TEAM: WE LOVE DEADLINES

Deblurring Image Using SVD

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Skoltech: Numerical Linear Algebra 2022

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PROBLEM STATEMENT

Problem Statement

Blurred images can occur when taking a picture with an out-of-focus lens or when snapping pictures a moving object with an excessively long exposure time. Blur occurs mathematically when pixel values from the original uncontaminated image are replaced by weighted averages of values from nearby pixels.

METHODOLOGY

Blurring Images by Toeplitz matrices

In digital image processing, an image is presented by a 2-D array. We blur the image matrix by multiplication by Toeplitz Matrix, since it represents convolution with blurring kernel

$$T = \begin{pmatrix} s_0 & s_1 & \cdots & s_{n-1} \\ s_{-1} & s_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ s_{-(m-1)} & s_{-(m-2)} & \cdots & s_{-(m-n+1)} \end{pmatrix}$$

At this stage the image is fully restorable since the transformation is non-degenerate.

Blurring Images by Toeplitz matrices

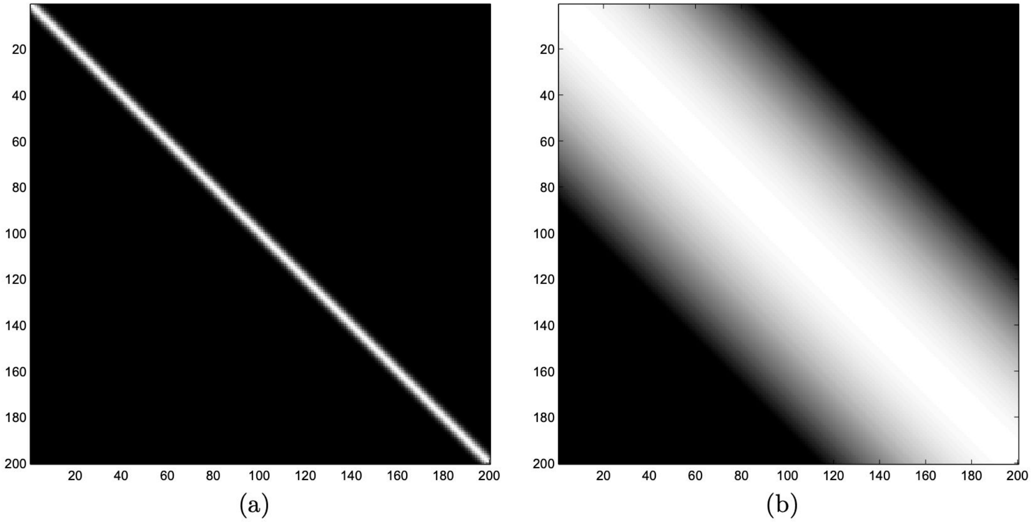


Figure1 : (a) Image of a blurring matrix with the gray-level proportional to the size of the entries, (b) image of the same blurring matrix the gray-level proportional to the logarithm of the size of the size of the entries.

Two-dimensional signals:

We consider deblurring of gray-scale images. Let X represent an image. Then the blurred image can be represented by

$$Y = T_1 X T_2.$$

Where the symmetric matrix $T \in \mathbb{R}^{256\times 256}$ is the blurring operator in the corresponding dimension

Let the available image also contaminated by noise. We represent the noise by the matrix $E \in \mathbb{R}^{256\times 256}$ with normally distributed random entries with zero mean. The available blur and noise contaminated image is given by

$$Z = TXT + E.$$

Our goal is to find such an algorithm that restores the image if the error is non-zero. Simple inverting of matrices won't work since error can become large.

Two-dimensional signals:

Let T_k be the rank-k approximation of T obtained by setting all but first k singular values to zero. Consider the approximations

