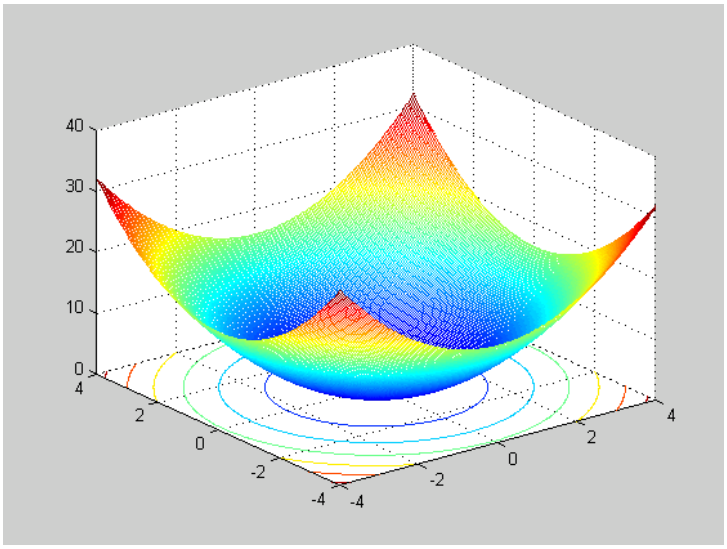
Plotting Lyapunov Functions Using MATLAB

updated Tuesday, February 04, 2003

Example 1 Quadratic Lyapunov Function

$V(x) = x^2 + y^2$

```
>> x=[-4:.04:4];
>> y=x;
>> [X,Y]=meshgrid(x,y);
>> z=X.^2 + Y.^2;
>> mesh(X,Y,z)
```

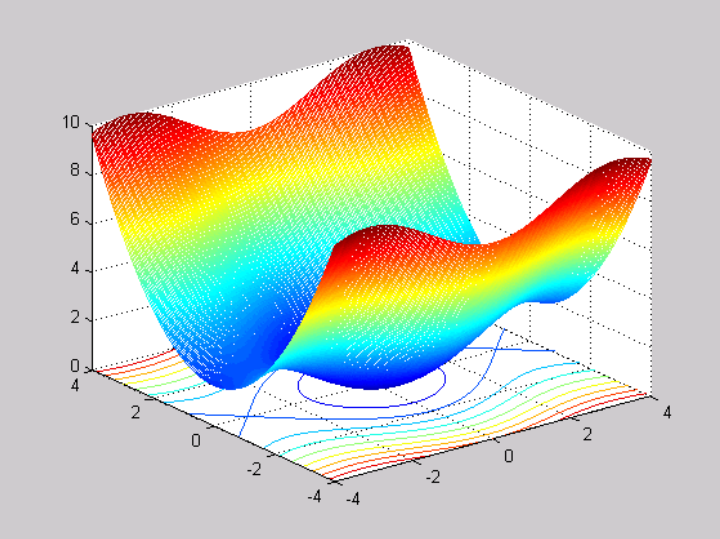


Example 2

Slotine and Li p. 63

$V(x) = (1 - \cos\theta) + \dot{\theta}^2/2$

```
>> x=[-4:.04:4];
>> y=x;
>> [X,Y]=meshgrid(x,y);
>> z=(1-cos(X))+(Y.^2)/2;
>> meshc(X,Y,z)
```

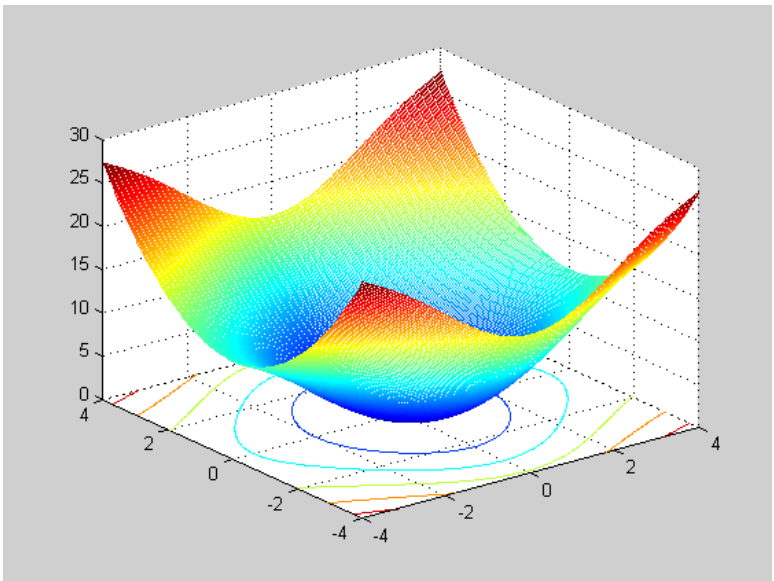


Example 3

Slotine and Li p. 67

$$V(x) = 2(1-\cos\theta) + \dot{\theta}^2/2 + \left(\dot{\theta}^2 + \theta^2\right)/2$$

```
>> x=[-4:.04:4];
>> y=x;
>> [X,Y]=meshgrid(x,y);
>> z=2*(1-cos(X))+(X.^2 + Y.^2)/2 + (Y.^2)/2;
>> meshc(X,Y,z)
```



For the nonlinear system:

$$\ddot{\theta} + \dot{\theta} + \sin\theta = 0$$

Using LCF:

$$V = \frac{\dot{\theta}^2}{2} + (1 - \cos\theta) + \frac{1}{2}\left(\dot{\theta} + \theta\right)^2$$

We found:

$$\dot{V} = -\dot{\theta}^2 - \theta\sin\theta$$

And concluded that the system is locally stable (SISL) within region:

$$-\pi < \theta < \pi$$

The phase plane plot for the system looks like this: