EE 5323 - HW06

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HW06 – Lyapunov Stability Analysis, LaSalle's Extension, and UUB
EE 5323 – Nonlinear Systems
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Exercise 1

LaSalle's Extension
Consider the system from HW05,

$$\begin{cases} \dot{x}_1 = x_2 + x_1(x_1^2 - 2) \\ \dot{x}_2 = -x_1 \end{cases}$$

We used a quadratic Lyapunov Function to show this system is locally SISL. And we found the region within which $\dot{V} \leq 0$. Use LaSalle's extension to verify that the system is AS. Find the equilibrium point.

Answer

Exercise 2

LaSalle's Extension
Consider the following system,

$$\begin{cases} \dot{x} = 4x^2y - f_1(x)\left(x^2 + 2y^2 - 4\right) \\ \dot{y} = 2x^3 - f_1(y)\left(x^2 + 2y^2 - 4\right) \end{cases}$$

where the continuous functions $f_1(x)$, $f_2(y)$ have the same sign as their argument. Show that the system tends towards a limit cycle independent of the explicit expressions of $f_1(x)$, $f_2(y)$.

Answer

Exercise 3

UUB of a System with Disturbance
Consider the system on S&L p. 66 with a disturbance d,

$$\dot{x} + c(x) + d = 0$$

Assume that $xc(x)ax^2$ with $a > 0$ a known positive constant.
a. Assume that d is unknown but is bounded by $||d|| < D$ with D a known positive constant. Prove that the system is UUB and find the bound on $x(t)$.
b. Assume that d is unknown but is bounded by $||d|| < D||x||$ with D a known positive constant. Prove that the system is UUB and find the bound on $x(t)$.

Answer

Exercise 4

Lyapunov Analysis
Use Lyapunov Equation to check the stability of the linear systems.

a. $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x$

b. $\dot{x} = Ax = \begin{bmatrix} -7 & 4 \\ -7 & 3 \end{bmatrix} x$

c. $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} x$

Answer