I/O FBlin. p. 216

p209 contr. canon. form

p. 216 ex FBL 1

$$\dot{x}_1 = \sin x_2 + (1+x_2)x_3$$
$$\dot{x}_2 = x_1^5 + x_3$$
$$\dot{x}_3 = x_1^2 + u$$
$$y = x_1$$

$$\dot{y} = \dot{x}_1 = \sin x_2 + (1+x_2)x_3$$

$$\ddot{y} = \left(\frac{\partial}{\partial x_2}\sin x_2\right)\dot{x}_2 + \dot{x}_2 x_3 + (1+x_2)\dot{x}_3$$

$$= \cos x_2 \left(x_1^5 + x_3\right) + \left(x_1^5 + x_3\right)x_3 + (1+x_2)(x_1^2 + u)$$

$$= \underbrace{(x_3 + \cos x_2)(x_1^5 + x_3) + x_1^2(1+x_2)}_{f_1(x)} + \underbrace{(1+x_2)}_{g_1(x)}u$$

$$\ddot{y} \equiv f_1(x) + g_1(x)u \equiv v$$

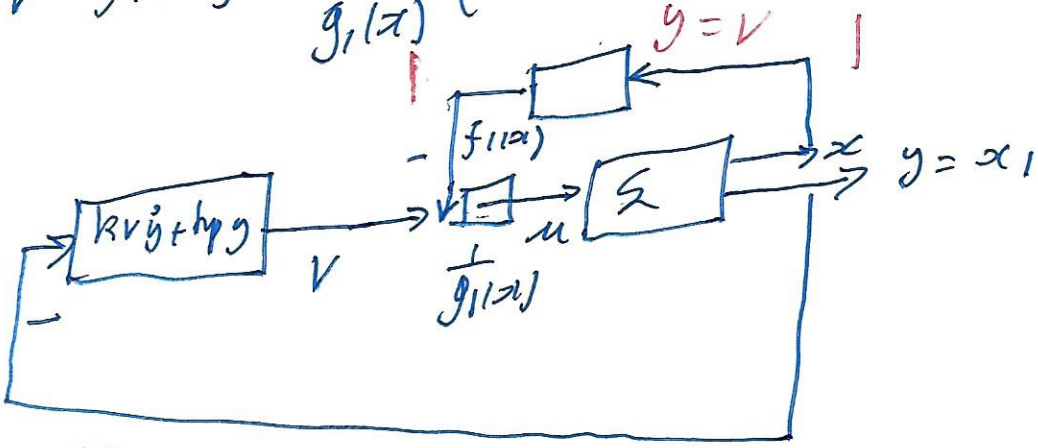
then $\ddot{y} = v$

a) Regulator

select $v = -k_v\dot{y} - k_p y$

then $\ddot{y} + k_v\dot{y} + k_p y = 0$. AS

$$u = \frac{1}{g_1(x)}(v - f_1(x)) = \frac{1}{g_1(x)}\left(-f_1(x) - k_v\dot{y} - k_p y\right)$$



outer reg loop. fblin inner loop

$(g_1 = 0$ if $x_2 = -1)$

tracker

$e = y_d - y$ not like s+L

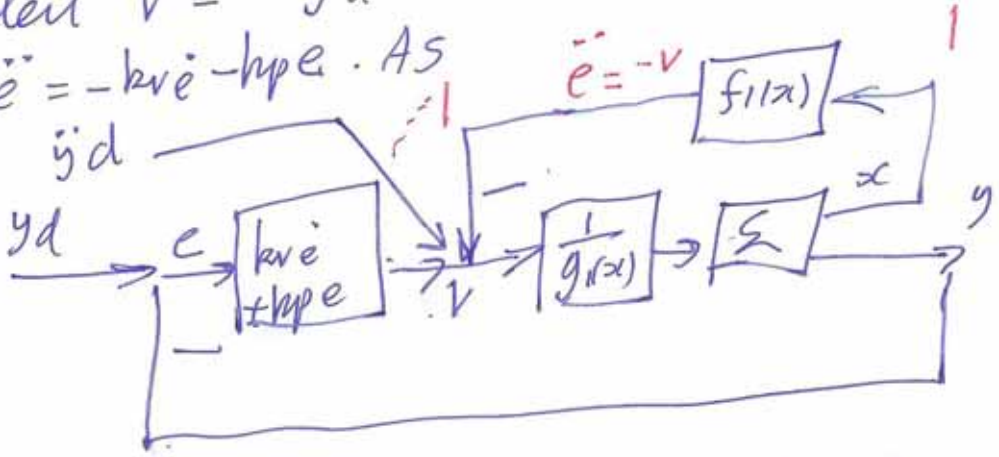
error dynam.

$\ddot{e} = \ddot{y}_d - \ddot{y}$

$\ddot{e} = \ddot{y}_d - (v)$

select $v = \ddot{y}_d + k_v \dot{e} + k_p e$

then $\ddot{e} = -k_v \dot{e} - k_p e$. AS



$$u = \frac{1}{g_1(x)} \left(-f_1(x) + \ddot{y}_d + k_v \dot{e} + k_p e \right)$$

