Angle inclination in Quadcopter using feedback linearization

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Abstract— **In this paper we will study the method of feedback linearization which can be used to maintain the position and to attain the angle of inclination to achieve specific position. Feedback linearization controller is a simple controller that can be directly applied to nonlinear dynamics. This controls the pitch, roll, and yaw motions required for movement along an arbitrary trajectory in space. The highly coupled nonlinear dynamics presented by the tilting quadcopter makes the use of back stepping, sliding mode, and adaptive controllers exceptionally challenging. Feedback linearization can efficiently deal with the nonlinearities in the complex equations of the tilting quadcopter. Feedback linearization can be combined with other controllers like PD controllers to achieve overall control of quadcopter to combine tilting and movement along a desired trajectory simultaneously. Simulation studies are presented based on the developed nonlinear dynamic model of the tilting rotor quadcopter to demonstrate the validity and effectiveness of the overall control system for an arbitrary trajectory tracking.**

I. INTRODUCTION AND OUTLINE

Unmanned vehicles (UAV) are gaining increasing interest worldwide in different sectors because of a wide area of possible applications. It is not only growing by intense research in military applications but also in miniaturization, mechatronics and microelectronics offering an enormous potential for small and inexpensive Micro-UAVs for package delivery, personal and targeting commercial use. These applications would be able to fly either in- or outdoor, leading to completely new applications.

The traditional copters that worked on principal of rotating motor, the tilting rotor studied in this paper is an extension of traditional copters that allow the rotors to tilt along the respective axes of the rotor arms allowing then to attain high speed.

Fig. 1. Civil aviation application Agusta Westland AW609

Quad-rotor is one of the most popular Unmanned Aerial Vehicles for indoor and outdoor applications having capability for Vertical Take-Off and Landing (VTOL)[1] [2]. The tilting rotor quadcopters uses an additional motor for each rotor that enables the rotor to rotate along the axis of the quadcopter arm. This is accomplished by eight inputs (four inputs for rotations and four for tilting actions), which allows us to have complete control over its position and the orientation. Indeed, the states of the system are coupled and highly non-linear. The dynamic modeling of this kind of quadcopter with tilting rotors is presented in [3].

There are number of advantages of tilting quadcopter, but the nonlinear dynamic behavior of the tilting quadcopter makes it difficult to design closed loop controls. To develop a good controller, the design must be robust for the bounded uncertainties like modeling error, sensor noise, external disturbances cause high challenge. In order to deal with these uncertainties with sensor noises and other disturbances, controller need to adapt itself to varying conditions.

Fig 2. Tilting Quadcopter

There are number of things to be taken into consideration while designing controller for the quadcopter. For example, one of the widely-considered controller is back-stepping controllers for non-linear systems like quadcopters [4, 5]. Usually, along with back-stepping, Lyapunov stability criteria, sliding mode or adaptive techniques which are derived from the dynamics of quadcopter are used [6, 7]. Robustness against disturbances is the main benefits of the back-stepping. Few other techniques include quaternion based feedback controller used for attitude stabilization [8].

Another way to design controller is using PID and LQ to compensate model imperfections with model averaging [9]. Some other methods include robust adaptive-fuzzy control used to minimize sinusoidal wind disturbance [10]. Feedback linearization is one of the techniques that can be used to design controller. It is usually combined with a high-order sliding mode or a PD controller [11]. can be related to determine the inertial coordinates of the UAV between successive images.

Feedback linearization controller is a simple controller that can be directly applied to nonlinear dynamics [12]. It becomes very challenging to use back-stepping or adaptive controllers because of highly coupled nonlinear dynamics caused by tilting quadcopter. It can be efficiently dealt using Feedback linearization. quadcopter. Also, it can be easily embedded in microcontrollers.

II. Dynamic Modeling

In traditional quad-rotor we have only four rotary propellers as vehicles input. In tilting rotor quadcopter, we have eight inputs, four additional inputs compared to traditional quadcopter which are attached to each arm of the propeller which adds additional degree of freedom, resulting in tilting quadcopter.