$$X_k = T_k^{\dagger} Z T_k^{\dagger}, \qquad k = 1, 2, \ldots$$

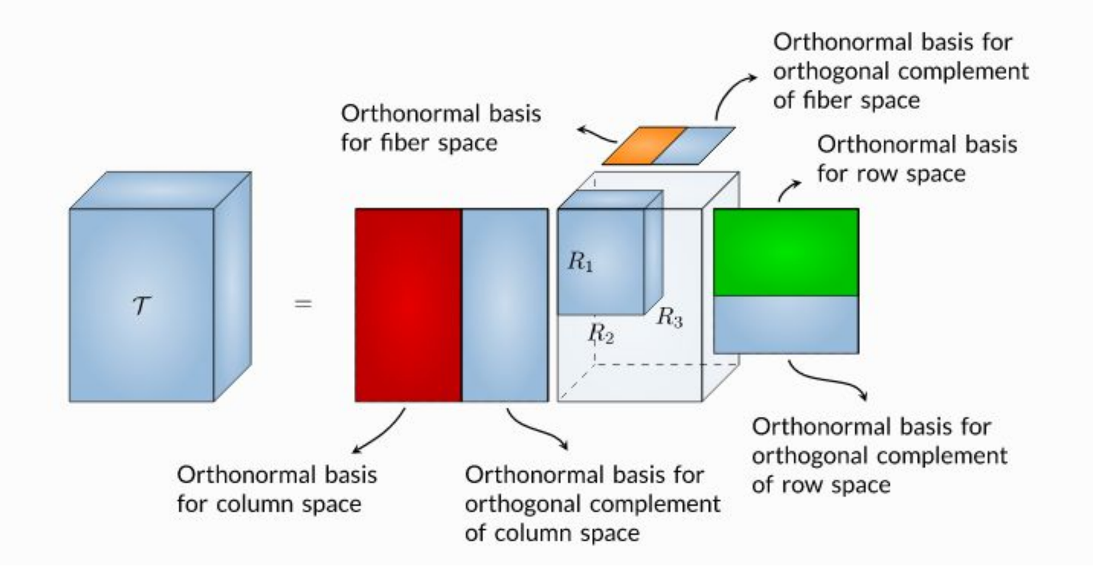
They can be computed fairly easily by computing the SVD of T.

We remark that color images can be deblurred in the same manner as gray-scale images. For each pixel three "channels" are provided to represent the colors red, green and blue.

Additional method: MLSVD for color image

Multilinear singular value decomposition (MLSVD)

The tensor can be represented as the form: $\mathcal{T} = \mathcal{S} \bullet_1 U^{(1)} \bullet_2 U^{(2)} \bullet_3 U^{(3)}$



Source : https://www.tensorlab.net/demos/mlsvd.html

Structure Similarity Index:

The structural similarity index(SSIM) metric extract 3 key features from an image:

- Luminance
- Contrast
- Structure

The comparison between the two images is performed on the basis of these 3 features.

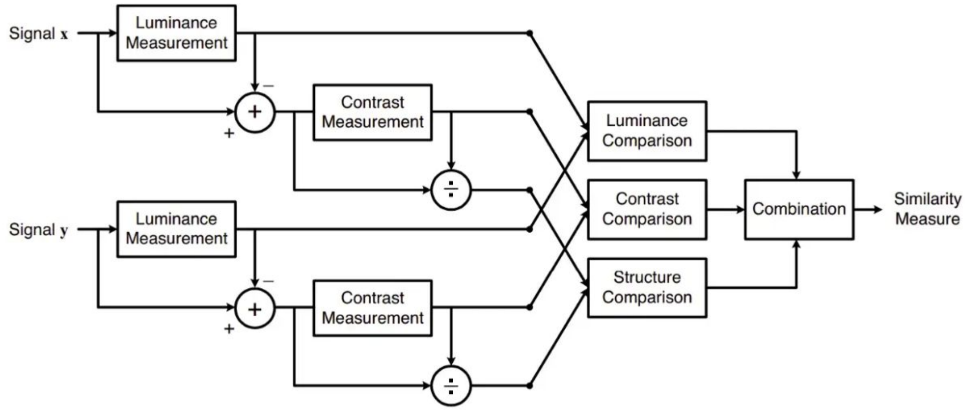
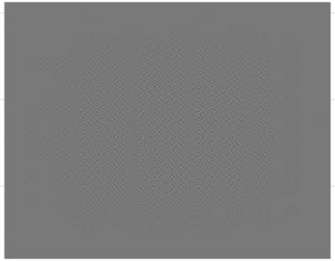
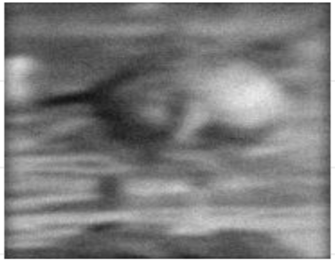
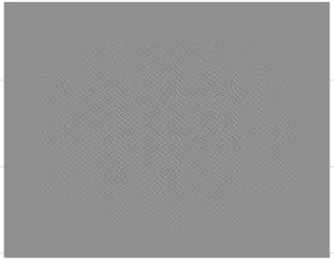
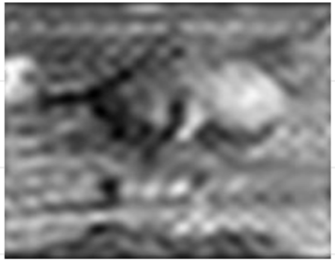
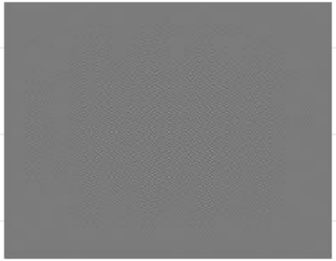
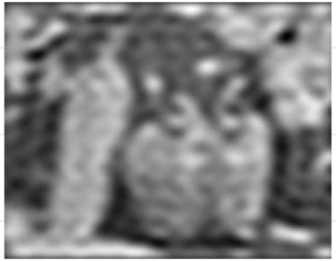