rel degree 2

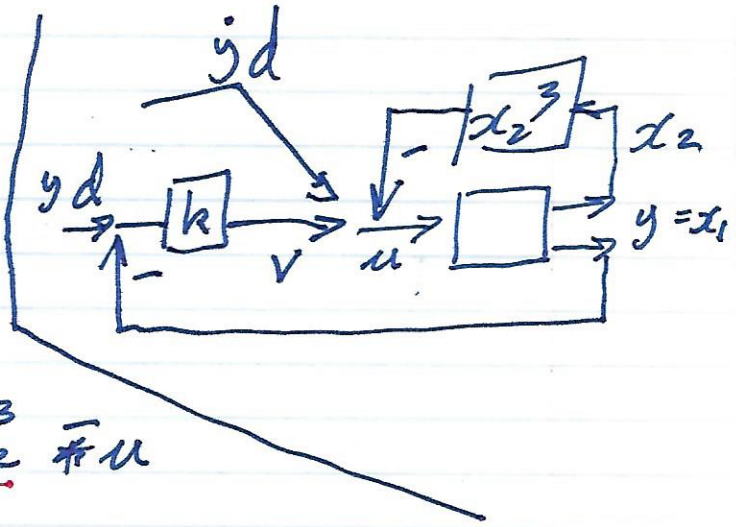
Internal dynam

$\dot{x}_3 = x_1^2 + u = x_1^2 + \frac{1}{1+x_2} \left(-f_1(x) + \ddot{y}_d - k_v \dot{e} - k_p e \right)$

if stable, NI is OK

p. 219 ex 6.3

$$\dot{x}_1 = x_2^3 + u$$
$$\dot{x}_2 = u$$
$$y = x_1$$
$$\dot{y} = \dot{x}_1 = x_2^3 + u.$$



$$e = y_d - y$$
$$\dot{e} = \dot{y}_d - \dot{y} = \dot{y}_d - x_2^3 \mp u$$

GROBS

$$u = -x_2^3 + \dot{y}_d - v$$
$$\dot{e} = v = -ke$$
$$\dot{e} = -ke$$

Internal Dynamics — not cool

$$\dot{x}_2 = u = -x_2^3 + \dot{y}_d + ke$$
$$\dot{x}_2 = -x_2^3 + \dot{y}_d + ke$$

Zero Dynamics — cool

set $e = 0$, $y_d = 0$ in Internal dyn. ($y = 0$)

$$\dot{x}_2 = -x_2^3$$
$$\dot{x}_2 + x_2^3 = 0$$

c.f. $\dot{x}_2 + c(x) = 0$ where $x\,c(x) > 0$

p220 LTI

ex1 $\dot{x}_1 = x_2 + u$, $\dot{x}_2 = u$, $y = x_1$ → $\dot{x} = \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix} x + \begin{bmatrix}1 \\ 1\end{bmatrix} u$; $Ax + Bu$

$y = x_1 = Cx$

$H(s) = C(sI - A)^{-1}B = \frac{s+1}{s^2}$

$\dot{y} = \dot{x}_1 = x_2 + u$

$e = y_d - y$, $\dot{e} = \dot{y}_d - \dot{y} = \dot{y}_d - x_2 - u = v$

$u = -x_2 + \dot{y}_d - v = -x_2 + \dot{y}_d + ke$ ($v = -ke$)

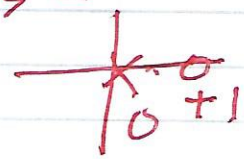
$\dot{e} = -ke$

TMT-int. dynam $\dot{x}_2 = -x_2 + \dot{y}_d + ke$

EZ- Zero dyn. $\dot{x}_2 + x_2 = 0$ ☹

$(s+1)x_2 = 0$

ex2 $\dot{x}_1 = x_2 + u$, $\dot{x}_2 = -u$, $y = x_1$ → $H(s) = \frac{s-1}{s^2}$



same $u = -x_2 + \dot{y}_d + ke$

int. dyn. $\dot{x}_2 = x_2 - \dot{y}_d - ke$

ZD $\dot{x}_2 - x_2 = 0$ ☺

$(s-1) = 0$

nonminimum phase

rel degree $r = 1$

I/O FB lin et Isidori p.221

FBL 2

$$\dot{x}_1 = x_1 + x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_2^2 + u$$

sei $y = x_2$

$$\dot{y} = \dot{x}_2 = x_3$$
$$\ddot{y} = \dot{x}_3 = x_2^2 + u$$
$$e = y_d - y$$
$$\dot{e} = \dot{y}_d - \dot{y}$$
$$\ddot{e} = \ddot{y}_d - x_2^2 - u$$

select $u = \ddot{y}_d + x_2^2 + k_d \dot{e} + k_p e$

then $\ddot{e} = -k_d \dot{e} - k_p e$

rel deg = 2

int. dyn. ZD $\dot{x}_1 = x_1 + x_2$

ZD $\dot{x}_1 = x_1$

not stable

NMP

ex ii

$y = x_1$

$\dot{y} = \dot{x}_1 = x_1 + x_2$

$\ddot{y} = \ddot{x}_1 = \dot{x}_1 + \dot{x}_2$

$= x_1 + x_2 + x_3$

$\dddot{y} = \dot{x}_1 + \dot{x}_2 + \dot{x}_3$

$= x_1 + x_2 + x_3 + x_2^2 + u$

$e = y_d - y$

$\dddot{e} = \dddot{y}_d - \dddot{y}$

$= y_d^{(3)} - (x_1 + x_2 + x_3 + x_2^2 + u)$

select $u = -(\underbrace{x_1 + x_2 + x_3 + x_2^2}_{f_1(x)}) + \dddot{y}_d + v$

GROBS

then

$e^{(3)} = v$

select $v = -(k_a \ddot{e} + k_d \dot{e} + k_p e)$

$e^{(3)} + k_a \ddot{e} + k_d \dot{e} + k_p e = 0$

select for stability

rel deg = 3

no int. dynam.