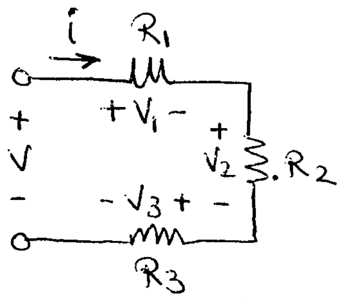
CHAPTER - 2 -

RESISTIVE CIRCUITS

Series Resistances

Consider the following circuit



$V_1 = R_1 i$

$V_2 = R_2 i$

$V_3 = R_3 i$

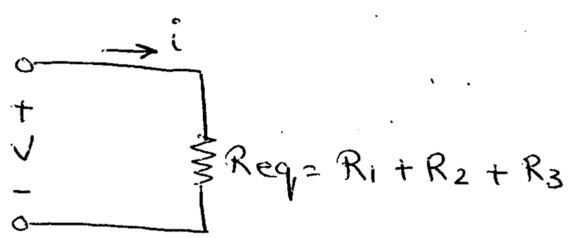
Using KVL $\Rightarrow V = V_1 + V_2 + V_3$

$V = R_1 i + R_2 i + R_3 i$

$= (R_1 + R_2 + R_3) i$

$R_{eq} = R_1 + R_2 + R_3$

$V = R_{eq} i$

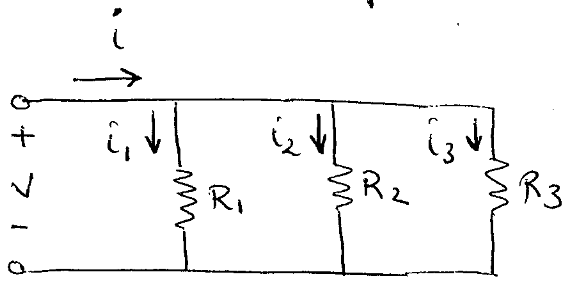


∴ A series combination of resistances has an equivalent resistance equal to the sum of the original resistances.

$$R_{eq} = \sum_{i=1}^{n} R_i$$

Parallel Resistances

Consider the following circuit.



$i_1 = V/R_1$

$i_2 = V/R_2$

$i_3 = V/R_3$

Using KCL at the top node $\Rightarrow$ $i = i_1 + i_2 + i_3$

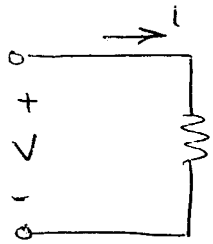
$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

$$i = \frac{1}{R_{eq}} V$$

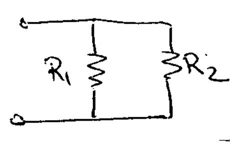


$R_{eq} = \dfrac{1}{1/R_1 + 1/R_2 + 1/R_3}$

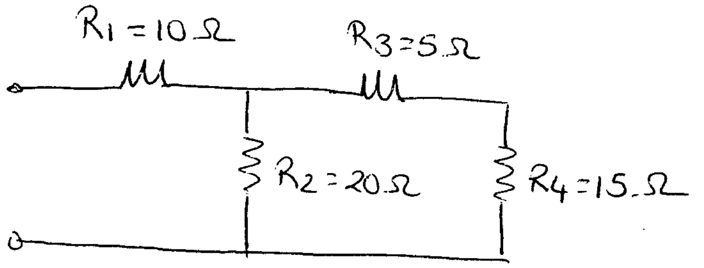
In general

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

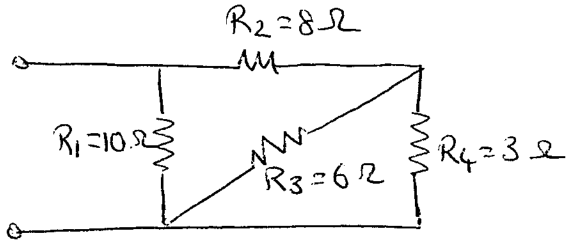
for two resistances: $R_{eq} = \dfrac{1}{1/R_1 + 1/R_2} = \dfrac{R_1 R_2}{R_1 + R_2}$



Example: Find a single equivalent resistance for the networks shown below.



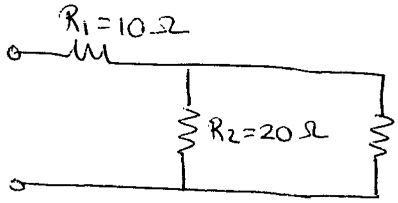
(a)



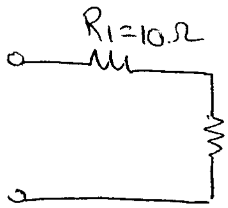
(b)

Solution:

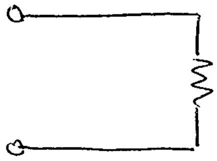
(a)



$R_{eq_1} = R_3 + R_4 = 20\,\Omega$

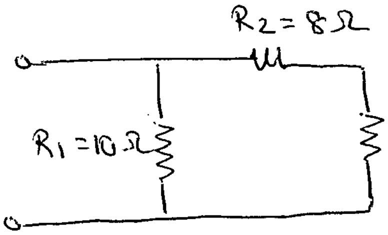


$$R_{eq_2} = \frac{1}{1/R_{eq_1} + 1/R_2} = \frac{1}{1/20 + 1/20} = 10\,\Omega$$

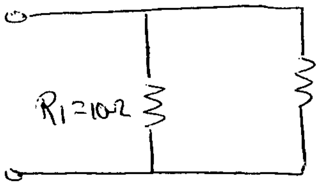


$R_{eq} = R_{eq_2} + R_1 = 10 + 10 = 20\,\Omega$

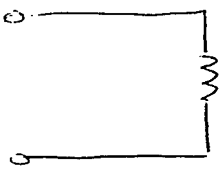
(b)



$$R_{eq_1} = \frac{1}{1/R_3 + 1/R_4} = \frac{1}{1/6 + 1/3} = \frac{1}{\frac{1+2}{6}} = \frac{6}{3} = 2\,\Omega$$

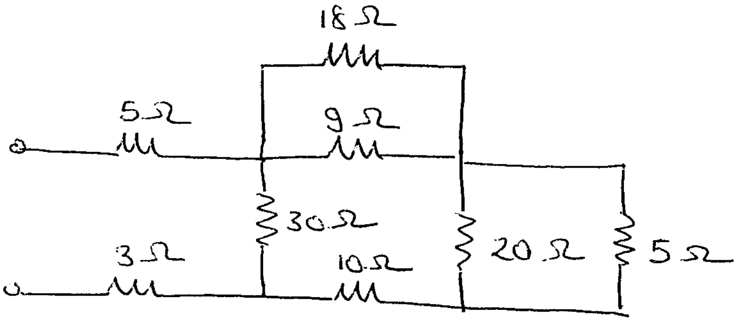


$R_{eq_2} = R_2 + R_{eq_1} = 8 + 2 = 10\,\Omega$

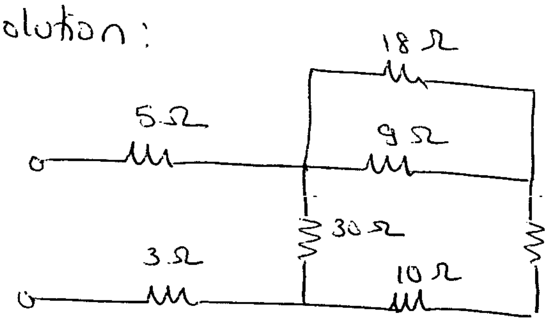


$$R_{eq} = R_1 // R_{eq_2} = \frac{R_1 R_{eq_2}}{R_1 + R_{eq_2}} = \frac{(10)(10)}{10+10} = 5\,\Omega$$

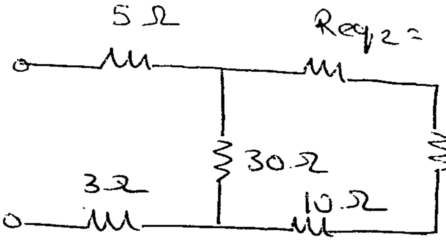
Exercise : Find the equivalent resistance.



Solution:

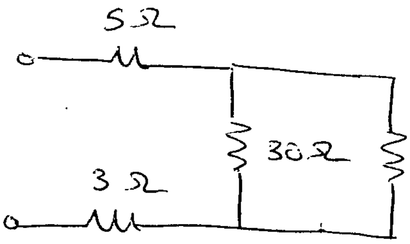


$$R_{eq_1} = \frac{(20)(5)}{20+5} = 4\,\Omega$$

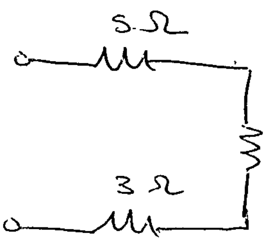


$$R_{eq_2} = \frac{18(9)}{18+9} = 6\,\Omega$$

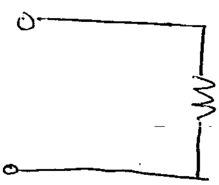
$$R_{eq_1} = 4\,\Omega$$



$$R_{eq_3} = R_{eq_1} + R_{eq_2} + 10 = 20\,\Omega$$

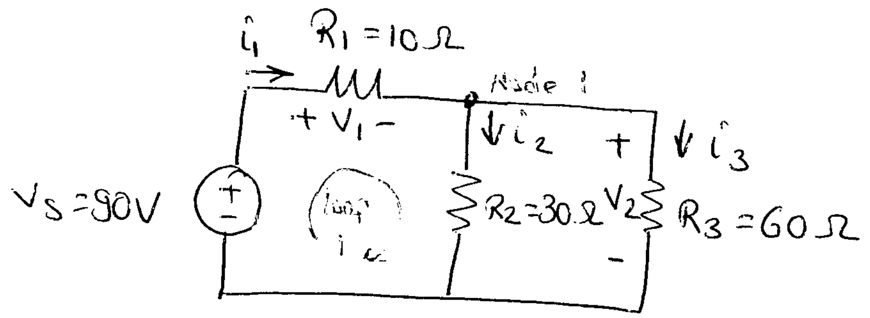


$$R_{eq_4} = \frac{(20)(30)}{20+30} = 12\,\Omega$$

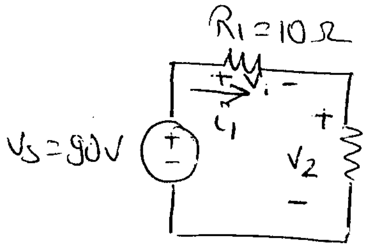


$$R_{eq} = 5 + 3 + R_{eq_4} = 20\,\Omega$$

Example: Find the current, voltage and power for each element of the circuit shown below.

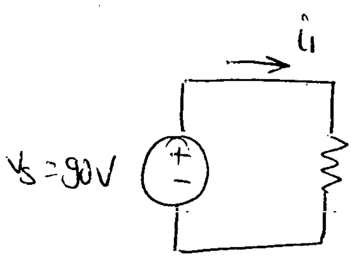


Solution:



$Req_1 = R_2 // R_3 = \dfrac{30\,(60)}{30+60} = 20\,\Omega$

(a)



$Req = Req_1 + R_1 = 20 + 10 = 30\,\Omega$

(b)

From (b) $\quad -V_s + Req\, i_1 = 0 \quad \Rightarrow i_1 = V_s / Req = 80/30 = 3A$

From (a) $\quad V_2 = i_1 Req_1 = (3)(20) = 60\,V$

$V_1 = i_1 R_1 = (3)(10) = 30V$

From original circuit:

$i_2 = V_2 / R_2 = 60/30 = 2A$

$i_3 = V_2 / R_3 = 60/60 = 1A$

$V_1 = R_1 i_1 = 10\,(3) = 30\,V$

check for KCL at node 1: $\quad i_1 \overset{?}{=} i_2 + i_3$

$3 = 2 + 1$

$3 = 3$ ✓

check for KVL for loop 1:

$$-V_S + V_1 + V_2 \stackrel{?}{=} 0$$

$$-90 + 30 + 60 = 0$$

$$0 = 0 \checkmark$$

$$P_{V_S} = -V_S i_1 = -(90)(3) = -270\ W$$

$$P_{R_1} = R_1 i_1^2 = (10)(3)^2 = 90\ W$$

$$P_{R_2} = R_2 i_2^2 = (30)(2)^2 = 120\ W$$

$$P_{R_3} = R_3 i_3^2 = (60)(1)^2 = 60\ W$$

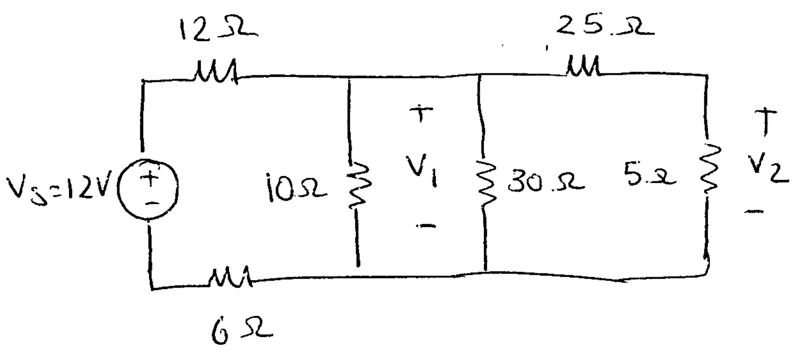
Power consumption check:

$$P_{S} + P_{R_1} + P_{R_2} + P_{R_3} \stackrel{?}{=} 0$$

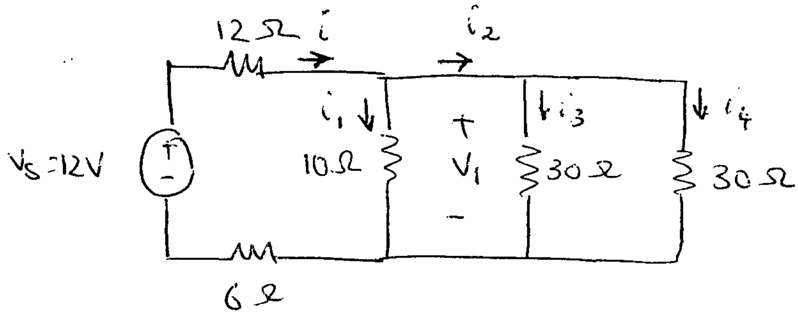
$$-270 + 90 + 120 + 60 \stackrel{?}{=} 0$$

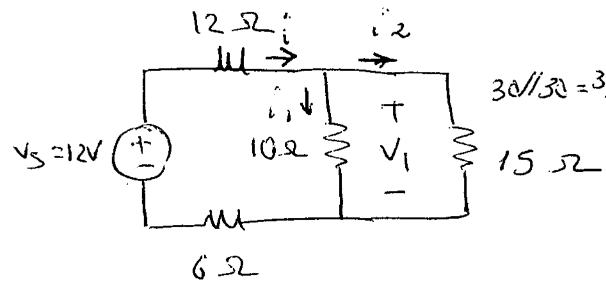
$$0 = 0 \checkmark$$

P2-9 Find V_1 and V_2 by combining resistances in series and parallel.

