

Problem 1 (Order statistics)

Cavit Cakir

- (a) I can sort the numbers using merge sort. It will take $(n \log n)$ worst case time. Take out k largest elements to the array which can be done in (k) . Worst case running time is $(n \log n + k) = (n \log n)$ ($i \leq n$).

Recurrence relation for worst case: $T(N) = 2T(N/2) + N$ $a > 1$ and $b > 1$ so we can use master theorem, It is case 2, since $\log_b a$ is 1 and $f(n) = n$ So it is $(n \log n)$

- (b) I can use **Selection Algorithm** which is an algorithm to find the k th smallest number in a list which will be in (n) , partition around k th number in (n) time. After that we have to sort a array which is k sized. If we use merge sort which we used in part a it will be in $(k \log k)$. All computations will be in $(n + k \log k)$.

Recurrence relation for Selection Algorithm; $T(n)$ SELECT(i, n) 1. Divide the n elements into groups of 5. Find the median of each 5-element group by rote.

2. Recursively SELECT the median x of the $n/5$ group medians to be the pivot.

3. Partition around the pivot x . Let $k = \text{rank}(x)$.

4. **if** $i = k$ **then** return x

else if $i < k$

then recursively SELECT the i th smallest element in the lower part

else recursively SELECT the $(i-k)$ th smallest element in the upper part

$$1 = (n)$$

$$2 = T(n/5)$$

$$3 = (n)$$

$$4 = T(3n/4)$$

if we add them up we will get $T(n) = T(3n/4) + T(n/5) + (n)$ which is (n)

I would prefer **method b** according to asymptotic runtime complexity. Only in worst case they will both run in same asymptotic complexity but in all other cases method b will be better. It will take at most $(n + k \log k)$ where $k \leq n$.

Problem 2 (Linear-time sorting)**Cavit Cakir**

- (a) I modified the line when we allocate an array for counting as $\text{count} = [0] * (27)$ because when we are using this count array for integers it was base (characters + 1) so i created it as (26+1) (I assume alphabet as uppercase english letters) Also we were adding 0s at beginning but now we are dealing with strings so we can not add 0's. I modified it as adding ' ' (blanks) to end.
- (b)
- | | |
|----------------------------------|-----------|
| 1 - EGE , SELIN , YASIN , VEYSEL | index = 5 |
| 2 - EGE , VEYSEL , SELIN , YASIN | index = 4 |
| 3 - EGE , SELIN , YASIN , VEYSEL | index = 3 |
| 4 - EGE , SELIN , YASIN , VEYSEL | index = 2 |
| 5 - YASIN , SELIN , VEYSEL , EGE | index = 1 |
| 6 - EGE , SELIN , VEYSEL , YASIN | index = 0 |
- (c) When we were dealing with integers, our running time was $O(nk)$ where n input size, k is longest digit number but now we are sorting strings. So our algorithms will do same computations for strings, then asymptotic runtime complexity will be $O(nl)$ where l is lenght of the longest string.