CHAPTER - 4 -

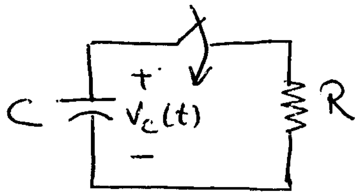
TRANSIENTS

The time varying currents and voltages resulting from the sudden application of sources, possibly due to switching, are called transients.

First-Order RC Circuits

Discharging of a capacitance

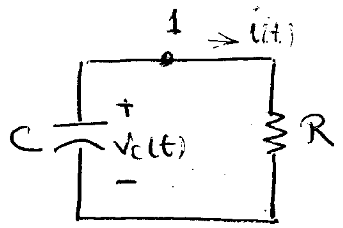
Consider the following circuit



Capacitance charged to V_i prior to $t=0$. ($V_c(0)=V_i$)

At $t=0$, the switch closes and current flows through the resistor, discharging the capacitor.

$t>0$



$$\frac{V_c(t)}{R} = -C\frac{dV_c(t)}{dt} \Rightarrow \frac{C\,dV_c(t)}{dt} + \frac{V_c(t)}{R} = 0$$

$$i(t) = \frac{V_c(t)}{R} \quad \text{(Ohm's law)}$$

$$i(t) = -C\frac{dV_c(t)}{dt}$$

$$RC \frac{dV_c(t)}{dt} + V_c(t) = 0 \qquad (1)$$

The above equation indicates that the solution for $V_c(t)$ must be function that has the same form as its first derivative.

$$V_c(t) = Ke^{st} \qquad (2)$$

in which K and s are constants to be determined

Substitute (2) into (1)

$$RC \frac{d}{dt}\left(Ke^{st}\right) + Ke^{st} = 0$$

$$RC\cancel{K}s\cancel{e^{st}} + \cancel{K}\cancel{e^{st}} = 0$$

$$s = -\frac{1}{RC}$$

$$V_c(t) = K\, e^{-t/RC}$$

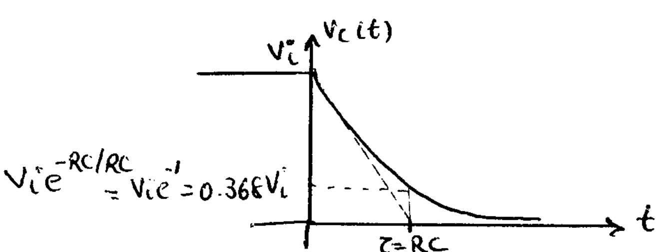
The voltage across the capacitor can not change instantenously when the switch closes. the voltage across the capacitor must be continuous. Thus

$$V_c(0^-) = \underbrace{V_c(0^+)} = V_i$$

↳ voltage immediately after the switch closes.

$$V_c(0^+) = V_i = Ke^0 = K$$

$$V_c(t) = V_i\, e^{-t/RC}$$



The time interval $\tau = RC$ is called the time constant.

Question:

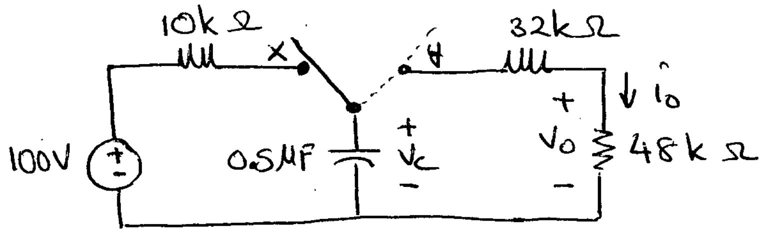
The switch in the following circuit has been in position x for a long time. At $t=0$, the switch moves instantaneously to position y. Find

(a) $V_c(t)$ for $t \geq 0$

(b) $V_o(t)$ for $t \geq 0^+$

(c) $i_o(t)$ for $t \geq 0^+$

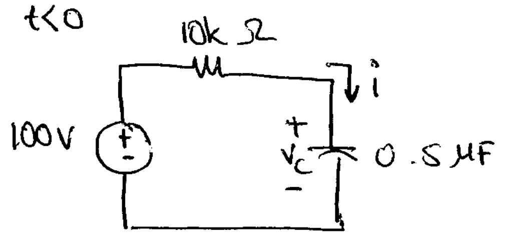
(d) the total energy dissipated in the $48 k\Omega$ resistor.



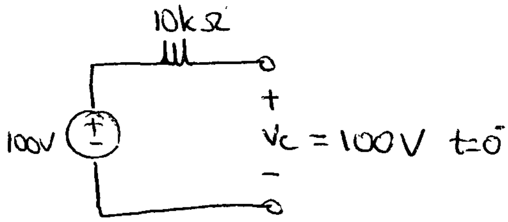
Solution:

(a) Because the switch has been in position x for a long time, the 0.5 μF capacitor will charge to 100V.

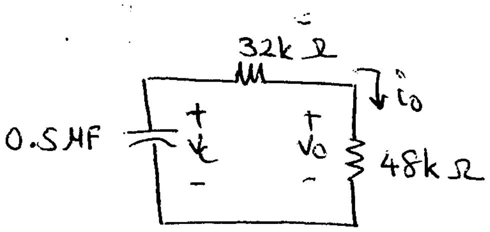
t<0

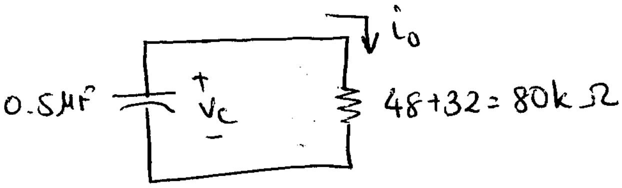


$$i = C\frac{dV_c}{dt} = 0$$



t≥0



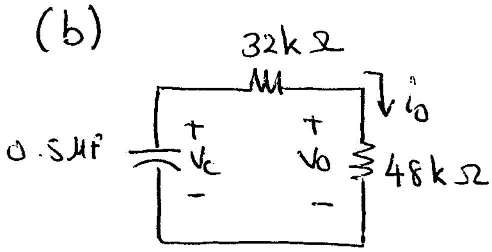


$$V_c(t) = V_i e^{-t/RC}$$

$$V_c(0^-) = V_c(0^+) = 100 = V_i e^{-0/RC} \implies V_i = 100$$

$$V_c(t) = 100 e^{-25t} \text{ V}, \quad t \geq 0 \qquad 1/RC = 1/(0.5\times10^{-6})(80\times10^{3}) = 25$$

(b)



$$v_o(t) = V_c(t)\ \frac{48k}{32k+48k}$$

$$= \frac{48}{80}\ 100e^{-25t}$$

$$= 60\ e^{-25t}\ V \quad t \geq 0^+$$

(c) $$i_o(t) = \frac{V_o(t)}{48k} = \frac{60\ e^{-25t}}{48\times10^3}$$

$$i_o(t) = 1.25\times10^{-3}\ e^{-25t}$$

$$i_o(t) = 1.25\ e^{-25t}\ mA \qquad t \geq 0^+$$

(d) The power dissipated in the 48 kΩ resistor

$$p_{48k\Omega}(t) = i_o^2(t)\ (48\times10^3) = (1.25\times10^{-3}e^{-25t})^2 (48\times10^3)$$

$$= 75\times10^{-3}\ e^{-50t}$$

$$= 75\ e^{-50t}\ mW$$

The total energy dissipated

$$w_{48k\Omega} = \int_0^\infty p_{48k\Omega}(t)\ dt$$

$$= \int_0^\infty 75\times10^{-3}\ e^{-50t}\ dt$$

$$= \frac{-75\times10^{-3}}{50}\left[e^{-50t}\right]_0^\infty$$

$$= -1.5\times10^{-3}\left(e^{-\infty} - e^{0}\right) = 1.5\times10^{-3} = 1.5\ mJ$$

Home Exercise

The switch in the following circuit has been closed for a long time and is opened at $t=0$. Find