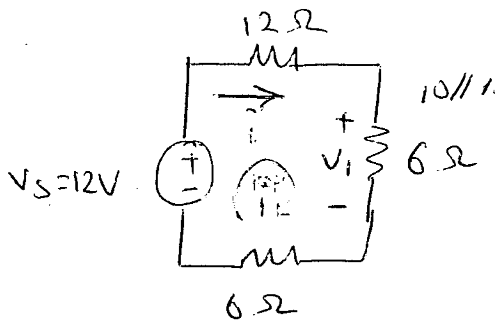


Solution:





$30 // 30 = \frac{30(30)}{30+30} = 15$

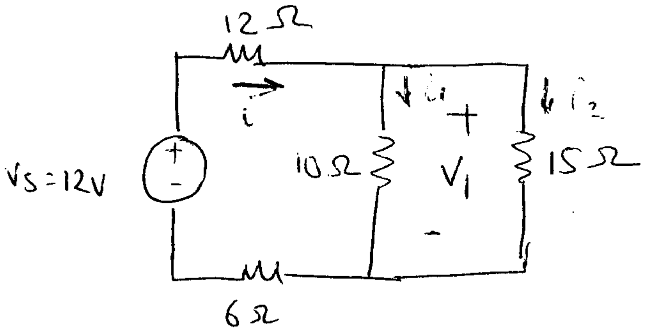


$10 // 15 = \frac{10(15)}{10+15} = 6\ \Omega$

KVL for loop 1: $-V_S + 12i + 6i + 6i = 0$

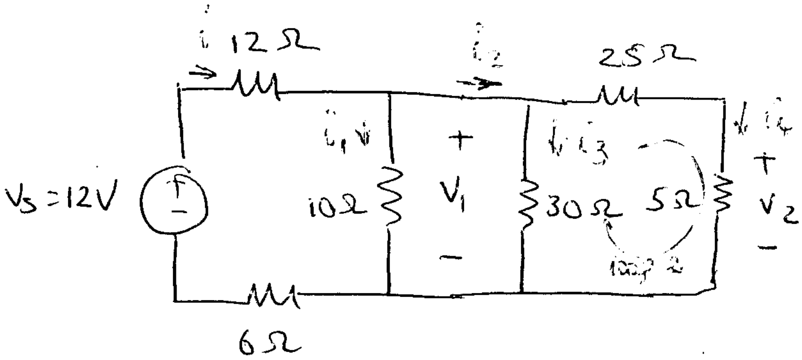
$i = \frac{V_S}{24} = \frac{12}{24} = 0.5A$

$V_1 = 6i = 6(0.5) = 3V$



$i_1 = \frac{V_1}{10} = \frac{3}{10} = 0.3\ A$

$i_2 = \frac{V_1}{15} = \frac{3}{15} = \frac{1}{5} = 0.2\ A$

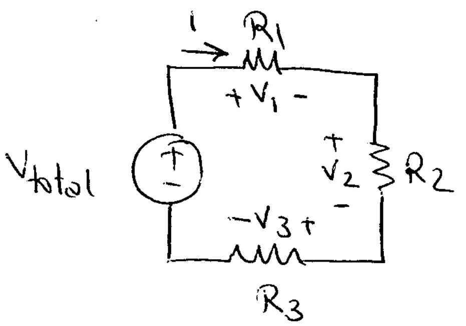


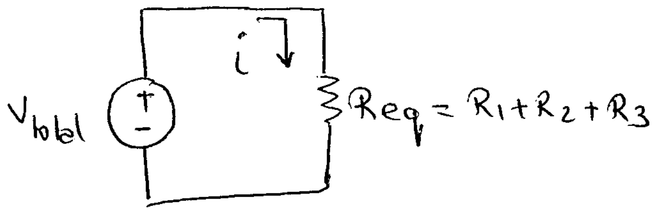
KVL for loop 2:

$-V_1 + 25i_4 + 5i_4 = 0 \Rightarrow i_4 = \frac{V_1}{30} = \frac{3}{30} = 0.1A$

$V_2 = 5i_4 = 5(0.1) = 0.5V$

Voltage Division





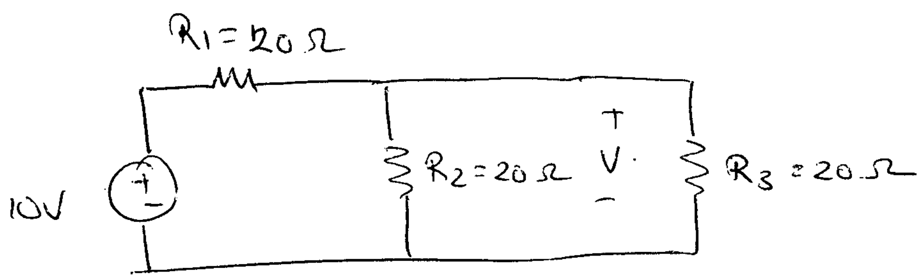
$$i = \frac{V_{total}}{R_{eq}} = \frac{V_{total}}{R_1 + R_2 + R_3}$$

$$V_1 = R_1 i = R_1 \frac{V_{total}}{R_1 + R_2 + R_3}$$

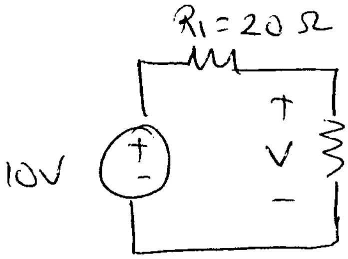
$$V_2 = R_2 i = R_2 \frac{V_{total}}{R_1 + R_2 + R_3}$$

$$V_3 = R_3 i = R_3 \frac{V_{total}}{R_1 + R_2 + R_3}$$

P2.22 Use the voltage-division principle to calculate v in the given circuit.



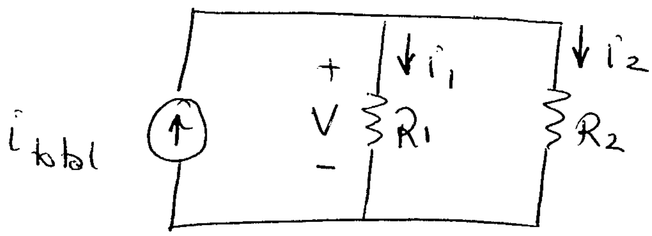
Solution:

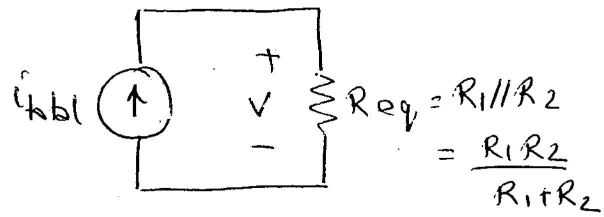


$$R_{eq} = 20 \parallel 20 = \frac{20(20)}{20+20} = 10\,\Omega$$

$$V = 10 \frac{R_{eq}}{R_1 + R_{eq}} = 10 \frac{10}{20+10} = \frac{100}{30} = \frac{10}{3}\text{ V}$$

Current Division





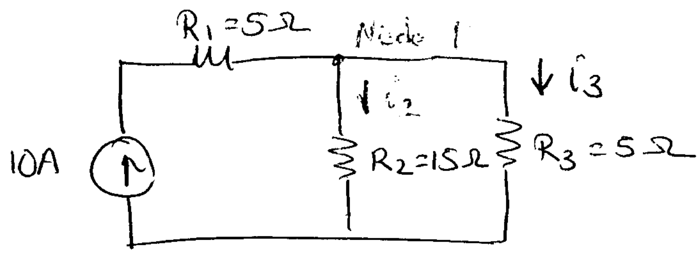
$$V = i_{total} R_{eq} = \frac{R_1 R_2}{R_1 + R_2} i_{total}$$

From the original circuit :

$$i_1 = \frac{V}{R_1} = \frac{R_2}{R_1 + R_2} i_{total}$$

$$i_2 = \frac{V}{R_2} = \frac{R_1}{R_1 + R_2} i_{total}$$

P2-23 Use current-division principle to calculate i_3.



Solution :

$$i_3 = 10 \frac{R_2}{R_2 + R_3} = 10 \frac{15}{15+5} = \frac{10\ (15)}{20} = 7.5\ A$$

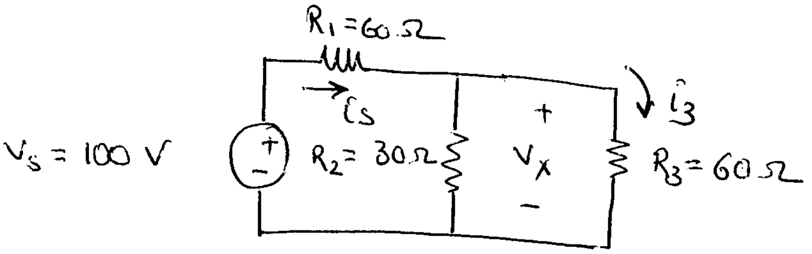
$$i_2 = 10 \frac{R_3}{R_2 + R_3} = 10 \frac{5}{15+5} = \frac{10\ (5)}{20} = 2.5 A$$

check for KCL at node 1 : $10 \overset{?}{=} i_2 + i_3$

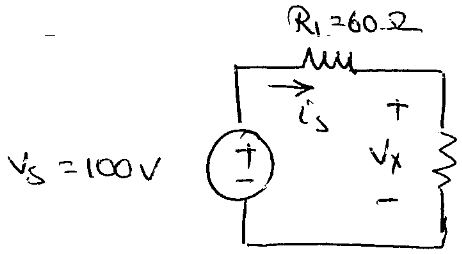
$10 = 2.5 + 7.5$

$10 = 10$ ✓

Example: Use the voltage-divison principle to find the voltage V_x in the given circuit. Then find the source current i_s and use the current-division principle to compute i_3.



Solution:



$$R_{eq_1} = R_2 // R_3 = \frac{30(60)}{30+60} = \frac{1800}{90} = 20\,\Omega$$

$$V_x = V_s \cdot \frac{R_{eq_1}}{R_1 + R_{eq_1}} = 100 \frac{20}{20+60} = 25\text{ V}$$

$$i_s = \frac{V_s}{R_1 + R_{eq_1}} = \frac{100}{60+20} = 1.25\text{ A}$$

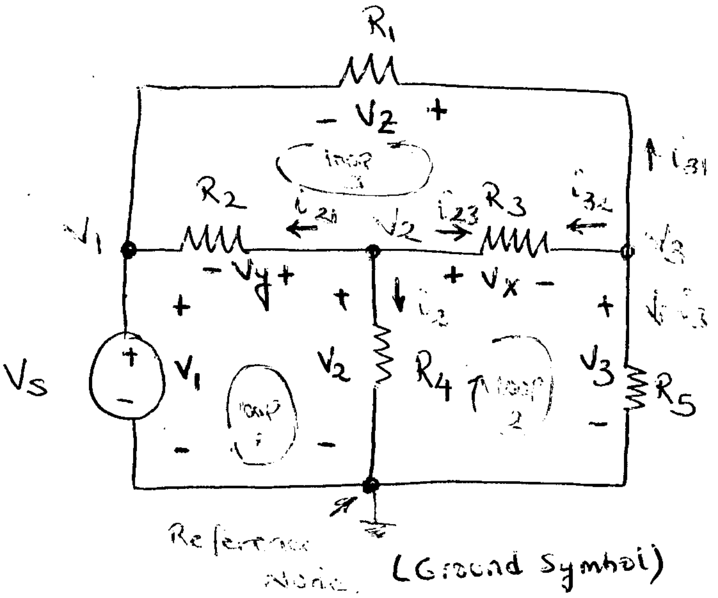
From the original circuit:

$$i_3 = i_s \frac{R_2}{R_2 + R_3} = 1.25 \frac{30}{30+90} = 0.417\text{ A}$$

Another way to find i_3:

$$i_3 = \frac{V_x}{R_3} = \frac{25}{60} = 0.417\text{ A}$$

Node Voltage Analysis



Step 1 : Select the reference node.

Step 2 : Define the node voltages on the circuit diagram. A node voltage is defined as the voltage rise from the reference node to a nonreference node.

KVL for loop 1:

$$-V_1 - V_y + V_2 = 0 \quad \Rightarrow \quad V_y = V_2 - V_1$$

KVL for loop 2:

$$-V_2 + V_x + V_3 = 0 \quad \Rightarrow \quad V_x = V_2 - V_3$$

KVL for loop 3:

$$V_z - V_y + V_x = 0 \quad \Rightarrow \quad V_z = V_y - V_x = V_2 - V_1 - (V_2 - V_3) = V_3 - V_1$$

Step 3: write KCL equations in terms of node voltages.

KCL at Node 2 (V_2) : $i_{21} + i_{23} + i_2 = 0$

$$\frac{V_y}{R_2} + \frac{V_x}{R_3} + \frac{V_2}{R_4} = 0$$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - V_3}{R_3} + \frac{V_2}{R_4} = 0$$

KCL at Node 3 (V_3) : $i_{32} + i_{31} + i_3 = 0$

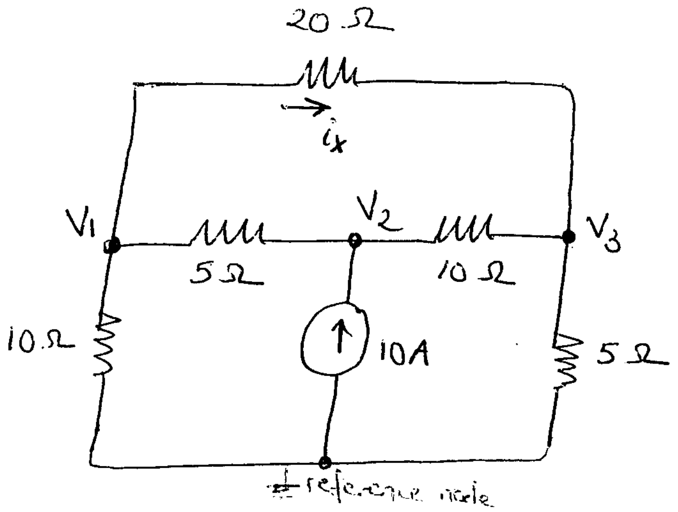
$$\frac{-V_x}{R_3} + \frac{V_z}{R_1} + \frac{V_3}{R_5} = 0$$

$$\frac{V_3 - V_2}{R_3} + \frac{V_3 - V_1}{R_1} + \frac{V_3}{R_5} = 0$$

$V_1 = V_S$ (Node 1)

So we have 3 equations with 3 unknown V_1, V_2 and V_3.