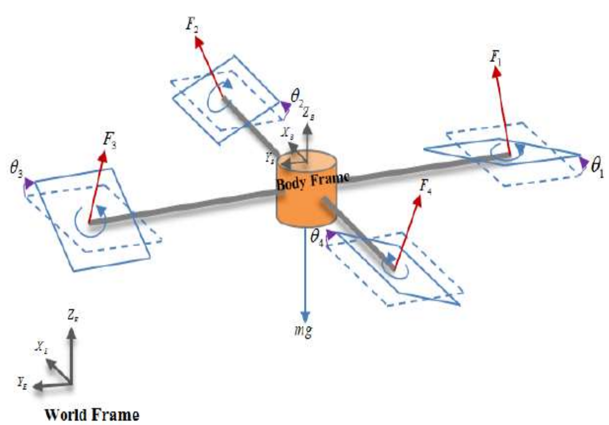

Fig. 3 Coordinate system and forces acting on quadcopter.

In the above figure, there are force for each rotor formed from the rotation of the motor. It also produces moment perpendicular to the plane of the propeller rotation. The moment produced is in opposite direction of the axis rotation. Hence to cancel out rotor 1 and 3 are set clockwise and other two in anticlockwise.

The tilting rotors in quadcopter is explained as, the propellers are free to tilt along their axes. The planes shown with dashed lines are the original planes of rotation with zero tilt angles for the respective propellers. Similarly, the planes shown with the rigid lines are the tilted planes of rotation for respective propellers. It may be noted that the forces generated by the propellers are perpendicular to these respective planes of rotations.

The Euler angle transformations are defined by Ψ, θ and Φ which respectively referred to as yaw, pitch and roll angles. The first rotation is about the X axis, second rotation is about the Y axis and the last rotation is about the Z axis.

$$R_{EB} = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \quad (1)$$

where cΨ and sΨ denote cos(Ψ) and sin(Ψ) respectively, and similarly for other angles.

$$\begin{aligned} m\ddot{x} &= F_1 s\theta_1 c\psi c\theta - F_3 s\theta_3 c\psi c\theta - F_4 s\theta_4 c\psi s\theta s\phi \\ &\quad + F_2 s\theta_2 c\psi s\theta s\phi + F_4 s\theta_4 s\psi c\phi - F_2 s\theta_2 s\psi c\phi \\ &\quad + F_1 c\theta_1 c\psi s\theta c\phi + F_2 c\theta_2 c\psi s\theta c\phi \\ &\quad + F_3 c\theta_3 c\psi s\theta c\phi + F_4 c\theta_4 c\psi s\theta c\phi + F_1 c\theta_1 s\psi s\phi \\ &\quad + F_2 c\theta_2 s\psi s\phi + F_3 c\theta_3 s\psi s\phi + F_4 c\theta_4 s\psi s\phi - C_1\dot{x} \\ m\ddot{y} &= F_1 s\theta_1 s\psi c\theta - F_3 s\theta_3 s\psi c\theta - F_4 s\theta_4 s\psi s\theta s\phi \\ &\quad + F_2 s\theta_2 s\psi s\theta s\phi - F_4 s\theta_4 c\psi c\phi + F_2 s\theta_2 c\psi c\phi \\ &\quad + F_1 c\theta_1 s\psi s\theta c\phi + F_2 c\theta_2 s\psi s\theta c\phi \\ &\quad + F_3 c\theta_3 s\psi s\theta c\phi + F_4 c\theta_4 s\psi s\theta c\phi - F_1 c\theta_1 c\psi s\phi \\ &\quad - F_2 c\theta_2 c\psi s\phi - F_3 c\theta_3 c\psi s\phi - F_4 c\theta_4 c\psi s\phi - C_2\dot{y} \\ m\ddot{z} &= -F_1 s\theta_1 s\theta + F_3 s\theta_3 s\theta - F_4 s\theta_4 c\theta s\phi \\ &\quad + F_2 s\theta_2 c\theta s\phi + F_1 c\theta_1 c\theta c\phi + F_2 c\theta_2 c\theta c\phi \\ &\quad + F_3 c\theta_3 c\theta c\phi + F_4 c\theta_4 c\theta c\phi - mg - C_3\dot{z} \end{aligned} \quad (2)$$