Figure 2 : Shows the arrangement and flow of the structural similarity from an image: Signal Y refer to the reference and sample images.

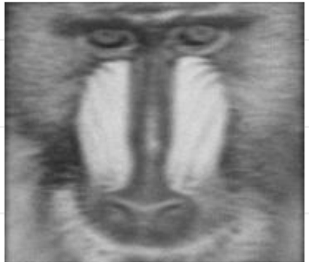
Source : https://www.cns.nyu.edu/pub/eero/wang03-reprint.pdf

RESULTS

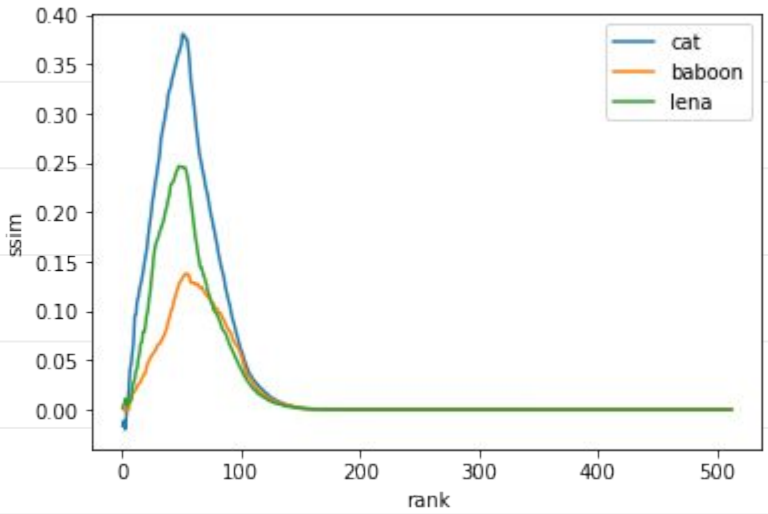
Result with images size 256:

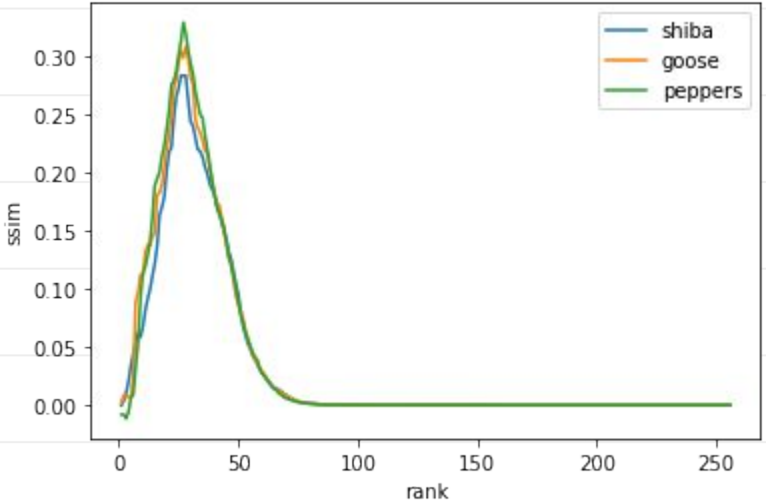
	Original	Noisy_blur	Full rank	Truncate_svd	SSIM
Shiba					0.2828
Goose					0.3077
Peppers					0.3283

Result with images size 512:

	Original	Noisy_blur	Full rank	Truncate_svd	SSIM
Cat					0.3804
Lena					0.1380
Baboon					0.2466

Comparing with SSIM:

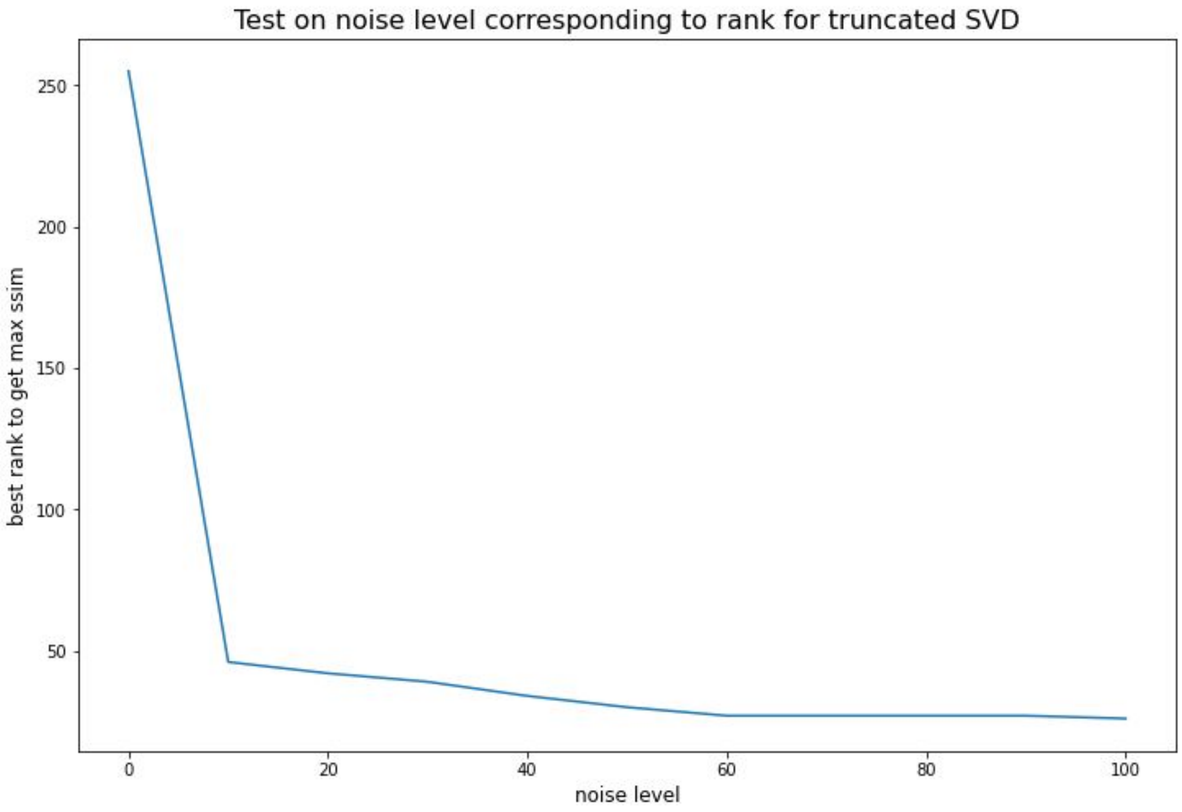




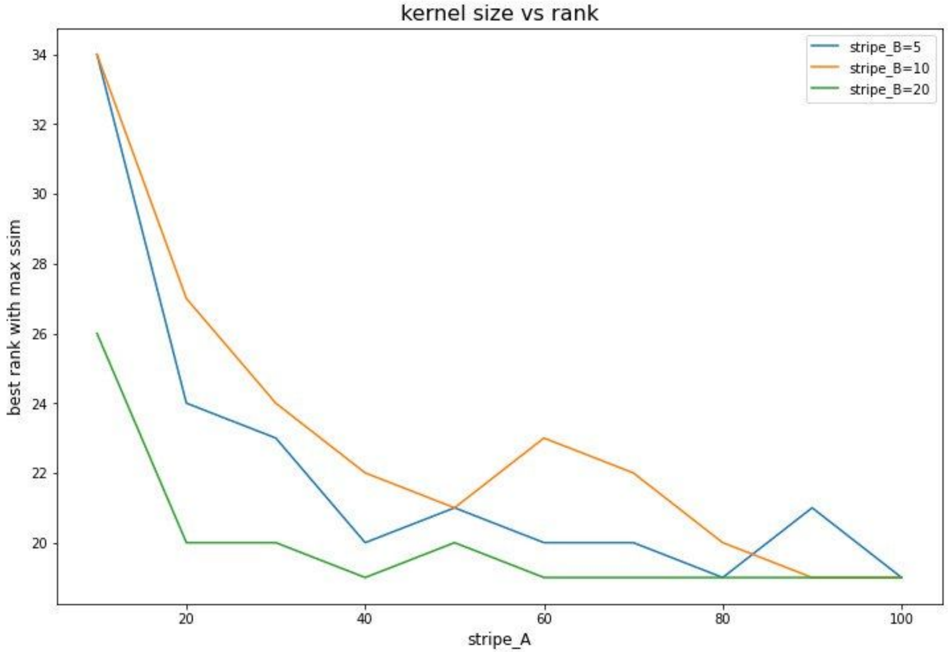

```
shiba: best ssim = 0.2827895072322408 at truncated rank = 25
goose: best ssim = 0.307735842568979 at truncated rank = 27
peppers: best ssim = 0.32831531883154524 at truncated rank = 26
```

