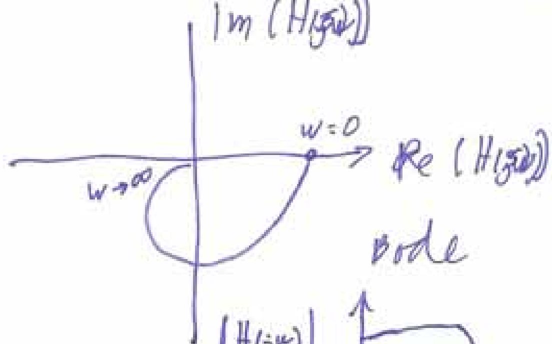
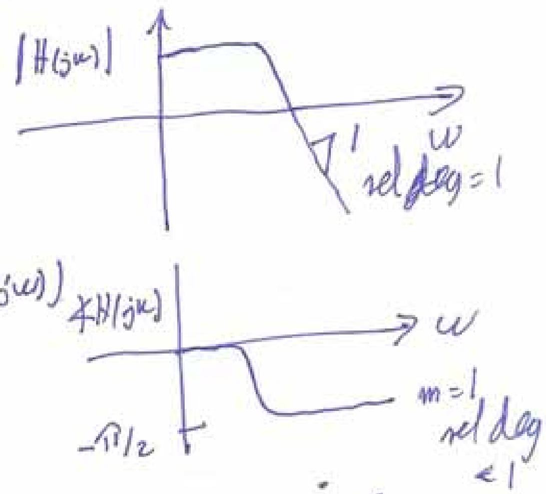
<u>Nyquist Plot</u>

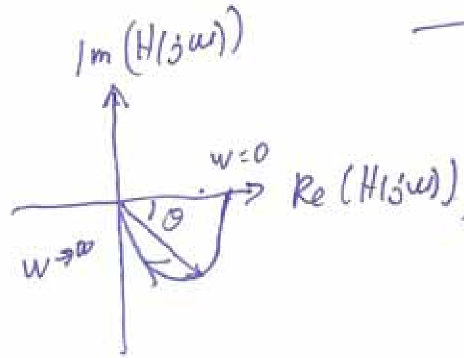
G&L p. 127



Bode



<u>pr</u>

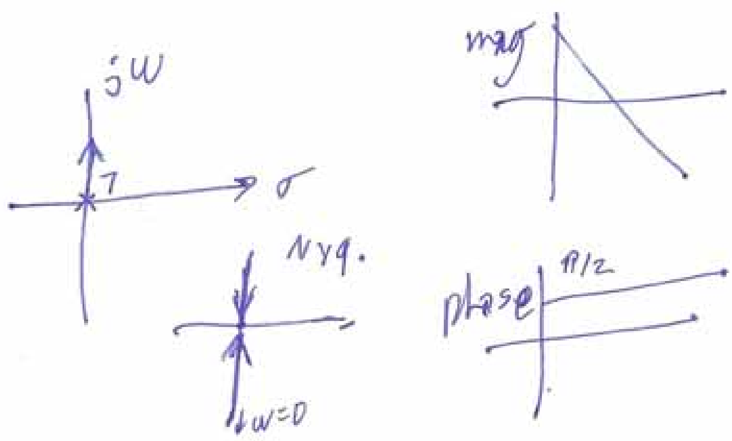


$\theta = \measuredangle H(j\omega)$

$|\theta| \leq \pi/2$ pr = passive

$|\theta| < \pi/2$ spr = dissip.

<u>ex</u> $\frac{1}{s}$



2

$$\dot V_1 = \overbrace{y_1^T v_1}^{\text{ext power input}} - \underbrace{g_1}_{\text{int. energy gen}} \quad \text{[K+L p. 133]}$$

$$\int_0^t \dot V_1 \, dt \le \int_0^t y_1^T v_1 \, dt - \int_0^t g_1 \, dt$$

$$\underbrace{V_1(t) - V_1(0)}_{\text{energy}} \le \underbrace{\int y_1^T v_1 \, dt}_{\int \text{power delivered} = \text{energy delivered}} - \underbrace{\int_0^t g_1 \, dt}_{\text{internal energy}}$$

passive if $\exists$ storage fn $V_1(t) \ni \Uparrow$

dissip. if $\int y_1^T v_1 \, dt \neq 0 \Rightarrow \int g_1 \, dt > 0$

Do linear Σ p. 137

passive iff $\forall \omega \geq 0 \ \ \mathrm{Re}\, H(j\omega) \geq 0$ p.r

dissip. iff $\forall \omega \geq 0 \ \ \mathrm{Re}\, H(j\omega) > 0$ spr

passivity extends pr to nonlin Σ

3

KY Lemma

S&L p-139

$\dot{x} = Ax + Bv \quad , y = Cx$

$V = \frac{1}{2} x^T P x$

$\dot{V} = \frac{1}{2}\left(x^T P \dot{x} + \dot{x}^T P x\right)$

$= \frac{1}{2}\left(x^T P(Ax + Bv) + (Ax + Bv)^T Px\right)$

$= \frac{1}{2} x^T (PA + A^T P) x + \underbrace{x^T \underbrace{PB}v}_{y^T v}$

$\dot{V} = -\frac{1}{2} x^T Q x + y^T v$

↑ power form

if $y = Cx = B^T P x$

$C = B^T P$

so SPR $\Leftarrow \exists\, Q, P \ni$

$A^T P + PA = -Q$

& $C = B^T P$

SJL p. 134 4

ex 4.17

$m\ddot{x} + x^2\dot{x}^3 + x^7 = f \quad , \; y = \dot{x}$

c.f. p. 74

$\ddot{x} + b(\dot{x}) + c(x) = 0$

$b(\dot{x})\dot{x} = x^2\dot{x}^4 > 0$

$c(x)x = x^8 > 0$

Lyap fn is $V = \frac{1}{2}\dot{x}^2 + \int_0^x c(y)\,dy$

chose $V_1 = \frac{1}{2}m\dot{x}^2 + \frac{1}{8}x^8$

then $\dot{V}_1 = \dot{x}m\ddot{x} + x^7\dot{x}$

$= -(x^2\dot{x}^3 + x^7 - f)\dot{x} + \cancel{\frac{1}{8}x^8}\; x^7\dot{x}$

$= -x^2\dot{x}^4 - x^7\dot{x} + f\dot{x} + x^7\dot{x}$

$\dot{V}_1 = \dot{x}f(x) - x^2\dot{x}^4$

Def $y = \dot{x}$, & $u = f(x)$, $g_1 = x^2\dot{x}^4$

then $\dot{V}_1 = yu - g_1(t)$

& system is passive

S&L p.134

~~e~~ 4.18

$$\dot{x} + \lambda x = u$$
$$y = h(x)$$

c.f. p.66

$$\dot{x} + c(x) = 0$$
$$x\,c(x) > 0$$

then $V = \frac{1}{2}x^2$

$$\& \ \dot{V} = \cancel{2}x\dot{x} = -x(\cancel{c(x)}\ \lambda x - u)$$
$$= -\lambda x^2 + xu$$

if $y = x$

Def $V = \int_0^x h(\xi)\, d\xi$

then $\dot{V} = \frac{d}{dt}\int_0^x h(\xi)\, d\xi = \dot{x}\, h(x)$

$$= h(x)(-\lambda x + u)$$

$$\dot{V} = \underbrace{-\lambda x h(x)}_{-g_1} + \underbrace{h(x) u}_{yu}$$

Σ is ~~pr~~ passive if $x h(x) > 0$
$\& \ \lambda > 0$

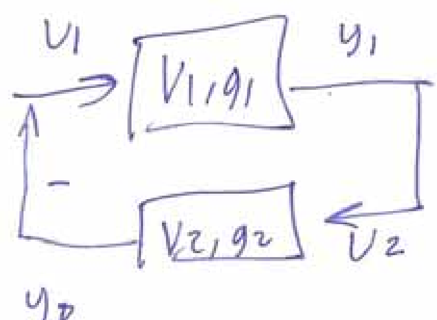
Σ is dissip if $\lambda \neq 0$
since $\int_0^\infty h u \, dt \neq 0 \Rightarrow \int_0^\infty \lambda x h(x)\, dt > 0$