Here m is the total mass, g is the acceleration due to gravity, and C1, C2 and C3 are the drag coefficients of the quadcopter, note that these coefficients are negligible at low speed. F1, F2, F3 and F4 are forces produced by the four rotors as given by the following equation:

$$F_i = K_f \omega_i^2 \quad (3)$$

where ω_i is the angular velocity of the i[th] rotor and K_f is a constant.
Above are the angular acceleration can be found as

$$\begin{aligned} I_x\ddot{\phi} &= l(F_3 c\theta_3 - F_1 c\theta_1 - C_1'\dot{\phi}) \\ &\quad + (M_1 s\theta_1 - M_3 s\theta_3) + (M_2' + M_4') \\ I_y\ddot{\theta} &= l(F_4 c\theta_4 - F_2 c\theta_2 - C_2'\dot{\theta}) \\ &\quad + (M_4 s\theta_4 - M_2 s\theta_2) + (M_1' + M_3') \\ I_z\ddot{\psi} &= l(F_1 s\theta_1 + F_2 s\theta_2 + F_3 s\theta_3 + F_4 s\theta_4 - C_3'\psi) \\ &\quad + (M_1 c\theta_1 - M_2 c\theta_2 + M_3 c\theta_3 - M_4 c\theta_4) \end{aligned} \quad (4)$$

where l is distance of each rotor from the vehicle's center of mass. I_x, I_y *and* I_z are the moments of inertia along x, y and directions respectively. C' is rotational drag coefficients, M' are the tilting moment motors attached to the end of each arm to cause a tilt angel and M_i are rotor moments produced due to rotor motions and are given by:

$$M_i = K_m \omega_i^2 \tag{5}$$

We are considering the tilting along just one direction, i.e, the roll direction just for simplicity. Hence, since the tilting happens in the roll direction, during a hovering flight, the quadcopter has zero positional accelerations and velocities but pitch angle, and angular velocities along the three directions will be zero. At this nominal hovering state, by considering the dynamics of the tilting rotor quadcopter given by Equations (2) and (4), and assuming the relationship between the tilting angles of the four rotors $\theta_1 = -\theta_3$ and $\theta_2 = \theta_1 = 0$ (it may be noted that tilting along roll is caused by tilting of the rotors 1 and 3 as shown in [3]), the motor speed needed for vehicle for hovering with a roll angle is given by:

$$\omega_i = \omega_h = \sqrt{\frac{mg}{4k_f c\frac{\theta_1}{2}}} \qquad when \quad \theta = 0$$

The input velocities to the rotors of the quadcopter is given by u_i, but in order to simplify the equations of motion which are described in 2 and 4, new artificial input variables are defined as the following. It may be noted that we assume that the tilting happens only along the roll direction.

$$\begin{aligned}
u_1 &= (F_1 + F_2 + F_3 + F_4)/m \\
u_2 &= l(F_3 - F_1)/I_x \\
u_3 &= l(F_4 - F_2)/I_y \\
u_4 &= k(F_1 cos\theta_1 - F_2 cos\theta_2 + F_3 cos\theta_3 - F_4 cos\theta_4)/I_z
\end{aligned} \tag{6}$$

where k is force/moment scaling factor. The equations of motion of the vehicle can be found as:

$$\begin{aligned}
\ddot{x} &= \frac{1}{2} sin\theta_1 cos\theta + u_1 cos\theta_1 cos\phi^h cos\phi\, sin\theta \\
\ddot{y} &= -u_1 cos\theta_1 sin\phi^h cos\phi - u_1 cos\theta_1 cos\phi^h sin\phi \\
\ddot{z} &= -mg - \frac{1}{2} u_1 sin\theta_1 sin\theta + u_1 cos\theta_1 cos\phi^h cos\phi\, cos\theta \\
&\quad - u_1 cos\theta_1 sin\phi^h sin\phi\, cos\theta \\
\ddot{\phi} &= (cos\theta_1 + k sin\theta_1) u_2 \\
\ddot{\theta} &= u_3 \\
\ddot{\psi} &= k u_4
\end{aligned} \tag{7}$$

III. Feedback Linearization Controller Design

A brief review of nonlinear control using feedback linearization method is presented here. Here Input - Output linearization technique used rather than that of Input-State linearization. Here the output of the systems differentiate is as many as times as needed so that the input of the system appears in the last derivative. It is a systematic way to linearize globally part of, or all, the dynamics of system [13, 14]. Here to the new input v, chosen to yield the following transfer function from the synthetic input to the output γ is explained:

$$\frac{Y(s)}{V(s)} = \frac{1}{s^\gamma}$$

where γ is the relative degree, is the last derivative of output so that the physical input appears in the equation. Here we consider a nonlinear system in the following form:

$$\begin{aligned}
\dot{\mathbf{x}} &= f(\mathbf{x}) + g(\mathbf{x})u \\
\mathbf{y} &= h(\mathbf{x})
\end{aligned} \tag{8}$$

where x is the state vector, u is the control inputs, and y is the outputs, f and g are vector fields, and h is scalar function. The control design process is to find an integer γ and a state feedback control law,

$$u = \alpha(\mathbf{x}) + \beta(\mathbf{x})v \tag{9}$$

Here α and β are functions.

