EE 5323 Nonlinear Control Systems

Homework Pledge of Honor

On all homeworks in this class - YOU MUST WORK ALONE.

Any cheating or collusion will be severely punished.

It is very easy to compare your software code and determine if you worked together
It does not matter if you change the variable names.

Please sign this form and include it as the first page of all of your submitted homeworks.

.....

Typed Name: Zhenlin Pei

Pledge of honor:

"On my honor I have neither given nor received aid on this homework."

e-Signature: Zhenlin Pei

EE 5323 Homework 4

Vector Fields, Flows, First Integrals

1. Consider the undamped oscillator
   $\ddot{x} + x = 0$

   a. Write position-velocity state space form $\dot{X} = f(X)$.
   b. Plot the trajectories $x(t), \dot{x}(t)$ vs. time. Use initial conditions of $x(0) = 0.1,\ \dot{x}(t) = 0$
   c. Plot the vector field $f(X) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix}$ in the phase plane $(x_1, x_2) = (x, \dot{x})$. Plot for points spaced in a uniform mesh in the box x1=[-10,10], x2=[-10,10].
   d. Plot the system trajectories (flows or orbits) in the phase plane. Take ICs spaced in a uniform mesh in the box x1=[-10,10], x2=[-10,10].
   e. Derive the First Integral of Motion $F(x_1, x_2)$ as done in class. Plot the FIM as a 3-D surface over the phase plane on the x1=[-10,10], x2=[-10,10].

2. Repeat for the unstable system
   $\ddot{x} - x = 0$

EE 5323 Homework4

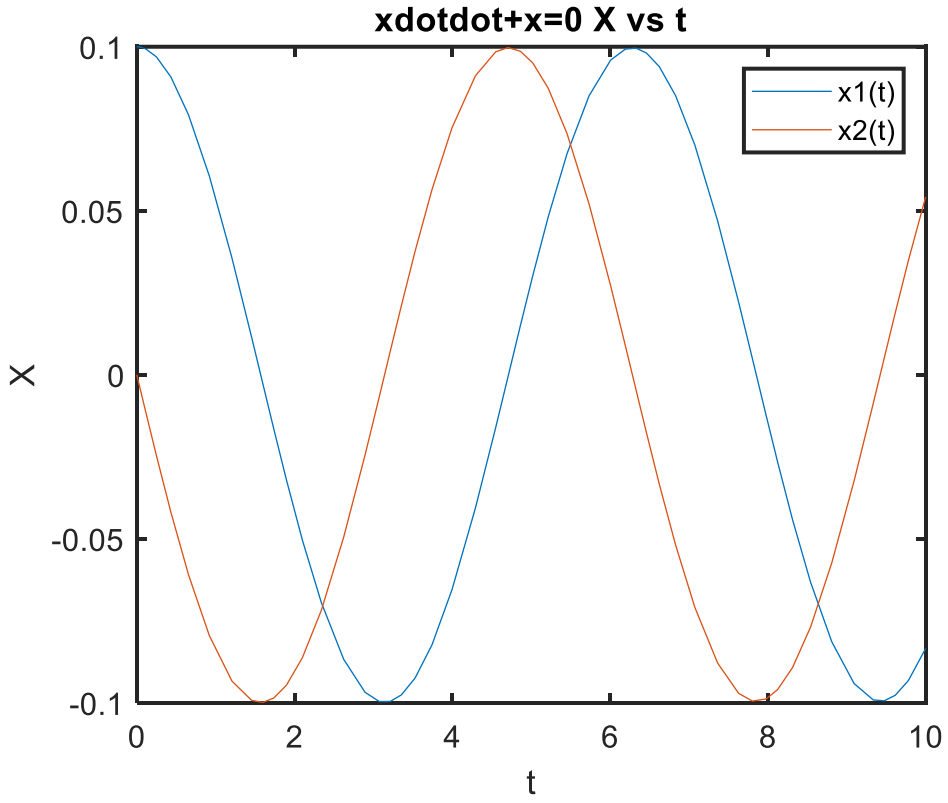
Name: Zhenlin Pei. Please refer to attached M-files.

1. Vector Fields, flows, first integrals. Undamped oscillator

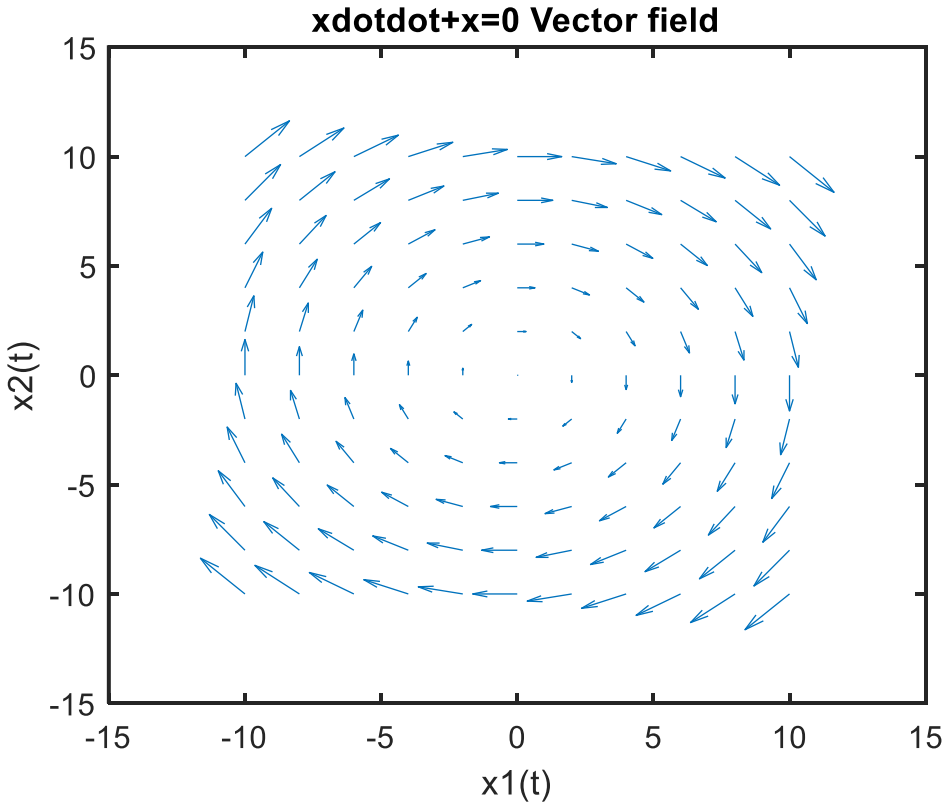
$$\ddot{x} + x = 0$$

(a, e) Please refer to attached scan pdf for handwriting.

