

EE 5323 - HW03

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HW03 – Nonlinear System Simulations

EE 5323 – Nonlinear Systems

Dr. Lewis

Exercise 1

Volterra Predator-Prey System

Consider the Volterra predator-prey system,

$$\dot{x}_1 = -x_1 + x_1 x_2$$

$$\dot{x}_2 = x_2 - x_1 x_2$$

Find the equilibrium points and their nature.

Answer

State variable is given as:

$$\dot{x}_1 = -x_1 + x_1 x_2$$

$$\dot{x}_2 = x_2 - x_1 x_2$$

The Volterra predator-prey system has limit cycles therefore the system is at equilibrium when the population of both predator and prey remain constant; thus, the derivative should be zero. To find the equilibrium, I set $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$. Solve the system for its roots.

$$\dot{x}_1 = 0 \Rightarrow 0 = -x_1 + x_1 x_2$$

$$\dot{x}_2 = 0 \Rightarrow 0 = x_2 - x_1 x_2$$

$$0 = x_1(\beta x_2 - \alpha) \Rightarrow x_1 = 0; x_2 = \alpha/\beta$$

$$0 = x_2(\gamma - \sigma x_1) \Rightarrow x_1 = \gamma/\sigma; x_2 = 0$$

There are two equilibrium points at (x_1, x_2) ,

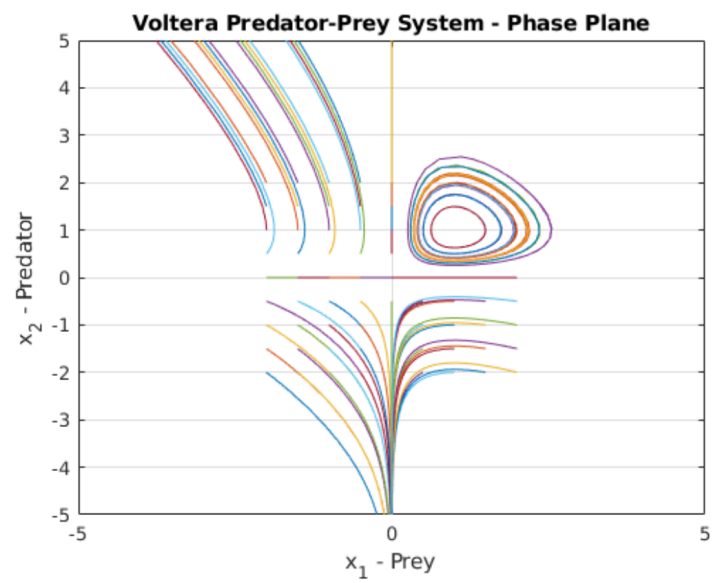
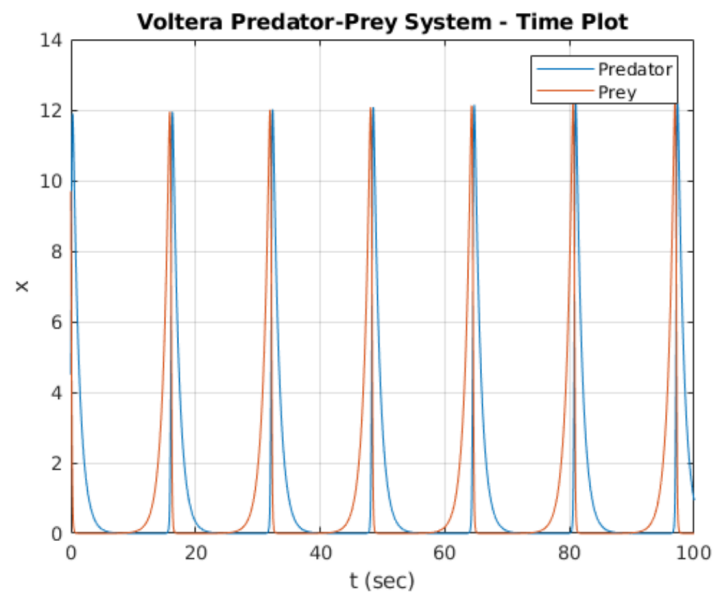
- At zero, $(0, 0)$,
- Any positive pair of integers $(\alpha/\beta, \gamma/\sigma)$

The equilibrium point nature of the zero is a stable center point that is a limit cycle. The other e.p. has a saddle point nature because it is stable in one dimension (goes to zero) and unstable in the other (goes to infinity).

Matlab Code

```
1 %% HW03 - Q01 - Voltera Predator-Prey System
2 % @author: Bardia Mojra
3 % @date: 09/28/2021
4 % @title HW03 - Q01 - Voltera Predator-Prey System
5 % @class ee5323 - Nonlinear Systems
6 % @professor - Dr. Frank Lewis
7
8 clc
9 close all
10 warning('off','all')
11 warning
12
13 x0_set = -2:.5:2;
14 t_intv= [0 100];
15 x_0= [4.5, 9.7]'; % initial conditions for x(t)
16
17 figure
18 [t,x]= ode23('Voltera', t_intv, x_0);
19 plot(t,x)
20 hold on;
21 grid on;
22 title('Voltera Predator-Prey System - Time Plot');
23 ylabel('x');
24 xlabel('t (sec)');
25 legend('Predator', 'Prey');
26 t_intv= [0 10];
27
28 figure
29 for i=x0_set
30     for j=x0_set
31         x0 = [i; j];
32         [t,x]= ode45('Voltera', t_intv, x0);
33         plot(x(:,1),x(:,2))
34         hold on;
35     end
36 end
37 title('Voltera Predator-Prey System - Phase Plane');
38 ylabel('x_2 - Predator');
39 xlabel('x_1 - Prey');
40 axis([-5 5 -5 5]);
41 grid on;
42
43 function xdot = Voltera(t,x)
44     xdot = [-x(1)+x(1)*x(2); x(2)-x(1)*x(2)];
45 end
```

Figures



Exercise 2

Equilibrium points and linearization

Consider the following system,

$$\dot{x}_1 = x_2(-x_1 + x_2 - 1)$$

$$\dot{x}_2 = x_1(x_1 + x_2 + 1)$$

- (a) Find all equilibrium points
- (b) Find Jacobian
- (c) Find the nature of all e.p.s

Answer

1. Find all e.p.s

At equilibrium points, all states reach their minimal energy state; therefore, the derivative of the state should equal zero. Then, we solve for the roots of the obtained characteristic equation.

$$\dot{X} = 0 \Rightarrow \dot{X} = AX \Rightarrow \dot{X} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad [2.1]$$

$$\begin{cases} \dot{x}_1 = 0 \Rightarrow x_2(-x_1 + x_2 - 1) = 0 \\ \dot{x}_2 = 0 \Rightarrow x_1(x_1 + x_2 + 1) = 0 \end{cases} \quad [2.2]$$

$$\dot{X} = \begin{cases} x_2 = 0; (-x_1 + x_2 - 1) = 0 \Rightarrow \\ x_1 = 0; (x_1 + x_2 + 1) = 0 \Rightarrow \end{cases} \quad [2.3]$$