LYAPUNOV STABILITY ANALYSIS

ex.1 p. p. 64

$$\dot{x}_1 = x_1(x_1^2 + x_2^2 - 2) - 4x_1x_2^2$$
$$\dot{x}_2 = x_2(x_1^2 + x_2^2 - 2) + 4x_1^2x_2$$

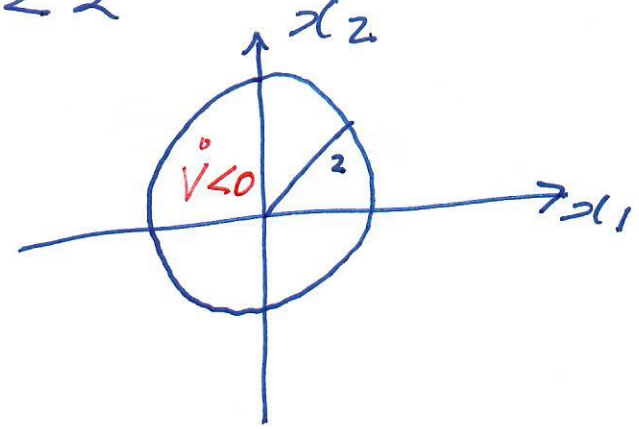
LFC $V = \frac{1}{2}(x_1^2 + x_2^2)$

check $\dot{V} = x_1\dot{x}_1 + x_2\dot{x}_2$

$$= x_1(x_1(x_1^2 + x_2^2 - 2) - 4x_1x_2^2)$$
$$+ x_2(x_2(x_1^2 + x_2^2 - 2) + 4x_1^2x_2)$$
$$= (x_1^2 + x_2^2)(x_1^2 + x_2^2 - 2)$$

$\dot{V}$ is < 0 if $x_1^2 + x_2^2 - 2 < 0$

$$x_1^2 + x_2^2 < 2$$



Locally AS with

Region of attraction

(or convergence region)

$$x_1^2 + x_2^2 < 2$$

Q2 p. 67

$$\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2)$$
$$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2)$$

LFC $V = \frac{1}{2}(x_1^2 + x_2^2)$

$$\dot{V} = x_1\dot{x}_1 + x_2\dot{x}_2$$
$$= x_1(x_2 - x_1(x_1^2 + x_2^2)) + x_2(-x_1 - x_2(x_1^2 + x_2^2))$$
$$= -(x_1^2 + x_2^2)(x_1^2 + x_2^2)$$
$$= -(x_1^2 + x_2^2)^2 \quad < 0$$

∴ Globally AS

Ex 3 UUB

Limit cycle p. 71

p.35
Ex. 2.7a

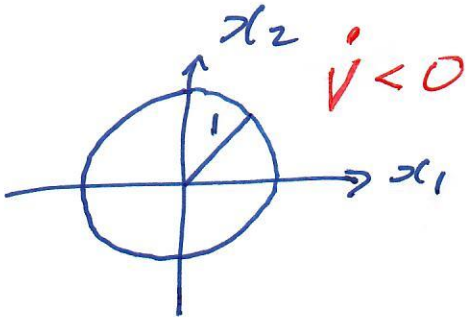
$$\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2 - 1)$$
$$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2 - 1)$$

LFC $V = \frac{1}{2}(x_1^2 + x_2^2)$

$$\dot{V} = x_1\dot{x}_1 + x_2\dot{x}_2$$
$$= x_1(x_2 - x_1(x_1^2 + x_2^2 - 1)) + x_2(-x_1 - x_2(x_1^2 + x_2^2 - 1))$$
$$= -(x_1^2 + x_2^2)(x_1^2 + x_2^2 - 1)$$

$\dot{V} < 0$ if $x_1^2 + x_2^2 - 1 > 0$

$x_1^2 + x_2^2 > 1$



UUB

c.f. p 71 Limit cycle

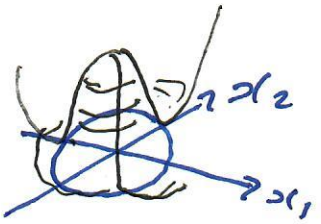
p. 35
ex 2.7 a)

ex 4

$$\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2 - 1)$$
$$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2 - 1)$$

Different LFC

$$V = \frac{1}{2}(x_1^2 + x_2^2 - 1)^2$$



Mexican Hat

$$\dot{V} = \frac{2}{2}(x_1^2 + x_2^2 - 1)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)$$

$$= (x_1^2 + x_2^2 - 1) \cdot$$
$$[2x_1(x_2 - x_1(x_1^2 + x_2^2 - 1))$$
$$+ 2x_2(-x_1 - x_2(x_1^2 + x_2^2 - 1))]$$
$$= -(x_1^2 + x_2^2 - 1)(x_1^2 + x_2^2 - 1))(2x_1^2 + 2x_2^2)$$
$$= -2(x_1^2 + x_2^2 - 1)^2(x_1^2 + x_2^2)$$

$\dot{V} < 0$ if $x_1^2 + x_2^2 - 1 \neq 0$

Stable Limit Cycle