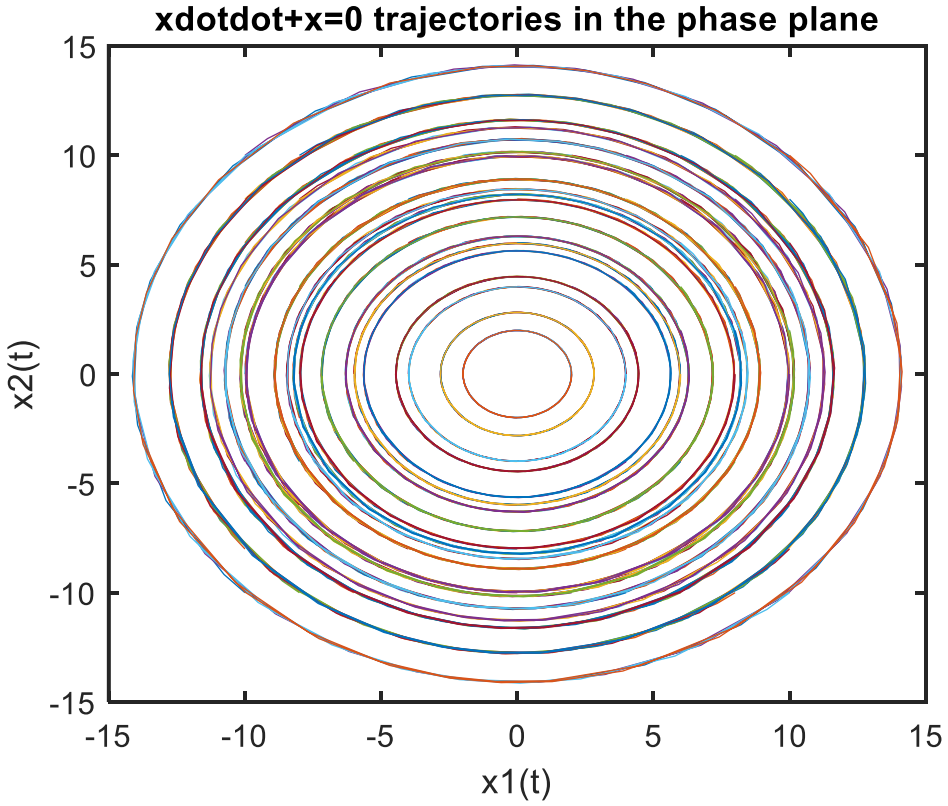
(b) plot x(t), xdot(t) vs t.



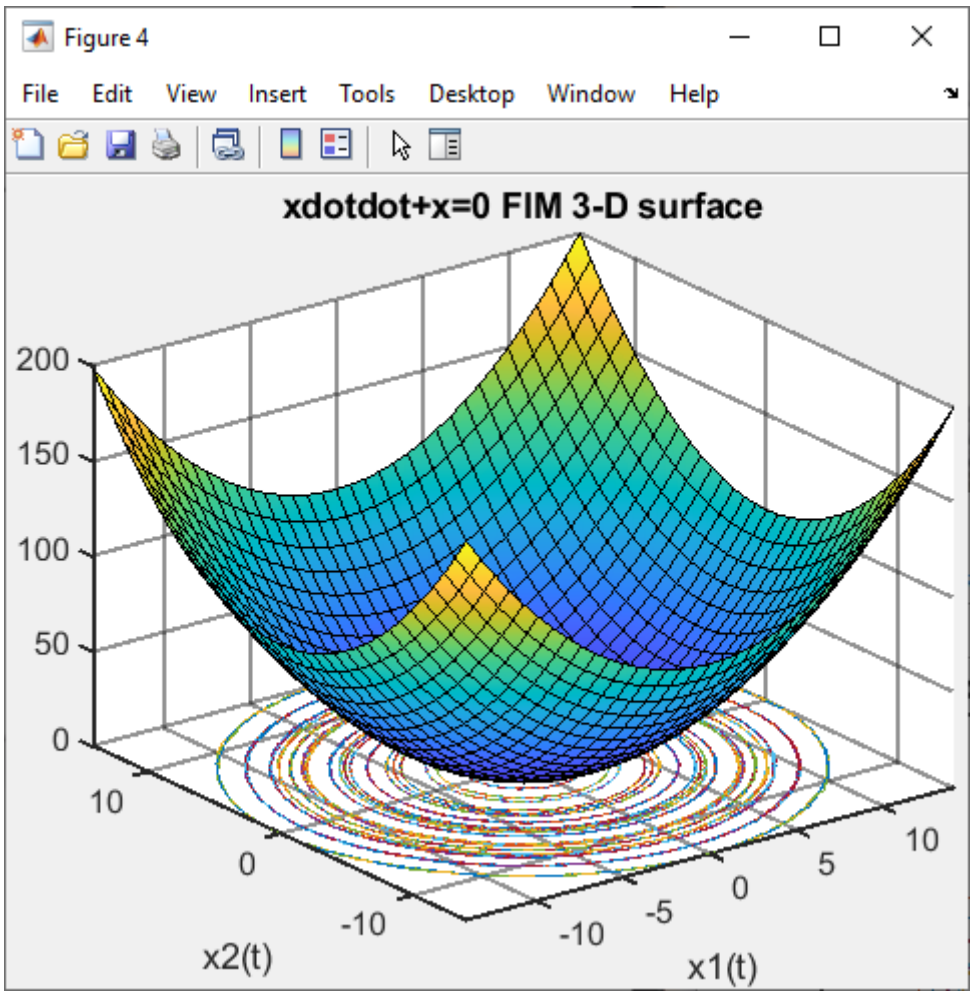
(c) plot vector field in the phase plane.



(d) plot trajectories in the phase plane.



(e) plot the FIM 3D surface.

