Backstepping (GAS)

First stabilize ZD

BS ex1

$$\dot{y} = v$$

$$\ddot{z} + \dot{z}^3 + yz = 0 \leftarrow ZD$$

add zero

$$\ddot{z} + \dot{z}^3 + yz + y_d z - y_d z = 0$$

$$\ddot{z} + \dot{z}^3 + z(y_d) = z(y_d - y) \equiv z\tilde{y}_d$$

select $y_d = z^2$, p. 74 ex. 3.14

then $\underline{\ddot{z} + \dot{z}^3 + z^3} = z\tilde{y}_d$

$$V = \underbrace{\left(\tfrac{1}{2}\dot{z}^2 + \tfrac{1}{4}z^4\right)}_{V_0} + \tfrac{1}{2}\tilde{y}_d^2 = V_0 + \tfrac{1}{2}\tilde{y}_d^2$$

↑ from p. 74

$$\dot{V} = \dot{z}\ddot{z} + z^3\dot{z} + \tilde{y}_d\dot{\tilde{y}}_d$$

$$= \dot{z}(-\dot{z}^3 - z^3 + z\tilde{y}_d) + z^3\dot{z} + \tilde{y}_d(\dot{y}_d - \dot{y})$$

$$= -\dot{z}^4 - \dot{z}z^3 + z\dot{z}\tilde{y}_d + z^3\dot{z} + \tilde{y}_d(2z\dot{z} - v)$$

$$= -\dot{z}^4 + \tilde{y}_d(z\dot{z} + 2z\dot{z} - v)$$

select $v = 3z\dot{z} + \tilde{y}_d$

then

$$\dot{V} = -\dot{z}^4 - \tilde{y}_d^2$$

Barbalat $\Rightarrow \dot{V} \to 0$

$\dot{V} \to 0 \Rightarrow \dot{z} \to 0 + \tilde{y}d \to 0$

$y \to yd = z^2$

to Dynamics

$\dot{y} = 3z\dot{z} + \tilde{y}d = 3z\dot{z} + (yd - y)$

$\ddot{z} + \dot{z}^3 + yz = 0$

goes to

$\dot{y} = 0$

$\ddot{z} + \dot{z}^3 + z^3 = 0 \Rightarrow z \to 0 \Rightarrow y \to 0$

$\dot{z} \to 0$

La Salle ect. $\Rightarrow y, z, \dot{z} \to 0$

BS ex.2 $\quad$ S&L + 260 $\quad$ NMP system $\quad$ 10/26/05

$$\dot{y} = v$$
$$\ddot{z} + \dot{z}^3 - z^5 + yz = 0 \quad \leftarrow ZD\text{, unstable}$$

add 0

$$\ddot{z} + \dot{z}^3 - z^5 + y_d z = z(y_d - y) \equiv z\tilde{y}_d$$

$$\tilde{y}_d = y_d - y$$

select $y_d = 2z^4$

then $\underbrace{\ddot{z} + \dot{z}^3 + z^5} = z\tilde{y}_d$

c.f. p.74 $\quad \ddot{x} + b(\dot{x}) + c(x) = 0$

where $\dot{x}\, b(\dot{x}) = \dot{x}^4 > 0$

$$x\, c(x) = x^6 > 0$$

select $V_0 = \frac{1}{2}\dot{z}^2 + \frac{1}{6}z^6$ from p.74

$$V = V_0 + \frac{1}{2}\tilde{y}_d^2 = \frac{1}{2}\dot{z}^2 + \frac{1}{6}z^6 + \frac{1}{2}\tilde{y}_d^2$$

$$\dot{V} = \dot{z}\ddot{z} + z^5\dot{z} + \tilde{y}_d\dot{\tilde{y}}_d$$
$$= \dot{z}(-\dot{z}^3 - z^5 + z\tilde{y}_d) + z^5\dot{z} + \tilde{y}_d(\dot{y}_d - \dot{y})$$
$$= -\dot{z}^4 - \dot{z}z^5 + z\dot{z}\tilde{y}_d + \dot{z}z^5 + \tilde{y}_d(8z^3\dot{z} - v)$$

v comes into $\dot{V}$

$$\dot{V} = -\dot{z}^4 + \tilde{y}_d(z\dot{z} + 8z^3\dot{z} - v)$$

10/26/p.#

Select $v = 8z^3\dot{z} + z\dot{z} + \underbrace{\dot{\tilde{y}}_d}_{+(2z^4-y)}$

$v = 8z^3\dot{z} + z\dot{z} + 2z^4 - y$

then

$\dot{V} = -\dot{z}^4 - \tilde{y}_d^2$

Barbalat $\Rightarrow \dot{V} = 0$ because $\ddot{V}$ bdd $\Rightarrow \dot{V}$ UUB