Example: Solve for the node voltages shown in the given circuit and determine i_x.



Solution :

KCL at node 1 (V_1) : $\frac{V_1}{10} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{20} = 0 \Rightarrow 0.35V_1 - 0.2V_2 - 0.05V_3 = 0$

KCL at node 2 (V_2): $\frac{V_2 - V_1}{5} - 10 + \frac{V_2 - V_3}{10} = 0 \Rightarrow -0.2V_1 + 0.3V_2 - 0.1V_3 = 10$

KCL at node 3 (V_3) : $\frac{V_3 - V_1}{20} + \frac{V_3 - V_2}{10} + \frac{V_3}{5} = 0 \Rightarrow -0.05V_1 - 0.1V_2 + 0.35V_3 = 0$

Apply Cramer's rule to solve V_1, V_2 and V_3

$V_1 = \frac{D_1}{D}$ $V_2 = \frac{D_2}{D}$ $V_3 = \frac{D_3}{D}$

where

$$D = \begin{vmatrix} 0.35 & -0.2 & -0.05 \\ -0.2 & 0.3 & -0.1 \\ -0.05 & -0.1 & 0.35 \end{vmatrix}$$

$$= 0.35 \begin{vmatrix} 0.3 & -0.1 \\ -0.1 & 0.35 \end{vmatrix} - (-0.2) \begin{vmatrix} -0.2 & -0.1 \\ -0.05 & 0.35 \end{vmatrix} + (-0.05) \begin{vmatrix} -0.2 & 0.3 \\ -0.05 & -0.1 \end{vmatrix}$$

$$= 0.35[(0.3)(0.35) - (-0.1)(-0.1)] + 0.2[(-0.2)(0.35) - (-0.1)(-0.05)]$$

$$- 0.05 \; [(-0.2)(-0.1) - (0.3)(-0.05)]$$

$$= 0.0165$$

Next we form the determinant D_1 by replacing the first column of the system determinant

$$D_1 = \begin{vmatrix} 0 & -0.2 & -0.05 \\ 10 & 0.3 & -0.1 \\ 0 & -0.1 & 0.35 \end{vmatrix} = 0.75$$

Similarly,

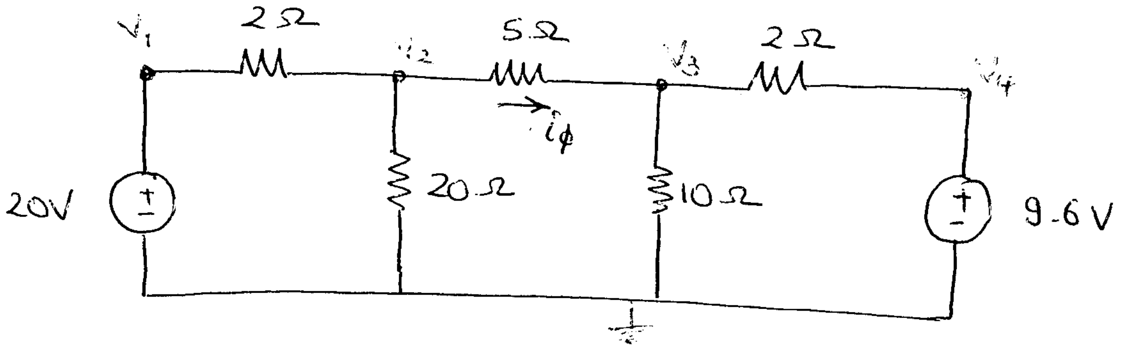
$$D_2 = \begin{vmatrix} 0.35 & 0 & -0.05 \\ -0.2 & 10 & -0.1 \\ -0.05 & 0 & 0.35 \end{vmatrix} = 1.2$$

$$D_3 = \begin{vmatrix} 0.35 & -0.2 & 0 \\ -0.2 & 0.3 & 10 \\ -0.05 & -0.1 & 0 \end{vmatrix} = 0.45$$

$$V_1 = \frac{0.75}{0.0165} = 45.45\,V \qquad V_2 = \frac{1.2}{0.0165} = 72.73\,V \qquad V_3 = \frac{0.45}{0.0165} = 27.27\,V$$

$$i_x = \frac{V_1 - V_3}{20} = \frac{45.45 - 27.27}{20} = 0.909\ A$$

Question: Use node-voltage method to find i_ϕ and the power dissipated in the 5-Ω resistor in the following circuit.



Solution:

$V_1 = 20V$

$V_4 = 9.6V$

Node 2 (V_2) :

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{5} + \frac{V_2}{20} = 0$$

$$\frac{V_2 - 20}{2} + \frac{V_2 - V_3}{5} + \frac{V_2}{20} = 0$$

$$15V_2 - 4V_3 = 200 \qquad (1)$$

Node 3 (V_3) :

$$\frac{V_3 - V_2}{5} + \frac{V_3 - V_4}{2} + \frac{V_3}{10} = 0$$

$$\frac{V_3 - V_2}{5} + \frac{V_3 - 9.6}{2} + \frac{V_3}{10} = 0$$

$$-V_2 + 4V_3 = 24 \qquad (2)$$

(1) + (2) =>

$$15V_2 - 4V_3 = 200$$

$$-V_2 + 4V_3 = 24$$

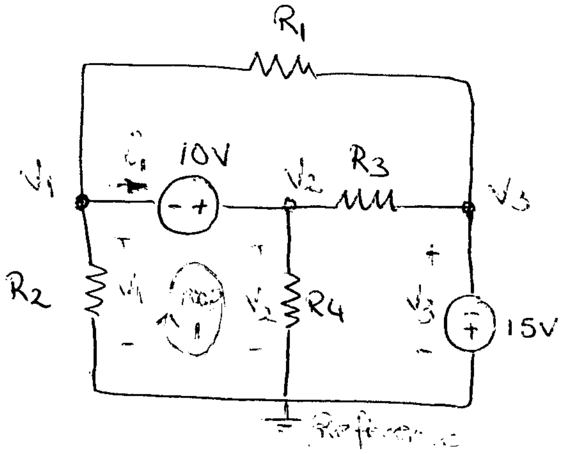
$$+ \overline{\qquad\qquad\qquad\qquad}$$

$$14V_2 = 224 \qquad \Rightarrow V_2 = 16V$$

$$V_3 = 10V$$

$$i_\phi = \frac{V_2 - V_3}{5} = \frac{16 - 10}{5} = 1.2A \qquad P = 5i_\phi^2 = 5(1.2)^2 = 7.2W$$

Circuits with voltage sources.



$V_3 = -15\,V$

There are 2 unknowns: V_1 and V_2

KCL at node 1: $\dfrac{V_1}{R_2} + \dfrac{V_1 - V_3}{R_1} + i_1 = 0 \implies i_1 = -\dfrac{V_1 - V_3}{R_1} - \dfrac{V_1}{R_2}$

KCL at node 2: $\dfrac{V_2}{R_4} + \dfrac{V_2 - V_3}{R_3} - i_1 = 0 \implies i_1 = \dfrac{V_2}{R_4} + \dfrac{V_2 - V_3}{R_3}$

$$-\frac{V_1 - V_3}{R_1} - \frac{V_1}{R_2} = \frac{V_2}{R_4} + \frac{V_2 - V_3}{R_3}$$

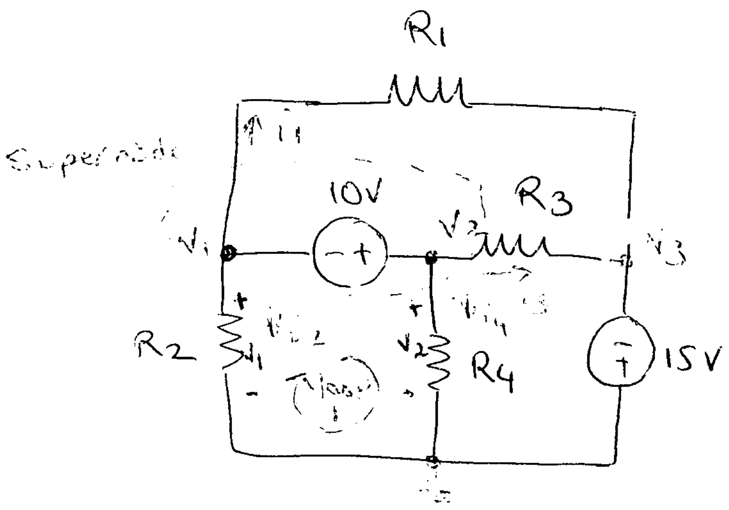
$$\frac{V_1}{R_2} + \frac{V_1 - V_3}{R_1} + \frac{V_2}{R_4} + \frac{V_2 - V_3}{R_3} = 0$$

KVL for loop 1:

$-V_1 - 10 + V_2 = 0$

$V_2 - V_1 = 10$

Another way to obtain a current equation is to form a supernode



$V_3 = -15\,V$

KCL at supernode: $i_1 + i_2 + i_3 + i_4 = 0$

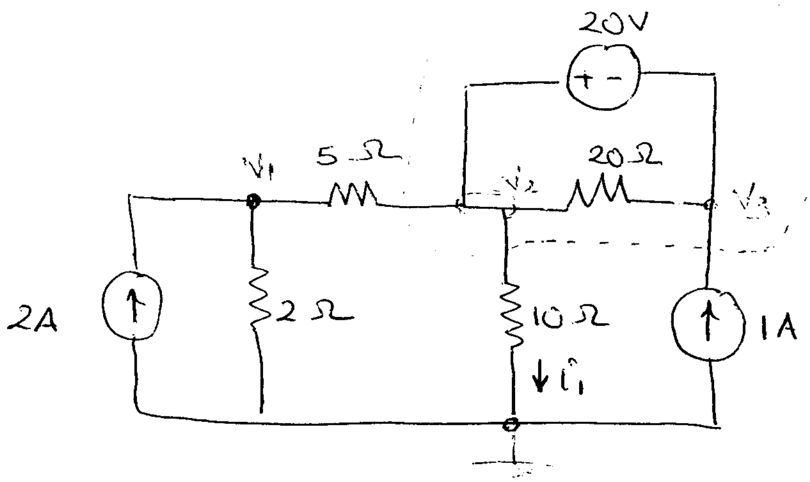
$$\frac{V_1 - V_3}{R_1} + \frac{V_1}{R_2} + \frac{V_2 - V_3}{R_3} + \frac{V_2}{R_4} = 0$$

KVL for loop 1:

$-V_1 - 10 + V_2 = 0$

$V_2 - V_1 = 10.$

P2.27 Use the node-voltage technique to find i_1.



Solution:

Node 1 (V_1): $\frac{V_1}{2} + \frac{V_1 - V_2}{5} - 2 = 0 \Rightarrow \frac{7}{10}V_1 - \frac{1}{5}V_2 = 2$

Supernode: $\frac{V_2 - V_1}{5} + \frac{V_2}{10} - 1 = 0$

$$\Rightarrow -\frac{1}{5}V_1 + \frac{3}{10}V_2 = 1$$

$\frac{7}{10}V_1 - \frac{1}{5}V_2 = 2 \quad \Rightarrow \quad \frac{7}{10}V_1 - \frac{1}{5}V_2 = 2$

$\frac{7}{2}\left(-\frac{1}{5}V_1 + \frac{3}{10}V_2 = 1\right) \quad \Rightarrow \quad -\frac{7}{10}V_1 + \frac{21}{20}V_2 = \frac{7}{2}$

$+$

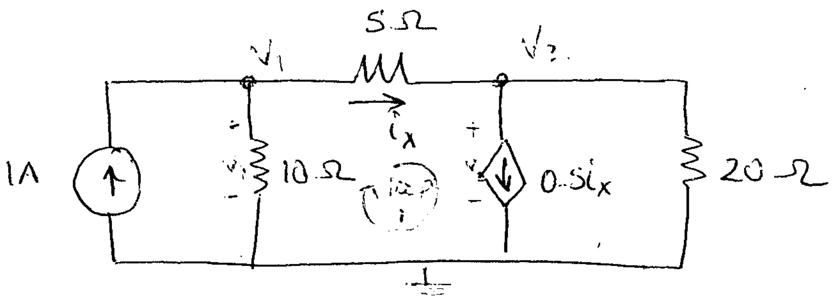
$$\frac{17}{20}V_2 = \frac{11}{2}$$

$$V_2 = \frac{110}{17}$$

$$i_1 = \frac{V_2}{10} = \frac{110/17}{10}$$

$$i_1 = \frac{11}{17}\ A$$

2.29 Solve for the values of node voltages and find i_x.



solution:

Node V_1: $\frac{V_1}{10} + \frac{V_1 - V_2}{5} - 1 = 0 \implies \frac{3}{10}V_1 - \frac{1}{5}V_2 = 1$ ①

Node V_2: $\frac{V_2 - V_1}{5} + 0.5 i_x + \frac{V_2}{20} = 0$ ②

KVL for loop 1: $-V_1 + 5 i_x + V_2 = 0 \implies i_x = \frac{V_1 - V_2}{5}$ ③

put ③ into ②

$$-\frac{1}{5}V_1 + V_2\left(\frac{1}{5} + \frac{1}{20}\right) + 0.5\left(\frac{V_1 - V_2}{5}\right) = 0$$

$$-\frac{1}{10}V_1 + \frac{3}{20}V_2 = 0 \quad ④$$

From ① and ④

$\frac{3}{10}V_1 - \frac{1}{5}V_2 = 1 \implies \frac{3}{10}V_1 - \frac{1}{5}V_2 = 1$

$3 \times \left(-\frac{1}{10}V_1 + \frac{3}{20}V_2 = 0\right) \implies -\frac{3}{10}V_1 + \frac{9}{20}V_2 = 0$

+

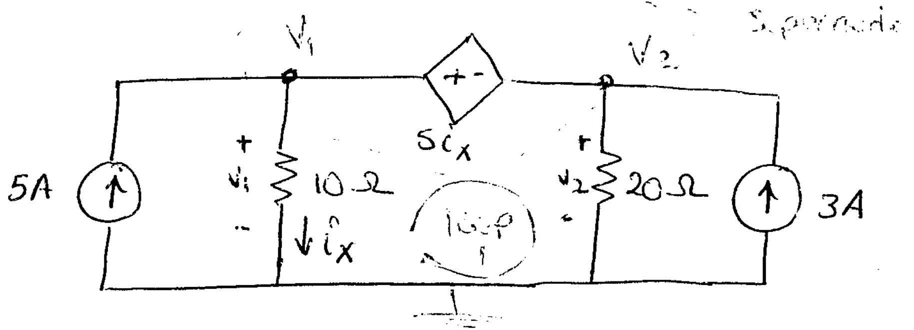
$\frac{5}{20}V_2 = 1 \implies V_2 = 4\text{ V}$ ⑤

substitute ⑤ into ④

$$-\frac{1}{10}V_1 + \frac{3}{20}(4) = 0 \implies V_1 = 6\text{V}$$

$$i_x = \frac{V_1 - V_2}{5} = \frac{6-4}{5} = 0.4\text{ A}$$

P2-30 Solve for node voltages and find i_x.



