COMP341 INTRODUCTION TO ARTIFICIAL INTELLIGENCE ASSIGNMENT-3

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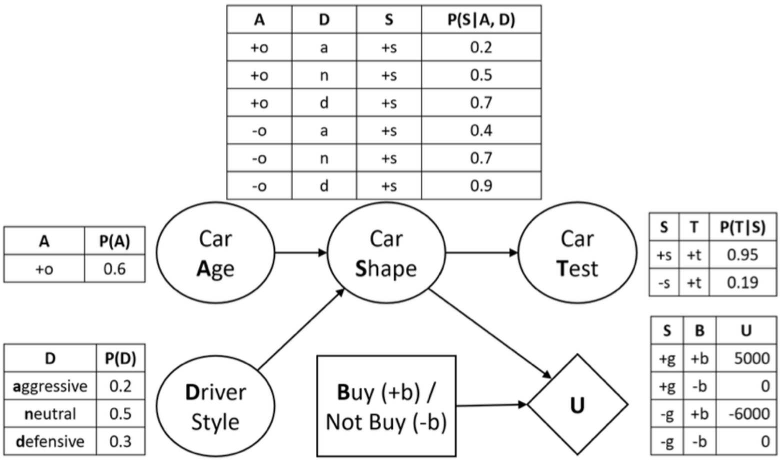
Q1-)

The joint distribution of this Bayesian Network is = P(A, D, S, T) = P(A) * P(D) * P(S|A,D) * P(T|S)

P(A, D, S, T) = P(A) * P(D) * P(S|A, D) * P(T|S)

Q2-)



A	D	S	P(S|A, D)
+o	a	+s	0.2
+o	n	+s	0.5
+o	d	+s	0.7
-o	a	+s	0.4
-o	n	+s	0.7
-o	d	+s	0.9

A	P(A)
+o	0.6

D	P(D)
aggressive	0.2
neutral	0.5
defensive	0.3

S	T	P(T|S)
+s	+t	0.95
-s	+t	0.19

S	B	U
+g	+b	5000
+g	-b	0
-g	+b	-6000
-g	-b	0

$$P(S \mid T = +t) = \sum_{a,\, d^*} P(a) * P(d^*) * P(S \mid a, d^*) * P(T = +t \mid S)$$

where $d^* \in D$, and $a \in A$.

Instead of the above summation, we can also write the following:

$$\mathbf{P(S = +s \mid T = +t) = \sum_{a,\, d^*} P(a) * P(d^*) * P(+s \mid a, d^*) * P(T = +t \mid S = +s)}$$

where $d^* \in D$, and $a \in A$.

If we write this summation, than the initial factor named fc3(S, A, D) will be equal to P(+s | A, D) instead of P(S | A, D). Moreover, the initial factor named fc4(t, S) will be equal to P(+t | +s) instead of P(+t | S). In this case, we will not need to calculate any probability including "-s" and we will not need to include any row which contains "-s" in the tables. All other processes will be the same with the first case. At the end table, in this case, we will get only the probability P(S = +s | T = +t) (which is the probability that I supposed to find) and not the probability P(S = -s | T = +t).

Note: For observing the names of initial factors, see below / next page!
In the above summation, 'a' represents each element in A (namely +o). Moreover, 'd*' represents each element in D (namely aggressive, neutral , and defensive). I have written d* instead of d to represent each element in D in order to not mix the d standing for the "defensive" and d representing each element in D. However, we can also write P(d) instead of P(d*) depending on our preference.

For solving this inference problem, we should seek objects which are called as "factors". Then, we should perform the necessary joining process for the factors.

From the probability formula I have written above, we can find the following initial factors:

- ⇨ fc1(A) = P(A)
- ⇨ fc2(D) = P(D)
- ⇨ fc3(S, A, D) = P(S | A, D)
- ⇨ fc4(t, S) = P(+t | S)

The Whole Process of Variable Elimination:

1-) Firstly, let's join fc2(D) and fc3(S,A,D). Then, lets say we obtain a factor named fc23(S,A,D).

A	D	S	fc23(S, A, D)
-o	a	-s	+ 0.12
-o	a	+s	+ 0.08
-o	d	-s	+ 0.03
-o	d	+s	+ 0.27
-o	n	-s	+ 0.15
-o	n	+s	+ 0.35
+o	a	-s	+ 0.16
+o	a	+s	+ 0.04
+o	d	-s	+ 0.09
+o	d	+s	+ 0.21
+o	n	-s	+ 0.25
+o	n	+s	+ 0.25

