HW 03 - Nonlinear Systems Simulation

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Document Information

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Voltera Predator-Prey System

Consider the Voltera predator-prey system

- $\dot{x}_1 = -x_1 + x_1 x_2$
- $\dot{x}_2 = x_2 - x_1 x_2$

1. Find the equilibrium points and their nature.

Answer

State variable is given as:

$$\dot{x}_1 = -x_1 + x_1 x_2 \quad \dot{x}_2 = x_2 - x_1 x_2$$

The Voltera predator-prey system has limit cycles therefore the system is at equilibrium when the population of both predator and prey remain constant; thus, derivative should be zero. To find the equilibrium, I set $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$. Solve the system for its roots.

$$\dot{x}_1 = 0 \Rightarrow 0 = -x_1 + x_1 x_2$$

$$\dot{x}_2 = 0 \Rightarrow 0 = x_2 - x_1 x_2$$

$$0 = x_1(\beta x_2 - \alpha) \;\Rightarrow\; x_1 = 0;\; x_2 = \alpha/\beta$$

$$0 = x_2(\gamma - \sigma x_1) \;\Rightarrow\; x_1 = \gamma/\sigma;\; x_2 = 0$$

There are two equilibrium points at $(x_1,\ x_2)$,

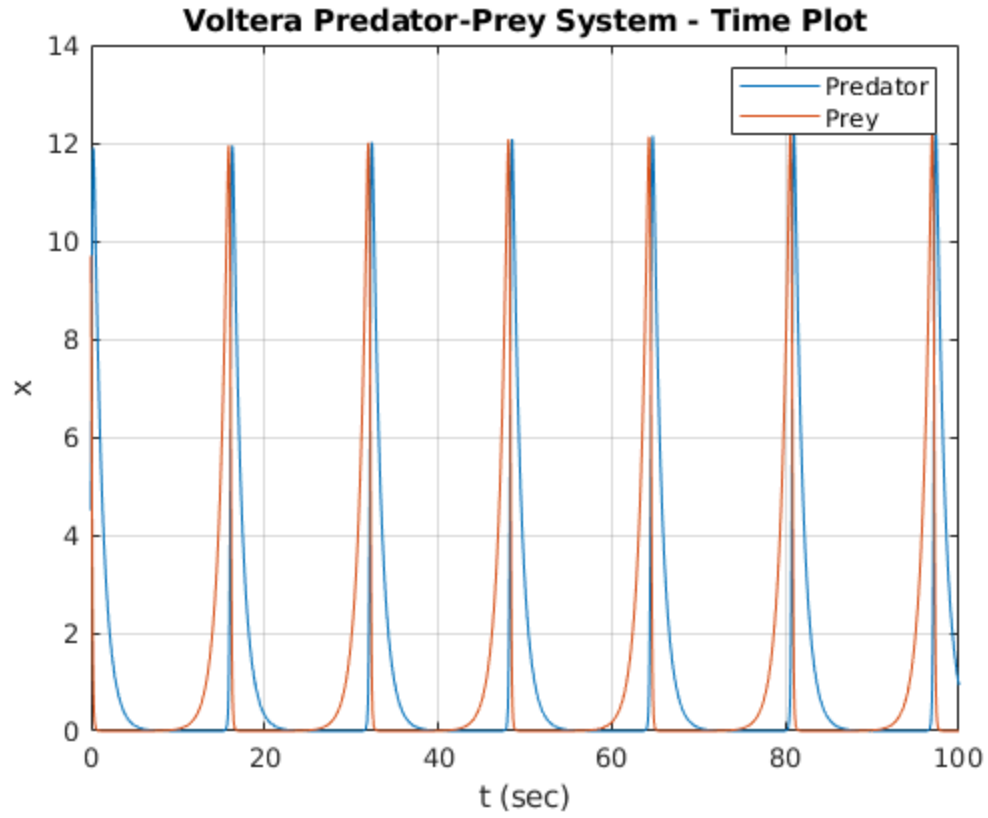
1. At zero, $(0,\ 0)$,
2. Any positive pair of integers $(\alpha/\beta,\ \gamma/\sigma)$