Solution:

KCL for supernode: $\frac{V_1}{10} - 5 + \frac{V_2}{20} - 3 = 0$ ①

KVL for loop 1 : $-V_1 + 5i_x + V_2 = 0 \Rightarrow V_1 - V_2 = 5i_x$ ②

$V_1 = 10 i_x \Rightarrow i_x = \frac{V_1}{10}$ ③

substitute ③ into ②

$V_1 - V_2 = \frac{5 V_1}{10} \Rightarrow V_2 = \frac{1}{2} V_1$ ④

substitute ④ into ①

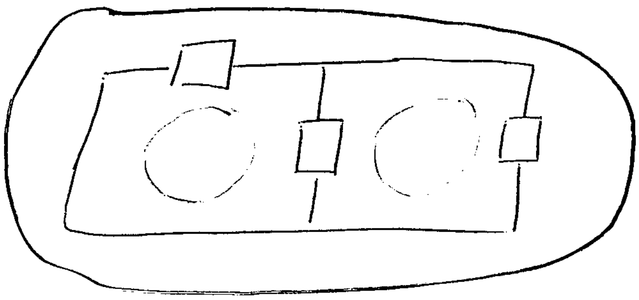
$\frac{V_1}{10} - 5 + \left(\frac{V_1}{2}\right)\frac{1}{20} - 3 = 0$

$V_1 = 64 \text{ V}$

$i_x = \frac{V_1}{10} = \frac{64}{10} = 6.4 \text{ A}$

Mesh Current Analysis

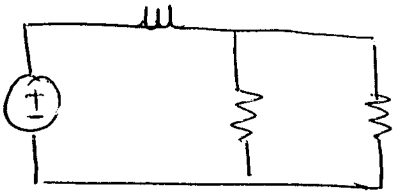
A mesh is a special type of loop, that is it does not contain any other loops within it.



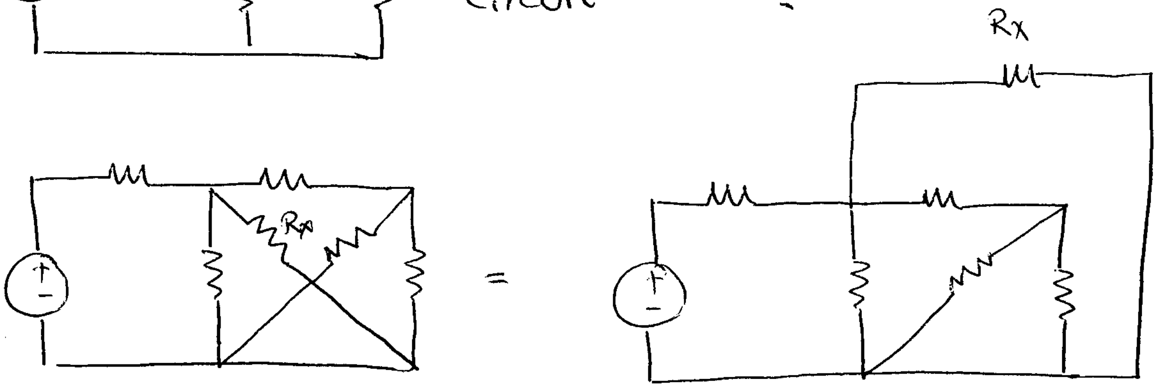
- mesh
- not mesh

Mesh current analysis can be used for planar circuits.

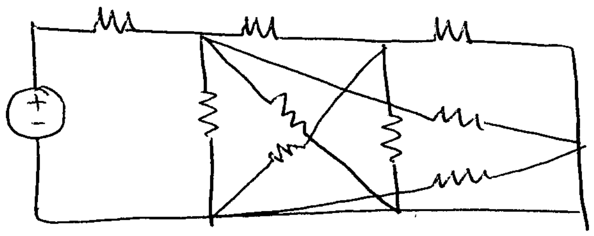
Planar circuit: They are the circuits that can be drawn on a plane with no crossing branches.



Planar circuit



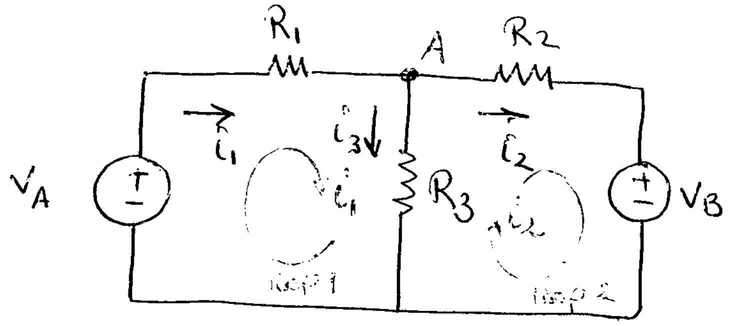
Planar circuit



Not planar circuit.

Node voltage analysis → Planar, Not Planar

Mesh current analysis → Planar.



KVL for loop 1 : $-V_A + R_1 i_1 + R_3 i_3 = 0$ (1)

KVL for loop2 : $-R_3 i_3 + R_2 i_2 + V_B = 0$ (2)

KCL at node A : $i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2$ (3)

substitute (3) into (1) : $-V_A + R_1 i_1 + R_3(i_1 - i_2) = 0$

$R_1 i_1 + R_3(i_1 - i_2) = V_A$

substitute (3) into (2) : $-R_3(i_1 - i_2) + R_2 i_2 + V_B = 0$

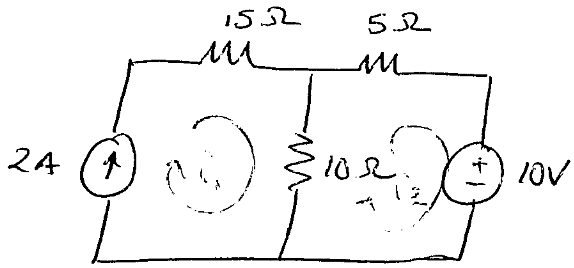
$-R_3(i_1 - i_2) + R_2 i_2 = -V_B$

Using Mesh current method :

KVL for mesh 1 : $-V_A + R_1 i_1 + R_3(i_1 - i_2) = 0$

KVL for mesh 2 : $R_3(i_2 - i_1) + R_2 i_2 + V_B = 0$

Consider the following circuit.

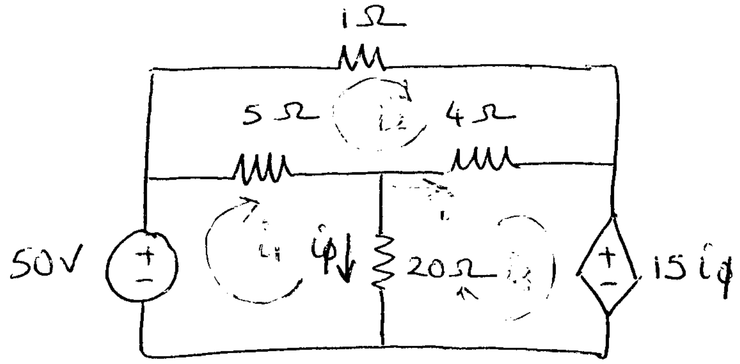


$i_1 = 2A$

$10(i_2 - i_1) + 5 i_2 + 10 = 0$ KVL for mesh 2

$i_2 = \frac{10}{15}$ A

Question: Use the mesh-current method to find the power dissipated in 4 Ω resistor.



Solution:

KVL for mesh 1: $-50+5(i_1-i_2)+20(i_1-i_3)=0 \Rightarrow 25i_1-5i_2-20i_3=50$ ①

KVL for mesh 2: $5(i_2-i_1)+1(i_2)+4(i_2-i_3)=0 \Rightarrow -5i_1+10i_2-4i_3=0$ ②

KVL for mesh 3: $20(i_3-i_1)+4(i_3-i_2)+15i_\phi=0 \Rightarrow -20i_1-4i_2+24i_3+15i_\phi=0$ ③

$i_\phi = i_1 - i_3$ ④

substitute ④ into ③

$-20i_1-4i_2+24i_3+15(i_1-i_3)=0 \Rightarrow -5i_1-4i_2+9i_3=0$

Using Cramer's Rule

$$i_1 = \frac{\begin{vmatrix} 50 & -5 & -20 \\ 0 & 10 & -4 \\ 0 & -4 & 9 \end{vmatrix}}{D} = \frac{50(90-16)}{125} = 29.6A$$

$$D = \begin{vmatrix} 25 & -5 & -20 \\ -5 & 10 & -4 \\ -5 & -4 & 9 \end{vmatrix}$$

$$= 25(90-16)-(-5)(-45-20)+(-20)(20+50)$$

$$= 125$$

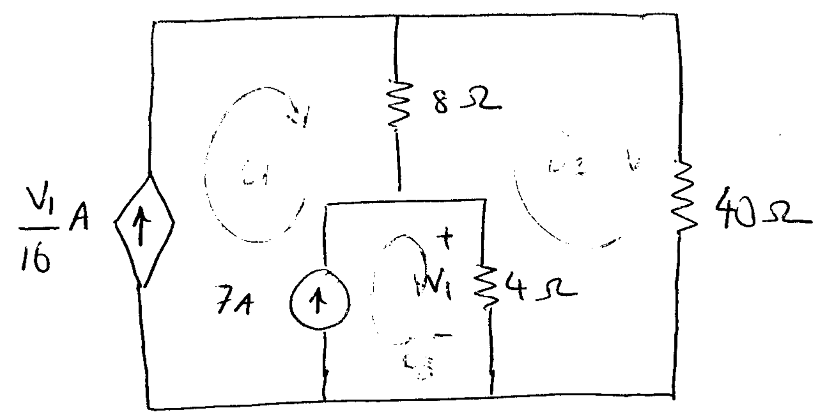
$$i_2 = \frac{\begin{vmatrix} 25 & 50 & -20 \\ -5 & 0 & -4 \\ -5 & 0 & 9 \end{vmatrix}}{D} = \frac{-50(-45-20)}{125} = 26A$$

$$i_3 = \frac{\begin{vmatrix} 25 & -5 & 50 \\ -5 & 10 & 0 \\ -5 & -4 & 0 \end{vmatrix}}{D} = \frac{50(20+50)}{125} = 28A$$

$i = i_3 - i_2 = 2A$

$P_{4\Omega} = i^2 4 = (2^2)4 = 16W$

Question: Using mesh analysis, find the mesh currents.



solution:

$$i_1 = \frac{V_1}{16} \quad (1)$$

$$V_1 = 4(i_3 - i_2) \quad (2)$$

substitute (2) into (1) $\Rightarrow i_1 = \frac{4(i_3 - i_2)}{16} \Rightarrow 16i_1 + 4i_2 - 4i_3 = 0$ (3)

KVL for mesh 2: $8(i_2 - i_1) + 40i_2 + 4(i_2 - i_3) = 0 \Rightarrow -8i_1 + 52i_2 - 4i_3 = 0$ (4)

$7 = i_3 - i_1 \Rightarrow i_3 = i_1 + 7$ (5)

substitute (5) into (3) $\Rightarrow 16i_1 + 4i_2 - 4(i_1 + 7) = 0 \Rightarrow 12i_1 + 4i_2 = 28$ (6)

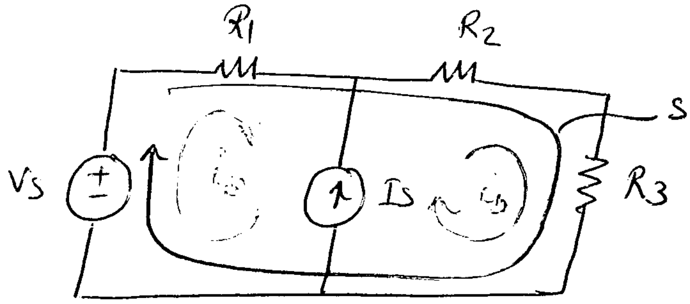
substitute (5) into (4) $\Rightarrow -8i_1 + 52i_2 - 4(i_1 + 7) = 0 \Rightarrow -12i_1 + 52i_2 = 28$ (7)

(6) + (7) $\Rightarrow 56i_2 = 56 \Rightarrow i_2 = 1A$ (8)

substitute (8) into (7) $\Rightarrow i_1 = \frac{28 - 52(1)}{-12} = 2A$ (9)

substitute (9) into (5) $\Rightarrow i_3 = 2 + 7 = 9A$

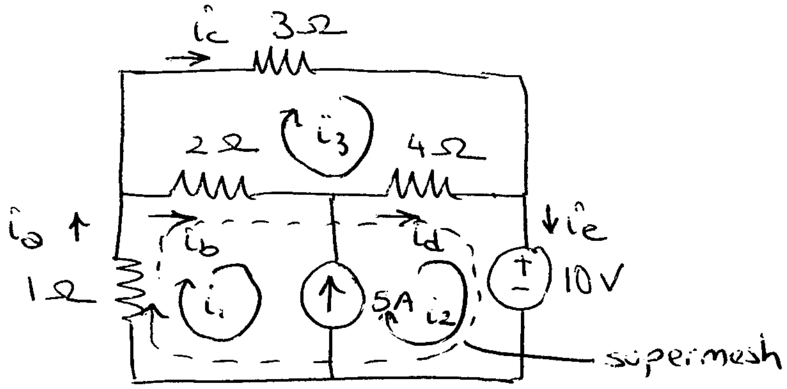
Supermesh



KVL for supermesh: $-V_S + R_1 i_a + R_2 i_b + R_3 i_b = 0$

$I_S = i_b - i_a$

Example: Find the currents in each branch.



Solution: A solution is to combine mesh 1 and 2 into a supermesh.

