Ch 10

Part one

Alternating Sinusoidal Volage

Casee of Resistor:

$$\text{Voltage across resistor}: U_R = RI$$
$$\text{Where}: R: \text{resistance of resistor and } I: \text{intensity of current}$$

U_R and i are in phase.

Casee of pure coil(L):

$$\text{Voltage across coil}: U_L = Z_L I;\ Z_L = Lw$$
$$\text{Where}: L: \text{inductanc of coil and } I: \text{intensity of current}$$

U_L leads above current i by 90^0 ($\varphi = \frac{\pi}{2} rd$)

Casee of Resistive coil (L, r):

$$\text{Voltage across coil}: U_L = Z_{coil} I;\ Z_{coil} = \sqrt{r^2 + (L\omega)^2}$$
$$\text{Where}: L: \text{inductanc of coil and } I: \text{intensity of current}$$
$$\omega: \text{angular frequency } (rd/s) \text{ and } \omega = 2\pi f \ (f: \text{frequency})$$
$$r: \text{resistance of coil}$$
$$\cos\varphi = \frac{r}{Z}$$

U_L leads above current i φ

Casee of capacitor C:

$$\text{Voltage across capacitor}: U_C = Z_c I;\ Z_c = \frac{1}{C\omega}$$
$$\text{Where}: C: \text{capacitanc of capacitor and } I: \text{intensity of current}$$
$$\omega: \text{angular frequency } (rd/s) \text{ and } \omega = 2\pi f \ (f: \text{frequency})$$

U_C lags behind current i by 90^0 ($\varphi = \frac{\pi}{2} rd$)

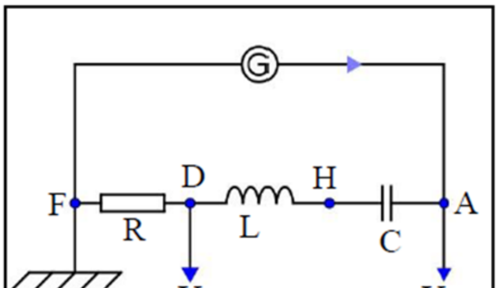
Case of RLC series circuit

$$\text{Voltage across generator}: U_G = ZI;\ Z = \sqrt{R^2 + (Z_L - Z_C)^2}$$

$$\cos\varphi = \frac{R}{Z}$$

The amplitude I_m of current *i* reaches a maximum value when the frequency f of the applied voltage is equal to a particular value f_0.

f_0: is called proper frequency. $f_0 = \frac{1}{2\pi\sqrt{LC}}$



(Doc 1)

When $f = f_0$, the current and voltage are in phase. This phenomenon is called current resonance. So, the proper frequency is also called resonance frequency.

At resonance: $Z_L = Z_C \Rightarrow L\omega_0^2 C = 1$

a. When $f < f_0$ (below resonance), the current leads the voltage by φ, then circuit is capacitive.

b. When $f = f_0$ (resonance), the current and voltage are in phase $\varphi=0$, then circuit is resistive.

c. When $f > f_0$ (above resonance), the current lags behind the voltage by φ, then circuit is inductive.

Remark:

$$In\ case\ of\ RLC\ series\ circuit: U_G = \sqrt{U_R^2 + (U_L - U_C)^2}$$
$$In\ case\ of\ RL\ series\ circuit: U_G = \sqrt{U_R^2 + U_L^2}$$
$$In\ case\ of\ RC\ series\ circuit: U_G = \sqrt{U_R^2 + U_C^2}$$

$$u_G = U_m \sin(\omega t)\ and\ \ i = I_m \sin(\omega t + \varphi)$$
$$Power: P = U.I.\cos\varphi$$
$$where: U = \frac{U_m}{\sqrt{2}}\ And\ I = \frac{I_m}{\sqrt{2}}$$

U_m is the max voltage and U is the effective voltage.

I_m is the max current, and I is the effective current.

Φ is the phase difference.