Differentiating the output equation we get:

$$\mathbf{y}^{(\gamma)} = L_f^\gamma h(\mathbf{x}) + L_g L_f^{\gamma-1} h(\mathbf{x})u \tag{10}$$

The control input can be found as:

$$u = \frac{1}{L_g L_f^{\gamma-1} h}(-L_f^\gamma h + v) \tag{11}$$

The functions α(x) and β(x) in (9) can be obtained as:

$$\begin{aligned}
\alpha(\mathbf{x}) &= \frac{-L_f^\gamma h(\mathbf{x})}{L_g L_f^{\gamma-1} h(\mathbf{x})} \\
\beta(\mathbf{x}) &= \frac{1}{L_g L_f^{\gamma-1} h(\mathbf{x})}
\end{aligned} \tag{12}$$

Let us consider:

$$\begin{aligned}
F(\mathbf{x}) &= L_f^\gamma h(\mathbf{x}) \\
G(\mathbf{x})^{-1} &= \frac{1}{L_g L_f^{\gamma-1} h(\mathbf{x})}
\end{aligned} \tag{13}$$

From eq. 11 we get:

$$u = G^{-1}(\mathbf{x})(v - F(\mathbf{x})) \tag{14}$$

$$\mathbf{y}^{(\gamma)} = v \tag{15}$$

After achieving linearization, further control objective like pole placement may be easily met using the linear controls theory.

In nonlinear quadcopter system under consideration, to make the system feedback linearizable, we may consider choosing x, y and z as the output variables. From eq. 6 it can be easily seen that the control inputs u_2 and u_3 which are the control inputs representing the pitch and roll angular accelerations of the vehicle, do not appear in the equation of the outputs. Differentiating of equations of motion successively, we can generate the new control input of the system. The derication can be found in [15]. It can be seen the new input terms appear in the fourth derivatives of the outputs (as obtained from (7):

$$\begin{aligned} x^{(4)} &= f(x)_x + \begin{bmatrix} g(x)_{x1} & g(x)_{x2} & g(x)_{x3} \end{bmatrix} u \\ y^{(4)} &= f(x)_y + \begin{bmatrix} g(x)_{y1} & g(x)_{y2} & g(x)_{y3} \end{bmatrix} u \\ z^{(4)} &= f(x)_z + \begin{bmatrix} g(x)_{z1} & g(x)_{z2} & g(x)_{z3} \end{bmatrix} u \end{aligned} \tag{16}$$