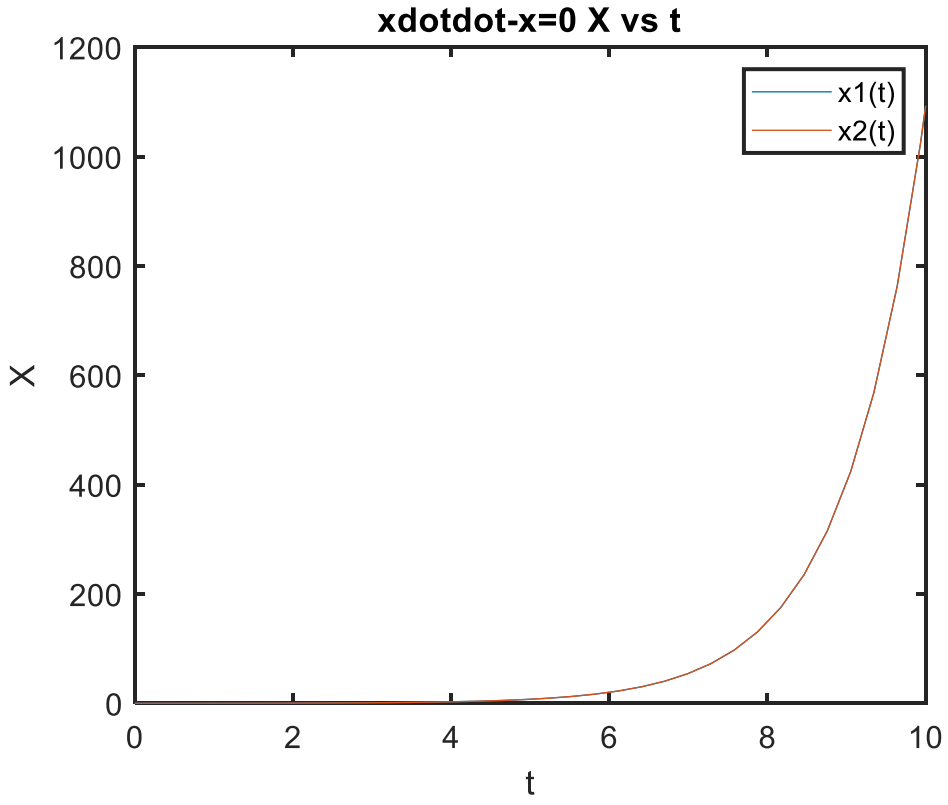


2. Vector Fields, flows, first integrals. Unstable system

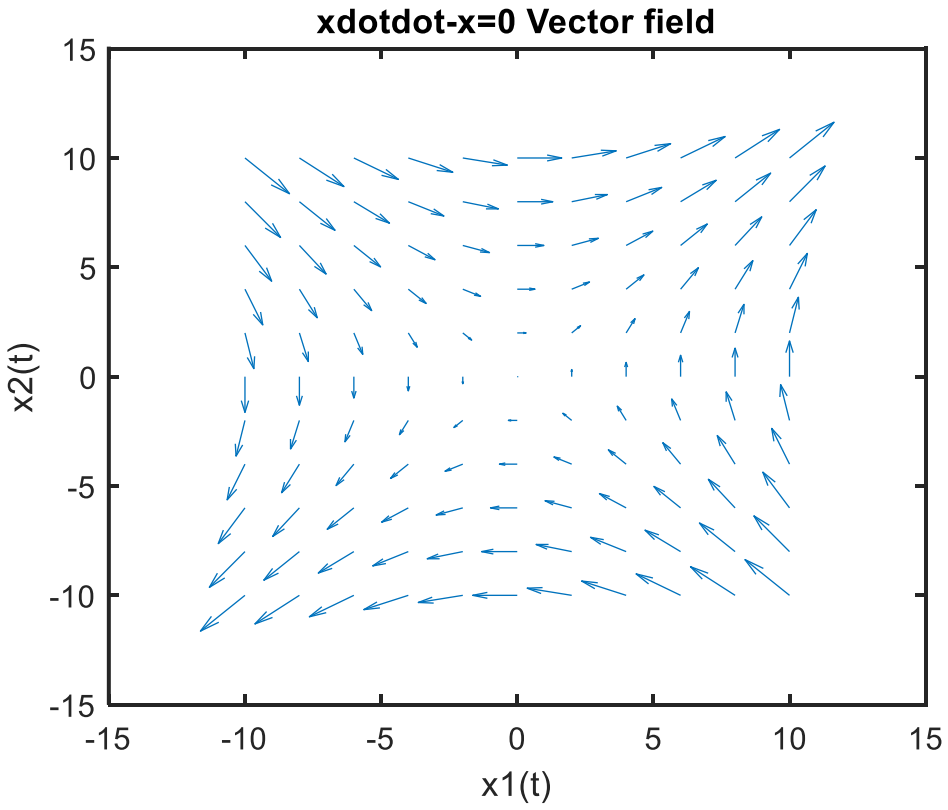
$$\ddot{x} - x = 0$$

(a, e) Please refer to attached scan pdf for handwriting.

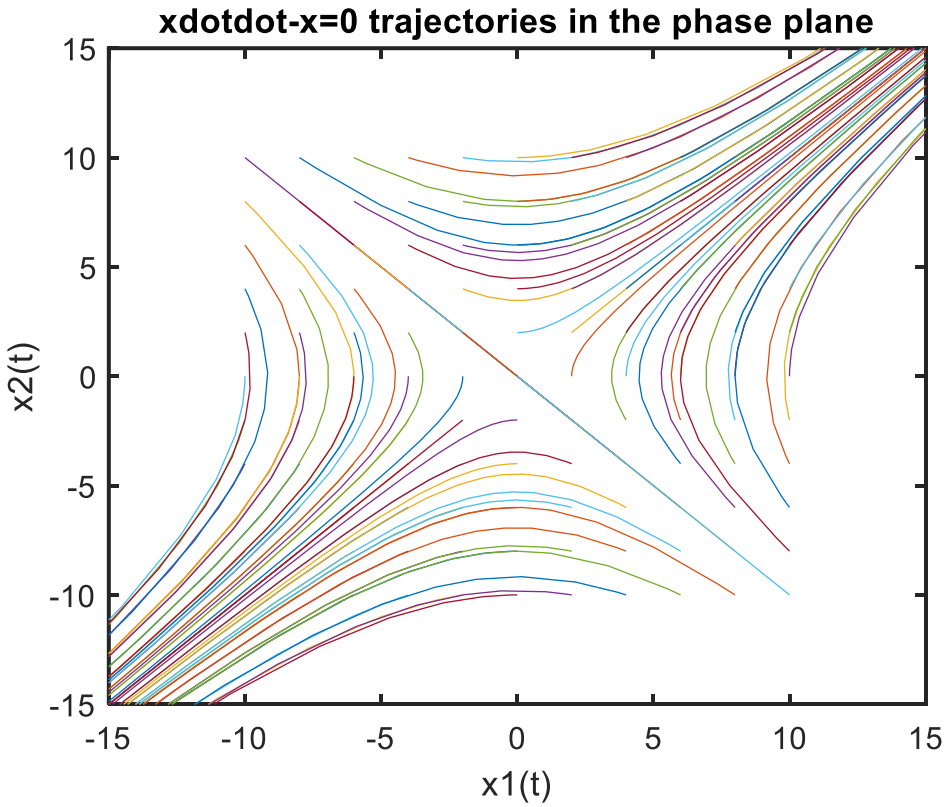
(b) plot x(t), xdot(t) vs t.



(c) plot vector field in the phase plane.



(d) plot trajectories in the phase plane.



(e) plot the FIM 3D surface.