Some example computations of some of above probabilities to better explain above table:

fc23(+s, +o, aggressive) = fc2(aggressive) * fc3(+s, +o, aggressive)
aggressive = a (already given in question)
fc23(+s, +o, a) = fc2(a) * fc3(+s, +o, a)
fc2(a) = + 0.2 (from the corresponding table)
fc3(+s, +o, a) = + 0.04
fc23(+s, +o, +a) = fc2(a) * fc3(+s, +o, a) = 0.2 * 0.2 = 0.04
fc23(+s, +o, +a) = 0.04

fc3(-s, +o, aggressive) + fc3(+s, +o, aggressive) = 1 (from the general probability summation axiom)
fc23(+s, +o, -a) = fc2(aggressive) * (1- fc3(+s, +o, aggressive))
aggressive = a (already given in question)
fc23(+s, +o, +a) = fc2(a) * (1 - fc3(+s, +o, +a))
fc2(a) = + 0.2 (from the corresponding table)
fc3(+s, +o, +a) = + 0.2 (from the corresponding table)
1 – fc3(+s, +o, +a) = 1 – 0.2 = 0.8
fc3(-s, +o, +a) = + 0.8
fc23(-s, +o, +a) = fc2(a) * fc3(-s, +o, +a) = (+ 0.2) * (+ 0.8) = + 0.16
fc23(-s, +o, +a) = 0.16

Note: I have followed the same pairwise multiplication approach for all other probabilities in the table. For calculating probabilities including "-o", I have used following general formula:

P(+s, +o, +a) + P(+s, -o, +a) = 1
For calculating probabilities including “-s”, I have used following general formula:
P(+s, +o, +a) + P(-s, +o, +a) = 1

2-) Given that D is in (a, n, d) set, obtain the summation of all probabilities within each different (S, A) tuple. In other words, eliminate D by summing over all possible elements in D for all different (S,A) tuples (Shortly, eliminate D). After all these, let’s say that we obtain a factor named fc6(S, A).

S	A	fc6(S, A)
-s	-o	+ 0.30
+s	-o	+ 0.70
-s	+o	+ 0.50
+s	+o	+ 0.50

General Formulas I used in the 2nd step:

P(-s, -o) = P(-s, -o, a) + P(-s, -o, n) + P(-s, -o, d)
P(+s, -o) = P(+s, -o, a) + P(+s, -o, n) + P(+s, -o, d)
P(-s, +o) = P(-s, +o, a) + P(-s, +o, n) + P(-s, +o, d)
P(+s, +o) = P(+s, +o, a) + P(+s, +o, n) + P(+s, +o, d)

Probability Calculations for the 2nd step:

P(-s, -o) = P(-s, -o, a) + P(-s, -o, n) + P(-s, -o, d) = (+ 0.12) + (+ 0.15) + (+ 0.03) = (+ 0.30)
P(+s, -o) = P(+s, -o, a) + P(+s, -o, n) + P(+s, -o, d) = (+ 0.08) + (+ 0.35) + (+ 0.27) = (+ 0.70)
P(-s, +o) = P(-s, +o, a) + P(-s, +o, n) + P(-s, +o, d) = (+ 0.16) + (+ 0.25) + (+ 0.09) = (+ 0.50)
P(+s, +o) = P(+s, +o, a) + P(+s, +o, n) + P(+s, +o, d) = (+ 0.04) + (+ 0.21) + (+ 0.25) = (+ 0.50)

3-) Let’s join fc1(A) and fc6(S, A). Subsequently, let’s say we obtain a factor named fc16(S, A).

Example Computation for fc16(S = +s, A = +o):

fc1(A) = +0.60
fc1(A = +o) = + 0.60
fc6(+s, +o) = + 0.50
fc16(S = +s, A = +o) = fc1(A = +o) * fc6(+s, +o)
fc16(S = +s, A = +o) = (+ 0.60) * (+ 0.50) = (+ 0.30)

Example Computation for fc16(S = +s, A = -o):

fc1(A) = +0.60
fc1(A = +o) = + 0.60
fc1(A = -o) = 1 – fc1(A = +o) = 1 – 0.60 = 0.40
fc6(+s, -o) = + 0.70
fc16(S = +s, A = -o) = fc1(A = -o) * fc6(+s, -o)
fc16(S = +s, A = -o) = (+ 0.40) * (+ 0.70) = (+ 0.28)

- ⇨ **I have applied again the pairwise multiplication strategy for calculating each of the new factors after the joining process.**

S	A	fc16(S, A)
-s	-o	+ 0.12
+s	-o	+ 0.28
-s	+o	+ 0.30
+s	+o	+ 0.30

4-) Take possible factor summations for A (Shortly, eliminate A). Let's say that we obtain a factor named fc10(S) at the end of this step.