Part 2

Charging and Discharging a Capacitor

$\boldsymbol{Charge\ of\ capacitor}: \boldsymbol{q = CV\ \ OR\ \ Q = CU}$

Where:

C: Capacitance of a capacitor (Unit in SI system is Farad: F)

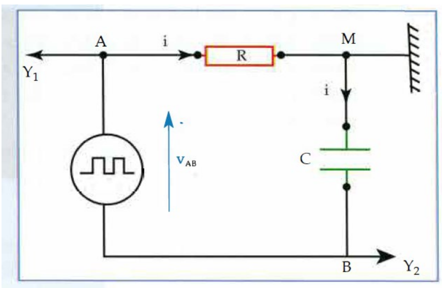
Q or q: charge (quantity of electricity) of the capacitor (Unit in SI system is Coulomb: C)

U or u: voltage across the terminals of a capacitor. (Volt: V)

Energy stored in a capacitor:

$\boldsymbol{W = \frac{1}{2}CU^2\ \ or\ \ W = \frac{1}{2}QU\ \ or\ \ W = \frac{1}{2}\frac{Q^2}{C}}$ **. W is expressed in Joules (J).**

❖ <u>**Charging of capacitor:**</u>



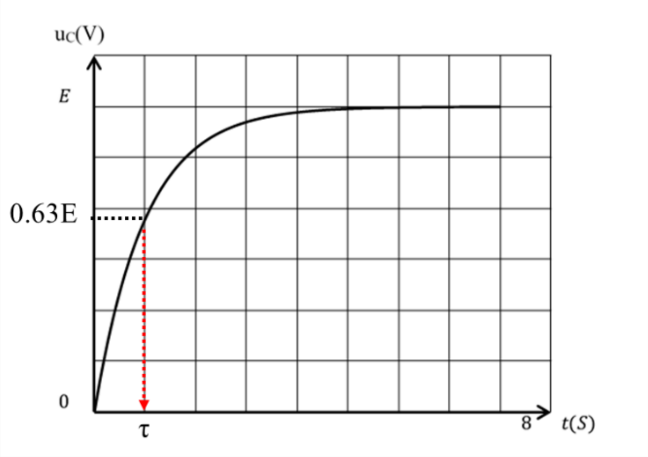
Differential equation: $E = RC\frac{du_C}{dt} + u_C$

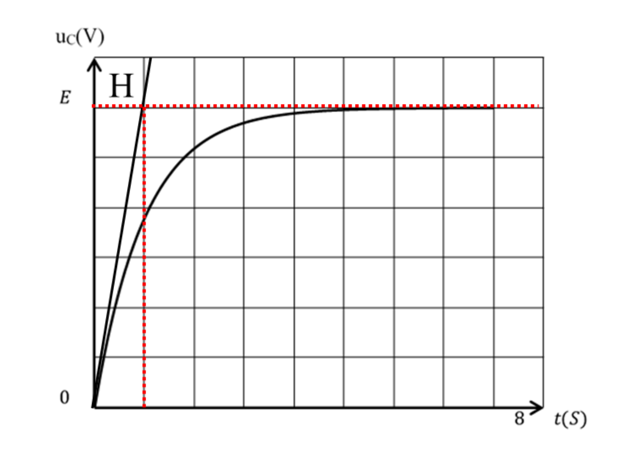
Solution of the differential equation:

$$u_C = E(1 - e^{-\frac{t}{\tau}})\ where\ \tau = RC\ is\ time\ constant.$$

$$after\ t = \tau \Rightarrow u_C = E(1 - e^{-\frac{\tau}{\tau}}) \Rightarrow u_C = E(1 - e^{-1}) = 0.63E$$

Time constant τ is the interval of time after which the voltage across the capacitor reaches 63% of its maximum value during charging of the capacitor.





$$after\ t = 5\tau \Rightarrow u_C = E(1 - e^{-\frac{5\tau}{\tau}}) \Rightarrow u_C = E(1 - e^{-5}) = 0.99E \simeq E$$

After t = 5τ, the capacitor is practically completely charged

The tangent to u_C at t=0, meets the asymptote in a point of abscissa τ.

❖ <u>Discharging of capacitor</u>

Differential equation: $RC\frac{du_C}{dt} + u_C = 0$

Solution of the differential equation: $u_C = E\,e^{-\frac{t}{\tau}}$ *where* $\tau = RC$ *is time constant.*

$$after\ t = \tau \Rightarrow u_C = Ee^{-\frac{\tau}{\tau}} \Rightarrow u_C = E\,e^{-1} = 0.37E$$

Time constant τ is the interval of time after which the voltage across the capacitor reaches 37% of its maximum value during discharging of the capacitor.

$$after\ t = 5\tau \Rightarrow u_C = Ee^{-\frac{5\tau}{\tau}} \Rightarrow u_C = E\,e^{-5} = 0.006E \simeq 0$$

After t = 5τ, the capacitor is practically completely discharged.

The tangent to u_C at t=0, meets the asymptote in a point of abscissa τ.