EE 5323 Homework 4

Vector Fields, Flows, First Integrals

1. Consider the undamped oscillator
   $$\ddot{x} + x = 0$$

   a. Write position-velocity state space form $\dot{X} = f(X)$.
   b. Plot the trajectories $x(t), \dot{x}(t)$ vs. time. Use initial conditions of $x(0) = 0.1, \ \dot{x}(t) = 0$
   c. Plot the vector field $f(X) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix}$ in the phase plane $(x_1, x_2) = (x, \dot{x})$. Plot for points spaced in a uniform mesh in the box x1=[-10,10], x2=[-10,10].
   d. Plot the system trajectories (flows or orbits) in the phase plane. Take ICs spaced in a uniform mesh in the box x1=[-10,10], x2=[-10,10].
   e. Derive the First Integral of Motion $F(x_1, x_2)$ as done in class. Plot the FIM as a 3-D surface over the phase plane on the x1=[-10,10], x2=[-10,10].

2. Repeat for the unstable system
   $$\ddot{x} - x = 0$$