Sliding mode control

ex 1

$$\dot{x}_1 = x_2$$

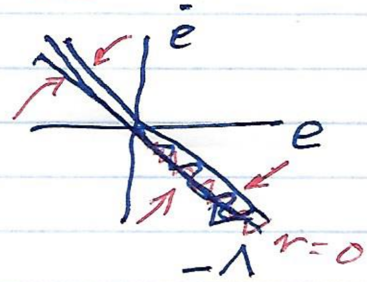
$$\ddot{x}_1 = \dot{x}_2 = f(x) + g(x)u$$

error dynamics

$$e = x_{d_1} - x_1 \qquad , \dot{e} = \dot{x}_{d_1} - \dot{x}_1 = x_{d_2} - x_2 \equiv e_2$$

$$\ddot{e} = \ddot{x}_{d_1} - \ddot{x}_1 = \dot{x}_{d_2} - \dot{x}_2 = \ddot{x}_{d_1} - (f + gu) \quad (i)$$

sliding mode error $\quad r = \dot{e} + \lambda e$



make $r(t) \to 0$

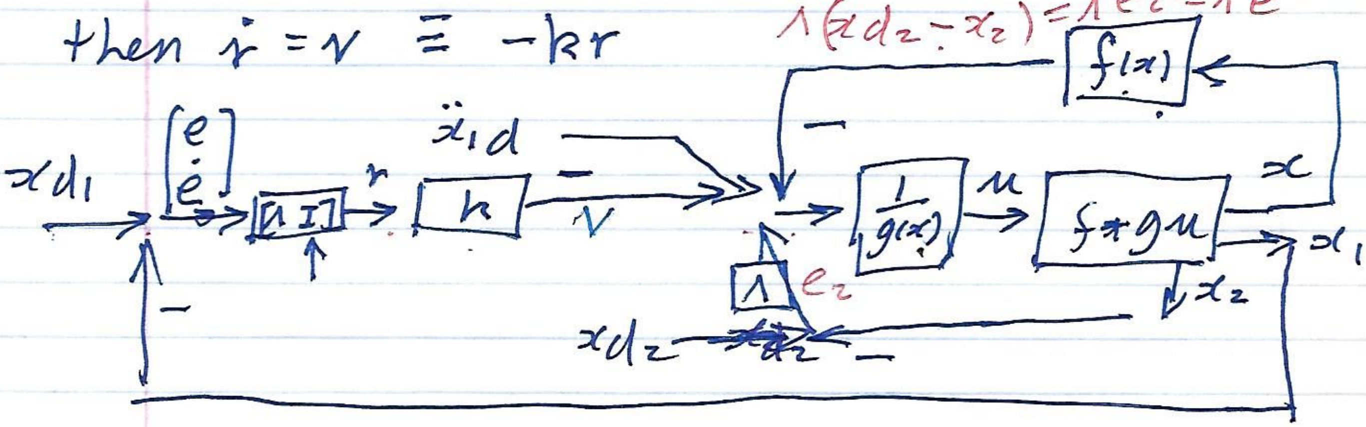
find $r(t)$ error dynamics

$$\dot{r} = \ddot{e} + \lambda \dot{e} = \ddot{x}_{1d} - (f + gu) + \lambda (\dot{x}_{d_1} - x_2) \equiv v$$

select

$$u = \frac{1}{g(x)} \left(-f + \ddot{x}_1 d + \underbrace{\lambda \dot{x}_{d_1} - \lambda x_2}_{\lambda (x_{d_2} - x_2) = \lambda e_2 = \lambda \dot{e}} + v \right) \pm d$$