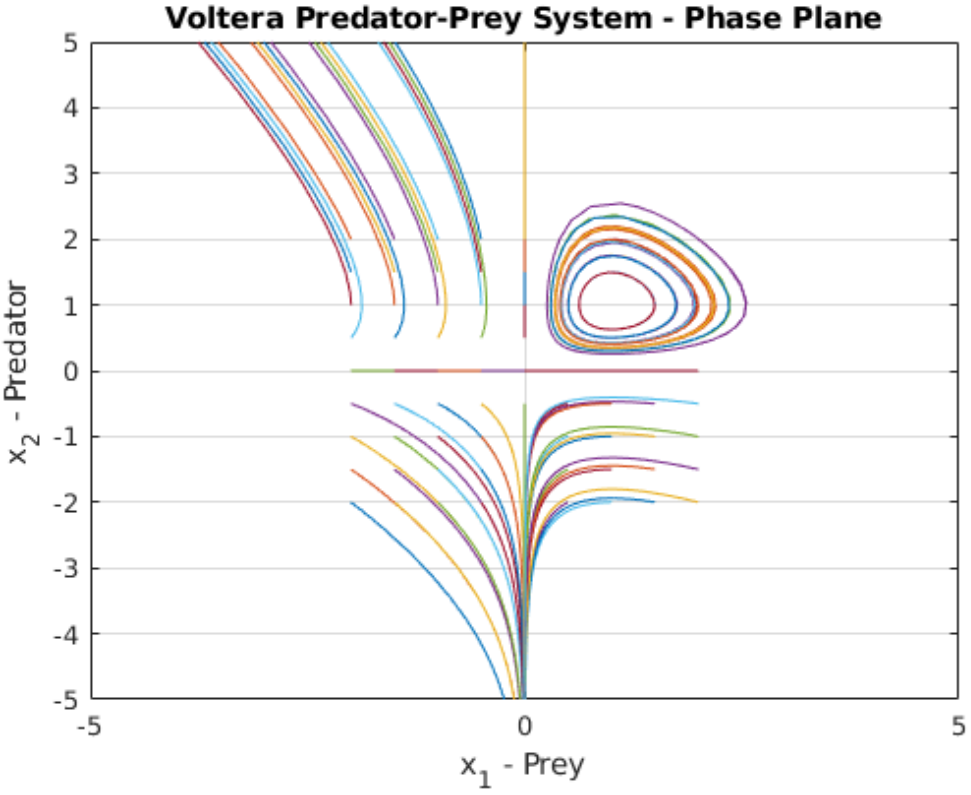
Equilibrium point nature of the zero is a stable center point that is a limit cycle. The other e.p. has saddle point nature because it is stable in one dimension (goes to zero) and unstable in the other (goes to infinity).

```
clc
close all
warning('off','all')
warning

x0_set = -2:.5:2;
t_intv= [0 100];
x_0= [4.5, 9.7]'; % initial conditions for x(t)
figure
[t,x]= ode23('Voltera', t_intv, x_0);
plot(t,x)
hold on;
grid on;
title('Voltera Predator-Prey System - Time Plot');
ylabel('x');
xlabel('t (sec)');
legend('Predator', 'Prey');
t_intv= [0 10];
figure
for i=x0_set
for j=x0_set
x0 = [i; j];
[t,x]= ode45('Voltera', t_intv, x0);
plot(x(:,1),x(:,2))
hold on;
end
end
title('Voltera Predator-Prey System - Phase Plane');
ylabel('x_2 - Predator');
xlabel('x_1 - Prey');
axis([-5 5 -5 5]);
grid on;

All warnings have the state 'off'.
```


Voltera Predator-Prey System - Time Plot
Predator
Prey
x
t (sec)
0
2
4
6
8
10
12
14
0
20
40
60
80
100


Voltera Predator-Prey System - Phase Plane
x2 - Predator
x1 - Prey
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
0
5

```
function xdot = Voltera(t,x)
  xdot = [-x(1)+x(1)*x(2); x(2)-x(1)*x(2)];
end
```

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