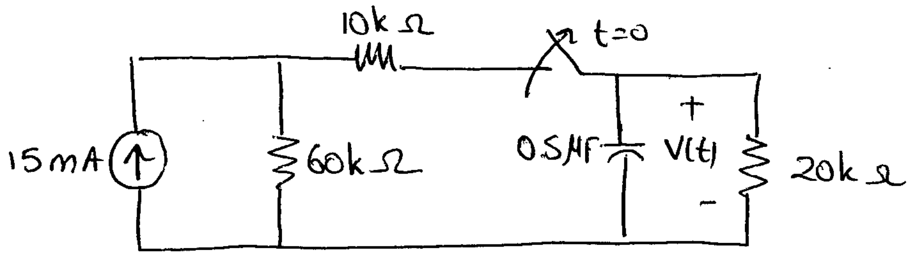
(a) the initial value of $v(t)$

(b) the time constant for $t>0$

(c) the numerical expression for $v(t)$ after the switch has been opened

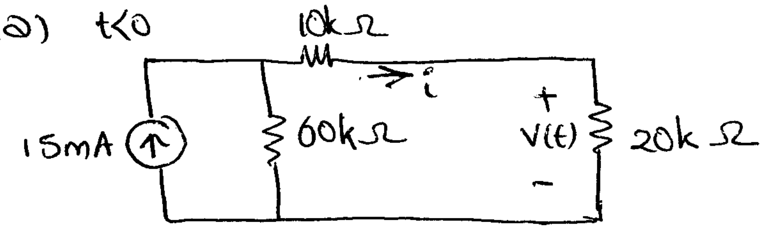
(d) the initial energy stored in the capacitor

(e) the length of time required to dissipate 75% of the initial stored energy.



Solution:

(a) $t<0$



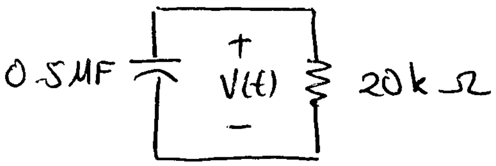
$$i = 15\times10^{-3}\ \frac{60\times10^{3}}{60\times10^{3}+10\times10^{3}+20\times10^{3}}$$

$$i = 10\times10^{-3}\ A$$

$$v(t) = 20k\, i = (20\times10^{3})(10\times10^{-3}) = 200\ V$$

$$v(0^{+}) = 200\ V$$

(b) $t>0$



$$\tau = RC = (20\times10^{3})(0.5\times10^{-6}) = 0.01\ s$$

$$\tau = 10\ ms$$

(c) $v(t) = Ke^{-t/RC}$ V $t>0$

$v(t) = Ke^{-t/0.01}$ V $t>0$

$v(t) = Ke^{-100t}$ V $t>0$

$v(0^+) = v(0^-) = 200 = Ke^{-0} \Rightarrow K = 200$

$v(t) = 200e^{-100t}$ V $t>0$

(d). $w_0 = \frac{1}{2} C v^2(0)$

$= \frac{1}{2}(0.5\times10^{-6})(200)^2$

$= 0.01$

$= 10\ mJ$

(e) $w_0 - 0.75w_0 = \frac{1}{2} C\, v^2(t)$

$10\times10^{-3} - 0.75\times10\times10^{-3} = \frac{1}{2}(0.5\times10^{-6})(200e^{-100t})^2$

$2.5\times10^{-3} = 10\times10^{-3}e^{-200t}$

$e^{-200t} = 0.25$

$\ln(e^{-200t}) = \ln(0.25)$

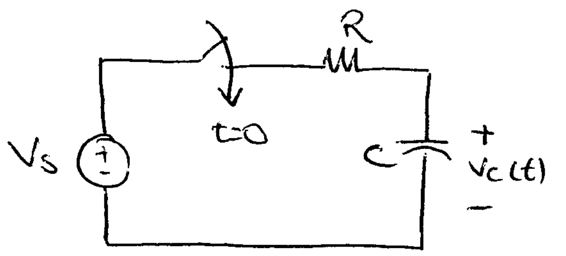
$-200t = \ln(0.25)$

$t = \frac{\ln(0.25)}{-200}$

$t = 6.93\ ms$

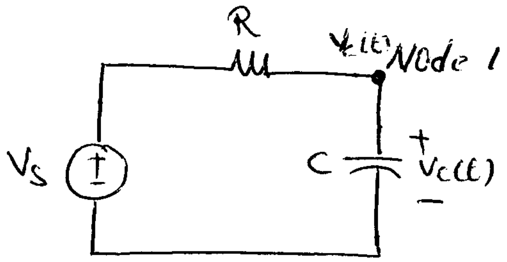
Charging a capacitor from a DC source

Consider the following circuit.



We assume that $V_c(0^-) = V_i$

$t \geq 0$



KCL at node 1:

$$C\frac{dV_c(t)}{dt} + \frac{V_c(t) - V_s}{R} = 0$$

$$RC\frac{dV_c(t)}{dt} + V_c(t) = V_s \qquad (1)$$

The above equation's solution:

$$V_c(t) = K_1 + K_2\, e^{st} \qquad (2)$$

in which K_1, K_2 and s are constants to be determined.

Substitute (2) into (1)

$$RC\frac{d}{dt}\left(K_1 + K_2\, e^{st}\right) + \left(K_1 + K_2\, e^{st}\right) = V_s$$

$$RCK_2se^{st} + K_1 + K_2e^{st} = V_s$$

$$\underbrace{(RCs+1)}_{0} K_2 e^{st} + K_1 = V_s$$

$$RCs + 1 = 0 \implies s = -1/RC$$

$$K_1 = V_s$$

$$V_c(t) = V_s + K_2 e^{-t/RC}$$

Now we use initial conditions ($V_c(0^-) = V_i$) to find K_2.

$$V_c(0^+) = V_c(0^-) = V_i = V_s + K_2 e^{-0/RC}$$

$$V_s + K_2 = V_i$$

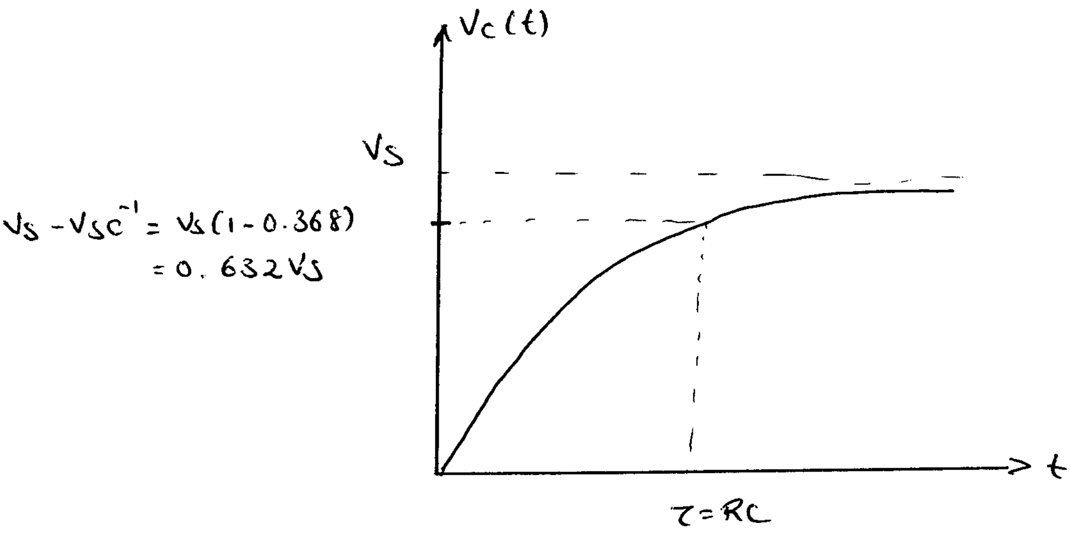
$$K_2 = V_i - V_s$$

$$V_c(t) = V_s + (V_i - V_s) e^{-t/RC} \qquad t > 0$$

If $V_c(0^+) = V_c(0^-) = 0$

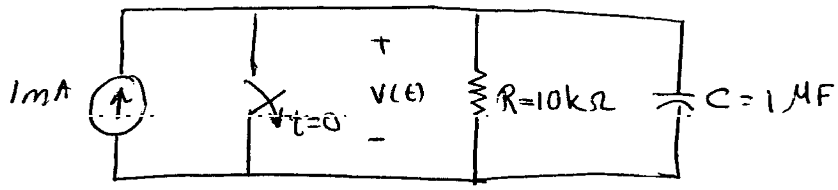
$$V_c(t) = V_s - V_s e^{-t/RC} \qquad t > 0$$

The second term on the right-hand side is called the transient response. The first term on the right hand side is steady-state response.