P(+s) = P(+s, -o) + P(+s, +o) , P(-s) = P(-s, -o) + P(-s, +o)

P(-s) = (+ 0.12) + (+ 0.30) = (+ 0.42)
P(+s) = (+ 0.28) + (+ 0.30) = (+ 0.58)

S	fc10(S)
-s	+ 0.42
+s	+ 0.58

5-) At this step, let's join the fc10(S) and fc4(t, S). Let's say that we obtain a factor named fc11(S) after the process of joining.

fc10(S = -s) = + 0.42 , fc10(S = +s) = + 0.58
fc4(+t, S = +s) = 0.95
fc4(+t, S = -s) = 0.19

fc11(S = +s) = fc4(+t, S = +s) * fc10(S = +s)
fc11(S = +s) = 0.95 * 0.58
fc11(S = +s) = 0.551

fc11(S = -s) = fc4(+t, S = -s) * fc10(S = -s)
fc11(S = +s) = 0.19 * 0.42
fc11(S = +s) = 0.0798

So, the table we obtained before normalization operation is as follows:

S	fc11(S)
-s	0.0798
+s	0.5510

When we normalize -s: ((0.0798) / (0.0798 + 0.5510)) = (0.0798 / 0.6308) = 798 / 6308 = 0.1265060241
When we normalizes +s: ((0.5510) / (0.0798 + 0.5510)) = (0.5510 / 0.6308) = 5510 / 6308 = **0.8734939759**

0.1265060241 is approximately 0.1265.
0.8734939759 is approximately 0.8735.

P(S = +s | T = +t) = 0.8735
P(S = -s | T = +t) = 0.1265

So, at the very end, we obtain the result of the probability P(S = +s| T = +t) approximately as 0.8735.

As the table which shows the normalized results, I have obtained the following table. Moreover; for each type of s value in S (+s and -s), I have obtained the value of probability formula P(S|T = +t).

S	**P(S|T = +t)**
-s	**0.1265**
+s	**0.8735**

Therefore, the value of the probability P(S = +s| T = +t) is approximately found as 0.8735.

P(S = +s| T = +t) is approximately 0.8735.

Q3-)

$$P(S) = \sum_{a,\,d,\,t} P(a) * P(d) * P(S|a, d) * P(t|S)$$

$$P(t|S) = P(-t|S) + P(+t|S) = \sum_{t \in T} P(t|S) = 1 \qquad (*)$$

NOTE: (*) formula/equality comes from the properties of the conditional probability.

Plug (*) formula to P(t|S). Moreover, since we go along all t variables (where each t variable is inside T), we can remove t from the outer summation.

$$P(S) = \sum_{a,\,d} P(a) * P(d) * P(S|a, d) * \sum_{t} P(t|S)$$

As we can wee in the (*) formula that the summation of all possible P(t|S) probabilities for all t variables inside T will give us 1 by the properties of the conditional probability. So, replace the inner summation with 1. Then, we will obtain the following summation formula.

$$P(S) = \sum_{a,\,d} P(a) * P(d) * P(S|a, d) * 1$$

$$P(S) = \sum_{a,\,d} P(a) * P(d) * P(S|\, a, d) \qquad (***)$$

In the 2nd question, initially, I have joined P(D) with P(S|A,D). Then, I have eliminated D variables by applying necessary summations within each (S,A) tuple for all elements in D (. At the third step, I have joined P(A) with P(S|A,D). Subsequently, I have eliminated A by applying necessary summations over all elements in O (namely +o and -o) for each different element in S (namely -s and +s). We can observe that this entire process which starts from the first step of the 2nd question and ends at the fourth step of the 2nd question is equivalent to the above summation formula (***). Therefore, for this question, we can consider the outputs of the fourth step in the 2nd question.

The outputs of the fourth step in the second question are as below:

S	fc10(S)
-s	+ 0.42
+s	+ 0.58

So, we can see from the above table that **the probability P(+s) is equal to (+ 0.58).**

P(S = +s) = P(+s) = P(S = +g) = P(+g) = (+ 0.58) = + 0.58 = 0.58

Q4-)

Expected Utility of the Buy Action (+b) Without Any Evidence:

- Expected Utility (EU) of an action

$$EU(action|evidence) = \sum_{y \in Y} P(y_i|evidence)U(y_i, action)$$

The general formula that I have followed for calculating the expected utility of the buy action (This screenshot is taken from the COMP341 Lecture Notes)

Since the question specifies that there is no evidence, this formula simplifies to the following formula (which is marked with (****)):

$$EU(action) = \sum_{y \in Y} P(yi) * U(yi, action) \qquad (****)$$

⇨ ExpectedUtility(+b) = P(S = -g) * U(-g, +b) + P(S = +g) * U(+g, +b)

From the third question, I have found that P(S = +g) is equal to the 0.58 and P(S = -g) is equal to the 0.42. So, plug 0.58 to P(S = +g) and 0.42 to P(S = -g).

⇨ ExpectedUtility(+b) = (0.42) * U(-g, +b) + (0.58) * U(+g, +b)

S	B	U
+g	+b	5000
+g	-b	0
-g	+b	-6000
-g	-b	0

Utilities for different cases (taken from the provided image in PDF)

From the above image which shows the utilities in different cases, we can observe that U(+g, +b) is equal to 5000 and U(-g, +b) is equal to -6000. Therefore, plug 5000 to U(+g, +b) and plug -6000 to U(-g, +b).