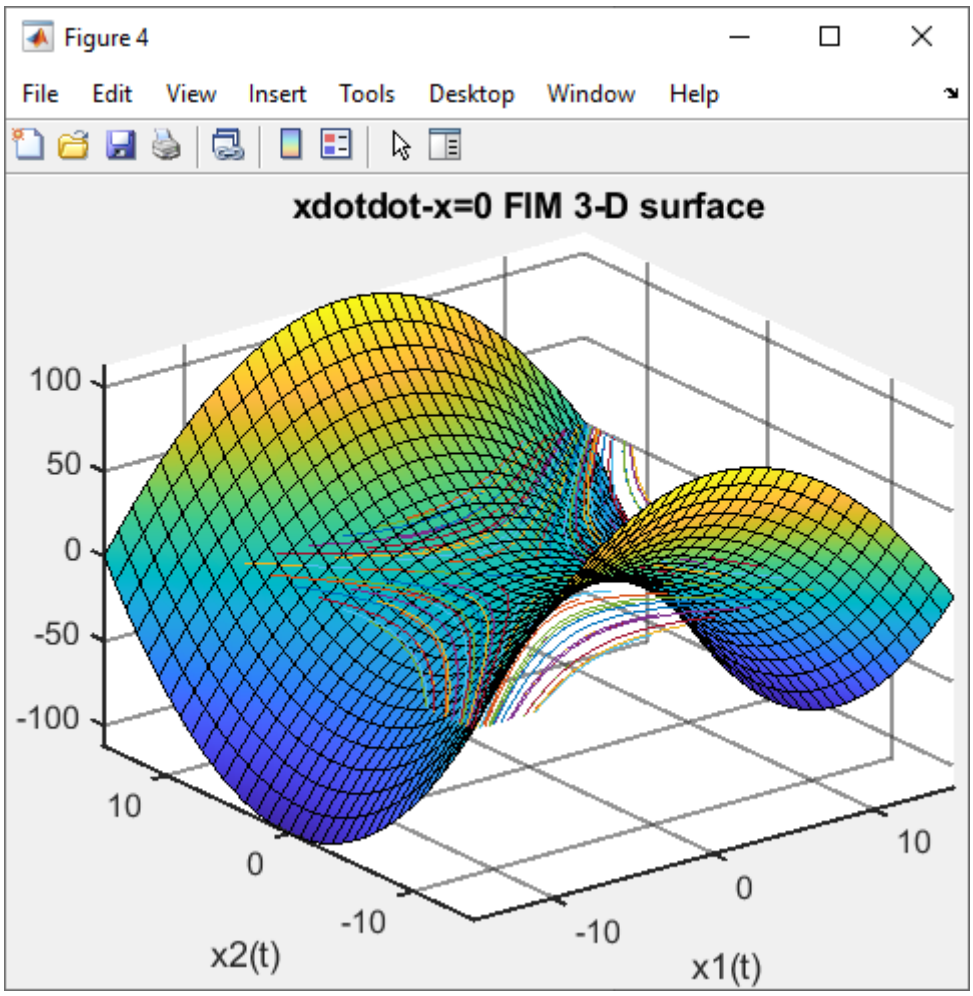


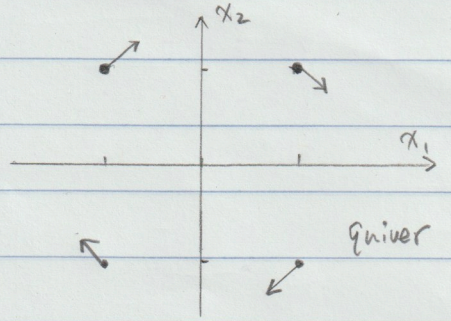
Figure 4
File Edit View Insert Tools Desktop Window Help
xdotdot-x=0 FIM 3-D surface
100
50
0
-50
-100
10
0
-10
x2(t)
-10
0
10
x1(t)

Zhenlin Pei HW4

55~1:00:00 10.5 First Integral of Motion – FIM $\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x$ ↙ 32 10.7 35

1. (a) $\ddot{x} + x = 0$,

$\ddot{x} = -x \Rightarrow x = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix} = f(x) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$



quiver

$f(1,1) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $f(1,-1) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$f(-1,-1) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $f(-1,1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(e)

18~21 10.7 / 21~25 both sides time $\dot{x}$

$\dot{x}\ddot{x} + \dot{x}x = 0 \Rightarrow \frac{d}{dt}\left(\frac{\dot{x}^2}{2} + \frac{x^2}{2}\right) = \left(\frac{2\dot{x}\ddot{x}}{2} + \frac{2x\dot{x}}{2}\right) = 0$

$\dot{x} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$

$V(x, \dot{x}) = \left(\frac{\dot{x}^2}{2} + \frac{x^2}{2}\right)$ is a FIM

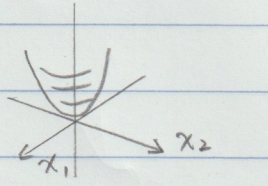
↳ $\left(\frac{x_2^2}{2} + \frac{x_1^2}{2}\right)$

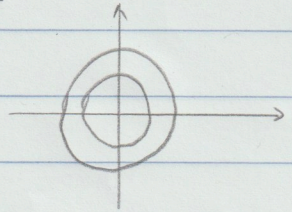
Double check

$= \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix}$

$\dot{V} = \frac{\partial V^T}{\partial x} f(x) = \begin{bmatrix} \frac{\partial V}{\partial x_1} & \frac{\partial V}{\partial x_2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix}$

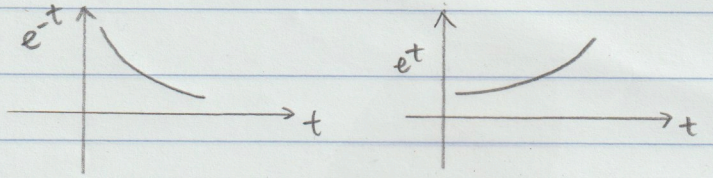
$= x_1 x_2 - x_1 x_2 = 0$

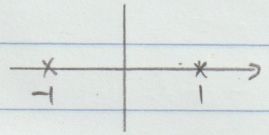




10.7 37~44 2. $\ddot{x} - x = 0 \Rightarrow X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} X = AX$

$A(s) = |sI - A| = \begin{vmatrix} s & -1 \\ -1 & s \end{vmatrix} = s^2 - 1 = 0 \Rightarrow s^2 = 1, \; s = \pm 1$





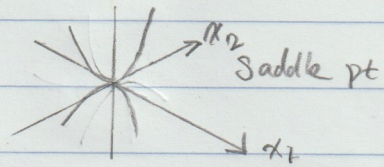
FIM $\quad V(x) > 0, \; \dot{V} = 0$

$\ddot{x} - x = 0$ both sides time $\dot{x}$

$\Rightarrow \dot{x}\ddot{x} - \dot{x}x = 0 \Rightarrow \frac{d}{dt}\left(\frac{\dot{x}^2}{2} - \frac{x^2}{2}\right) = 0$