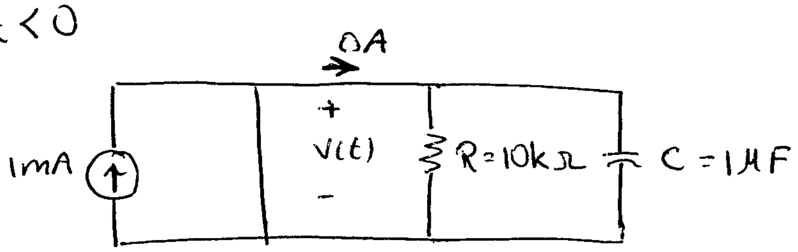
P4.5

The switch for the following circuit opens at $t=0$. Find an expression for $v(t)$ and sketch to scale versus time.



Solution:

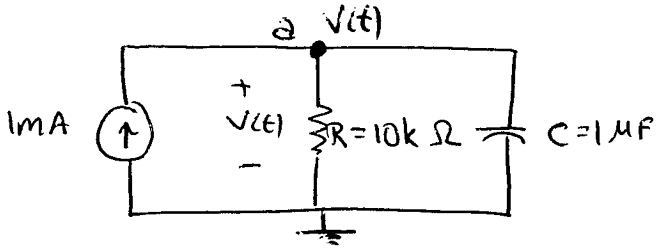
$t<0$



$V(t)=0 \qquad t<0$

$V(0^-)=0$

$t>0$



KCL at node a:

$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} - 1\times10^{-3} = 0$$

$$RC\frac{dv(t)}{dt} + v(t) = 1\times10^{-3}R$$

$$(10\times10^{3})(1\times10^{-6})\frac{dv(t)}{dt} + v(t) = (1\times10^{-3})(10\times10^{3})$$

$$0.01\frac{dv(t)}{dt} + v(t) = 10 \qquad (1)$$

$$v(t) = K_1 + K_2e^{st}$$

$$S = \frac{-1}{RC}$$

$$S = \frac{-1}{(10 \times 10^{3})(1 \times 10^{-6})}$$

$$S = -100$$

$$v(t) = K_1 + K_2 e^{-100t} \qquad (2)$$

Substitute (2) into (1)

$$0.01 \frac{d}{dt}\left(K_1 + K_2 e^{-100t}\right) + \left(K_1 + K_2 e^{-100t}\right) = 10$$

$$0.01 (K_2)(-100)\, e^{-100t} + K_1 + K_2 e^{-100t} = 10$$

$$-K_2 \cancel{e^{-100t}} + K_1 + K_2 \cancel{e^{-100t}} = 10$$

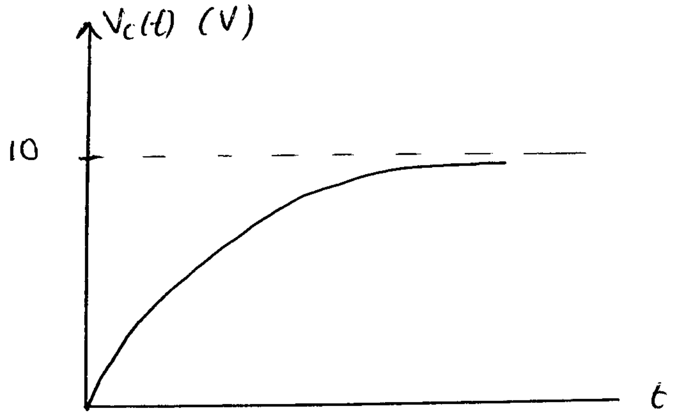
$$K_1 = 10$$

$$V_c(t) = 10 + K_2 e^{-100t}$$

$$V_c(0^+) = V_c(0^-) = 0 = 10 + K_2 e^{-0}$$

$$K_2 = -10$$

$$V_c(t) = 10 - 10 e^{-100t} \ \text{V} \qquad t \geq 0$$



DC Steady State

The response that exist a long time after the switch has taken place is called the steady-state response. For DC sources, the steady state current and voltages are constant.

Current through a capacitance

$$i_c(t) = C \frac{dV_c(t)}{dt}$$

If $V_c(t) =$ constant , $i_c(t) = 0$

For steady state conditions with dc sources, capacitances behaves as open circuits.

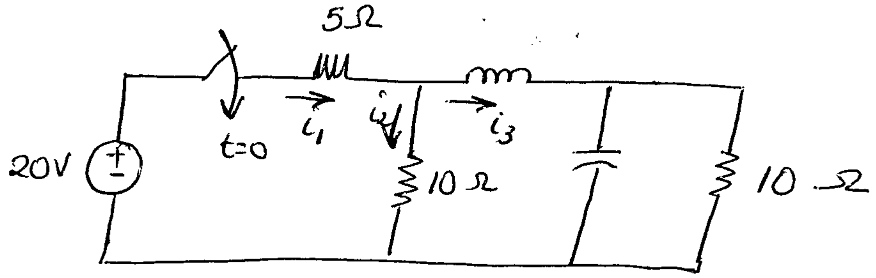
Similarly for an inductance

$$V_L(t) = L \frac{di_L(t)}{dt}$$

If $i_L(t) =$ constant , $V_L(t) = 0$

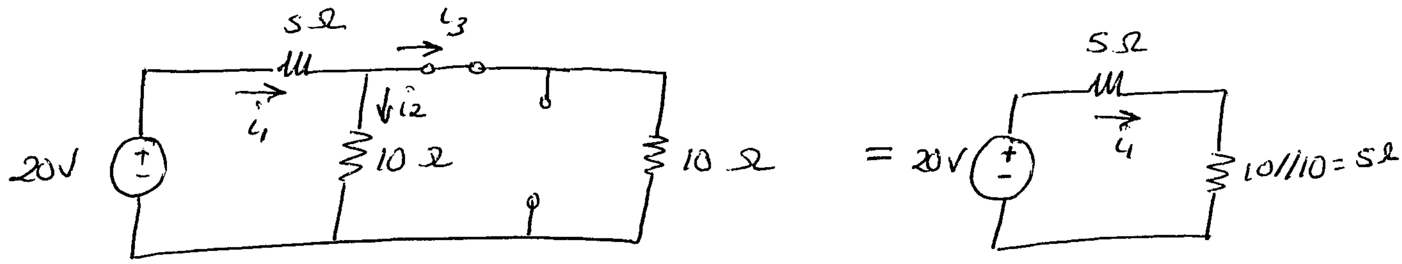
For steady state conditions with dc sources, inductances behaves as short circuit.

Exercise : Find i_1, i_2 and i_3 for $t >> 0$.