⇨ ExpectedUtility(+b) = (0.42) * (-6000) + (0.58) * (+5000)
⇨ (0.42) * (-6000) = -2520
⇨ (0.58) * (+5000) = +2900
⇨ ExpectedUtility(+b) = -2520 + 2900
⇨ ExpectedUtility(+b) = +380
⇨ ExpectedUtility(+b) = 380

Therefore, the expected utility of the buy action without any evidence is equal to 380.

Note = In this assignment, the "Buy" action is represented as +b. So, we can write ExpectedUtility(+b) instead of ExpectedUtility(buy action).

ExpectedUtility(buy action) = ExpectedUtility(+b) = EU(+b) = 380

Q5-)

$$\text{VPI}(E'|e) = \left(\sum_{e'} P(e'|e)\text{MEU}(e, e')\right) - \text{MEU}(e)$$

The value of information (VPI) formula that I have followed in this question (Screenshot taken from the COMP341 Lectures Slides; Figure 1 - Question5)

$$\text{MEU}(e) = \max_a \sum_s \boxed{P(s|e)} U(s,a)$$

$$\text{MEU}(e, e') = \max_a \sum_s \boxed{P(s|e, e')} U(s,a)$$

$$\text{MEU}(e, E') = \sum_{e'} \boxed{P(e'|e)} \text{MEU}(e, e')$$

$$\text{VPI}(E'|e) = \text{MEU}(e, E') - \text{MEU}(e)$$

Some additional formulas that I have followed in this question (Screenshot taken from the COMP341 Lectures Slides; Figure2 – Question5)

From the formula in Figure 1 – Question5, we can write the following summation and condition:

$$\text{VPI(T)} = \left(\sum_{t \in T} \text{MEU(Buying Action}|t) * P(t) \right) - \text{MEU(Buying Action)}$$

where $t \in T$

- ⇨ ExpectedUtility(Buying Action | T) = ExpectedUtility(+b | T)
- ⇨ ExpectedUtility(+b | T) = P(S = -g | T) * U(-g, +b) + U(+g, +b) * P(S = +g | T)
- ⇨ U(-g, +b) = -6000 (coming from the screenshot which shows the different utilities in different cases and which is at the beginning of the 2nd question).
- ⇨ U(+g, +b) = +5000 (coming from the screenshot which shows the different utilities in different cases and which is at the beginning of the 2nd question).
- ⇨ ExpectedUtility(+b | T) = P(S = -g | T) * (-6000) + (+5000) * P(S = +g | T)
- ⇨ ExpectedUtility(+b | T) = 5000 * P(S = +g | T) – 6000 * P(S = -g | T)

+g = +s, -g = -s (from the screenshot at the beginning of 2nd question)

<u>Calculation of MEU Without Any Evidence</u>

- ⇨ ExpectedUtility(+b) = 5000 * P(S = +s) – 6000 * P(S = -s)
- ⇨ ExpectedUtility(+b) = 5000 * 0.58 – 6000 * 0.42 **(This comes from Question 4)**
- ⇨ ExpectedUtility(+b) = EU(+b) = 2900 – 2520 = 380 **(This comes from Question 4)**
- ⇨ EU(+b) = 380 **(This comes from Question 4)**
- ⇨ ExpectedUtility(-b) = EU(-b) = P(S = -g) * U(-g, -b) + U(+g, -b) * P(S = +g)
- ⇨ U(-g, -b) = 0 **(Utility Equality 1)**
- ⇨ U(+g, -b) = 0 **(Utility Equality 2)**
- ⇨ **(Utility Equality 1 and Utility Equality 2 comes from the screenshot at the beginning of the question2 and provided in the PDF Description of this assignment).**
- ⇨ When we plug 0 to U(-g, -b) and 0 to U(+g, -b), we obtain the following:
- ⇨ EU(-b) = P(S = -g) * 0 + P(S = +g) * 0 = 0 + 0 = 0
- ⇨ EU(-b) = 0 (I found this above this line) and EU(+b) = 380 (from the result of Q4).
- ⇨ Since 380>0 (380 is greater than 0), the maximum expected utility (MEU) of the buy action without any evidence is 380.