$\Rightarrow \dot{z} \to 0$, $\tilde{y}_d \to 0 \Rightarrow y \to 2z^4$
$\Rightarrow v \to 0$

<u>backto dynamics</u>

$\dot{y} \Rightarrow 0$

$\ddot{z} + \dot{z}^3 - z^5 + 2z^5 = 0$

$\ddot{z} + \dot{z}^3 + z^5 = 0 \Rightarrow z, \dot{z} \to 0$
$\Rightarrow y \to 0$

<u>GAS</u>

ex BS3 **Recursive Backstep** SJL p. 261

1) $\dot{x} + x^2 y^5 z e^{xy} = (x^4 + 2) u$

2) $\dot{y} + y^3 z^2 - x = 0$

3) $\ddot{z} + \dot{z}^3 - z^5 + yz = 0$

fb lin 1) $\dot{x} = (x^4+2) u - x^2 y^5 z e^{xy} \equiv v_1$

$$u = \frac{1}{x^4+2}\left(v_1 + x^2 y^5 z e^{xy} \right)$$

inner fb lin. loop.

1) then $\dot{x} = v_1$

2) $\dot{y} + y^3 z^2 = x \quad + x_d - x_d$

3) $\ddot{z} + \dot{z}^3 - z^5 + yz = 0$

} same as ex BS2

select $x_d = y^3 z^2 - y + 2z^4 + 8z^3 \dot{z} + z\dot{z}$

$= y^3 z^2 + v_0$ ← from ex BS 2

then

2) $\dot{y} = -y + 2z^4 + 8z^3 \dot{z} + z\dot{z} + \underbrace{(x - x_d)}_{= \tilde{x}}$

$=$

select $V_\phi = \underbrace{\frac{1}{2}\dot{z}^2 + \frac{1}{6} z^6}_{v_0 \text{ from ex BS2}} + \frac{1}{2}\left(y - 2z^4 \right)^2$

$(y - y_d)^2 = \tilde{y}_d^2$

$V = V_\phi + \frac{1}{2} \tilde{x}_d^2$

$$\dot{V}_1 = \dot{z}\ddot{z} + z^5\dot{z} + \tilde{y}_d\dot{\tilde{y}}_d + \tilde{x}_d\dot{\tilde{x}}_d$$

$$= -\dot{z}^4 - \dot{z}z^5 + z\dot{z}\tilde{y}_d + \dot{z}z^5 + \tilde{y}_d(\dot{y}_d - \dot{y}) + \tilde{x}_d(\dot{x}_d - \dot{x})$$

$$= -\dot{z}^4 - \dot{z}z^5 + z\dot{z}\tilde{y}_d + \dot{z}z^5 + \tilde{y}_d(8z^3\dot{z} - v_0 + \tilde{x}_d) + \tilde{x}_d(\dot{x}_d - v_1)$$

$$= -\dot{z}^4 + \tilde{y}_d(z\dot{z} + 8z^3\dot{z} - v_0) + \tilde{x}_d(\dot{x}_d - v_1 + \tilde{y}_d)$$

$$= -\dot{z}^4 - \tilde{y}_d^2 + \tilde{x}_d(\dot{x}_d - v_1 + \tilde{y}_d)$$

recall $v_0 = 8z^3\dot{z} + \dot{z}z + \underbrace{\tilde{y}_d}_{(\underbrace{xz^4}_{y_d} - y)}$ from ex BS2

$x_d = y^3 z^2 + v_0$

set $v_1 = \tilde{y}_d + \underset{-\tilde{x}_d}{\dot{x}_d} = \dot{x}_d - \tilde{x}_d + \tilde{y}_d$

$$v_1 = \dot{x}_d - (x_d - x) + (xz^4 - y)$$

then,

$$\dot{V}_1 = -\dot{z}^4 - \tilde{y}_d^2 - \tilde{x}_d^2$$

now use LaSalle