Solution :

$t >> 0$: steady state



$$i_1 = \frac{20}{10} = 2\ A$$

$$i_2 = i_1 \frac{10}{10+10} =$$

$$= 2 \frac{10}{20}$$

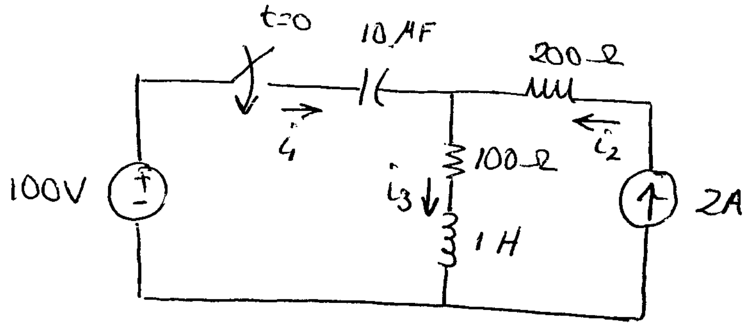
$$= 1\ A$$

$$i_3 = 2 \frac{10}{10+10}$$

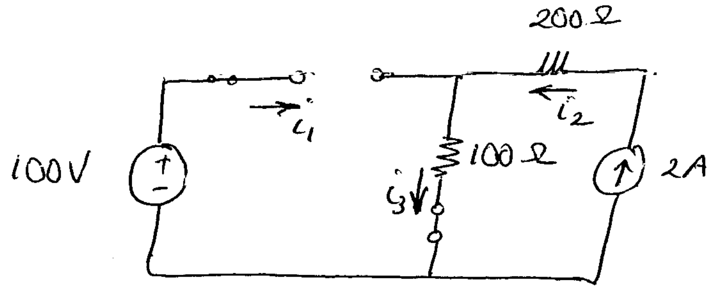
$$= 2 \frac{10}{20}$$

$$= 1\ A.$$

P4.10 Find the steady state values of i_1, i_2 and i_3.



solution:

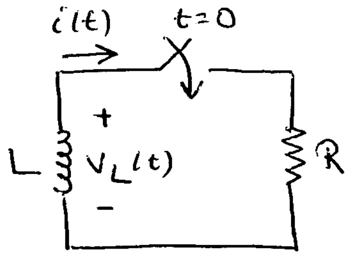


$$i_1 = 0$$

$$i_2 = i_3 = 2\ A$$

RL Circuits

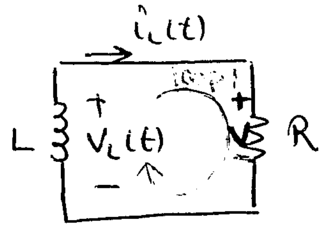
Consider the following circuit



Assume that

$i(0^-) = i_0$

$t>0$



KVL for loop 1 :

$-V_L(t) + V = 0$

$$V_L(t) = -L\frac{di_L(t)}{dt}$$

$V = Ri_L(t)$

$$L\frac{di_L(t)}{dt} + Ri_L(t) = 0$$

$$\frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = 0 \qquad (1)$$

The solution of (1) :

$$i_L(t) = Ke^{st} \qquad (2)$$

in which K and s are constants to be determined.

Substitute (2) into (1)

$$\frac{L}{R}\frac{d}{dt}\left(Ke^{st}\right) + Ke^{st} = 0$$

$$\frac{L}{R}Kse^{st} + Ke^{st} = 0$$

$$\left(\frac{L}{R}s + 1\right)\underbrace{Ke^{st}}_{\neq 0} = 0$$

$$\frac{L}{R}s + 1 = 0$$

$$s = -\frac{R}{L} \quad \Rightarrow \tau = \frac{L}{R}$$ Time constant of inductor

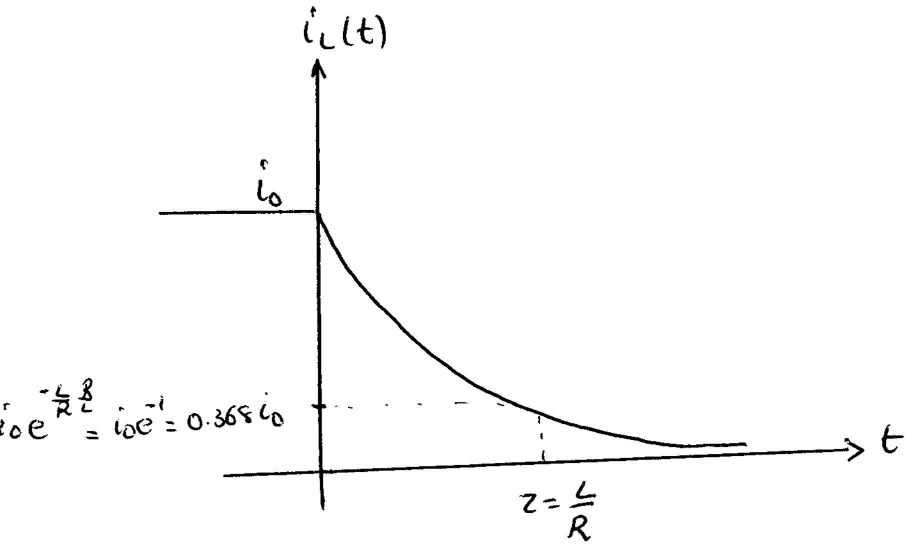
$$i_L(t) = Ke^{-tR/L}$$

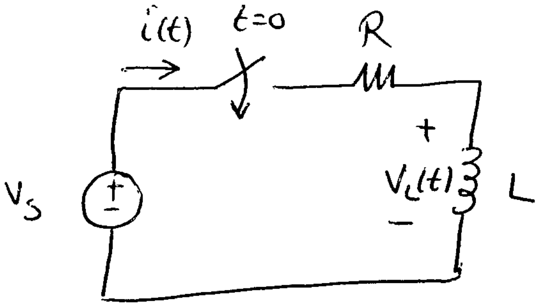
The inductor current must be continuous

$$i_L(0^+) = i_L(0^-) = i_0 = Ke^{-0}$$

$$K = i_0$$

$$i_L(t) = i_0 e^{-tR/L} \quad A, \quad t > 0$$

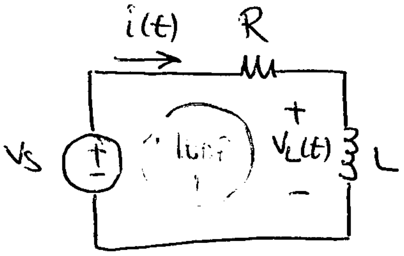




We assume that $i(t) = 0$, $t < 0$

After the switch is closed, the current increases in valve eventually reaching a steady-state value.

$t > 0$



KVL for loop 1:

$$-V_S + Ri(t) + V_L(t) = 0 \quad (1)$$

$$V_L(t) = L\frac{di(t)}{dt} \quad (2)$$

substitute (2) into (1)

$$-V_S + Ri(t) + L\frac{di(t)}{dt} = 0$$

$$\frac{L}{R}\frac{di(t)}{dt} + i(t) = \frac{V_S}{R} \quad (3)$$

The above equation's solution:

$$i(t) = K_1 + K_2 e^{st} \quad (4)$$

Substitute (4) into (3)

$$\frac{L}{R}\frac{d}{dt}\left(K_1+K_2 e^{st}\right)+\left(K_1+K_2 e^{st}\right)=\frac{V_s}{R}$$

$$\frac{L}{R}K_2 s e^{st}+K_1+K_2 e^{st}=\frac{V_s}{R}$$

$$K_1+\left(\frac{L}{R}s+1\right)K_2 e^{st}=\frac{V_s}{R}$$

$$\frac{L}{R}s+1=0 \implies s=-\frac{R}{L}$$

$$K_1=\frac{V_s}{R}$$

$$i(t)=\frac{V_s}{R}+K_2 e^{-tR/L}$$

Now we use initial conditions ($i(0^-)=i_0$) to find K_2

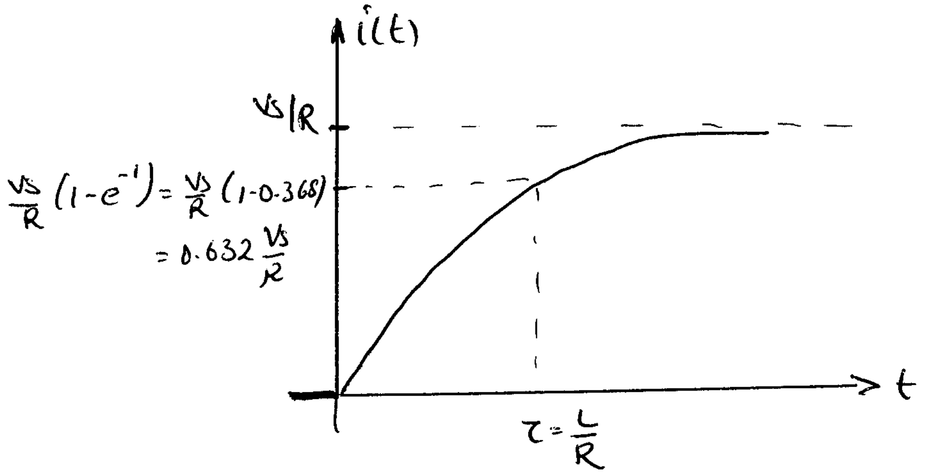
$$i(0^+)=i(0^-)=i_0=\frac{V_s}{R}+K_2 e^{-0}$$

$$K_2=i_0-\frac{V_s}{R}$$

$$i(t)=\frac{V_s}{R}+\left(i_0-\frac{V_s}{R}\right)e^{-tR/L} \qquad t>0$$

If $i(0^+)=i(0^-)=0$

$$i(t)=\frac{V_s}{R}-\frac{V_s}{R}e^{-tR/L} \qquad t>0$$



Question: The switch for the following circuit has been closed for a long time and is opened at $t=0$

