EE 5323 - HW03

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HW03 – Nonlinear System Simulations
EE 5323 – Nonlinear Systems
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Exercise 1

Voltera Predator-Prey System
Consider the Voltera predator-prey system,

$$\dot{x}_1 = -x_1 + x_1 x_2 \quad [1.1]$$

$$\dot{x}_2 = x_2 - x_1 x_2 \quad [1.2]$$

Find the equilibrium points and their nature.

Answer
State variable is given as:

$$\dot{x}_1 = -x_1 + x_1 x_2 \quad [1.3]$$

$$\dot{x}_2 = x_2 - x_1 x_2 \quad [1.4]$$

The Voltera predator-prey system has limit cycles therefore the system is at equilibrium when the population of both predator and prey remain constant; thus, the derivative should be zero. To find the equilibrium, I set $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$. Solve the system for its roots.

$$\dot{x}_1 = 0 \Longrightarrow 0 = -x_1 + x_1 x_2 \quad [1.5]$$

$$\dot{x}_2 = 0 \Longrightarrow 0 = x_2 - x_1 x_2 \quad [1.6]$$

$$0 = x_1(\beta x_2 - \alpha) \implies x_1 = 0;\ x_2 = \alpha/\beta \quad [1.7]$$

$$0 = x_2(\gamma - \sigma x_1) \implies x_1 = \gamma/\sigma;\ x_2 = 0 \quad [1.8]$$

There are two equilibrium points at $(x_1,\ x_2)$,

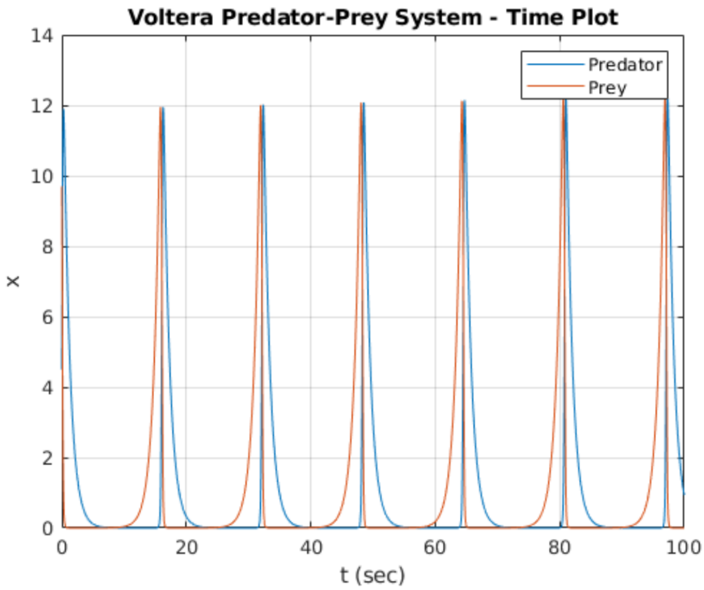
- At zero, $(0,\ 0)$,
- Any positive pair of integers $(\alpha/\beta,\ \gamma/\sigma)$

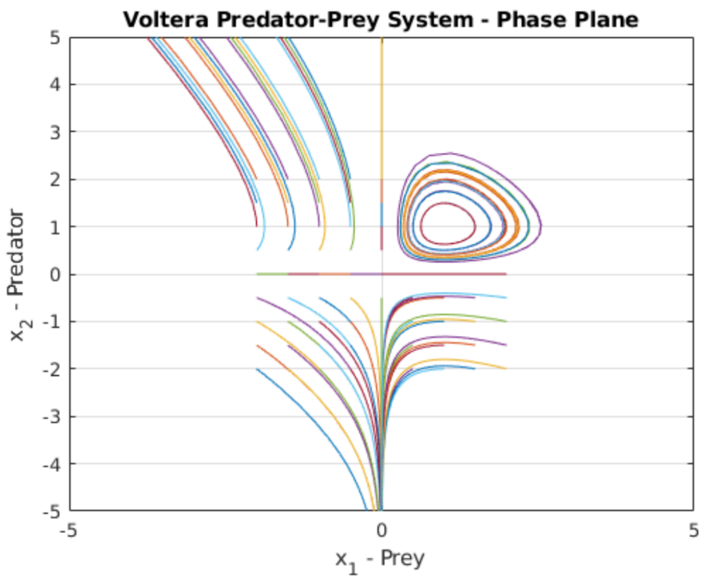
The equilibrium point nature of the zero is a stable center point that is a limit cycle. The other e.p. has a saddle point nature because it is stable in one dimension (goes to zero) and unstable in the other (goes to infinity).