KVL for supermesh: $1(i_1) + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0$

$$3i_1 + 4i_2 - 6i_3 = -10 \qquad (1)$$

KVL for mesh 3: $3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$

$$-2i_1 - 4i_2 + 9i_3 = 0 \qquad (2)$$

$i_2 - i_1 = 5 \implies i_2 = 5 + i_1 \qquad (3)$

substitute (3) into (1) $\implies 3i_1 + 4(5 + i_1) - 6i_3 = -10 \implies 7i_1 - 6i_3 = -30 \quad (4)$

substitute (3) into (2) $\implies -2i_1 - 4(5 + i_1) + 9i_3 = 0 \implies -6i_1 + 9i_3 = 20 \quad (5)$

$6 \times (4) + 7 \times (5) \implies$

$$\begin{array}{r} 42i_1 - 36i_3 = -180 \\ + \; -42i_1 + 63i_3 = 140 \\ \hline 27i_3 = -40 \end{array}$$

$i_3 = -\frac{40}{27} = -1.48\,A$ (6)

Substitute (6) into (4) $\Rightarrow$ $i_1 = \frac{-30+6(-40/27)}{7} = -5.55\,A$ (7)

Substitute (7) into (5) $\Rightarrow$ $i_2 = 5+(-5.55) = -0.55\,A$

$i_a = i_1 = -5.55\,A$

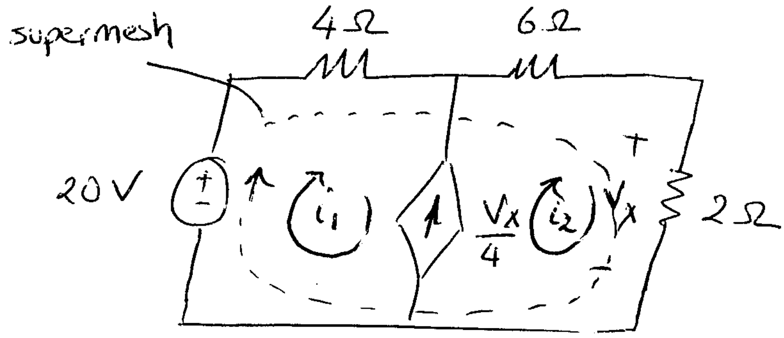
$i_b = i_1 - i_3 = -5.55-(-1.48) = -4.07\,A$

$i_c = i_3 = -1.48\,A$

$i_d = i_2 - i_3 = -0.55-(-1.48) = 0.93\,A$

$i_e = i_2 = -0.55\,A$

Example: Solve for the mesh currents.



Solution: Apply a supermesh:

KVL for supermesh: $-20 + 4i_1 + 6i_2 + 2i_2 = 0 \Rightarrow 4i_1 + 8i_2 = 20$ ①

$i_2 - i_1 = \frac{V_x}{4}$ ②

ohm's law: $V_x = 2i_2$ ③

Substitute ③ into ② $\Rightarrow$ $i_2 - i_1 = 2i_2$

$i_1 = \frac{i_2}{2}$ ④

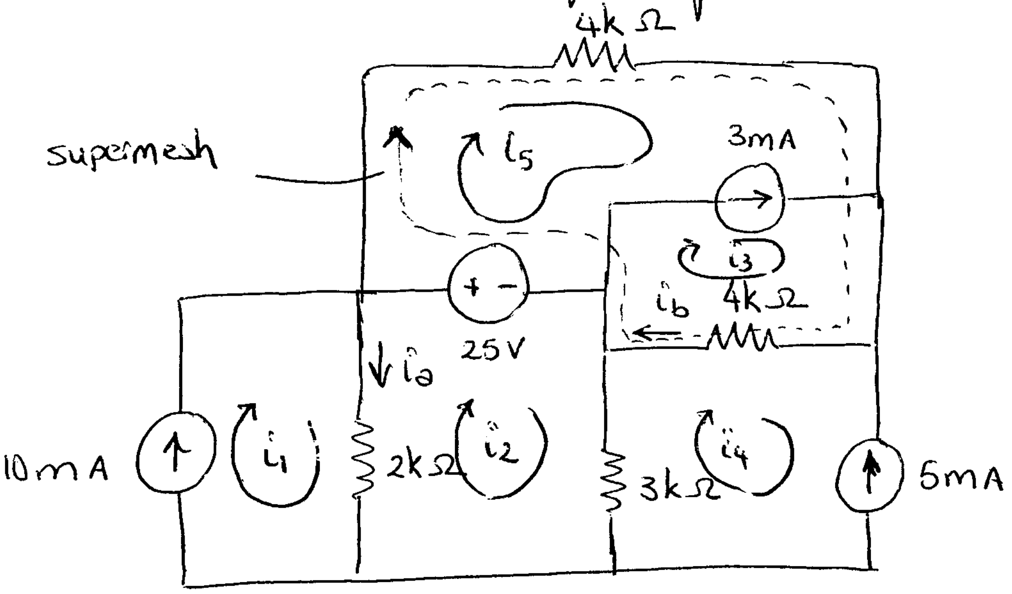
Substitute ④ into ① $\Rightarrow$ $4\left(\frac{i_2}{2}\right) + 8i_2 = 20$

$2i_2 + 8i_2 = 20$

$i_2 = 2\,A$ ⑤

Substitute ⑤ into ④ $\Rightarrow$ $i_1 = \frac{2}{2} = 1\,A$

Question: Find i_a and i_b by using mesh current analysis.



Solution:

$i_1 = 10\,mA$

$i_4 = -5\,mA$

KVL for mesh 2: $2k(i_2 - i_1) + 25 + 3k(i_2 - i_4) = 0$

$-2k i_1 + 5k i_2 - 3k i_4 = -25$

$-2k(10m) + 5k i_2 - 3k(-5m) = -25$

$i_2 = -4\,mA$

$i_a = i_1 - i_2 = 10m - (-4m) = 14\,mA$

KVL for supermesh: $4k i_5 + 4k(i_3 - i_4) - 25 = 0$

$4k i_3 + 4k i_5 = 25 + 4k i_4$

$= 25 + 4k(-5m)$

$= 25 - 20$

$4k i_3 + 4k i_5 = 5$ ①

$i_3 - i_5 = 3m \Rightarrow i_3 = 3m + i_5$ ②

Substitute ② into ① $\Rightarrow$ $4k(3m + i_5) + 4k i_5 = 5$

$i_5 = -7/8\,m = -0.875\,mA$ ③

Substitute ③ into ② $\Rightarrow$ $i_3 = 3m + (-0.875) = 2.125\,mA$

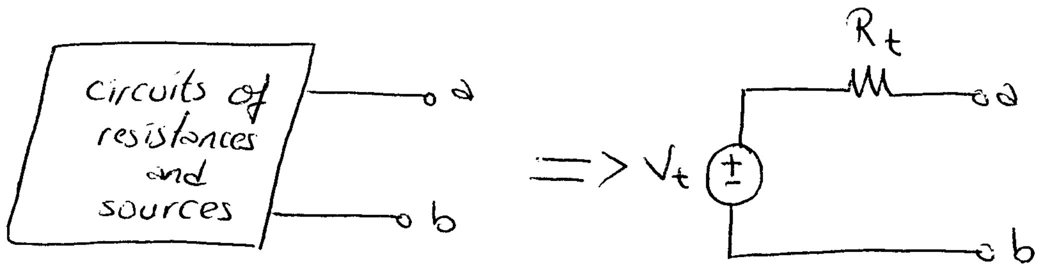
$i_b = i_3 - i_4$

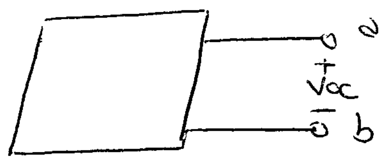
$= (2.125m) - (-5)$

$= 7.125\,mA$

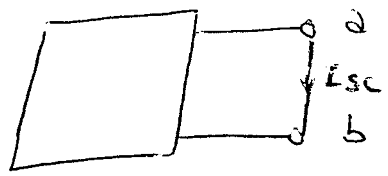
THEVENIN AND NORTON EQUIVALENT CIRCUITS

Thevenin Equivalent Circuits.



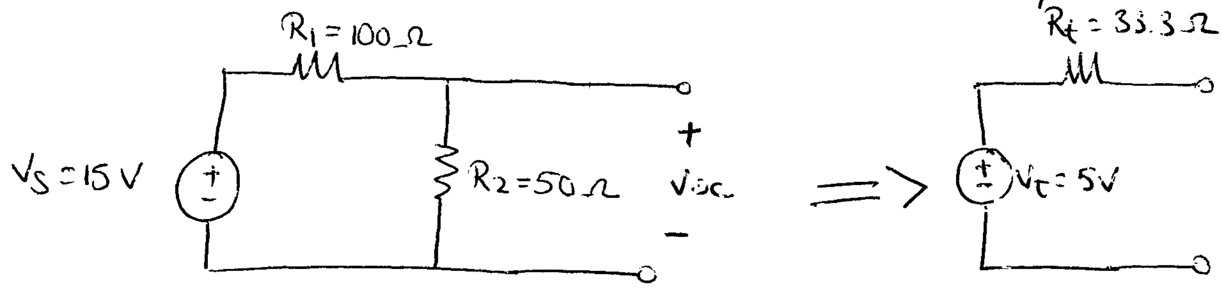


$$V_t = V_{oc}$$



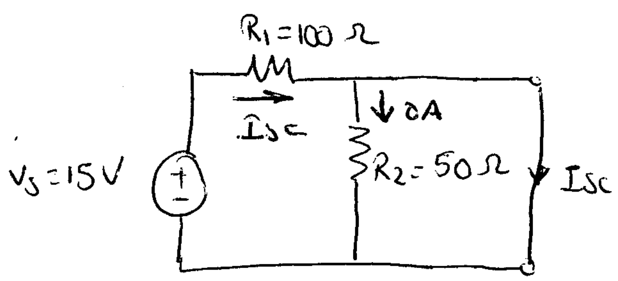
$$R_t = \frac{V_t}{I_{sc}}$$

Example: Find the Thevenin equivalent for the following circuit.



Solution:

$$V_{oc} = V_s \frac{R_2}{R_1 + R_2} = 15 \frac{50}{100+50} = 5V$$



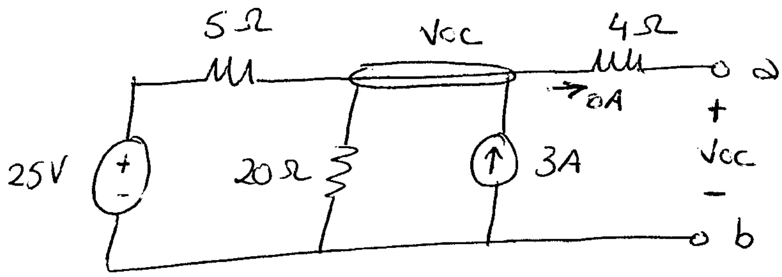
$$I_{sc} = \frac{V_s}{R_1} = \frac{15}{100} = 0.15A$$

$$R_t = \frac{V_{oc}}{I_{sc}} = \frac{5}{0.15} = 33.3\,\Omega$$

Finding R_t Directly

If there is no dependent source in the circuit, short circuit the voltage sources, replace current sources with open circuits and calculate the equivalent resistance between the terminals.

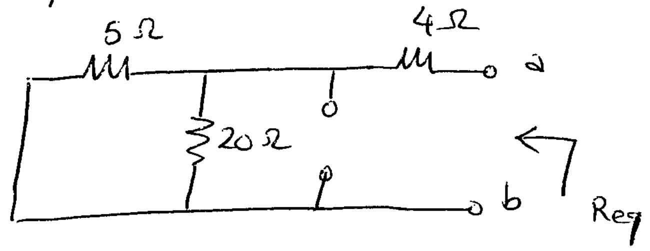
Example: Find the Thevenin equivalent for the following circuit.

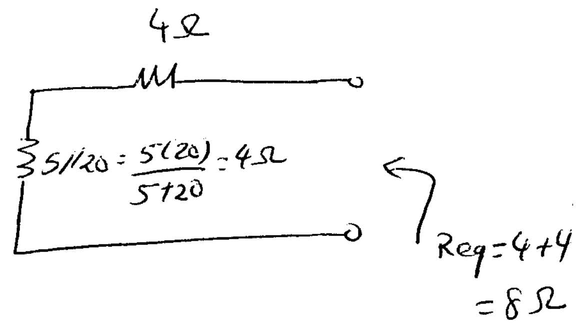


Solution.

$$\frac{V_{oc}-25}{5}+\frac{V_{oc}}{20}-3=0 \quad \Rightarrow V_{oc}=32\,V \quad \Rightarrow V_t=V_{oc}=32\,V$$

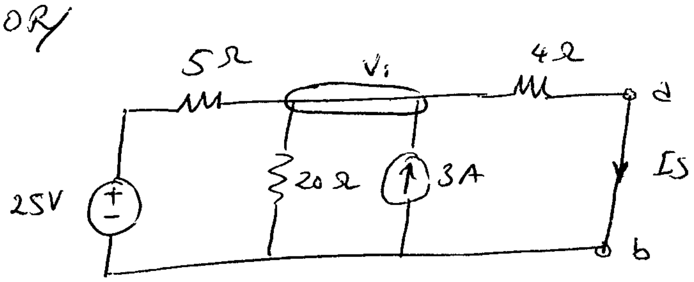
Finding R_t directly





$R_t = R_{eq} = 8\,\Omega$

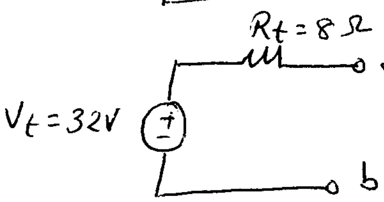
OR



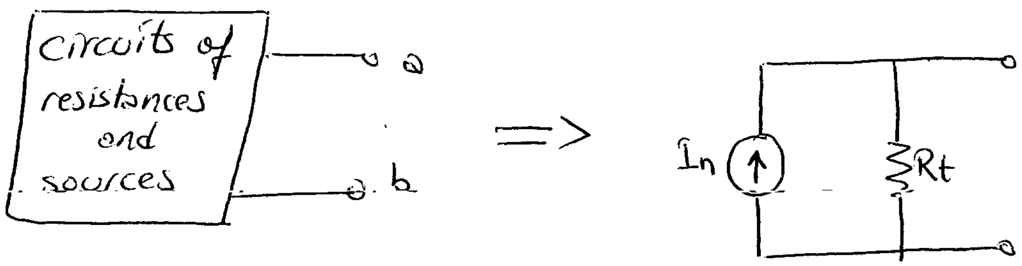
$$\frac{V_1-25}{5}+\frac{V_1}{20}-3+\frac{V_1}{4}=0 \quad \Rightarrow V_1=16V$$

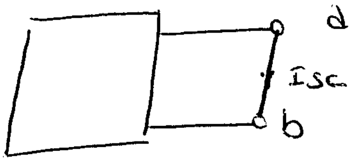
$$I_S=\frac{V_1}{4}=\frac{16}{4}=4A$$

$$R_t=\frac{V_t}{I_{sc}}=\frac{32}{4}=8\,\Omega$$

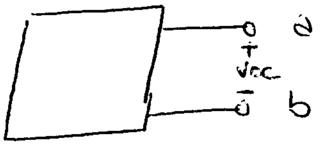


Norton Equivalent Circuits.