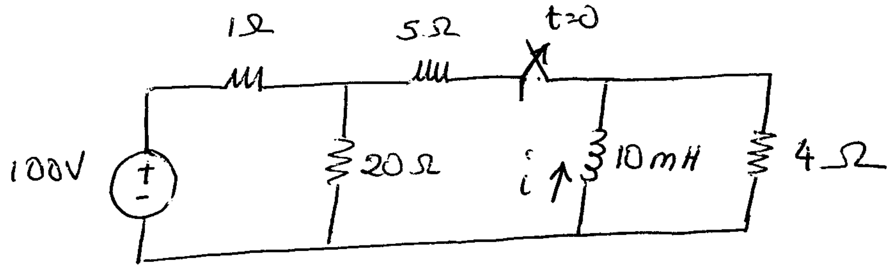
(a) Calculate the initial value of i.

(b) What is the initial energy stored in the inductor?

(c) What is the time constant of the circuit for $t>0$?

(d) What is $i(t)$ for $t \geq 0$?

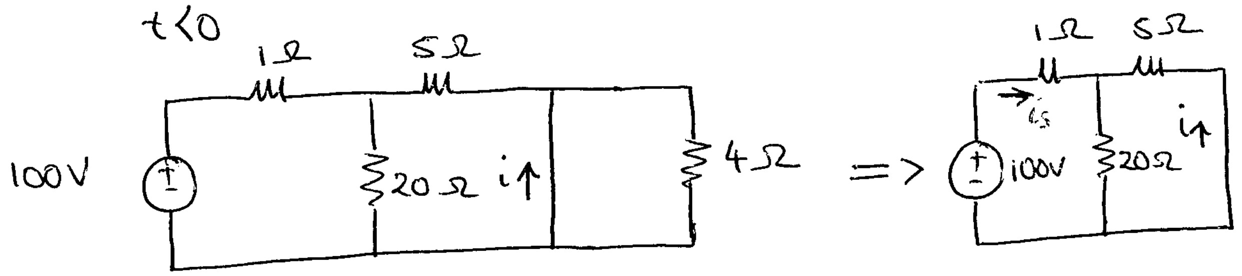
(e) What percentage of the initial energy stored has been dissipated in 4Ω resistor 5 ms after the switch has been opened?

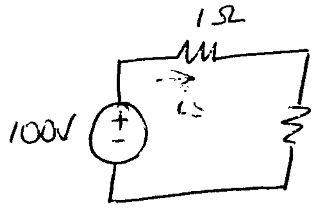


Solution:

(a) $i(0) = ?$

$t<0$





$20 // 5 = \frac{(20)(5)}{20+5} = 4\Omega$

$i_s = \frac{100}{1+4} = 20\text{ A}$

$i = -i_s \cdot \frac{20}{20+5} = -20 \frac{20}{20+5} = -16\text{ A}$

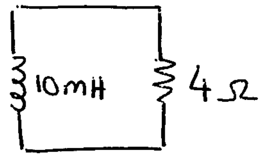
$i(0) = -16\text{ A}$

(b) $W_0 = \frac{1}{2} L \, i(0)^2$

$= \frac{1}{2} (10 \times 10^{-3})(-16)^2$

$= 1.28 \text{ J}$

(c) $t > 0$



$\tau = \frac{L}{R} = \frac{10 \times 10^{-3}}{4} = 2.5 \times 10^{-3} = 2.5 \text{ ms}$

(d) $i(t) = i_0 e^{-tR/L}$

$i(t) = -16 \, e^{-t/2.5 \times 10^{-3}}$

$i(t) = -16 \, e^{-400t} \text{ A} \quad t \geq 0$

(e) $w(t) = \frac{1}{2} L \, i(t)^2$

$w(5 \times 10^{-3}) = \frac{1}{2} (10 \times 10^{-3}) \, i(5 \times 10^{-3})^2$

$= \frac{1}{2} (10 \times 10^{-3}) \left(-16 \, e^{-400(5 \times 10^{-3})}\right)^2$

$= \frac{1}{2} (10 \times 10^{-3}) (-2.165)^2$

$= 0.0234 \text{ J}$

$w_0 \qquad w(5 \times 10^{-3})$

$100 \qquad x$

$x = \frac{100 \times w(5 \times 10^{-3})}{w_0} = \frac{100 \times 0.0234}{1.28} = 1.83 \%$

Dissipated : $100 - 1.83 = 98.17 \%$

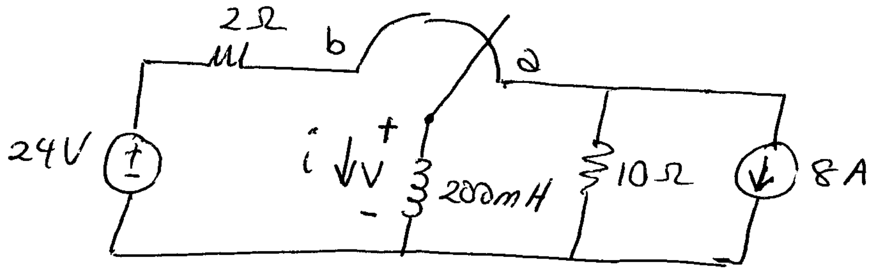
Home Exercise:

The switch in the circuit has been in position a for a long time. At $t=0$, the switch moves from position a to position b.

(a) Find the expression for $i(t)$ for $t \geq 0$.

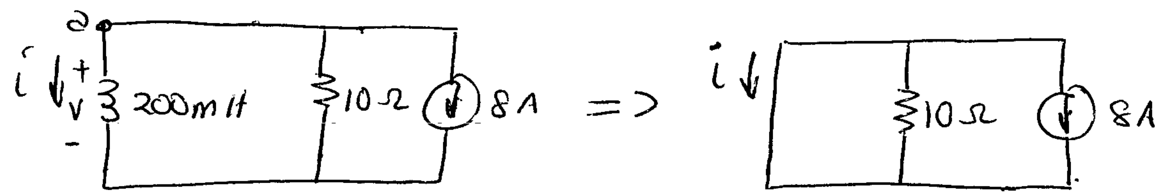
(b) What is the initial voltage across the inductor just after the switch has been moved to position b?

(c) How many milliseconds after the switch has been moved to position b does the inductor voltage equal 24 V?