$V(x)$

$\Rightarrow$ FIM $V(x) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2 = \frac{1}{2}x_2^2 - \frac{1}{2}x_1^2$



Double check

$\dot{V} = \frac{\partial V^T}{\partial x} f(x) = \begin{bmatrix} \frac{\partial V}{\partial x_1} & \frac{\partial V}{\partial x_2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} -x_1 & x_2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = -x_1x_2 + x_1x_2 = 0$

Contents

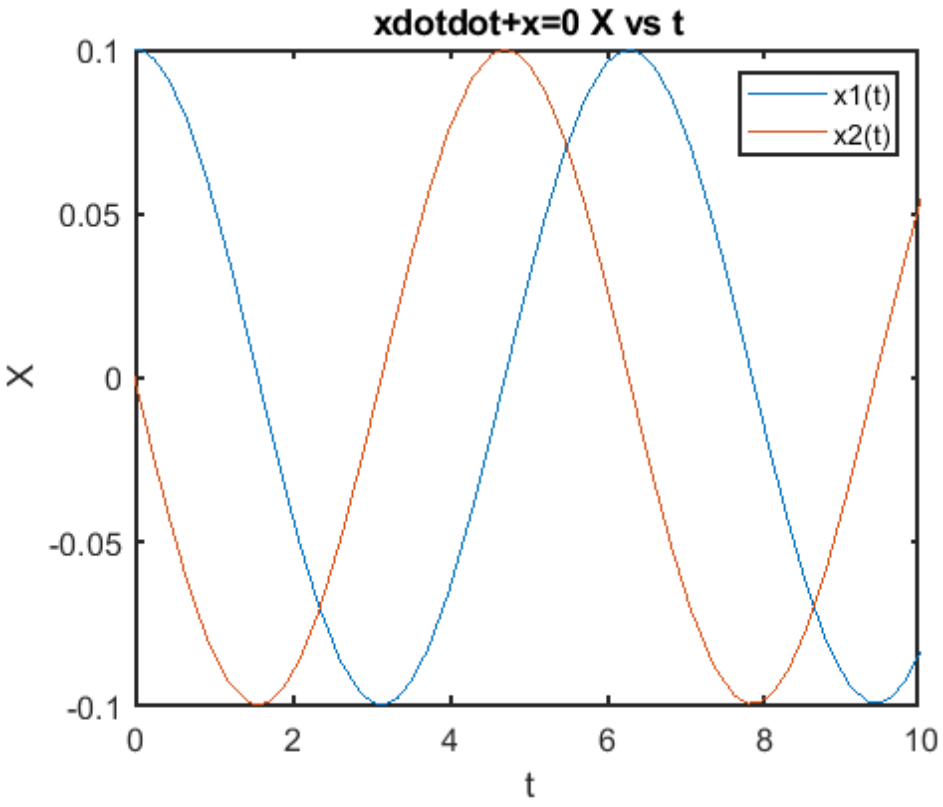
Zhenlin Pei hw4 q1 undamped oscillator

```
function [t,x] = hw4_q1()
```

```
close all; clc; addpath('..\func');
time = [0 10];   % interval time
```

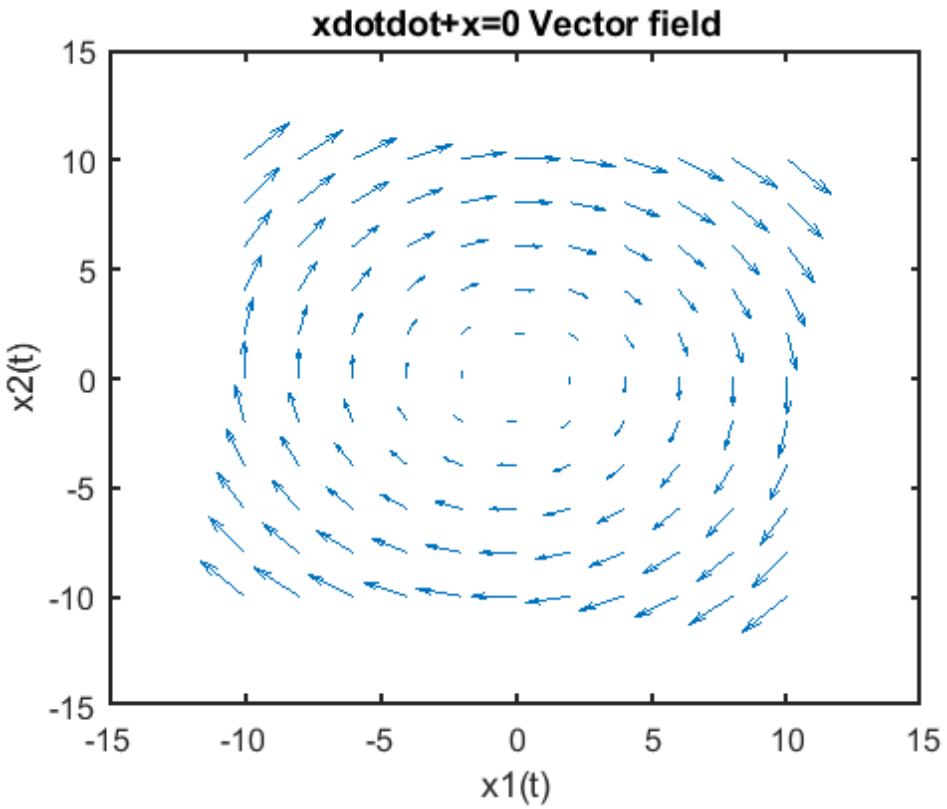
solve the system by ode23(time,state). plot X vs t.

```
x0 = [.1 0]; % ICs, % initial state
[t,x] = ode23(@control,time,x0);
figure();plot(t,x);
xlabel('t'); ylabel('X'); title('xdotdot+x=0 X vs t');
legend('x1(t)','x2(t)');
set(gca,'FontSize',12,'Linewidth',1.5,'GridAlpha',.5);
set(gcf,'position',[1630 539 480 406]);tightfig;
```