MEU (Buying Action) = 380
Plug this into the VPI(T) equation, then we obtain the following:

$$\text{VPI(T)} = \left(\sum_{t \in T} \text{MEU}(B|t) * P(t) \right) - 380$$

P(T = +t | S = +s) = 0.95 **(comes from the screenshot at the beginning of the question 2)**
P(T = +t | S = -s) = 0.19 **(comes from the screenshot at the beginning of the question 2)**
P(T = -t | S = +s) = 1 – P(T = +t | S = +s) = 1 – 0.95 = 0.05 **(comes from the conditional probability properties)**

<u>Calculations of the table entries which are P(-s, -t), P(+s, +t), P(+s, -t), and P(-s, +t)</u>
P(S, -t) = P(-t|S) * P(S) **(From the Bayes' Rule)**
P(S = +s, T = -t) = P(T = -t | S = +s) * P(S = +s) **(From the Bayes' Rule)**
P(T = -t | S = +s) + P(T = +t | S = +s) = 1
P(T = -t | S = +s) = 1 – P(T = +t | S = +s)
P(T = -t | S = +s) = 1 – 0.95 = 0.05
P(T = -t | S = +s) = 0.05
P(S = +s) = 0.58
P(S = +s, T = -t) = 0.58 * 0.05
P(S = +s, T = -t) = (58/100) * (5/100)
P(S = +s, T = -t) = ((58*5)/(100*100))
P(S = +s, T = -t) = ((290)/(10000)) = 290/10000 = 29 / 1000
P(S = +s, T = -t) = 29 / 1000 = 0.029
P(S = +s, T = -t) = 0.029

P(S = -s, T = -t) = P(T = -t | S = -s) * P(S = -s) **(From the Bayes' Rule)**
P(T = +t | S = -s) = 0.19 **(From the screenshot at the beginning of the question-2)**
P(T = -t | S = -s) = 1 – P(T = + t | S = -s)
P(T = -t | S = -s) = 1 – 0.19 = 0.81
P(S = -s, T = -t) = 0.81 * 0.42
P(S = -s, T = -t) = (81/100) * (42/100) = ((81*42)/(100*100))
P(S = -s, T = -t) = (3402 / 10000)
P(S = -s, T = -t) = 0.3402

P(S = +s, T = +t) = P(T = +t | S = +s) * P(S = +s) **(From the Bayes' Rule)**
P(S = +s, T = +t) = 0.95 * 0.58
P(S = +s, T = +t) = (95/100) * (58/100) = ((95*58) / (100*100))
P(S = +s, T = +t) = (5510/10000) = (551/1000) = 0.551
P(S = +s, T = +t) = 0.551

P(S = -s, T = +t) = P(T = +t | S = -s) * P(S = -s) **(From the Bayes' Rule)**
P(S = -s, T = +t) = 0.19 * 0.42
P(S = -s, T = +t) = (19/100) * (42/100) = ((19*42)/(100*100))
P(S = -s, T = +t) = (798/10000) = 0.0798
P(S = -s, T = +t) = 0.0798

S	T	P(S, T)
-s	-t	+ 0.3402
+s	+t	+ 0.551
+s	-t	+ 0.029
-s	+t	+ 0.0798

<u>Equations for finding P(T)</u>
P(T) = P(S, +t) + P(S, -t) , P(+t) = P(-s, +t) + P(+s, +t) , P(-t) = P(-s, -t) + P(+s, -t)
When we apply the above equations, we will obtain the following table for P(T):

<u>Calculations for P(+t) and P(-t)</u>
P(+t) = (+ 0.0798) + (+ 0.5510) = (+ 0.6308) = + 0.6308 = 0.6308
P(-t) = (+ 0.3402) + (+ 0.0290) = (+ 0.3692) = + 0.3692 = 0.3692

T	P(T)
-t	0.3692
+t	0.6308

⇨ ExpectedUtility(+b | T) = 5000 * P(S = +s | T) – 6000 * P(S = -s | T)

If T = +t, then ExpectedUtility(+b | T) = 5000 * P(S = +s | T = +t) – 6000 * P(S = -s | T = +t)

P(S = +s | T = +t) = 0.8735
P(S = -s | T = +t) = 0.1265
When we plug the above probabilities to the equation of ExpectedUtility(+b | T) for which T = +t, we will obtain the following:

5000 * 0.8735 – 6000 * 0.1265 = 4367.5 – 759 = 3608.5
ExpectedUtility(+b | T = +t) = 3608.5

⇨ ExpectedUtility(+b | T) = 5000 * P(S = +s | T) – 6000 * P(S = -s | T)

If T = -t, then ExpectedUtility(+b | T) = 5000 * P(S = +s | T = -t) – 6000 * P(S = -s | T = -t)

P(S = +s, T = -t) = 0.029 (I found this in this question. For observing calculations, see above.)
P(S = -s, T = -t) = 0.3402 (I found this in this question. For observing calculations, see above.)