Where,

$$\begin{aligned} f(x)_x &= -\dot{u}_1 \dot{\theta} sin\theta_1 sin\theta + \dot{u}_1 \dot{\theta} cos\theta_1 cos\phi^h cos\phi cos\theta \\ &\quad - \dot{u}_1 \dot{\phi} cos\theta_1 cos\phi^h sin\phi sin\theta \\ &\quad - \dot{u}_1 \dot{\phi} \dot{\theta} cos\theta_1 cos\phi^h sin\phi cos\theta) \\ g(x)_{x1} &= \frac{1}{2} sin\theta_1 cos\theta + cos\theta_1 cos\phi^h cos\phi sin\theta \\ g(x)_{x2} &= -\frac{1}{2} u_1 sin\theta_1 cos\theta - u_1 cos\theta_1 cos\phi^h cos\phi sin\theta \\ g(x)_{x3} &= -u_1 cos\theta_1 cos\phi^h cos\phi sin\theta \\ f(x)_y &= 2\dot{u}_1 \dot{\phi} cos\theta_1 sin\phi^h sin\phi - 2\dot{u}_1 \dot{\phi} cos\theta_1 cos\phi^h cos\phi \\ g(x)_{y1} &= -cos\theta_1 sin\phi^h cos\phi - cos\theta_1 sin\phi^h sin\phi \\ g(x)_{y2} &= 0 \\ g(x)_{y3} &= u_1 cos\theta_1 sin\phi^h cos\phi + u_1 cos\theta_1 sin\phi^h sin\phi \\ f(x)_z &= 2 - \dot{u}_1 \dot{\theta} sin\theta_1 cos\theta - 2\dot{u}_1 \dot{\theta} cos\theta_1 cos\phi^h cos\phi sic\theta \\ &\quad - 2\dot{u}_1 \dot{\phi} cos\theta_1 cos\phi^h sin\phi cos\theta \\ &\quad + 2u_1 \dot{\theta} \dot{\phi} cos\theta_1 cos\phi^h sin\phi sin\theta \\ &\quad + 2\dot{u}_1 \dot{\theta} cos\theta_1 sin\phi^h sin\phi sin\theta \\ &\quad - 2\dot{u}_1 \dot{\phi} cos\theta_1 sin\phi^h cos\phi cos\theta \\ &\quad + 2u_1 \dot{\theta} \dot{\phi} cos\theta_1 sin\phi^h cos\phi sin\theta \\ g(x)_{z1} &= -\frac{1}{2} sin\theta_1 sin\theta + cos\theta_1 cos\phi^h cos\phi cos\theta \\ &\quad + cos\theta_1 sin\phi^h sin\phi cos\theta \\ g(x)_{z2} &= \frac{1}{2} u_1 sin\theta_1 sin\theta - u_1 cos\theta_1 cos\phi^h cos\phi cos\theta \\ &\quad + u_1 cos\theta_1 sin\phi^h sin\phi cos\theta \\ g(x)_{z3} &= -u_1 cos\theta_1 cos\phi^h cos\phi cos\theta + u_1 cos\theta_1 sin\phi^h sin\phi cos\theta \\ u &= \begin{bmatrix} \ddot{u}_1 & u_2 & u_3 \end{bmatrix}^T \end{aligned}$$

where u is the control inputs which control the x, y and z. We choose u as:

$$u = \begin{bmatrix} g(x)_{x1} & g(x)_{x2} & g(x)_{x2} \\ g(x)_{y1} & g(x)_{y2} & g(x)_{y3} \\ g(x)_{z1} & g(x)_{z2} & g(x)_{z3} \end{bmatrix}^{-1} \cdot \begin{bmatrix} -f(x)_x + v_1 \\ -f(x)_y + v_2 \\ -f(x)_z + v_3 \end{bmatrix} \tag{17}$$

The output equation is given by:

$$\begin{bmatrix} x^{(4)} \\ y^{(4)} \\ z^{(4)} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{18}$$

Setting pseudo inputs terms as:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} x_d^{(4)} - k_{x1} e_x^{(3)} - k_{x2} e_x^{(2)} - k_{x3} \dot{e}_x - k_{x4} e_x \\ y_d^{(4)} - k_{y1} e_y^{(3)} - k_{y2} e_y^{(2)} - k_{y3} \dot{e}_y - k_{y4} e_y \\ z_d^{(4)} - k_{z1} e_z^{(3)} - k_{z2} e_z^{(2)} - k_{z3} \dot{e}_z - k_{z4} e_x \end{bmatrix} \tag{19}$$

Where e is the error in specific direction, $e_x = x-x_d$, $e_y = y-y_d$ and $e_z = z-z_d$, k_x, ky and k_z are gains, x_d, y_d and z_d are desired outputs. From (19), the error dynamics are given by:

$$\begin{aligned} e_x^{(4)} - k_{x1} e_x^{(3)} - k_{x2} e_x^{(2)} - k_{x3} \dot{e}_x - k_{x4} e_x &= 0 \\ e_y^{(4)} - k_{y1} e_y^{(3)} - k_{y2} e_y^{(2)} - k_{y3} \dot{e}_y - k_{y4} e_y &= 0 \\ e_z^{(4)} - k_{z1} e_z^{(3)} - k_{z2} e_z^{(2)} - k_{z3} \dot{e}_z - k_{z4} e_x &= 0 \end{aligned} \tag{20}$$

IV. Conclusion

In this paper, an Input-Output feedback linearization technique is proposed for tilting-rotor quadcopter. In this kind of quadcopter, rotors can tilt along their respective arms. This tilting mechanism makes the vehicle fully-controlled which not only can track any arbitrary trajectory, but also can adjust the pitch or roll of the vehicle with the desired orientation during flight. A dynamic model of the tilting rotor quadcopter is presented which is used to design the control laws in the framework of feedback linearization method. Future work would involve extending the proposed feedback linearization method to allow the desired orientation to be achieved in any arbitrary and simultaneous combination of pitch and roll angles.

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