Do S+L p-133 Block combs.

p.133 if

$$\frac{d}{dt}[V_1+V_2] = y_1^T u_1 - g_1 + y_2^T u_2 - g_1$$

$$= y_1^T u_1 - g_1 - y_1^T u_1 - g_1$$



$u_1 = -y_2$

$u_2 = y_1$

$$+\frac{d}{dt}(V_1+V_2) = -(g_1+g_2)$$

Then $V = V_1 + V_2$ is a Lyap fn

$$\frac{d}{dt}(V) < 0$$

then if $\ddot{V}$ is unif cont.

$$(g_1+g_2) \to 0$$

Robot Manipulator is passive.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} + g(q) = \tau$$

$$V = \frac{1}{2}\dot{q}^T M \dot{q} + P(q) \quad = \text{total energy}$$

$$\dot{V} = \cancel{\tfrac{1}{2}}\,\dot{q}^T M \ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}\dot{q} + \dot{P}(q)$$

$$\dot{V} = \dot{q}^T(-C\dot{q} - D\dot{q} - g + \tau) + \frac{1}{2}\dot{q}^T \dot{M}\dot{q} + \dot{P}$$

$$= -\dot{q}^T C\dot{q} - \dot{q}^T D\dot{q} - \dot{q}^T g + \dot{q}^T\tau + \frac{1}{2}\dot{q}^T \dot{M}\dot{q} + \dot{P}$$

$$\dot{V} = -\dot{q}^T D\dot{q} + \dot{q}^T\left(\frac{1}{2}\dot{M} - C\right)\dot{q} \quad = 0 \text{ skew symm}$$

$$- \dot{q}^T g + \dot{P} + \dot{q}^T\tau$$

$$= -\dot{q}^T D\dot{q} + \dot{q}^T\tau \quad \text{if} \quad \dot{P} = \dot{q}^T g$$

$$P = \int_0^t \dot{q}^T g(q)\, dt$$

pot energy

$$\dot{V} = -\dot{q}^T D\dot{q} + \dot{q}^T\tau$$

if $y = \dot{q}$ → passive

dissip if $D > 0$

8

nm autom chap 4

EE 5323 Homework 5
Fall 2009, Slotine and Li

Lyapunov

X 1. S&L p. 103, Example 4.2.

   a. For the 3 systems given, prove the stability claimed by verifying the 3 conditions given.

   b. Integrate the state equations to find the solutions x(t) of the three systems.

X 2. S&L p. 105, Example 4.3. Integrate the state equation to verify the solution given.

X 3. S&L p. 155 problem 4.9, parts a and b.

4. Consider the nonlinear dynamics for an m-link robot manipulator,

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+D\dot{q}+g(q)=\tau,$$

where $q,\tau\in R^m$. $M(q)$ accounts for the robot inertia. $C(q,\dot{q})\dot{q}$ accounts for centrifugal and Coriolis forces. $D\dot{q}$ accounts for viscous damping. $g(q)$ accounts for gravity forces. In addition, we have the following properties:

   i. $M(q)$ is a symmetric positive definite matrix of q.
   ii. $\dot{M}-2C$ is a skew symmetric matrix of $q,\dot{q}$.
   iii. $g(q)=\partial W(q)/\partial q$, where $W(q)$ is a positive definite function of q.

Show that for $D=0$, the map from τ to $\dot{q}$ is passive *lossless*. And when D is positive definite, the map from τ to $\dot{q}$ is passive *dissipative*.

Hint: Use the total energy $V=\frac{1}{2}\dot{q}^T M(q)\dot{q}+P(q)$ as a storage function. Select an appropriate $P(q)$, a positive definite function of q.