

ENGR-421 HOMEWORK-8

Name-Surname: Barış KAPLAN

KU ID Number: 69054

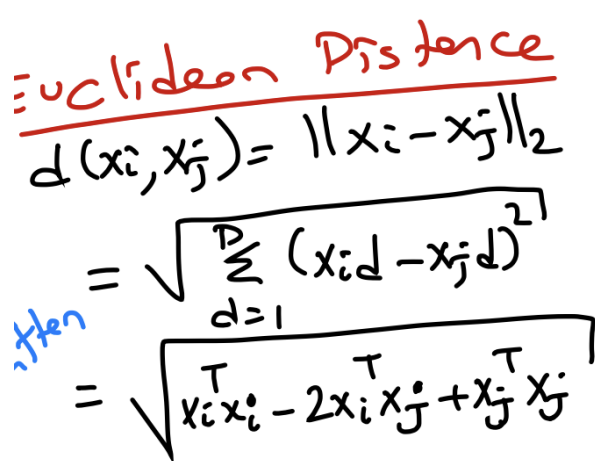
KU Email Address: bkaplan18@ku.edu.tr

Initially, I have imported the necessary libraries. The libraries I imported are as follows:

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.spatial as spa
import math
```

After that, as we did in the previous labs and as I did in the previous homeworks; I have read the data by using the np.genfromtxt function of the numpy library.

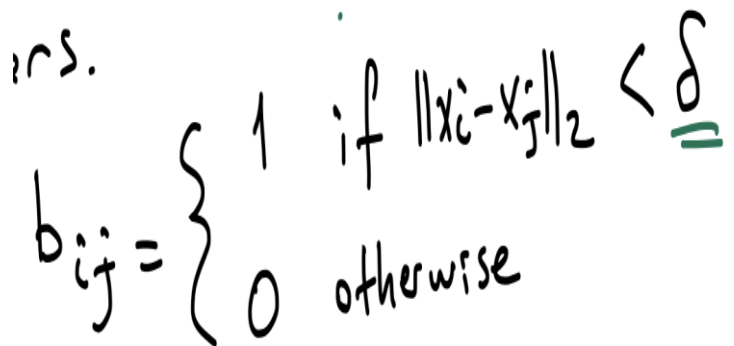
Then, I have calculated the euclidean distances between the data points. While calculating the euclidean distances between the data points, I have used the formula in Figure 1.



Handwritten formula for Euclidean Distance:

$$\begin{aligned} \text{Euclidean Distance} \\ d(x_i, x_j) &= \|x_i - x_j\|_2 \\ &= \sqrt{\sum_{d=1}^D (x_{id} - x_{jd})^2} \\ &= \sqrt{x_i^T x_i - 2x_i^T x_j + x_j^T x_j} \end{aligned}$$

Figure 1: The formula I used to calculate the euclidean distances between the data points (from Lecture 23-Clustering)



Handwritten rule for constructing the B matrix:

$$b_{ij} = \begin{cases} 1 & \text{if } \|x_i - x_j\|_2 < \underline{\delta} \\ 0 & \text{otherwise} \end{cases}$$

Figure 2: The rule I have used to construct the B matrix (from Lecture-23 Clustering)

After that, by using the rules in Figure 2 & Figure 3 ; I have constructed the B matrix. If the distance between the data points is smaller than the threshold value, I have assigned 1 to the entries of B matrix. Otherwise, I have assigned 0 to the entries of the B matrix. For the case where i is equal to j , I have assigned 0 to the B_{ij} values for every i (assigning 0 to diagonals). Then, I have plotted the connectivity matrix by checking whether an entry of B matrix is equal to 1 (indicates connection between data points). You can see the connectivity matrix I obtained in Figure 5.

$$b_{ii} = 0 \quad \forall i$$

Figure 3: The second rule I used while constructing B matrix

Next, by using the rules in Figure 4, I constructed the D matrix. For constructing the D matrix, I have summed up the number of 1's in each row and assigned the resulting sum for each row to the diagonal entries in each row. I have assigned 0 to all other entries.

$$d_{ii} = \sum_{j \neq i} b_{ij} \quad \forall i$$