Matlab Code

```matlab
%% HW03 - Q01 - Voltera Predator-Prey System
% @author: Bardia Mojra
% @date: 09/28/2021
% @title HW03 - Q01 - Voltera Predator-Prey System
% @class ee5323 - Nonlinear Systems
% @professor - Dr. Frank Lewis

clc
close all
warning('off','all')
warning

x0_set = -2:.5:2;
t_intv= [0 100];
x_0= [4.5, 9.7]'; % initial conditions for x(t)

figure
[t,x]= ode23('Voltera', t_intv, x_0);
plot(t,x)
hold on;
grid on;
title('Voltera Predator-Prey System - Time Plot');
ylabel('x');
xlabel('t (sec)');
legend('Predator', 'Prey');
t_intv= [0 10];

figure
for i=x0_set
  for j=x0_set
    x0 = [i; j];
    [t,x]= ode45('Voltera', t_intv, x0);
    plot(x(:,1),x(:,2))
    hold on;
  end
end
title('Voltera Predator-Prey System - Phase Plane');
ylabel('x_2 - Predator');
xlabel('x_1 - Prey');
axis([-5 5 -5 5]);
grid on;

function xdot = Voltera(t,x)
  xdot = [-x(1)+x(1)*x(2); x(2)-x(1)*x(2)];
end
```

Figures


Voltera Predator-Prey System - Time Plot
Predator
Prey
x
t (sec)
0
20
40
60
80
100
0
2
4
6
8
10
12
14


Voltera Predator-Prey System - Phase Plane
x_2 - Predator
x_1 - Prey
-5
0
5
-5
-4
-3
-2
-1
0
1
2
3
4
5

Exercise 2

Equilibrium points and linearization
Consider the following system,

$$\dot{x}_1 = x_2(-x_1 + x_2 - 1) \quad [2.1]$$

$$\dot{x}_2 = x_1(x_1 + x_2 + 1) \quad [2.2]$$

(a) Find all equilibrium points

(b) Find Jacobian

(c) Find the nature of all e.p.s

Answer
1. Find all e.p.s
At equilibrium points, all states reach their minimal energy state; therefore, the derivative of the state should equal zero. Then, we solve for the roots of the obtained characteristic equation.

$$\dot{x}_1 = x_2(-x_1 + x_2 - 1) = -x_1 \cdot x_2 + x_2^2 - x_2 \quad [2.3]$$

$$\dot{x}_2 = x_1(x_1 + x_2 + 1) \;=\; x_1^2 + x_1 \cdot x_2 + x_1 \quad [2.4]$$

$$\dot{X} = 0 \Longrightarrow \quad \dot{X} = AX \Longrightarrow \quad \dot{X} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 & x_2 - 1 \\ x_1 + 1 & x_1 \end{bmatrix}_{2x2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2x1} \quad [2.5]$$

$$\begin{cases} \dot{x}_1 = 0 \Longrightarrow & x_2(-x_1 + x_2 - 1) = 0 \\ \dot{x}_2 = 0 \Longrightarrow & x_1(x_1 + x_2 + 1) = 0 \end{cases} \quad [2.6]$$

$$\dot{X} = \begin{cases} x_2 = 0; & (-x_1 + x_2 - 1) = 0 \Longrightarrow \\ x_1 = 0; & (x_1 + x_2 + 1) = 0 \Longrightarrow \end{cases} \quad [2.7]$$