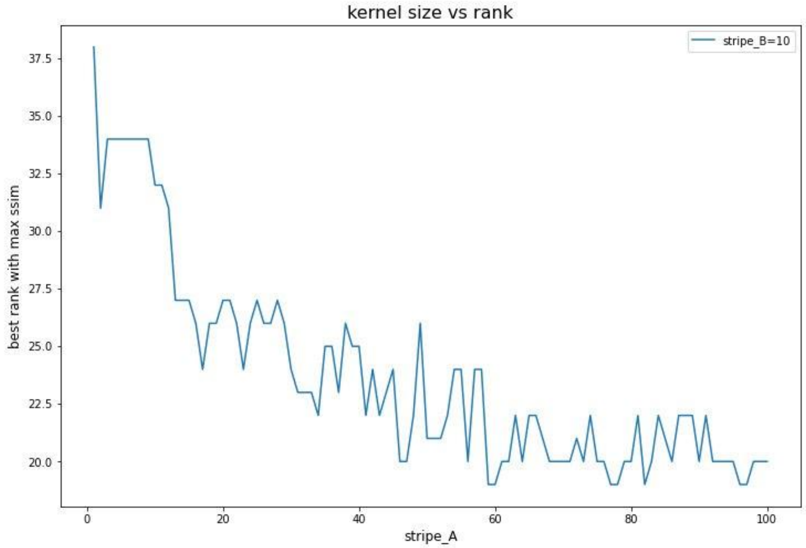
```
cat: best ssim = 0.38041476769184923 at truncated rank = 50
baboon: best ssim = 0.1380380031258109 at truncated rank = 53
lena: best ssim = 0.24656765862833527 at truncated rank = 47
```

Corresponding with rank:



Toeplitz size and restoration rank:





Result with RGB image:

Original image

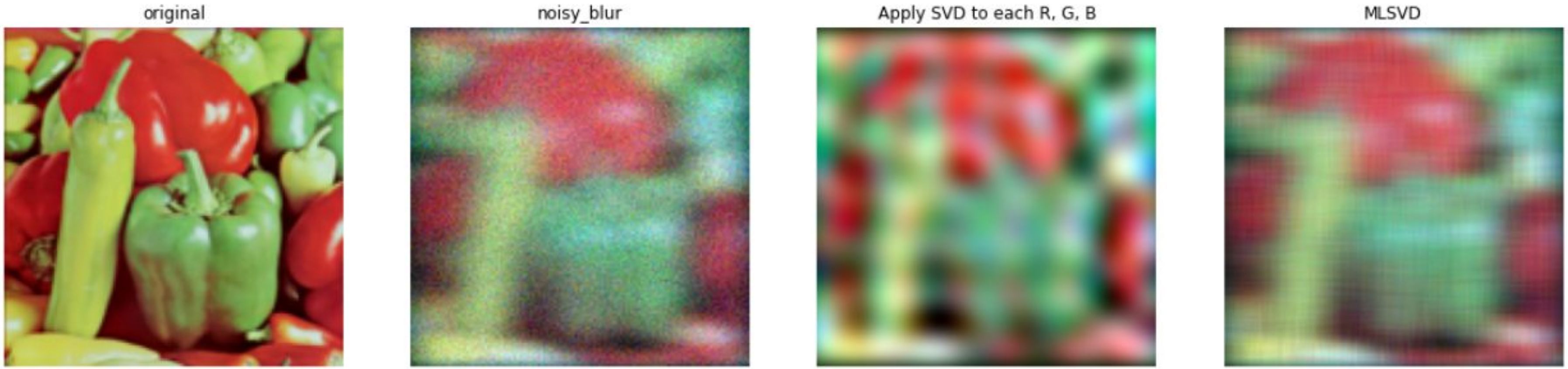
Initial blurred image

Noise in blure

Restoration without rank trancation

Restoration with rank trancation

Result with RGB image (additional method):