Solution:

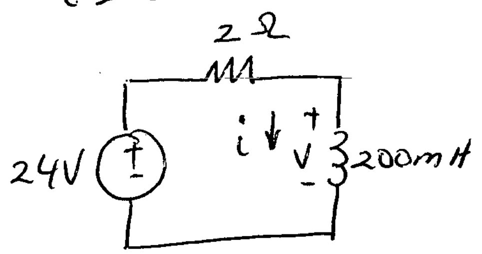
(a) $t<0$



$i = -8A \quad t<0$

$i(0^-) = -8A$

$t>0$



$$i(t) = \frac{V_S}{R} + \left(i(0^+) - \frac{V_S}{R} \right) e^{-tR/L} \quad A \quad t \geq 0$$

$$= \frac{24}{2} + \left(-8 - \frac{24}{2} \right) e^{-t\, 2/200\times10^{-3}} \quad A \quad t \geq 0$$

$$= 12 + (-8 - 12)\, e^{-10t} \quad A \quad t \geq 0$$

$$= 12 - 20\, e^{-10t} \quad A \quad t \geq 0$$

(b) $$v(t) = L \frac{di(t)}{dt}$$

$$= (200\times10^{-3}) \frac{d}{dt} \left(12 - 20 e^{-10t} \right) \quad V,\ t \geq 0$$

$$= (200\times10^{-3})(-20)(-10)\, e^{-10t} \quad V,\ t \geq 0$$

$$= 40 e^{-10t} \quad V,\ t \geq 0$$

$$v(0^+) = 40\, e^{-10(0)} = 40\ V$$

(c) $$v(t) = 40 e^{-10t} \quad V \quad t \geq 0$$

$$24 = 40 e^{-10t}$$

$$e^{-10t} = \frac{24}{40}$$

$$\ln(e^{-10t}) = \ln\left(\frac{24}{40}\right)$$

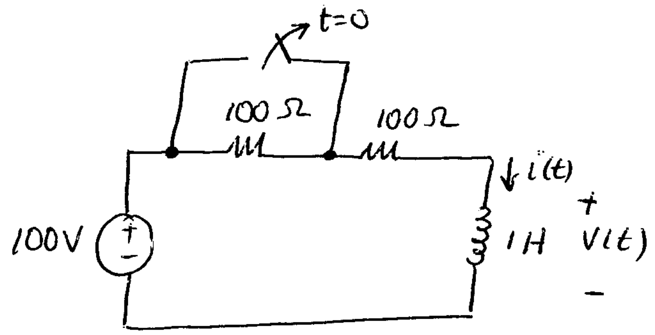
$$-10t(\ln e) = \ln\left(\frac{24}{40}\right)$$

$$t = \frac{\ln(24/40)}{-10}$$

$$t = 51.08\ ms$$

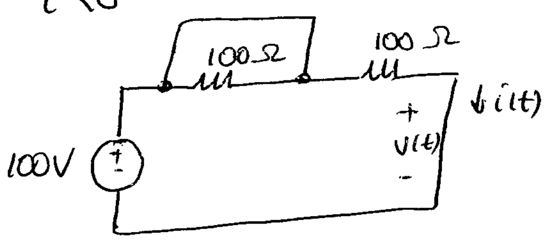
Exercise 4-6

Consider the following circuit. Assume that the switch has been closed for a very long time prior to $t=0$. Find expressions for $i(t)$ and $v(t)$.



Solution:

$t < 0$



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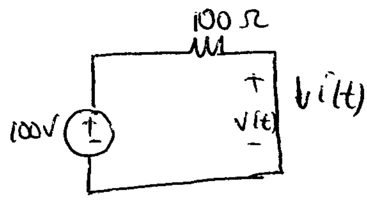
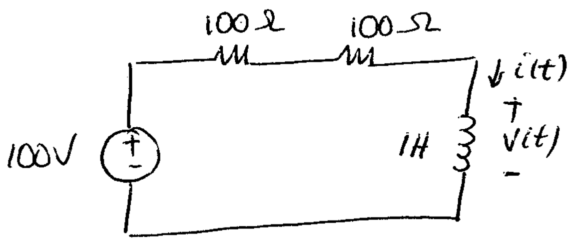


$i(t) = \frac{100}{100} = 1\ A \quad t<0$

$v(t) = 0\ V \quad t<0$

$t > 0$



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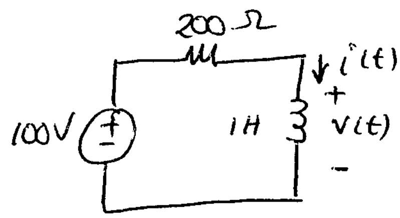


$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-tR/L} \quad A, \quad t \geq 0$$

$$= \frac{100}{200} + \left(1 - \frac{100}{200}\right) e^{-t\,200/1}$$

$$= 0.5 + 0.5\, e^{-200t} \quad A \quad t \geq 0$$

$$V(t) = L \frac{di(t)}{dt}$$

$$= 1 \frac{d}{dt}\left(0.5 + 0.5\, e^{-200t}\right)$$

$$= +0.5(-200)\, e^{-200t}$$

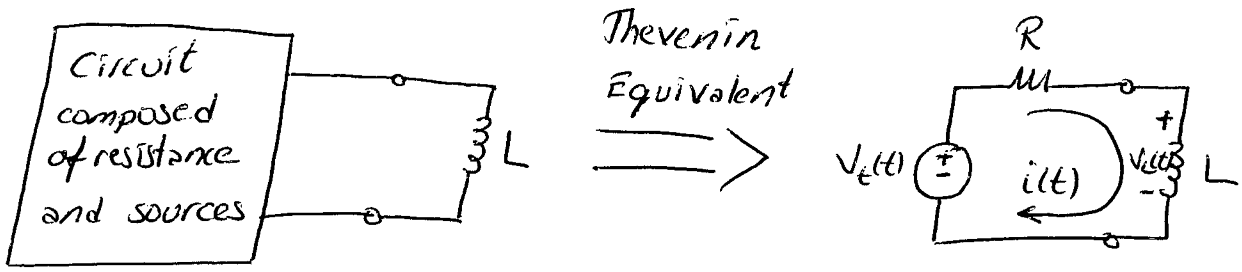
$$= -100 e^{-200t} \quad V \quad t \geq 0$$

$$i(t) = \begin{cases} 1 \text{ A} & , t < 0 \\ 0.5 + 0.5 e^{-200t} \text{ A} & , t \geq 0 \end{cases}$$

$$v(t) = \begin{cases} 0 \text{ V} & , t < 0 \\ -100 e^{-200t} \text{ V} & , t \geq 0 \end{cases}$$

RC and RL Circuits with General Sources

Consider the following circuit.



KVL : $\quad -V_t(t) + Ri(t) + V_L(t) = 0$

$$V_L(t) = L\frac{di(t)}{dt}$$

$$-V_t(t) + Ri(t) + L\frac{di(t)}{dt} = 0$$

$$\frac{L}{R}\frac{di(t)}{dt} + i(t) = \frac{V_t(t)}{R}$$

In general, the equation for any circuit containing one inductance or one capacitance can be put into the form

$$\tau\frac{dx(t)}{dt} + x(t) = f(t)$$

Time constant (τ)

$f(t)$ → Forcing function (If we have a circuit without sources $f(t)=0$. For DC sources $f(t)=$ constant)

Solution of the differential equation:

General solution consists two parts : $x(t) = x_p(t) + x_c(t)$

(1) Particular solution (force response) $x_p(t)$ is any expression that satisfies

$$\tau\frac{dx_p(t)}{dt} + x_p(t) = f(t)$$

(2) Complementary solution (Natural response) $x_c(t)$ is the solution of the homogeneous equation

$$\tau \frac{dx_c(t)}{dt} + x_c(t) = 0$$

$$\frac{dx_c(t)/dt}{x_c(t)} = -\frac{1}{\tau}$$

Integrating both sides

$$\int \frac{dx_c(t)}{x_c(t)} = \int -\frac{1}{\tau}\, dt$$

$$\ln(x_c(t)) = -\frac{t}{\tau} + c \leftarrow \text{the constant of integration}$$

$$x_c(t) = e^{\left(-\frac{t}{\tau}+c\right)}$$

$$= e^{c}\, e^{-t/\tau} \qquad K = e^{c}$$

$$= K e^{-t/\tau}$$

Step-by-step Solution

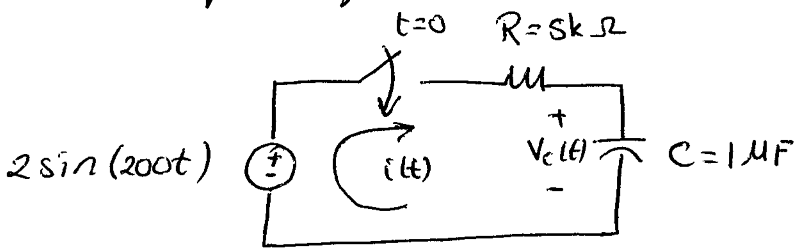
1- Write the circuit equation and reduce it to a first order differential equation.