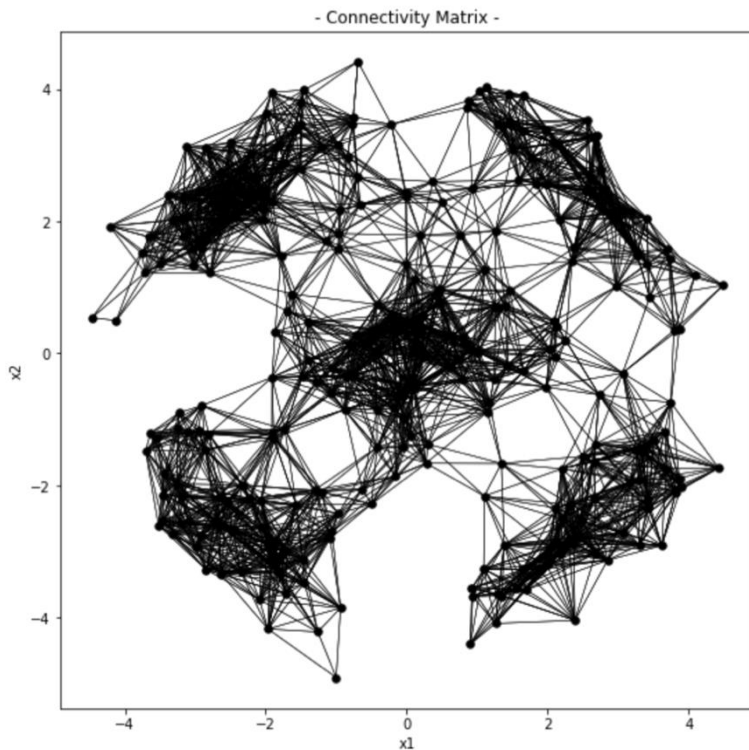
↪ # of neighbors
of data point i

$$d_{ij} = 0 \quad \forall (i, j \neq i)$$

Figure 4: The rules I used while constructing the D matrix

After that, by using the formula in figure 7; I have normalized the L matrix that I have calculated. Subsequently, I have obtained the eigenvalues and eigenvectors of the Lsymmetric matrix by using the `np.linalg.eig` function of the numpy library. Then, I have sorted the eigenvalues according to their indices by using `np.argsort` function of the numpy library. Moreover, I have obtained the 2nd, 3rd, 4th, 5th, and 6th eigenvalues. I have constructed the Z matrix by firstly making an array (by using `np.array` function of the numpy library of Python) with 2nd, 3rd, 4th, 5th, and 6th eigenvalues; and then by taking the transpose of that array (by using `“.”` abbreviation) . Next, I have obtained initial centroids by vertically stacking (with `np.vstack` function of the numpy library) of the desired rows of the Z matrix. Then, I have implemented the K-means clustering algorithm, and executed the K-means clustering algorithm on the Z matrix. While implementing the K-means clustering algorithm, I have benefitted from “Lab11-Clustering”. While initializing the centroids in the `update_cens` function, I have assigned the centroids to the initial centroids which are obtained from the desired rows of the Z matrix. The logic of plotting the clustering results is same with “Lab11-Clustering”. While plotting the clustering results, I have used the `plt.plot`, `plt.xlabel`, `plt.ylabel`, `plt.title` functions of the Python’s **matplotlib.pyplot** library. In Figure 6, you can see the plot which displays the clustering results.

CONNECTIVITY MATRIX :



Please see the 1st and 2nd slides (the Spectral Clustering part) of the “Lecture-23-Clustering” to see the formulas I used in this homework.

$$\mathbf{L}_{\text{symmetric}} = \mathbf{I} - \mathbf{D}^{-1/2} \mathbf{B} \mathbf{D}^{-1/2}$$

Figure 7: The formula I used to normalize the L matrix

Figure 5: The connectivity matrix that I found (It is same with the expected result given in pdf)

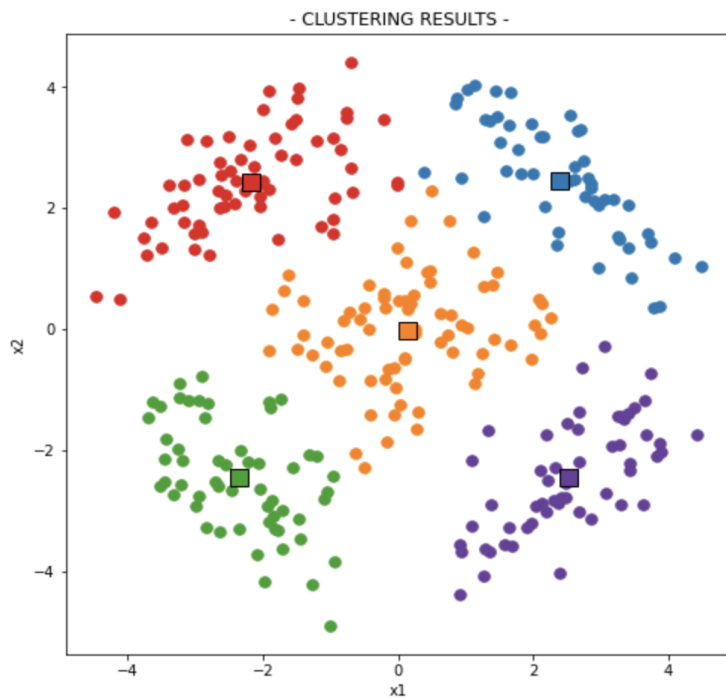


Figure 6: The plot which displays the clustering results (It is the same with the expected result given in pdf)