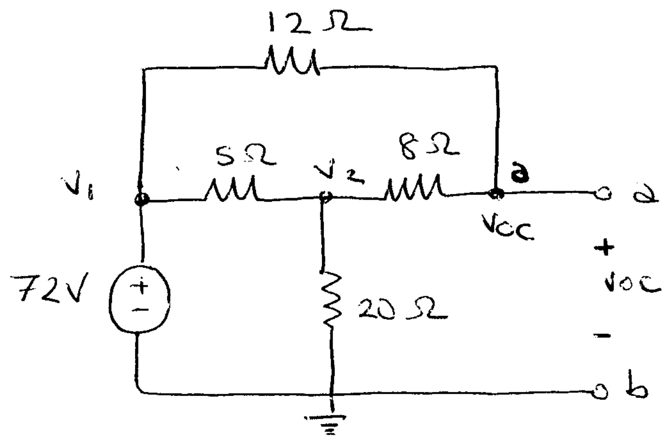


$I_n = I_{sc}$



$R_t = \dfrac{V_{oc}}{I_n}$

Example: Find the Thevenin and Norton equivalent circuit with respect to the terminals a, b for the following circuit.



Solution:

$V_1 = 72V.$

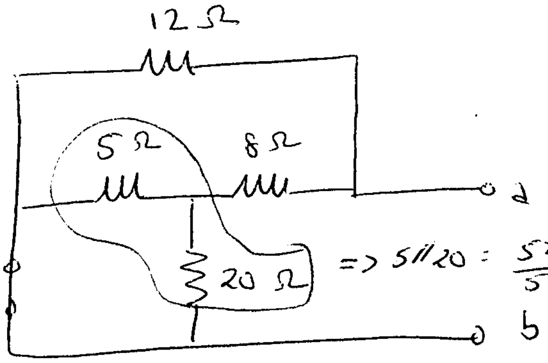
KCL at node 2 (V_2): $\dfrac{V_2 - V_1}{5} + \dfrac{V_2}{20} + \dfrac{V_2 - V_{oc}}{8} = 0$

$3V_2 - V_{oc} = 115.2$

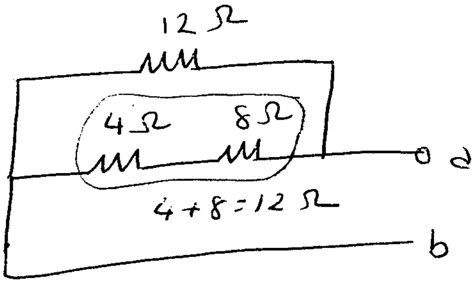
KCL at node a ; $\dfrac{V_{oc} - V_2}{8} + \dfrac{V_{oc} - V_1}{12} = 0$

$$-3V_2 + 5V_{oc} = 144$$

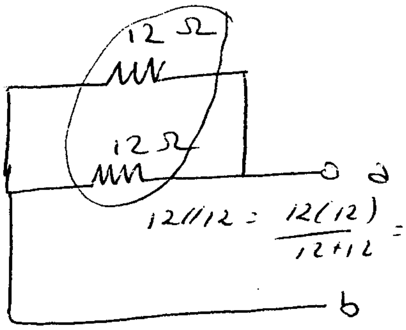
$$\begin{array}{rl} & 3V_2 - V_{oc} = 115.2 \\ + & -3V_2 + 5V_{oc} = 144 \\ \hline & 4V_{oc} = 259.2 \end{array} \implies V_{oc} = 64.8\,V \implies V_t = V_{oc} = 64.8\,V$$



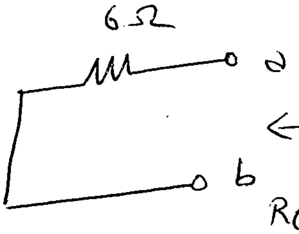
$\implies 5 /\!/ 20 = \dfrac{5 \times 20}{5+20} = 4\,\Omega$



$4 + 8 = 12\,\Omega$

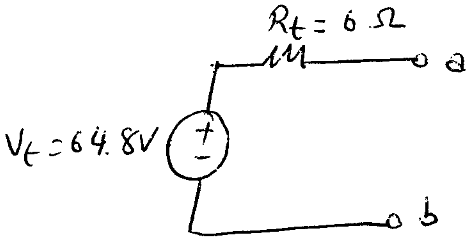


$12 /\!/ 12 = \dfrac{12(12)}{12+12} = 6\,\Omega$ $\implies$

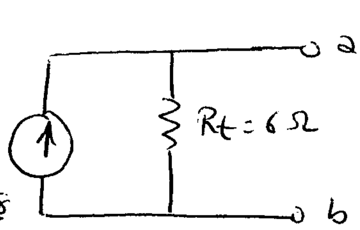


$R_t = R_{eq} = 6\,\Omega$

Thevenin



Norton

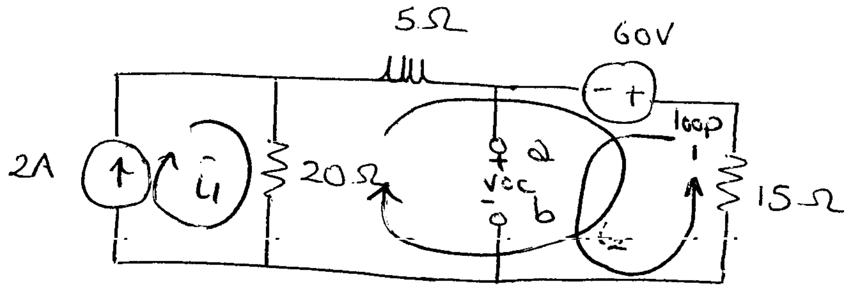


$I_n = \dfrac{V_t}{R_t} = \dfrac{64.8}{6} = 10.8\,A$

Question: (1st midterm - 1997)

(a) Find the open circuit voltage (V_{oc}) and short circuit current (i_{sc}) and thevenin resistance (R_t) with respect to terminals a-b.

(b) Draw the Thevenin and Norton equivalent circuits as seen at a-b.



Solution: $i_1 = 2A$

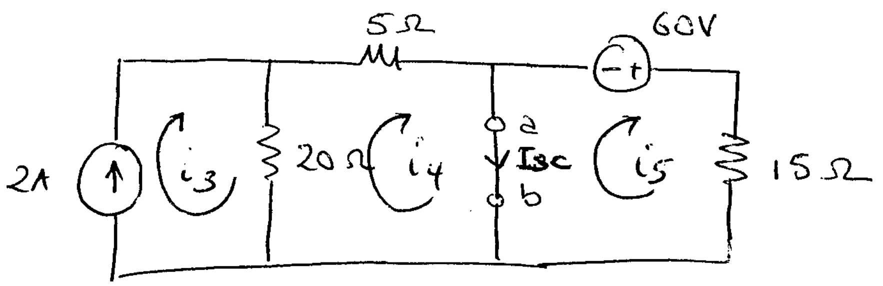
KVL for mesh 2: $20(i_2 - i_1) + 5i_2 - 60 + 15i_2 = 0$

$$40i_2 = 60 + 20i_1$$

$$i_2 = \frac{100}{40} = 2.5A$$

KVL for loop 1: $60 + V_{oc} - 15i_2 = 0$

$$V_{oc} = 15i_2 - 60$$
$$= 15(2.5) - 60$$
$$= -22.5\ V$$



$i_3 = 2A$

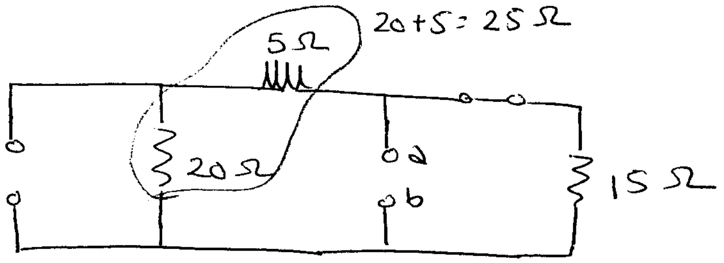
KVL for mesh 4: $20(i_4 - i_3) + 5i_4 = 0 \Rightarrow 25i_4 = 20i_3 \Rightarrow i_4 = 1.6A$

KVL for mesh 5: $-60 + 15i_5 = 0 \Rightarrow i_5 = 60/15 = 4A$

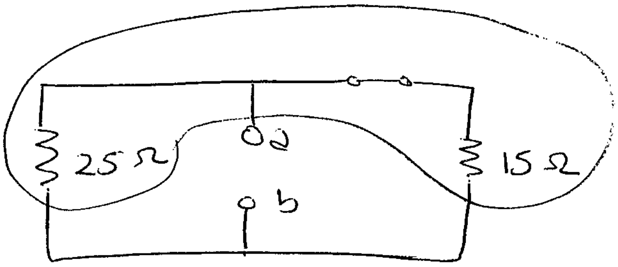
$I_{sc} = i_4 - i_5 = 1.6 - 4 = -2.4\ A$

$$R_t = \frac{V_t}{I_{sc}} = \frac{-22.5\,V}{-2.4\,A} = 9.375\,\Omega$$

or

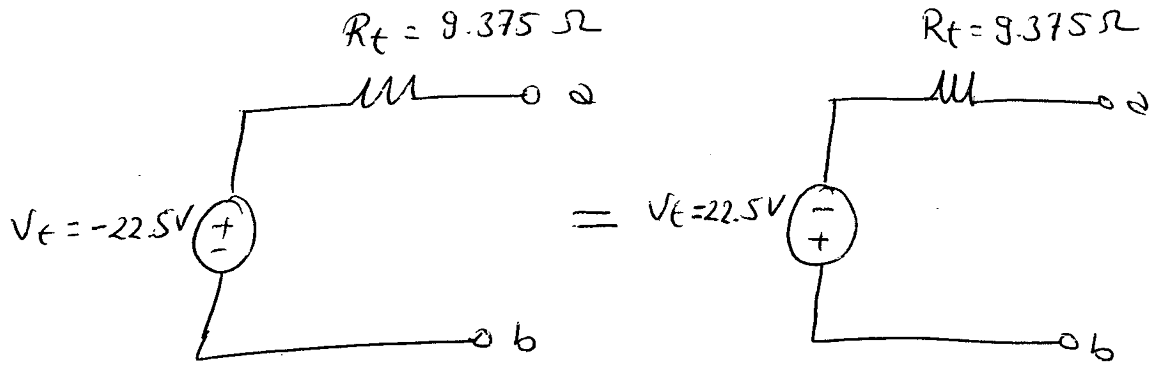


$20+5 = 25\,\Omega$

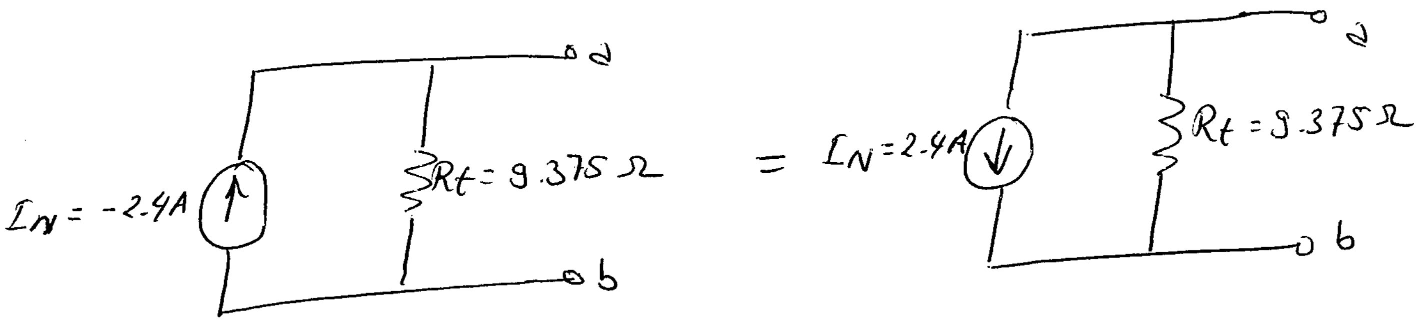


$$25 // 15 = \frac{25(15)}{25+15} = 9.375\,\Omega$$

(b) *Thevenin Equivalent*

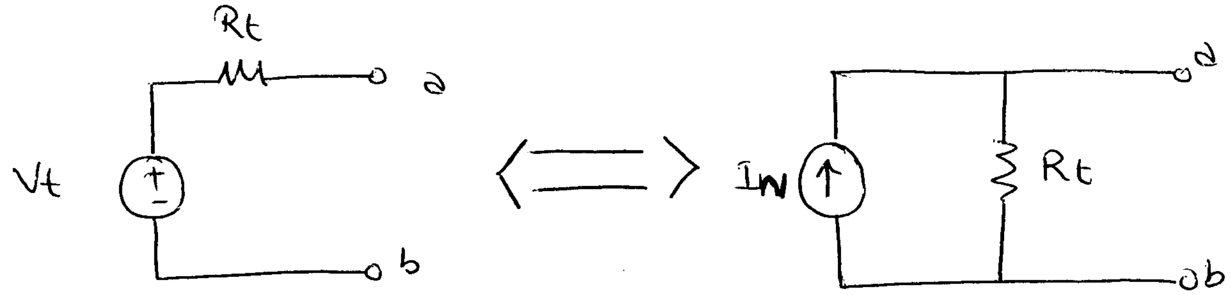


Norton Equivalent



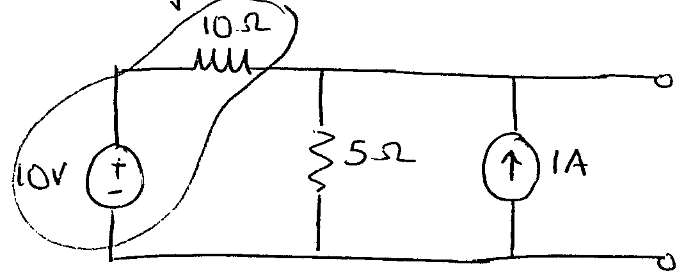
Source Transformation

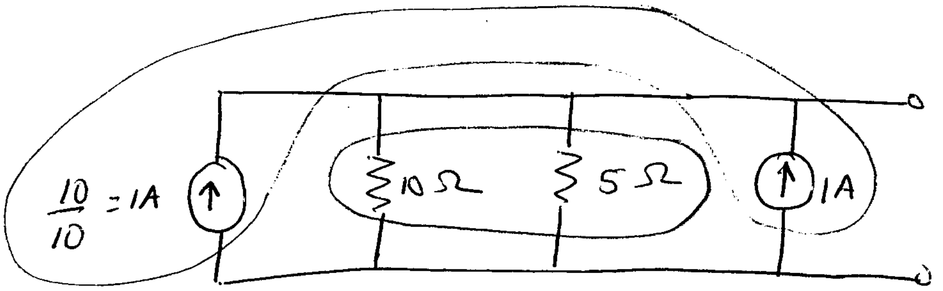
We can replace a voltage source in series with a resistance by their Norton equivalent, which consists of a current source in parallel with the resistance.



