \end{bmatrix} + \begin{bmatrix} -3P_2 & P_1 - 4P_2 \\ -3P_3 & P_2 - 4P_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

$$\begin{bmatrix} -6P_2 & P_1 - 3P_3 - 4P_2 \\ P_1 - 3P_3 - 4P_2 & 2P_2 - 8P_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$-6P_2 = -1 \therefore P_2 = \frac{1}{6}$.

$P_1 - 3P_3 - 4P_2 = 0$.

$2P_2 - 8P_3 = -1 \therefore \frac{2}{6} - 8P_3 = -1$.

$\therefore \frac{1}{6} - 4P_3 = -1/2$.

$\therefore 4P_3 = 8/12 \quad \therefore P_3 \Rightarrow 1/6$.

$P_1 = 4P_2 + 3P_3$.

$\therefore P_1 = 4\left(\frac{1}{6}\right) + 3\left(1/6\right)$.

$= 7/6$.

$$P = \begin{bmatrix} 7/6 & 1/6 \\ 1/6 & 1/6 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 7 & 1 \\ 1 & 1 \end{bmatrix} > 0.$$

It is positive definite as $P > 0$ for $Q = I$.

$\therefore$ System is A.S.

Q4 . $\dot{x}_1 = x_1 x_2^2 - x_1(x_1^2 + x_2^2 - 3)$

$\dot{x}_2 = -x_1^2 x_2 - x_2(x_1^2 + x_2^2 - 3)$.

Lyapunov Function candidate

$V(x) = \frac{1}{2}(x_1^2 + x_2^2)$

Now,

$\dot{V} = x_1 \dot{x}_1 + x_2 \dot{x}_2$

Put in system Dynamics.

$\dot{V} = x_1[x_1 x_2^2 - x_1(x_1^2 + x_2^2 - 3)] + x_2[-x_1^2 x_2 - x_2(x_1^2 + x_2^2 - 3)]$

$\dot{V} = -x_1(x_1^2 + x_2^2 - 3) - x_2(x_1^2 + x_2^2 - 3)$.

$= -(x_1 + x_2)(x_1^2 + x_2^2 - 3)$.

$\dot{V}$ is negative if $x_1^2 + x_2^2 > 3$.

$\therefore$ radius $= \sqrt{3}$.

System is UUB.