P(T = -t) = P(S = +s, T = -t) + P(S = -s, T = -t)

Therefore, P(T = -t) = **0.029 + 0.3402 = 0.3692**
P(T = -t) = 0.3692

P(S = +s | T = -t) = (P(S = +s, T = -t)) / (P(T = -t))
P(S = +s | T = -t) = (0.029) / (0.3692)
P(S = +s | T = -t) = 0.07854821235

P(S = -s | T = -t) = (P(S = -s, T = -t)) / (P(T = -t))
P(S = -s | T = -t) = (0.3402) / (0.3692)
P(S = -s | T = -t) = 0.9214517876

If T = -t => ExpectedUtility(+b | T = -t) = 5000 * 0.07854821235 – 6000 * 0.9214517876
ExpectedUtility(+b | T = -t) = 392.74106175 – 5528.7107256
ExpectedUtility(+b | T = -t) = -5135.96966385

ExpectedUtility(+b | T = -t) = -5135.96966385

<u>For the action of “Not Buying”</u>

<u>If T = +t:</u>
U(+g, -b) = 0 , U(-g, -b) = 0 **(comes from the screenshot at the beginning of question2 and the provided image at the PDF Description of this assignment)**

ExpectedUtility(-b | T = +t) = P(S = +g | T = +t) * U(+g, -b) + U(-g, -b) * P(S = -g | T = +t)
ExpectedUtility(-b | T = +t) = (P(S = +s | T = +t) * 0) + (0 * P(S = -g | T = +t))
ExpectedUtility(-b | T = +t) = 0 + 0
ExpectedUtility(-b | T = +t) = 0

<u>If T = -t:</u>
U(+g, -b) = 0 , U(-g, -b) = 0 **(comes from the screenshot at the beginning of question2 and the provided image at the PDF Description of this assignment)**

ExpectedUtility(-b | T = -t) = P(S = +g | T = -t) * U(+g, -b) + U(-g, -b) * P(S = -g | T = -t)
ExpectedUtility(-b | T = -t) = (P(S = +s | T = -t) * 0) + (0 * P(S = -g | T = -t))
ExpectedUtility(-b | T = -t) = 0 + 0
ExpectedUtility(-b | T = -t) = 0

So, consequently, I have found the following expected utilities:

ExpectedUtility(-b | T = +t) = 0
ExpectedUtility(-b | T = -t) = 0
ExpectedUtility(+b | T = +t) = 3608.5
ExpectedUtility(+b | T = -t) = -5135.96966385

Note: For the expected utilities above, '-b' represents the action of "Not Buying". Furthermore, "+b" represents the action of "Buying".

When the T is equal to -t (T = -t):

ExpectedUtility(-b | T = -t) = 0
ExpectedUtility(+b | T = -t) = -5135.96966385

Since 0 is bigger than -5135.96966385 (0 > -5135.96966385), the maximum expected utility for T being equal to -t is equal to 0. "ExpectedUtility(-b | T = -t)" is the maximum expected utility for T being equal to -t. Therefore, since the action is represented with the "-b" symbol in this expected utility, we can conclude that the optimal move/action that can be taken (for the case where T is equal to -t) is the "Not Buying" action.

MaximumExpectedUtility(B | T = -t) = MaximumExpectedUtility(-b | T = -t) = 0
MEU(B|T = -t) = MEU(-b | T = -t) = 0

When the T is equal to +t (T = +t):

ExpectedUtility(+b | T = +t) = 3608.5
ExpectedUtility(-b | T = +t) = 0

Since 3608.5 is bigger than 0 (3608.5>0), the maximum expected utility for T being equal to +t is equal to 3608.5. "ExpectedUtility(+b | T = +t)" is the maximum expected utility for T being equal to +t. Therefore, since the action is represented with the "+b" symbol in this expected utility, we can conclude that the optimal move/action that can be taken (for the case where T is equal to +t) is the "Buying" action.

MaximumExpectedUtility(B | T = +t) = MaximumExpectedUtility(+b | T = +t) = 3608.5
MEU(B | T = +t) = MEU(+b | T = +t) = 3608.5

Equations for finding P(T)
P(T) = P(S, +t) + P(S, -t) , P(+t) = P(-s, +t) + P(+s, +t) , P(-t) = P(-s, -t) + P(+s, -t)
When we apply the above equations, we will obtain the following table for P(T):

Calculations for P(+t) and P(-t)
P(+t) = (+ 0.0798) + (+ 0.5510) = (+ 0.6308) = + 0.6308 = 0.6308
P(-t) = (+ 0.3402) + (+ 0.0290) = (+ 0.3692) = + 0.3692 = 0.3692

T	P(T)
-t	0.3692
+t	0.6308

Note: For observing where the values of the probabilities P(-s, +t), P(+s, +t), P(-s, -t), and P(+s, -t) come from, please see the part called "**Calculations of the table entries which are P(-s, -t), P(+s, +t), P(+s, -t), and P(-s, +t)". (Please see above for finding that part. This part is within my answer to this question (Question-5)).**

By combining all the related values I find; we can obtain the value of the maximum amount of money a person wants to pay, which is equal to the value of the VPI(T).