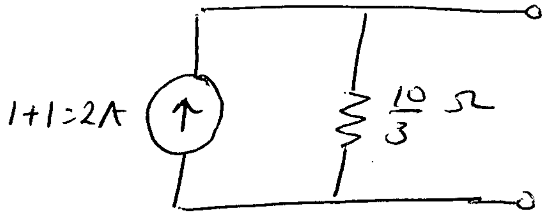
$$I_N = \frac{V_t}{R_t}$$

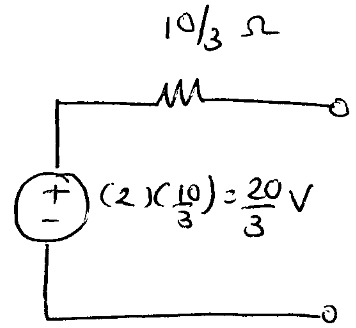
P2.40 Find the Thevenin and Norton equivalent circuits of the following circuit.





$$20 // 5 = \frac{10(5)}{10+5} = \frac{10}{3}\ \Omega$$

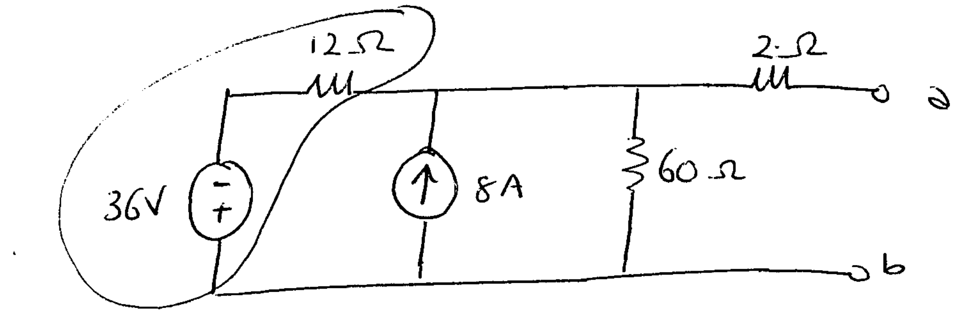




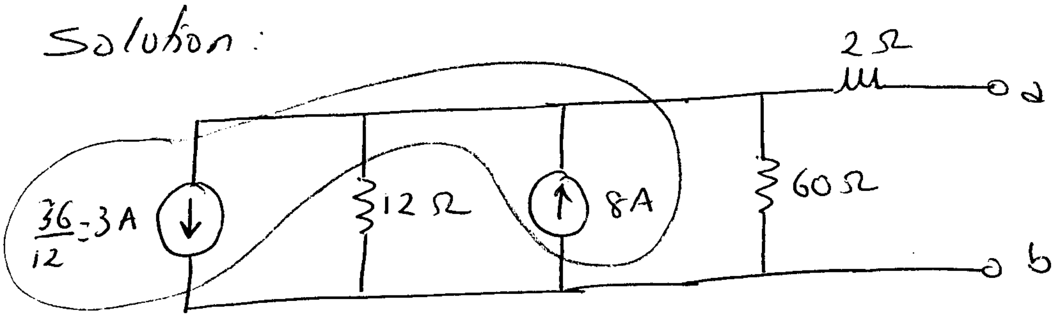
Norton equivalent

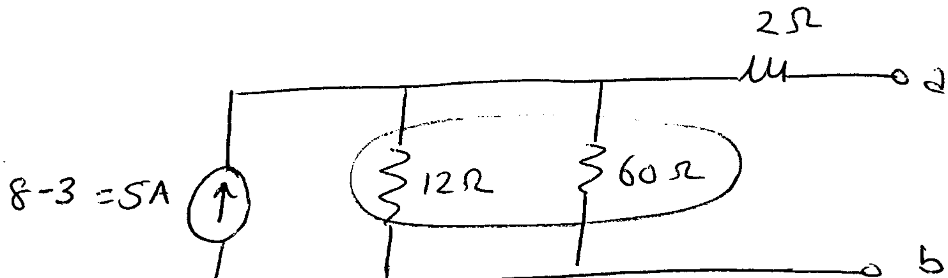
Thevenin equivalent

Question: Use source transformation to find the Thevenin equivalent with respect to terminals a, b.

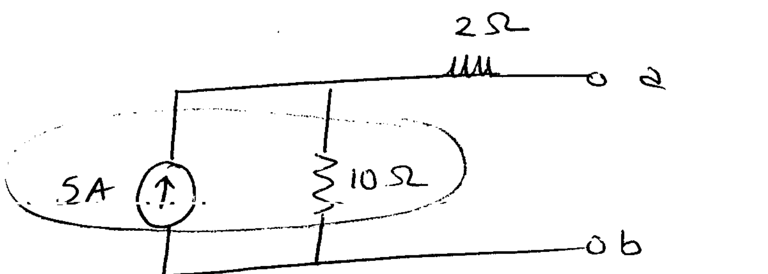


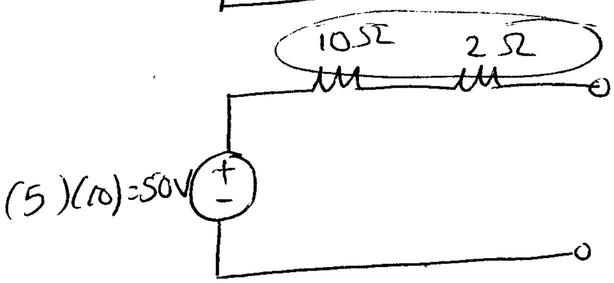
Solution:



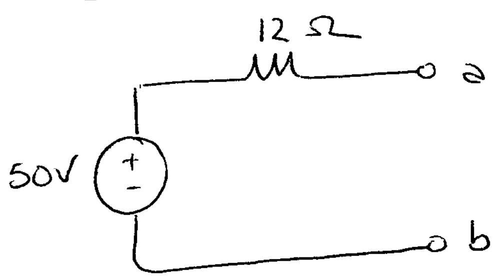


$$12 // 60 = \frac{12(60)}{12+60} = 10\ \Omega$$

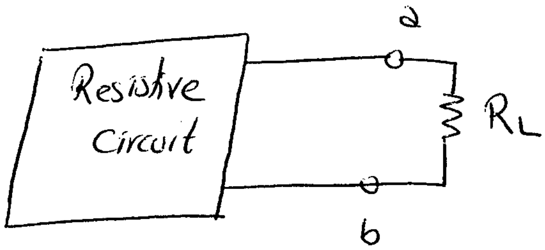




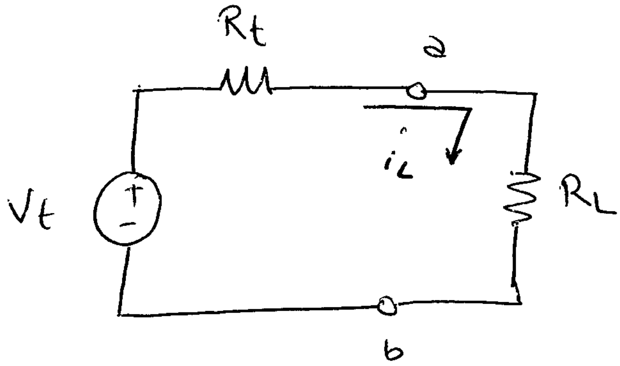
$10 + 2 = 12\ \Omega$



Maximum Power Transfer



what is the value of R_L (load resistor) to have maximum power transfer.



$$i_L = \frac{V_t}{R_t + R_L}$$

$$P_L = i_L^2 R_L$$

$$= \frac{V_t^2}{(R_t + R_L)^2} R_L$$

For maximum power

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{V_t^2 (R_t + R_L)^2 - 2V_t^2 (R_L)(R_t + R_L)}{(R_t + R_L)^4} = 0$$

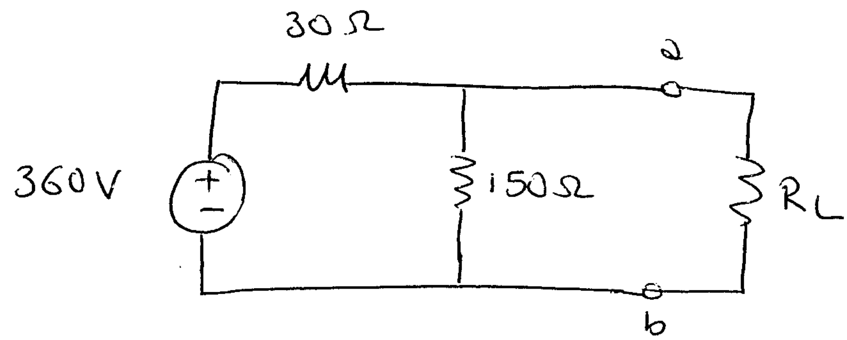
$$\underset{\neq 0}{V_t^2} \left[\frac{(R_t + R_L)^2 - 2R_L (R_t + R_L)}{(R_t + R_L)^4} \right] = 0$$

$$(R_t + R_L)^2 - 2R_L (R_t + R_L) = 0 \implies (R_t + R_L)^{\cancel{2}} = 2R_L \cancel{(R_t + R_L)}$$

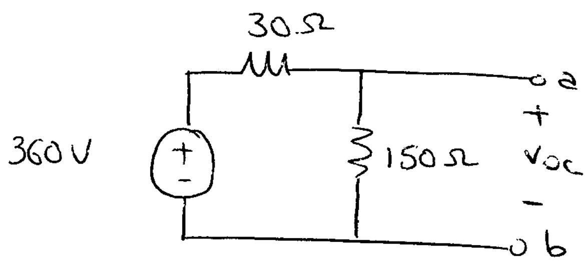
$$R_t + R_L = 2R_L \implies R_L = R_t$$ for maximum power transfer

$$P_L = \frac{V_t^2}{(R_t + R_L)^2} R_L \implies P_{max} = \frac{V_t^2}{(R_L + R_L)^2} R_L = \frac{V_t^2}{4R_L}$$

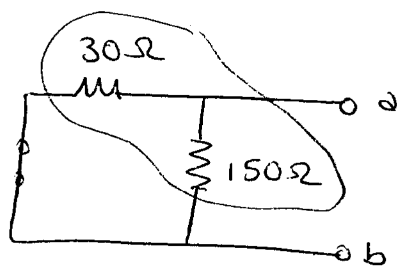
Question: Find the value of R_L that results in maximum power transfer, then calculate the maximum power transferred to R_L.



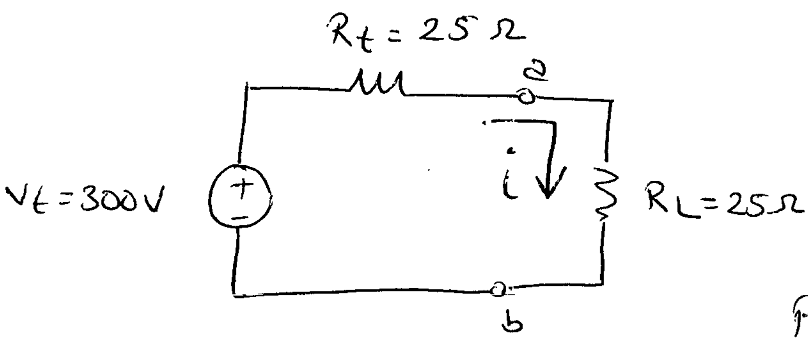
Solution:



$$V_{oc} = 360 \cdot \frac{150}{30+150} = 300V = V_t$$



$$30 // 150 = \frac{30(150)}{30+150} = 25\,\Omega = R_t$$



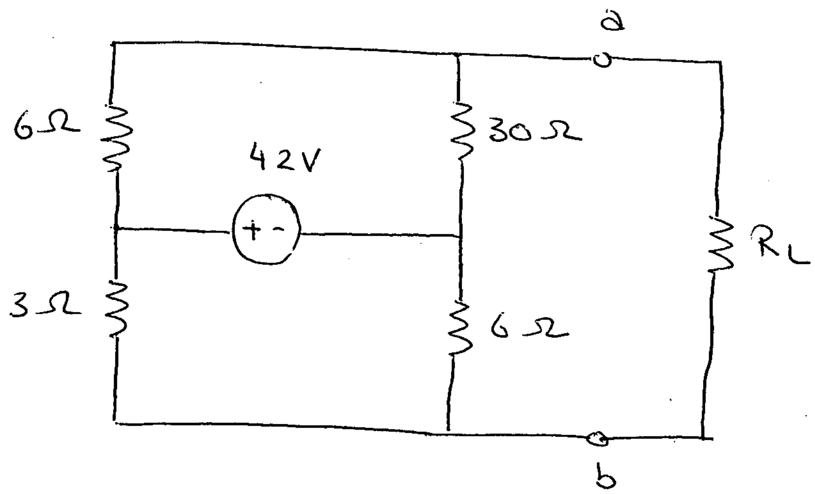
$$i = \frac{V_t}{R_t + R_L} = \frac{300}{25+25} = 6A$$

$$P_L = i^2 R_L = (6)^2 25 = 900W$$

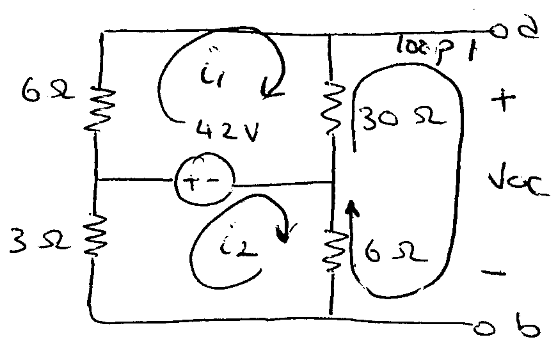
$R_L = R_t = 25\,\Omega$

Question:

Find the value of R_L that will draw the maximum power from the rest of the circuit. Also find the maximum power drawn by R_L.