2- Find a particular solution depending on the form of forcing function

3- Obtain the complete solution by adding the particular solution to the complementary solution which contains the arbitrary constant

4- Use initial conditions to find K.

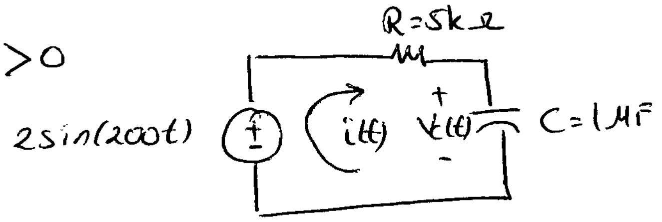
Example: Solve for the current in the following circuit. The capacitor is initially charged so that $V_c(0^+) = 1V$



Solution:

step 1: obtain first order derivative equation.

$t>0$



KVL : $\quad -2\sin(200t) + Ri(t) + V_c(t) = 0 \qquad (1)$

$$V_c(t) = \frac{1}{C}\int_0^t i(t)\,dt + V_c(0) \qquad (2)$$

Substitute (2) into (1)

$$-2\sin(200t) + R\,i(t) + \frac{1}{C}\int_0^t i(t)\,dt + V_c(0) = 0 \qquad (3)$$

Take the derivative of each term in (3)

$$-2(200)\cos(200t) + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = 0$$

$$RC\frac{di(t)}{dt} + i(t) = 400\,C\,\cos(200t)$$

$$(5\times10^{3})(1\times10^{-6})\frac{di(t)}{dt} + i(t) = (400)\times(1\times10^{-6})\cos(200t)$$

$$5\times10^{-3}\frac{di(t)}{dt} + i(t) = 400\times10^{-6}\cos(200t) \qquad (4)$$

Step 2: Find $i_p(t)$

Since the derivatives of $\sin(200t)$ and $\cos(200t)$ are $200\cos(200t)$ and $-200\sin(200t)$ respectively we try a particular solution of the form

$$i_p(t) = A\cos(200t) + B\sin(200t) \qquad (5)$$

Substitute (5) into (4)

$$5\times10^{-3}\frac{d}{dt}\left(A\cos(200t) + B\sin(200t)\right) + A\cos(200t) + B\sin(200t) = 400\times10^{-6}\cos(200t)$$

$$5\times10^{-3}\left(-A\,200\sin(200t) + B\,200\cos(200t)\right) + A\cos(200t) + B\sin(200t) = 400\times10^{-6}\cos(200t)$$

$$(-A+B)\sin(200t) + (B+A)\cos(200t) = 400\times10^{-6}\cos(200t) \qquad (6)$$

LHS should be equal to RHS for (6)

$$-A+B = 0$$
$$B+A = 400\times10^{-6}$$
$$+ \overline{\quad 2B = 400\times10^{-6} \quad} \Longrightarrow B = 200\times10^{-6} = 200\ \mu A$$
$$A = 200\ \mu A$$

Substitute A and B in (5)

$$i_p(t) = 200\cos(200t) + 200\sin(200t)\ \mu A$$

Step 3: Find $\hat{i}_c(t)$ and add it to $\hat{i}_p(t)$

$$RC\frac{d\hat{i}_c(t)}{dt}+\hat{i}_c(t)=0$$

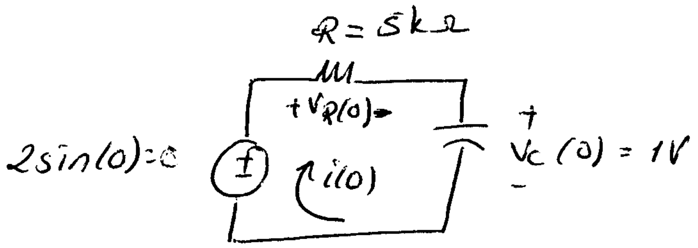
$$\hat{i}_c(t)=Ke^{-t/RC}$$

$$\hat{i}(t)=\hat{i}_p(t)+\hat{i}_c(t)$$

$$=200\cos(200t)+200\sin(200t)+Ke^{-t/RC}$$

Step 4: Find K by initial conditions.

The voltages and currents immediately after the switch closes are shown below.



KVL : $V_R(0^+)+V_c(0^+)-0=0$

$$V_R(0^+)=-V_c(0^+)=-1$$

$$\hat{i}(0^+)=\frac{V_R(0^+)}{R}=\frac{-1}{5000}=-200\ \mu A$$

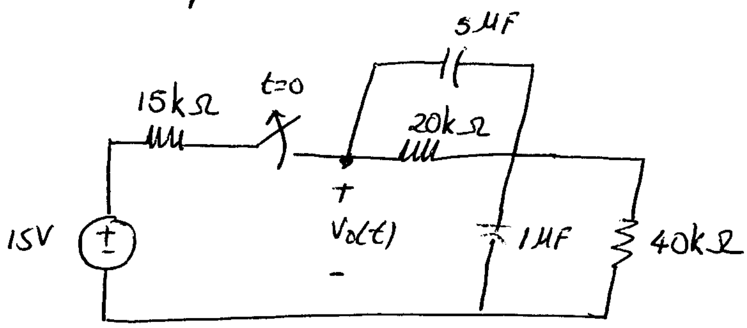
$$\hat{i}(0^+)=-200=200\cos(200(0))+200\sin(200(0))+Ke^{-0}$$

$$-200=200+K$$

$$K=-400\ \mu A$$

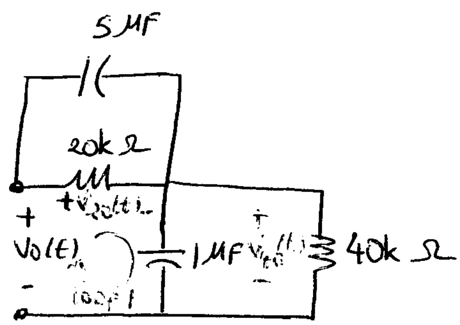
$$\hat{i}(t)=200\cos(200t)+200\sin(200t)-400e^{-t/RC}\quad \mu A$$

Question: The switch in the following circuit has been closed for a long time before being opened at $t=0$. Find $V_o(t)$ for $t \geq 0$.



Solution:

$t \geq 0$



KVL for loop1:

$$-V_o(t) + V_{20}(t) + V_{40}(t) = 0 \qquad t \geq 0$$

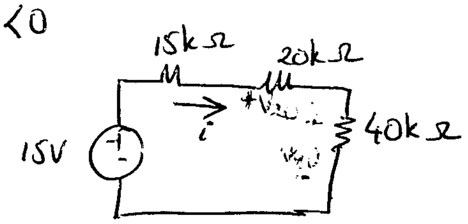
$$V_o(t) = V_{20}(t) + V_{40}(t)$$

$$V_{20}(t) = V_{20}(0)\, e^{-t/(5\times10^{-6})(20\times10^{3})} = V_{20}(0)\, e^{-10t} \qquad t \geq 0$$

$$V_{40}(t) = V_{40}(0)\, e^{-t/(1\times10^{-6})(40\times10^{3})} = V_{40}(0)\, e^{-25t} \qquad t \geq 0$$

$$V_o(t) = V_{20}(0)\, e^{-10t} + V_{40}(0)\, e^{-25t} \qquad t \geq 0$$

$t < 0$



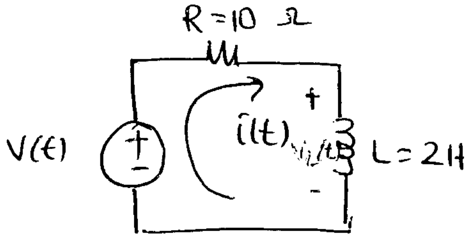
$$i = \frac{15}{(15+20+40)\times10^{3}} = 0.2\,mA$$

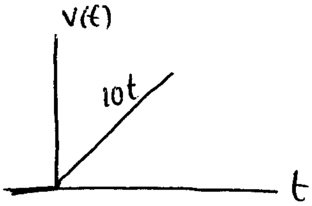
$V_{20} = 20k\,i = (20k)(0.2m) = 4V \implies V_{20}(0) = 4V$