VPI(T) = The maximum amount of money that a person wants to pay = ($\sum_{t \in T}$ MEU(B|t) * P(t)) - 380

⇨ **MEU(-b | T = -t) * P(-t) = 0 * (0.3692) = 0**

⇨ **MEU(-b | T = -t) * P(-t) = 0 (1st multiplication)**

⇨ MEU(+b | T = +t) * P(+t) = (3608.5) * (0.6308) = (+ 2276.2418) = 2276.2418

⇨ MEU(+b | T = +t) * P(+t) = 2276.2418 **(2nd multiplication)**

⇨ **VPI(T) = (1st multiplication) + (2nd multiplication) - 380**
⇨ VPI(T) = (0) + (2276.2418) – 380
⇨ VPI(T) = 0 + 2276.2418 – 380
⇨ VPI(T) = 2276.2418 – 380
⇨ VPI(T) = 1896.2418. (The maximum amount of money that a person wants to pay)

Therefore, at the end of the question5, I have found that the maximum amount of money that a person wants to pay is **1896.2418**

Q6-)

$$w(\mathbf{z}, \mathbf{e}) = \prod_{i=1}^{m} P(e_i | \text{Parents}(E_i))$$

The formula I have followed while calculating the weights of the samples provided in the description of Q6 in this assignment
(Screenshot taken from the COMP341 Lecture Notes)

- Given a BN and the evidence as before

```
w = 1.0
for i = 1 to n do
  if X_i is an evidence variable then
    X_i = x_i, where x_i ∈ E is the observation for X_i
    w = w × P(X_i|Parents(X_i))
  else
    x_i = sample(P(X_i|Parents(X_i))
return (x_1, ..., x_n), w
```

The likelihood weighting strategy that I have followed in this question
(Screenshot taken from the COMP341 Lecture Notes)

A	D	S	T	Weight
+o	d	+s	+t	0.57 (see below for the calculation of this)
+o	d	-s	+t	0.114 (see below for the calculation of this)

For the first sample; the evidence variables are +o, +s, and +t in the provided order.
For the second sample; the evidence variables are +o, -s, and +t in the provided order.

We want to find the probability P(d|+o, +t) after performing the process of likelihood weighting.

w(+o, d, +s, +t) = w1 = P(A = +o) * P(T = +t | S = +s)

P(A = +o) = (+ 0.60) = + 0.60 = 0.60 **(coming from the screenshot at the beginning of Q2 and provided in PDF Description of this assignment)**

if (T = +t) and (S = +s), then P(T | S) = P(T = +t | S = +s) = 0.95
elif (T = +t) and (S = -s), then P(T | S) = P(T = +t | S = -s) = 0.19

(0.95 and 0.19 comes from the screenshot at the beginning of Q2 and provided in PDF Description of this assignment)

w(+o, d, -s, +t) = w2 = P(A = +o) * P(T = +t | S = -s)
P(A = +o) = (+ 0.60) = (0.60) = 0.60

If we plug these values,

We can obtain the following for w(+o, d, +s, +t):

w(+o, d, +s, +t) = w1 = 0.60 * 0.95 = (60 / 100) * (95 / 100) = (95 * 60 / (100 * 100))
w(+o, d, +s, +t) = w1 = (95 * 60 / (100 * 100)) = (95 * 60 / 10000) = (5700 / 10000)
w(+o, d, +s, +t) = w1 = (5700 / 10000) = (57 / 100) = 0.57

We can obtain the following for w(+o, d, -s, +t):

w(+o, d, -s, +t) = w2 = 0.60 * 0.19 = (60 / 100) * (19 / 100) = (19 * 60 / (100 * 100))
w(+o, d, -s, +t) = w2 = (19 * 60 / (100 * 100)) = (19 * 60 / 10000) = (1140 / 10000)
w(+o, d, -s, +t) = w2 = (1140 / 10000) = (114 / 1000) = 0.114

So, I have found the following weights at the end of this question:

w(+o, d, +s, +t) = w1 = **0.57**
w(+o, d, -s, +t) = w2 = **0.114**

Q7-)

- Given a BN, the evidence as before and a full instantiation $(x_1, \dots, x_n)$

Pick a random non-evidence variable, X_i
$x_i = sample(P(X_i | MarkovBlanket(X_i)))$
return $(x_1, \dots, x_n)$

The Gibbs Sampling Pseudocode that I have followed while doing this question (Screenshot taken from the COMP341 Lecture Notes)

⇨ {A = −o, D = n, S = +s, T = −t} is given.