then $\dot{r} = v \equiv -kr$



ex 2 Lyapunov for sliding mode

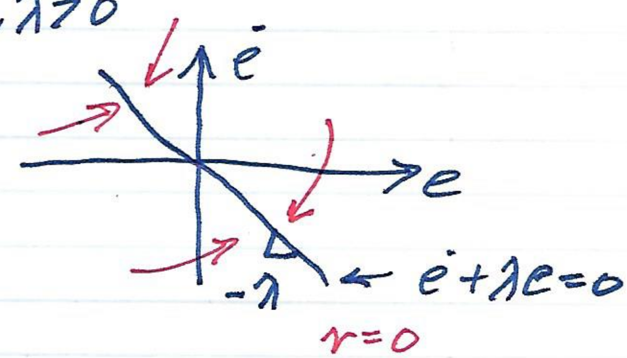
ex 3.23 S+L p. 94

$$\ddot{x} - \dot{x}^3 + x^2 = u \quad , \quad \ddot{x} = \dot{x}^3 - x^2 + u$$

$$e = x_d - x$$

slide mode error

$$r = \dot{e} + \lambda e \quad , \lambda > 0$$



error dynamics

$$\dot{r} = \ddot{e} + \lambda \dot{e}$$

$$= \ddot{x}_d - \ddot{x} + \lambda \dot{e}$$

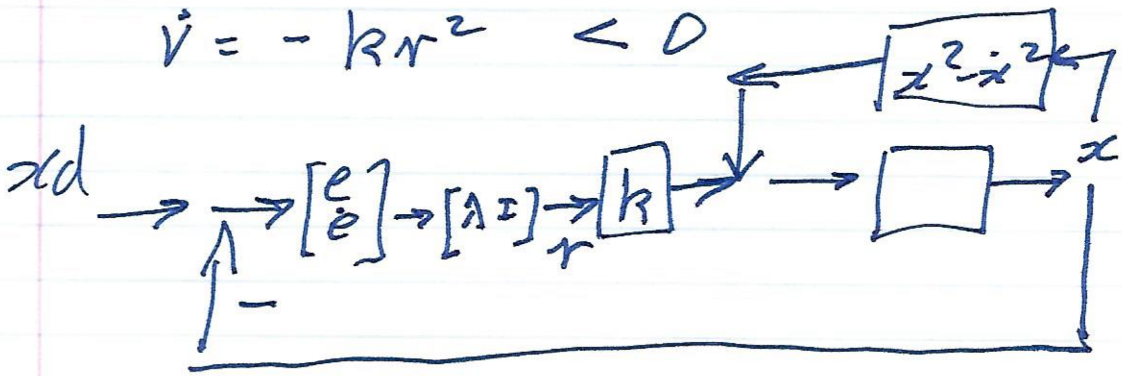
$$\dot{r} = \ddot{x}_d + \lambda \dot{e} - \dot{x}^3 + x^2 - u$$

$$V = \frac{1}{2} r^2$$

$$\dot{V} = r\dot{r} = r\left(\ddot{x}_d + \lambda \dot{e} - \dot{x}^3 + x^2 - u\right)$$

$$u = \underbrace{\ddot{x}_d + \lambda \dot{e} - \dot{x}^3 + x^2}_{\text{GROBS}} + \underbrace{kr.}_{\text{PIGS}}$$

$$\dot{V} = -kr^2 < 0$$



$$kr = k(\dot{e} + \lambda e) = k\dot{e} + k\lambda e$$

$\dot{e} + \Lambda \underline{e} = \underline{r}$

$(sI+\Lambda)E(s) = R(s)$ — $H(s)$

$E(s) = (sI+\Lambda)^{-1} R(s)$

$\|e\|_2$

L_2 system norm

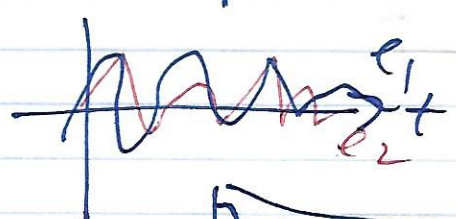
$\|e(t)\|_2 \le \sigma_{max}(sI+\Lambda)^{-1} \|r(t)\|_2$

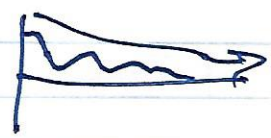
$r(t) \to \boxed{(sI+\Lambda)^{-1}} \to e(t)$

L_2 norm

$$\|e(t)\|_p = \left(\int_0^\infty |e(t)|^p \, dt\right)^{1/p}$$

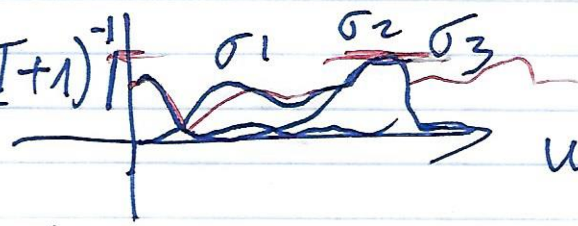
$$\|e(t)\|_2 = \left(\int_0^\infty |e(t)|_p^p \, dt\right)^{1/2}$$





Weird

$|j\omega I+\Lambda|^{-1}$



$|H(j\omega)|_\infty$

H_∞ gain

SVD

$U S(\omega) V^T$