

Problem 1

i) SMP is a matching problem. This problem aims to find stable couples between finite number of men (n) and finite number of women (n) according to their preferences order. A couple called stable if there are no two people of opposite sex who would both rather have each other than their current partners.

- Input : A set of women, men and their preferences.
- Output : Stable marriages.

ii) There are 3 men and 3 women. $[m1, m2, m3, w1, w2, w3]$

$m1$'s list of preferences : $[w1, w2, w3]$

$m2$'s list of preferences : $[w2, w1, w3]$

$m3$'s list of preferences : $[w1, w2, w3]$

$w1$'s list of preferences : $[m2, m1, m3]$

$w2$'s list of preferences : $[m1, m2, m3]$

$w3$'s list of preferences : $[m1, m2, m3]$

stable couples : $m1-w1, m2-w2, m3-w3$

Problem 2

i)

Starts as all men and women to free

while (there exist a unmatched men/women and not proposed)

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{
  w = m's most preferred w which he has not yet proposed
  if (w is free)
    engage (m, w)
  else if (w is engaged before)
    if (w prefers m to current fellow)
      engage (m, w)
      w's old husband became free
    else
      w and current husband stays engaged
}
```

ii) Gale-Shapley algorithm claims that the algorithm works in $O(n^2)$.

As a proof;

Firstly, there is no man proposes to a woman more than once. Worst case is n women proposes to n men which causes n^2 proposes.

Furthermore, every propose takes $O(1)$ time.

According to these complexities, algorithm works in $O(n^2)$.