P(S | A, D, T) = P(S ∩ (A, D, T)) / (P(A, D, T)) = P(S, A, D, T) / P(A, D, T)

- ⇨ **We can observe that the probability in the denominator is the version of P(S, A, D, T) where all s values in S are summed up.**
- ⇨ **By considering this and by using the joint distribution of this Bayesian Network (which I found in Q1), we can write the following:**

P(S | A, D, T) = P(A, D, S, T) / P(A, D, T)
P(A, D, S, T) = P(A) * P(D) * P(S|A, D) * P(T|S) **(from the first question)**

P(S | A, D, T) = (P(A) * P(D) * P(S|A, D) * P(T|S)) / P(A, D, T)

P(S | A, D, T) = (P(A) * P(D) * P(S|A, D) * P(T|S)) / $\sum_{s*}$ **P(A) * P(D) * P(s*|A, D) * P(T|s*)**

At this step, since s* ∈ S already depends on A and D, we can remove the P(A) and P(D) from both numerator and denumerator. After that, we will obtain the following:

P(S | A, D, T) = (P(S|A, D) * P(T|S)) / $\sum_{s*}$ **P(s*| A, D) * P(T|s*)**

Now, plug -o to A, n to D, and -t to T. After that, we will obtain the following:

P(S | -o, n, -t) = (P(S | -o, n) * P(-t | S)) / $\sum_{s*}$ **P(s* | -o, n) * P(-t | s*) (*******)**

The Calculation Process of the denumerator of (*******):

- ⇨ **P(-s | -o, n) * P(-t | -s) + P(+s | -o, n) * P(-t | +s)**
- ⇨ **P(-s | -o, n) + P(+s | -o, n) = 1 (From the properties of the conditional probability)**
- ⇨ **P(-s | -o, n) = 1 - P(+s | -o, n)**
- ⇨ **P(+s | -o, n) = (+ 0.7) = + 0.7 = 0.7 (From the screenshot at the beginning of Q2 and provided in the PDF Description of this assignment)**
- ⇨ **P(-s | -o, n) = 1 – 0.7 = 0.3**
- ⇨ **P(-s | -o, n) * P(-t | -s) + P(+s | -o, n) * P(-t | +s) = 0.3 * P(-t | -s) + 0.7 * P(-t | +s)**
- ⇨ **P(T = +t | S = +s) = P(+t | +s) = 0.95 (From the screenshot at the beginning of Q2)**
- ⇨ **P(T = +t | S = +s) + P(T = -t | S = +s) = 1 (From the properties of the conditional probability)**
- ⇨ **P(+t | +s) + P(-t | +s) = 1**
- ⇨ **P(-t | +s) = 1 – P(+t | +s) = 1 – 0.95 = 0.05**
- ⇨
- ⇨ **P(T = +t | S = -s) = P(+t | -s) = 0.19 (From the screenshot at the beginning of Q2)**
- ⇨ **P(T = +t | S = -s) + P(T = -t | S = -s) = 1 (From the properties of the conditional probability)**
- ⇨ **P(+t | -s) + P(-t | -s) = 1**
- ⇨ **P(-t | -s) = 1 – P(+t | -s) = 1 – 0.19 = 0.81**
- ⇨ **P(-s | -o, n) * P(-t | -s) + P(+s | -o, n) * P(-t | +s) = 0.3 * P(-t | -s) + 0.7 * P(-t | +s)**
- ⇨ **0.30 * 0.81 + 0.70 * 0.05 = (30/100) * (81/100) + (70/100) * (5/100)**
- ⇨ **((81*30)/(100*100)) + ((70*5)/(100*100))**
- ⇨ **(2430/10000) + (350/10000)**
- ⇨ **(243/1000) + (35/1000) = (278/1000)**
- ⇨ **(278/1000) = 278/1000 = 0.278**

So, **the denominator of the probability** marked with (*******) is equal to **0.278**.

Numerator of the probability marked with (*******) = **(P(S | -o, n) * P(-t | S))**

Numerator for S = +s => P(S = +s | -o, n) * P(-t | S = +s)
P(S = +s | -o, n) = 0.70 (From the screenshot at the beginning of Q2)
P(-t | S = +s) = 0.05 (As I found above)
Numerator for S = +s => 0.70 * 0.05 = 0.035

Numerator for S = -s => P(S = -s | -o, n) * P(-t | S = -s)
P(S = -s | -o, n) = 1 - P(S = +s | -o, n) = 1 – 0.70 = 0.30
P(-t | S = -s) = 0.81 (As I found above)
Numerator for S = -s => 0.30 * 0.81 = 0.243

Calculate P(S | -o, n, -t) for S = +s:
P(S | -o, n, -t) = (0.035) / (0.278) = 35 / 278 = 0.1258992806

Calculate P(S | -o, n, -t) for S = -s:
P(S | -o, n, -t) = (0.243) / (0.278) = 243 / 278 = 0.87410071942446

0.1258992806 is approximately equal to 0.1259
0.87410071942446 is approximately equal to 0.8741

So, I have found the following:

P(S = +s | A = -o, D = n, T = -t) = 0.1259
P(S = -s | A = -o, D = n, T = -t) = 0.8741

So, in a table, we can show this gibbs sample distribution as follows:

S	The Value Of Gibbs Sample Distribution
-s	0.8741
+s	0.1259

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