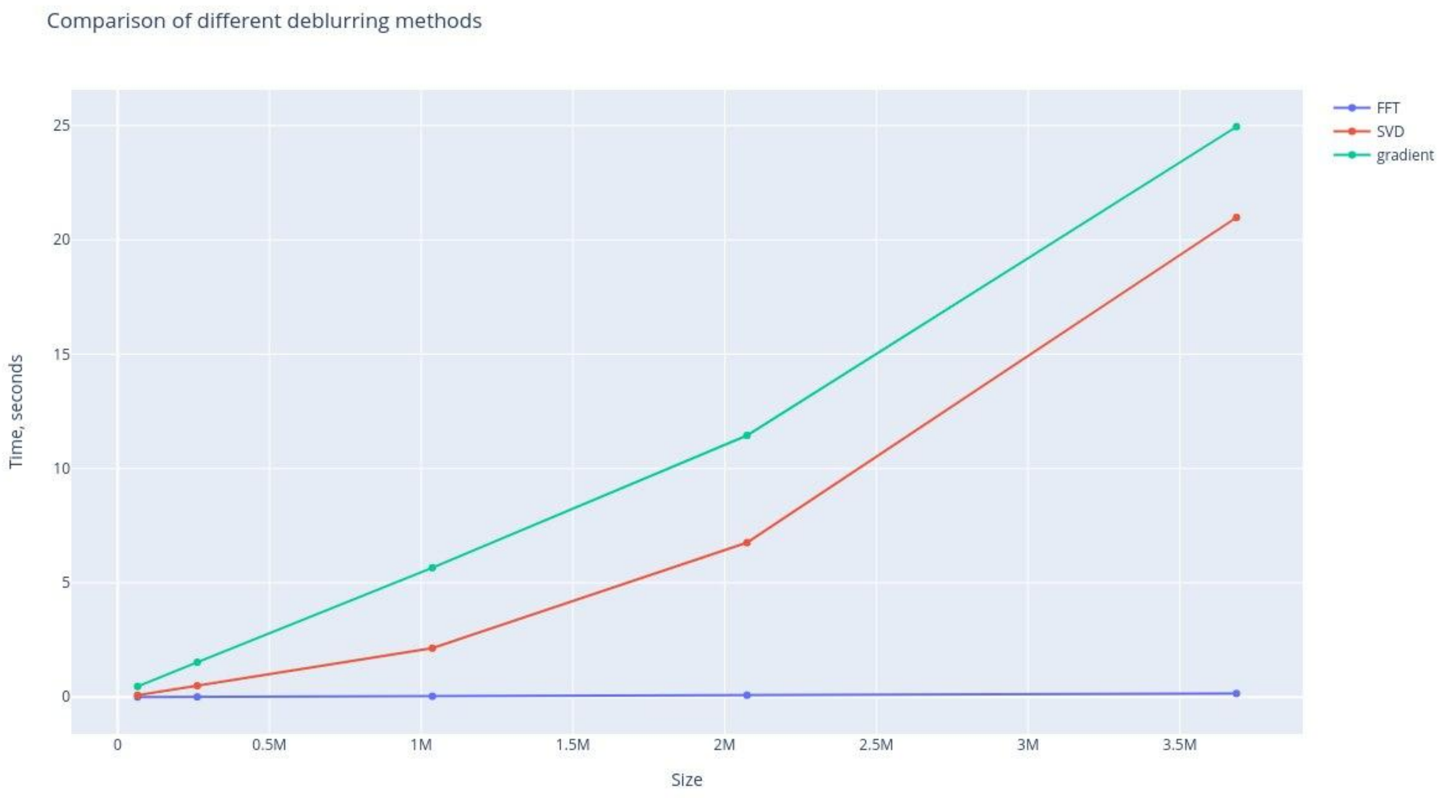


SSIM = 0.2308

SSIM = 0.2241

We apply MLSVD to the blur image and find the best truncated rank and compare the result with our method

Comparison of algorithms:



THANK YOU

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TEAM CONTRIBUTION!

Anastasia Gavrish

- RGB generalization

Artem Chuprov

- Efficiency experiments

Bari Khairullin

- coordination

Sudarut Kasemsuk

- presentation

Waralak Pariwatphan

- rank-SSIM experiments