Solution:



KVL for mesh 1:

$6i_1 + 30i_1 - 42 = 0 \Rightarrow i_1 = \frac{7}{6}$ A

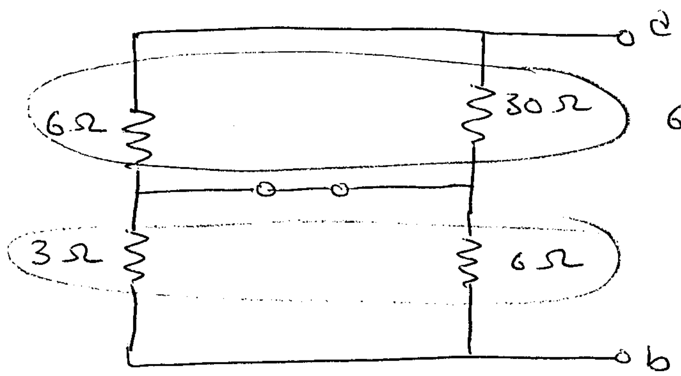
KVL for mesh 2:

$3i_2 + 42 + 6i_2 = 0 \Rightarrow i_2 = -\frac{14}{3}$ A

KVL for loop 1:

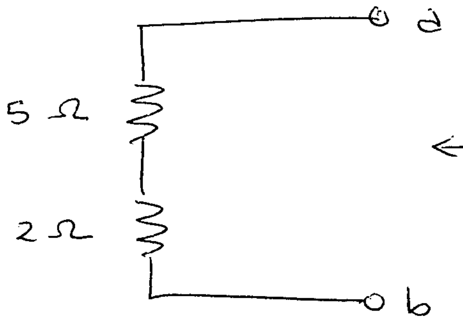
$V_{oc} - 6i_2 - 30i_1 = 0 \Rightarrow V_{oc} = 6i_2 + 30i_1 = 6\left(-\frac{14}{3}\right) + 30\left(\frac{7}{6}\right) = 7V$

$V_t = V_{oc} = 7V$



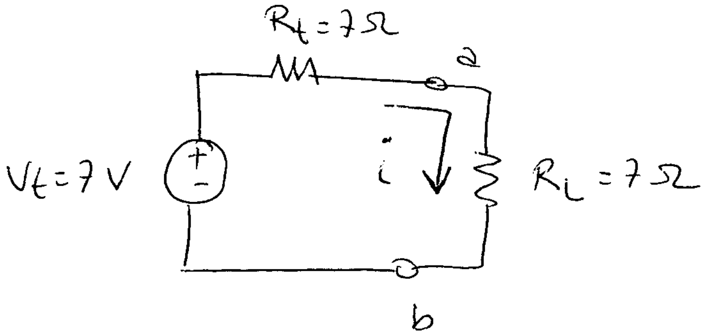
$6 // 30 = \frac{6(30)}{6+30} = 5\ \Omega$

$3 // 6 = \frac{3(6)}{3+6} = 2\ \Omega$



$R_{eq} = 5+2 = 7\,\Omega = R_t$

For maximum power: $R_L = R_t = 7\,\Omega$



$$i = \frac{V_t}{R_t + R_L} = \frac{7}{7+7} = 0.5\,A$$

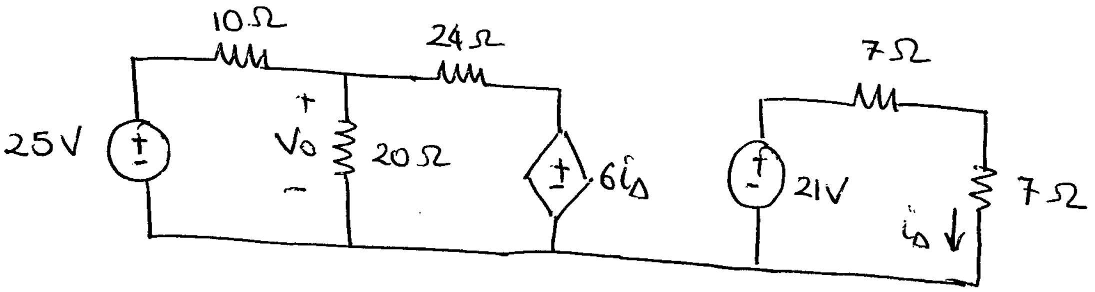
$$P_{max} = i^2 R_L = (0.5)^2\, 7 = 1.75\,W$$

Superposition Principle

Superposition is a method of analyzing a circuit containing multiple independent sources by activating one source at a time and summing the resulting voltages and currents that exists when all the independent sources are active.

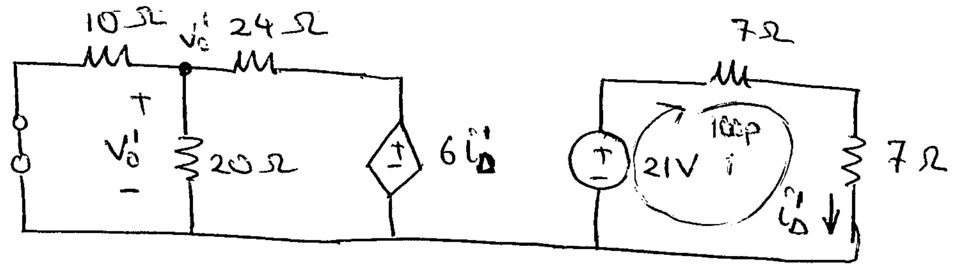
Note: Dependent sources are not deactivated when applying superposition.
The superposition principle does not apply to nonlinear circuits.
Deactivating a voltage source is to make it short circuit and deactivating a current source is to make it open circuit.

Question: Use superposition to find V_o.



Solution:

Deactivate 25V source.



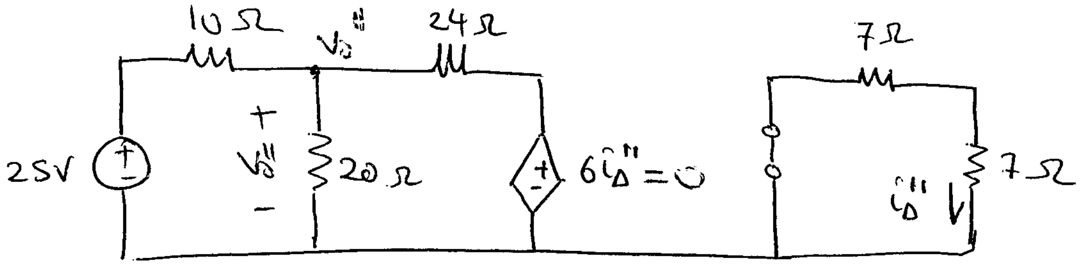
KVL for loop 1: $-21 + 7i_\Delta' + 7i_\Delta' = 0 \implies i_\Delta' = 21/14 = 1.5A$

KCL at node V_o': $\frac{V_o'}{10} + \frac{V_o'}{20} + \frac{V_o' - 6i_\Delta'}{24} = 0$

$$\frac{V_o'}{10} + \frac{V_o'}{20} + \frac{V_o' - 6(1.5)}{24} = 0$$

$V_o' = 1.96V$

Deactivate 21V source :



$i_\Delta'' = 0$

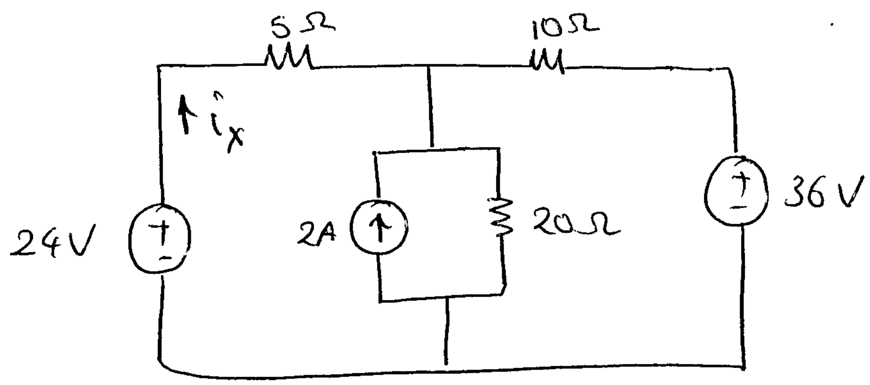
KCL at node V_o'' : $\frac{V_o'' - 25}{10} + \frac{V_o''}{20} + \frac{V_o''}{24} = 0$

$V_o'' = 13.04V$

$V_o = V_o' + V_o'' = 1.96 + 13.04 = 15V$

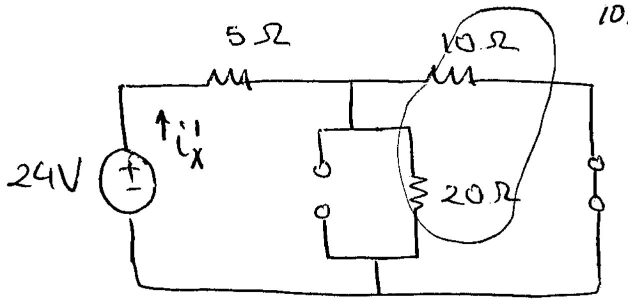
Question : (1st midterm 1997)

Find i_x using superposition principle.



Solution: Deactivating 2A and 36V sources.

$10 // 20 = \frac{10(20)}{10+20} = \frac{20}{3}$



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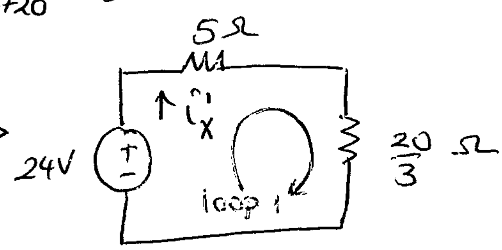
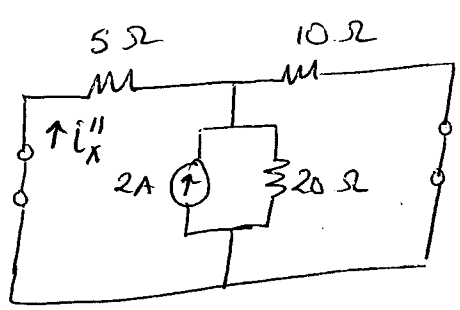


KVL for loop 1:

$-24 + 5 i_x' + \frac{20}{3} i_x' = 0 \implies i_x' = \frac{72}{35}$ A

Deactivating 24 V and 36V sources.



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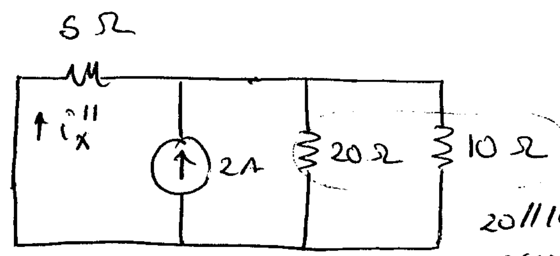
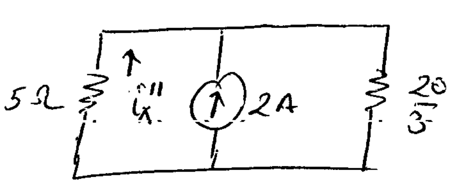


$20 // 10 = \frac{20(10)}{20+10} = \frac{20}{3}$



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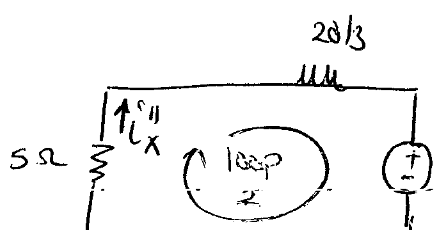
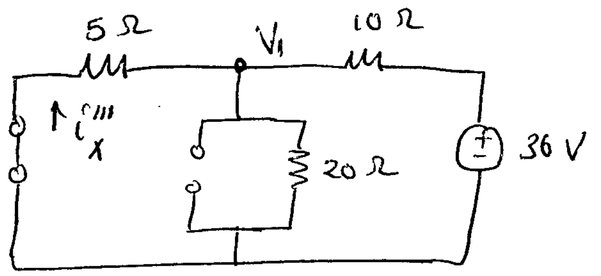


$(\frac{20}{3})(2) = \frac{40}{3}$ V

KVL for loop 2: $5 i_x'' + \frac{20}{3} i_x'' + \frac{40}{3} = 0$

$i_x'' = -8/7$ A

Deactivating 24V and 12A sources.



KCL at node V_1 : $\frac{V_1}{5} + \frac{V_1}{20} + \frac{V_1 - 36}{10} = 0$

$$V_1 = \frac{72}{7}$$

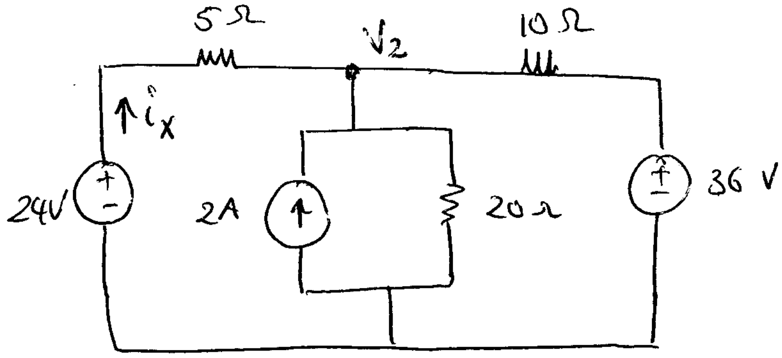
$$i_X''' = -\frac{V_1}{5} = -\frac{72/7}{5} = -\frac{72}{35} \text{ A}$$

$$i_X = i_X' + i_X'' + i_X'''$$

$$= \frac{72}{35} + \left(-\frac{8}{7}\right) + \left(-\frac{72}{35}\right)$$

$$= -\frac{8}{7} \text{ A}$$

Without superposition



KCL at node V_2 : $\frac{V_2 - 24}{5} + \frac{V_2}{20} - 2 + \frac{V_2 - 36}{10} = 0 \Rightarrow V_2 = \frac{208}{7} \text{ V}$

$$i_X = -\left(\frac{V_2 - 24}{5}\right) = -\left(\frac{208/7 - 24}{5}\right) = -\frac{8}{7} \text{ A}.$$