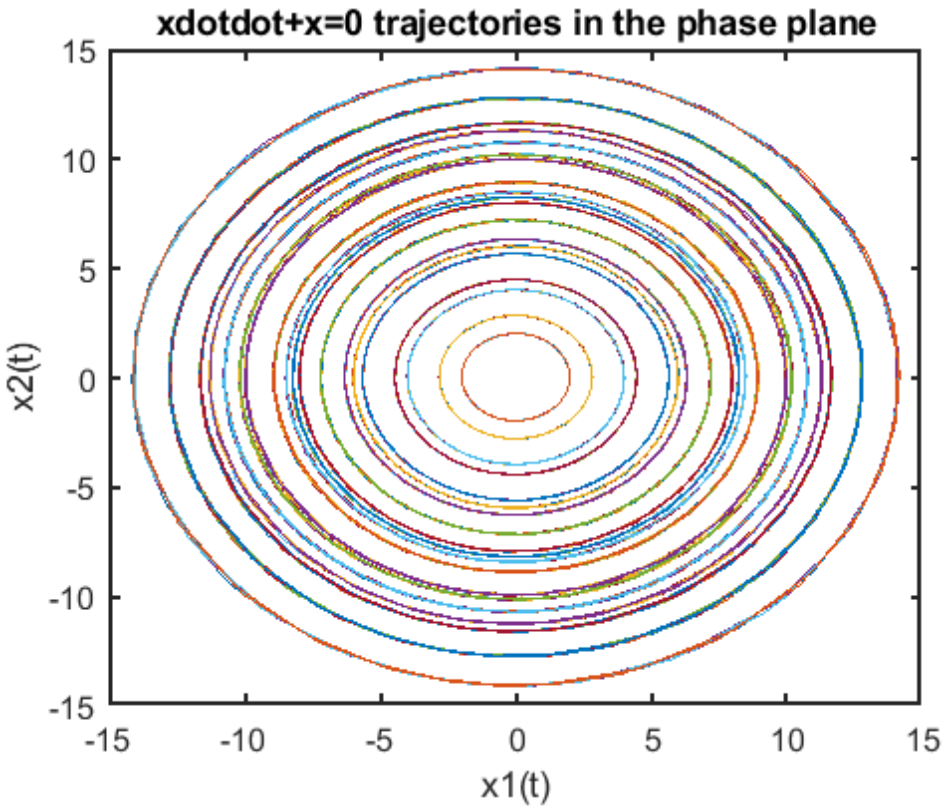
solve the system. plot vector field under x2 vs x1.

```
uniform_mesh = -10:2:10;
[x1, x2] = meshgrid(uniform_mesh,uniform_mesh);
dx1 = []; dx2 = [];
for i = 1:length(x1)
    for j = 1:length(x2)
        dx = control(0,[x1(i,j);x2(i,j)]); %initial time.
        dx1(i,j) = dx(1);
        dx2(i,j) = dx(2);
    end
end
figure(); quiver(x1,x2,dx1,dx2);
xlabel('x1(t)'); ylabel('x2(t)'); title('xdotdot+x=0 Vector field');
set(gca,'FontSize',12,'Linewidth',1.5,'GridAlpha',.5);
set(gcf,'position',[1630 539 480 406]);tightfig;
```



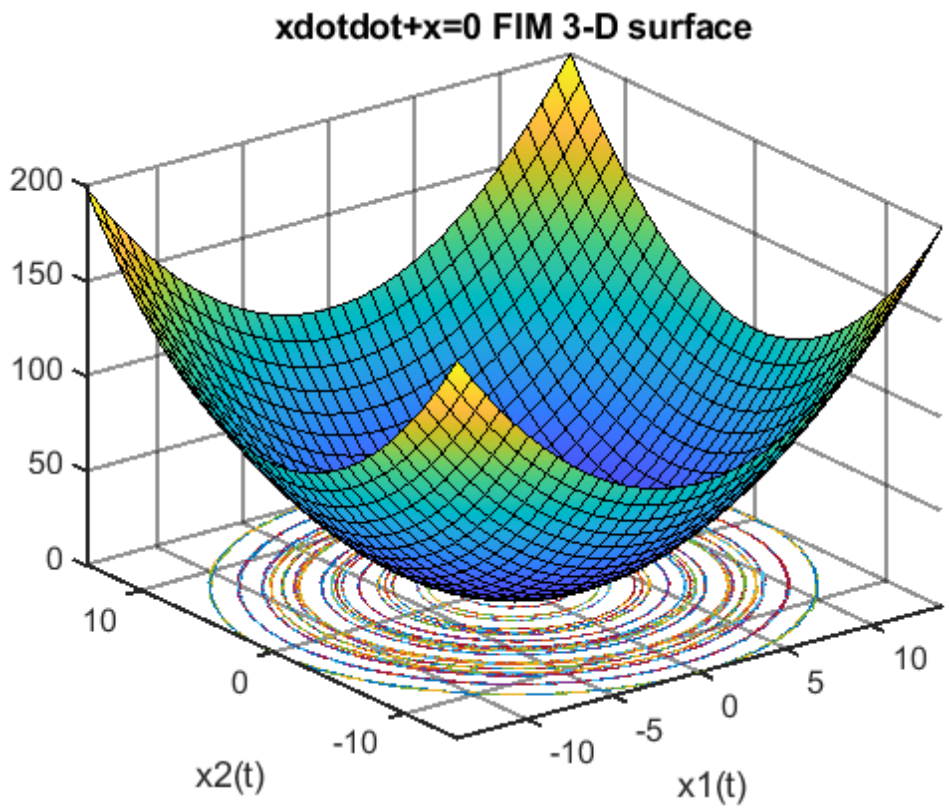
solve the system. plot x2 vs x1. phase plane.

```
figure();
for i = uniform_mesh
    for j = uniform_mesh
        x0 = [i,j]; % ICs, % initial state
        [t,x] = ode23(@control,time,x0);
        plot(x(:,1),x(:,2)); hold on;
    end
end
xlabel('x1(t)'); ylabel('x2(t)');
title('xdotdot+x=0 trajectories in the phase plane');
set(gca,'FontSize',12,'Linewidth',1.5,'GridAlpha',.5);
set(gcf,'position',[1630 539 480 406]);tightfig;
```



solve the system. plot the FIM as a 3-D surface over the phase plane

```
figure();
for i=uniform_mesh
    for j=uniform_mesh
        x0 = [i,j]; % ICs, % initial state
        [t,x] = ode23(@control,time,x0);
        plot3(x(:,1),x(:,2),zeros(length(x),1));
        syms x1 x2; fsurf(.5*x2^2+.5*x1^2);
        hold on;
    end
end
xlabel('x1(t)'); ylabel('x2(t)'); title('xdotdot+x=0 FIM 3-D surface');
set(gca,'FontSize',12,'Linewidth',1.5,'GridAlpha',.5);
set(gcf,'position',[1630 539 480 406]);tightfig;
```



define the system.

```
function dx = control(t,x);
    dx = [ x(2);    %    x1dot
          -x(1)];   %    x2dot
end
```

```
end
```

```
ans =

         0
    0.0800
    0.3634
    0.6198
    0.7780
    0.9363
    1.0909
    1.2880
    1.5245
    1.8035
    2.0778
    2.2652
    2.3979
    2.5307
    2.6933
    2.8974
    3.1411
    3.4285
    3.6958
    3.8721
```

```
    4.0485
    4.1921
    4.3798
    4.6072
    4.8756
    5.1591
    5.4037
    5.5407
    5.6776
    5.8420
    6.0477
    6.2929
    6.5824
    6.8481
    7.0216
    7.1952
    7.3410
    7.5305
    7.7597
    8.0301
    8.3118
    8.5546
    8.6947
    8.8348
    9.0040
    9.2140
    9.4638
    9.7588
   10.0000
```

