EE 5323 Homework 6

Lyapunov Stability Analysis, LaSalle, UUB

1. LaSalle extension.

   You studied and simulated this system on homework 5.

$$\dot{x}_1 = x_2 + x_1(x_1^2 - 2)$$
$$\dot{x}_2 = -x_1$$

You used a quadratic Lyapunov Function to show this system is locally SISL. And you found a region within which $\dot{V} \le 0$.

For this problem, Use LaSalle's extension to verify that the system is actually AS. Find the equilibrium point.

2. Consider the system

$$\dot{x} = 4x^2 y - f_1(x)\left(x^2 + 2y^2 - 4\right)$$
$$\dot{y} = -2x^3 - f_2(y)\left(x^2 + 2y^2 - 4\right)$$

where the continuous functions $f_1(x), f_2(y)$ have the same sign as their argument. Show that the system tends towards a limit cycle independent of the explicit expressions of $f_1(x), f_2(y)$.

3. UUB of system with disturbance.

   Consider the system on S&L p. 66 with a disturbance d

$$\dot{x} + c(x) + d = 0$$

Assume that $xc(x) > ax^2$ with $a > 0$ a known positive constant

   a. Assume that d is unknown but is bounded by $\|d\| < D$ with D a known positive constant. Prove that the system is UUB and find the bound on *x(t)*.

   b. Assume that d is unknown but is bounded by $\|d\| < D\|x\|$ with D a known positive constant. Prove that the system is UUB and find the bound on *x(t)*.

4. Use Lyapunov Equation to check the stability of the linear systems

   a. $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x$

   b. $\dot{x} = Ax = \begin{bmatrix} -7 & 4 \\ -7 & 3 \end{bmatrix} x$

   c. $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} x$