EE 5323- Take Home Exam 2

Fall 2021

This exam has 6 pages in all. There are 4 problems.

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Almost all questions require numerical calculations to arrive at the answers. To obtain full credit, show all your work. No partial credit will be given without the supporting work.

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Name: _____

Pledge of honor:

"On my honor I have neither given nor received aid on this examination."

Signature: _____

1. Lyapunov Function

Use Lyapunov function to examine the stability of the following systems. Be clear and show all steps.

a.

$$\dot{x}_1 = x_2 \sin x_1 - x_1$$
$$\dot{x}_2 = -x_1 \sin x_1 - x_2$$

b.

$$\dot{x}_1 = x_2 \sin x_1 - x_1$$
$$\dot{x}_2 = -x_1 \sin x_1$$

2. LaSalle's Extension

$$\ddot{x} + k_1\dot{x} + k_2\dot{x}^3 + k_3x^5 = 0$$

a. Use Lyapunov to check the stability. Hint: Use the energy as the Lyapunov function. Take the potential energy as

$$PE = \int_0^x (k_2\dot{x}^3 + k_3x^5)dx$$

b. Use LaSalle's extension to find a stronger type of stability for the system.

3. Lyapunov Equation for Linear Systems

Use Lyapunov Equation to check the stability of the linear systems

a. $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ 0 & -6 \end{bmatrix} x$

b. $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x$

4. UUB

Use Lyapunov to show that the system

$$\dot{x}_1 = x_1 x_2{}^2 - x_1(x_1{}^2 + x_2{}^2 - 3)$$

$$\dot{x}_2 = -x_1{}^2 x_2 - x_2(x_1{}^2 + x_2{}^2 - 3)$$

is uniformly ultimately bounded UUB. That is, show that the Lyapunov derivative is NEGATIVE OUTSIDE A BOUNDED REGION. Find the radius of the bounded region outside which $\dot{V}$ <0. Any states outside this region are attracted towards the origin.