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Contents

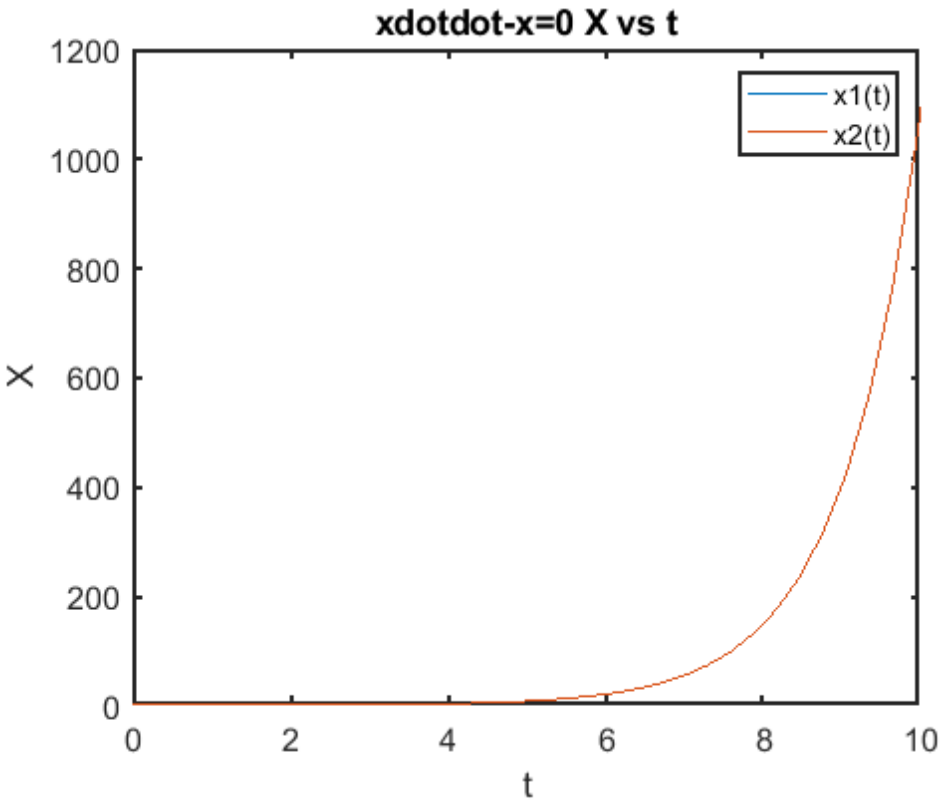
Zhenlin Pei hw4 q2 unstable

```
function [t,x] = hw4_q2()
```

```
close all; clc; addpath('..\func');
time = [0 10];   % interval time
```

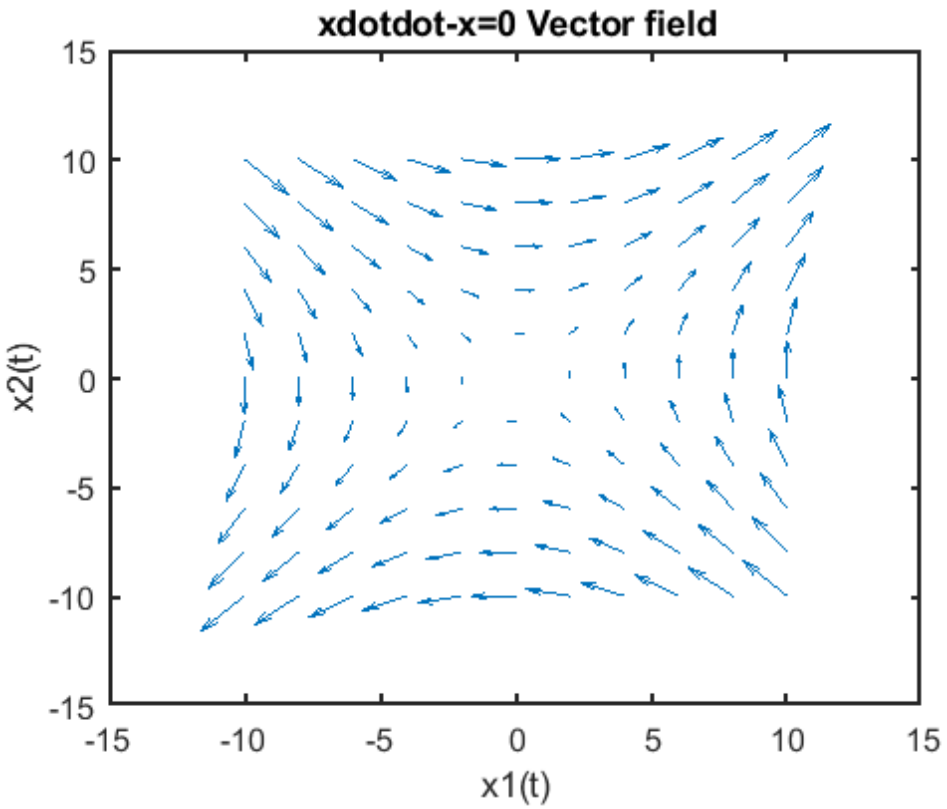
solve the system by ode23(time,state). plot X vs t.

```
x0 = [.1 0]; % ICs, % initial state
[t,x] = ode23(@control,time,x0);
figure();plot(t,x);
xlabel('t'); ylabel('X'); title('xdotdot-x=0 X vs t');
legend('x1(t)','x2(t)');
set(gca,'FontSize',12,'Linewidth',1.5,'GridAlpha',.5);
set(gcf,'position',[1630 539 480 406]);tightfig;
```