$V_{40} = 40k\,i = (40k)(0.2m) = 8V \implies V_{40}(0) = 8V$

$$V_o(t) = 4e^{-10t} + 8e^{-25t} \qquad t \geq 0$$

P4.23 The voltage source shown below is called a ramp function. Assume that $i(0)=0$. Write the differential equation for $i(t)$ and find the complete solution. (Hint: Try $i_p(t)=A+Bt$)





Solution:

KVL : $-V(t)+Ri(t)+V_L(t)=0$

$$V_L(t)=L\frac{di(t)}{dt}$$

$$-V(t)+Ri(t)+L\frac{di(t)}{dt}=0$$

$$\frac{L}{R}\frac{di(t)}{dt}+i(t)=\frac{V(t)}{R}$$

$$\frac{2}{10}\frac{di(t)}{dt}+i(t)=\frac{10t}{10}$$

$$0.2\frac{di(t)}{dt}+i(t)=t \qquad (1)$$

$$i(t)=i_p(t)+i_c(t)$$

$$i_p(t)=A+Bt \qquad (2)$$

substitute (2) into (1)

$$0.2\frac{d}{dt}(A+Bt)+(A+Bt)=t$$

$$0.2B+A+Bt=t$$

$$0.2B+A=0$$

$$B=1 \quad \Rightarrow A=-0.2$$

$$i_p(t)=-0.2+t$$

$$i_c(t) = K e^{-tR/L}$$

$$i_c(t) = K e^{-t\,10/2}$$

$$i_c(t) = K e^{-5t}$$

$$i(t) = i_p(t) + i_c(t)$$

$$i(t) = -0.2 + t + K e^{-5t}$$

$$i(0^+) = i(0^-) = 0 = -0.2 + 0 + K e^{-0}$$

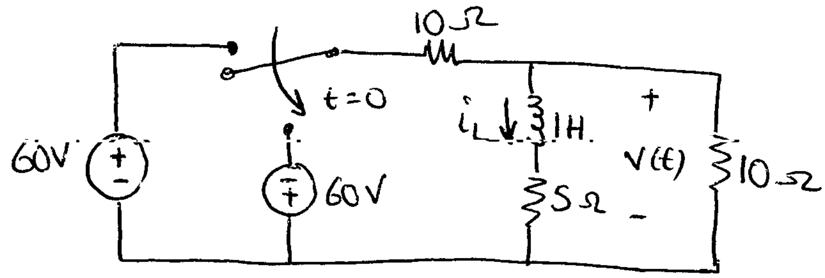
$$K = 0.2$$

$$i(t) = -0.2 + t + 0.2 e^{-5t} \quad A \qquad t > 0$$

$$i(t) = \begin{cases} 0\,A & , \; t < 0 \\ -0.2 + t + 0.2 e^{-5t} \quad A & , \; t > 0 \end{cases}$$

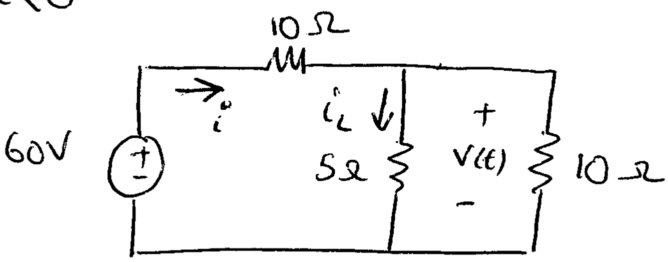
Question: (Midterm II - 1996)

The circuit is in steady state at $t=0^-$. The switch changes position at $t=0$. Find $i_L(t)$ for $t>0$. and $v(t)$ for $t>0$.



Solution:

$t<0$



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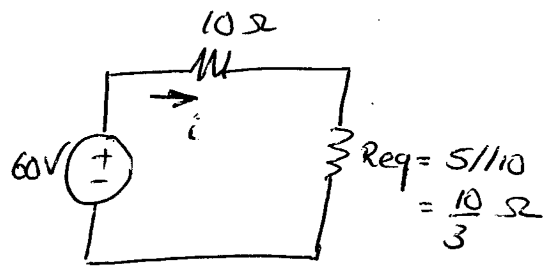
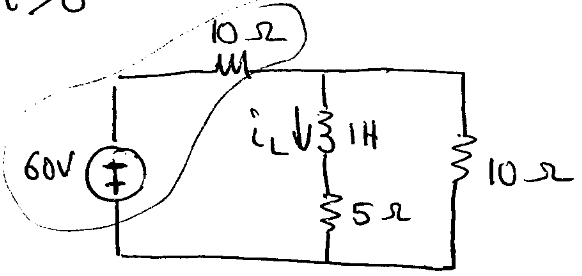


$$i = \frac{60}{10+\frac{10}{3}} = 4.5\,A$$

$$i_L = i\,\frac{10}{10+5} = 4.5\,\frac{10}{15} = 3\,A \quad t<0 \quad \Rightarrow \quad i_L(0) = 3A.$$

$t>0$



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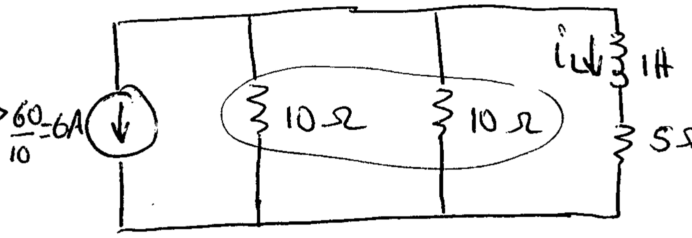

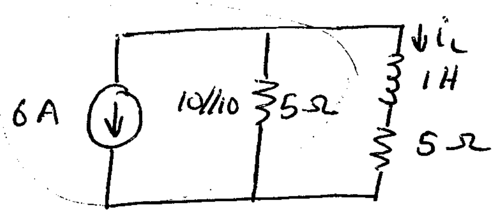

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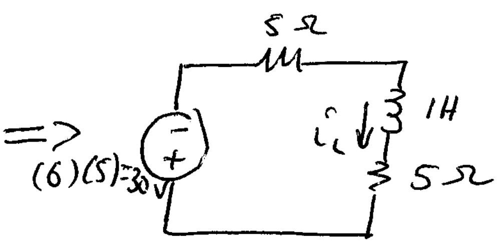
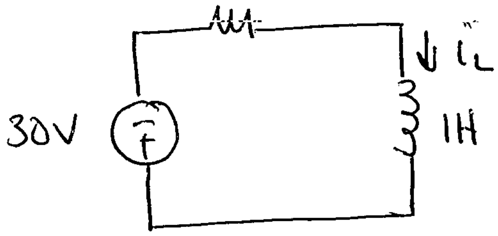


$5+5 = 10\,\Omega$

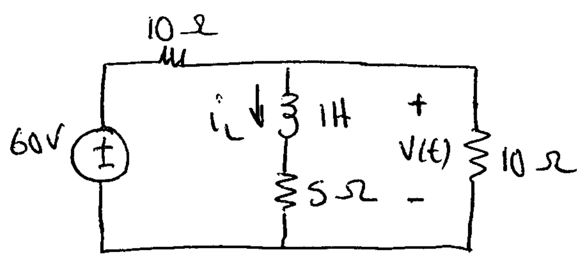


$$i_L(t) = \frac{V_s}{R} + \left(i_L(0) - \frac{V_s}{R}\right) e^{-tR/L} \quad A$$

$$i_L(t) = \frac{-30}{10} + \left(3 - \left(\frac{-30}{10}\right)\right) e^{-t\,10/1}$$

$$i_L(t) = -3 + (3+3)e^{-10t}$$

$$i_L(t) = -3 + 6e^{-10t} \quad A, \quad t > 0$$



$$V(t) = V_L(t) + V_R(t)$$

$$= L\frac{di_L(t)}{dt} + 5\,i_L(t)$$

$$= 1\,\frac{d}{dt}\left(-3 + 6e^{-10t}\right) + 5\left(-3 + 6e^{-10t}\right)$$

$$= -60\,e^{-10t} - 15 + 30\,e^{-10t}$$

$$= -15 - 30\,e^{-10t} \quad V, \quad t > 0$$

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