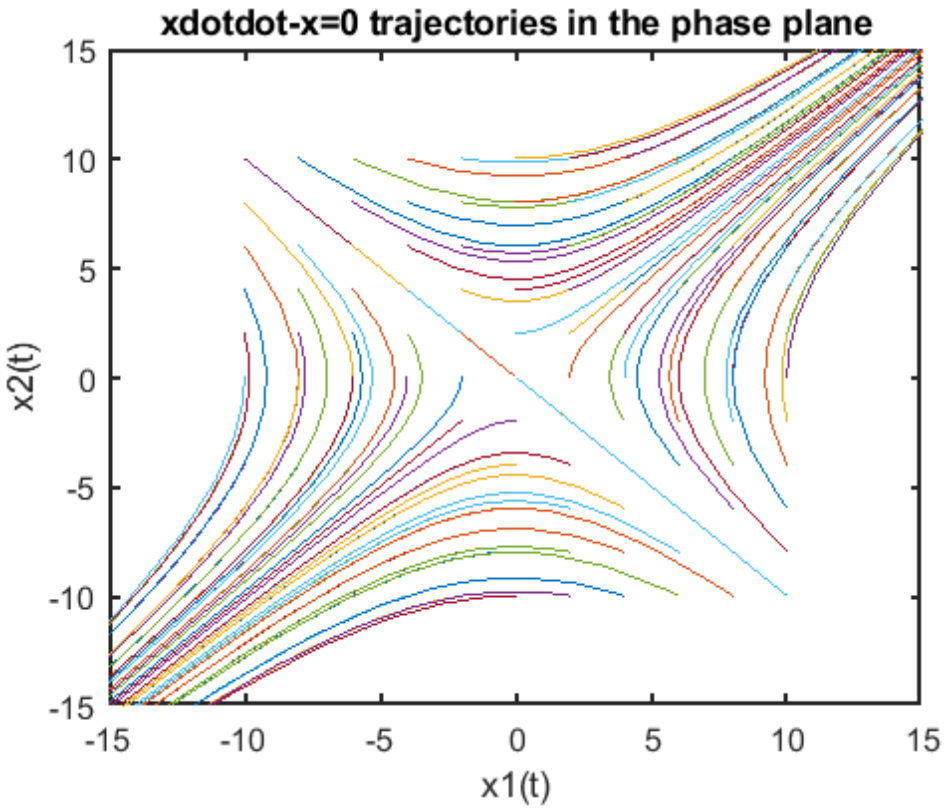
solve the system. plot vector field under x2 vs x1.

```
uniform_mesh = -10:2:10;
[x1, x2] = meshgrid(uniform_mesh,uniform_mesh);
dx1 = []; dx2 = [];
for i = 1:length(x1)
    for j = 1:length(x2)
        dx = control(0,[x1(i,j);x2(i,j)]); %initial time.
        dx1(i,j) = dx(1);
        dx2(i,j) = dx(2);
    end
end
figure(); quiver(x1,x2,dx1,dx2);
xlabel('x1(t)'); ylabel('x2(t)'); title('xdotdot-x=0 Vector field');
set(gca,'FontSize',12,'Linewidth',1.5,'GridAlpha',.5);
set(gcf,'position',[1630 539 480 406]);tightfig;
```



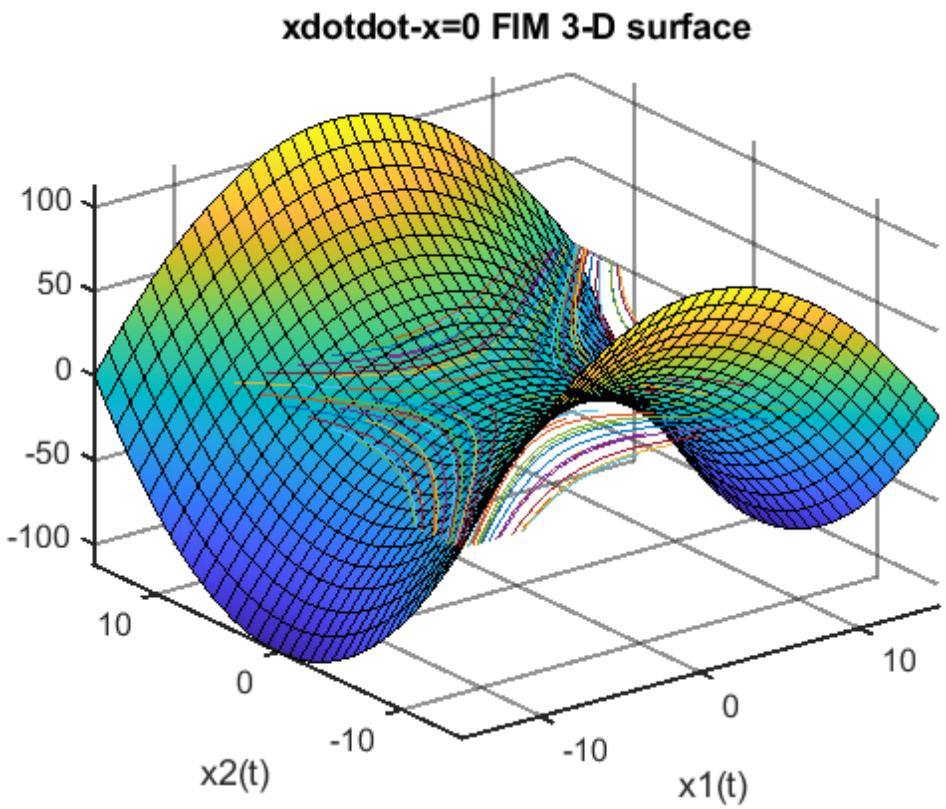
solve the system. plot x2 vs x1. phase plane.

```
figure();
for i = uniform_mesh
    for j = uniform_mesh
        x0 = [i,j]; % ICs, % initial state
        [t,x] = ode23(@control,time,x0);
        plot(x(:,1),x(:,2)); hold on;
    end
end
xlim([-15 15]); ylim([-15 15]);
xlabel('x1(t)'); ylabel('x2(t)');
title('xdotdot-x=0 trajectories in the phase plane');
set(gca,'FontSize',12,'Linewidth',1.5,'GridAlpha',.5);
set(gcf,'position',[1630 539 480 406]);tightfig;
```



solve the system. plot the FIM as a 3-D surface over the phase plane

```
figure();
for i=uniform_mesh
    for j=uniform_mesh
        x0 = [i,j]; % ICs, % initial state
        [t,x] = ode23(@control,time,x0);
        plot3(x(:,1),x(:,2),zeros(length(x),1));xlim([-15 15]); ylim([-15 15]);
        syms x1 x2; fsurf(.5*x2^2-.5*x1^2,[-15 15 -15 15]);
        hold on;
    end
end
xlabel('x1(t)'); ylabel('x2(t)'); title('xdotdot-x=0 FIM 3-D surface');
set(gca,'FontSize',12,'Linewidth',1.5,'GridAlpha',.5);
set(gcf,'position',[1630 539 480 406]);tightfig;
```



define the system.

```
function dx = control(t,x);
    dx = [ x(2);    %    x1dot
           x(1)];   %    x2dot
end
```

```
end
```

```
ans =

         0
    0.0800
    0.3710
    0.6652
    0.9595
    1.2537
    1.5480
    1.8422
    2.1365
    2.4308
    2.7250
    3.0193
    3.3135
    3.6078
    3.9020
    4.1963
    4.4906
    4.7848
    5.0791
    5.3733
```

```
    5.6676
    5.9618
    6.2561
    6.5503
    6.8446
    7.1389
    7.4331
    7.7274
    8.0216
    8.3159
    8.6101
    8.9044
    9.1987
    9.4929
    9.7872
   10.0000
```

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