CS 404 – Artificial Intelligence
HW 4 – 2020 Spring

150pt

Objective: Learning about Propositional Logic

1) 20 pt - Decide whether each of the following sentences is valid, unsatisfiable, or satisfiable (not valid, but only satisfiable). **Show how you come to that decision using truth table enumeration or logical equivalence rules.**

- Smoke => Smoke 10pt

Smoke	Smoke	Smoke => Smoke
TRUE	TRUE	TRUE
FALSE	FALSE	TRUE

Circle the correct answer: Valid, Satisfiable, Unsatisfiable

- Smoke => Fire 10pt

Smoke	Fire	Smoke => Fire
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	TRUE
FALSE	FALSE	TRUE

Circle the correct answer: Valid, Satisfiable, Unsatisfiable

2) 40pt – Truth Table Enumeration for Inference

Assume we have the knowledge base **KB: Rain ∧ (Rain ⇒ Wet) ∧ (Snow ⇒ Cold)** and the given propositions are the only ones in the KB.

a) **25 pts – How many *possible worlds*** (truth value assignments to the propositions) **are *models* of the KB?** Show your work by filling the truth table for the KB.

In order to be model of KB, that possible world should be true. So there are 3 worlds are models of the KB.

Rain	Wet	Snow	Cold	KB
TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	TRUE	FALSE	FALSE
TRUE	TRUE	FALSE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE
TRUE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	FALSE
TRUE	FALSE	FALSE	FALSE	FALSE
FALSE	TRUE	TRUE	TRUE	FALSE
FALSE	TRUE	TRUE	FALSE	FALSE
FALSE	TRUE	FALSE	TRUE	FALSE
FALSE	TRUE	FALSE	FALSE	FALSE
FALSE	FALSE	TRUE	TRUE	FALSE
FALSE	FALSE	TRUE	FALSE	FALSE
FALSE	FALSE	FALSE	TRUE	FALSE
FALSE	FALSE	FALSE	FALSE	FALSE

b) **15pts – Extend the above truth table** (use truth table enumeration method) to show whether the knowledge base entails α**=Wet.**

State your answer here: Entails because Wet is true whenever KB is true.

Rain	Wet	Snow	Cold	KB
TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE

3) 90pt – AIMA 3rd Ed. Q. 7.2

7.2 (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is homed.

a) **20pt – First state the English paragraph as a set of Propositional Logic sentences. Please use the given two-letter proposition names below.** Be careful about what propositions to use (especially for the the statement "mortal mammal"), considering all of the paragraph.

- P1 = Mythical => Immortal
- P2 = ¬mythical => (¬Immortal ∧ Mammal)
- P3 = (Immortal ∨ Mammal) => Horned
- P4 = Horned => Magical
- KB = P1 ∧ P2 ∧ P3 ∧ P4

b) **20pt – Convert the above KB into Conjunctive Normal Form.** Show your work clearly.

- P1:
  Step 1 -> Mythical => Immortal
  Step 2 -> ¬Mythical ∨ Immortal
  Step 3 -> ¬Mythical ∨ Immortal
  Step 4 -> ¬Mythical ∨ Immortal

- P2:
  Step 1 -> ¬mythical => (¬Immortal ∧ Mammal)
  Step 2 -> mythical ∨ (¬Immortal ∧ Mammal)
  Step 3 -> mythical ∨ (¬Immortal ∧ Mammal)
  Step 4 -> (mythical ∨ ¬Immortal) ∧ (mythical ∨ Mammal)

- P3:
  Step 1 -> (Immortal ∨ Mammal) => Horned
  Step 2 -> ¬(Immortal ∨ Mammal) ∨ Horned
  Step 3 ->(¬Immortal ∧ ¬Mammal) ∨ Horned
  Step 4 -> (¬Immortal ∨ Horned) ∧ (¬Mammal ∨ Horned)

- P4:
  Step 1 -> Horned => Magical
  Step 2 -> ¬ Horned ∨ Magical
  Step 3 -> ¬ Horned ∨ Magical
  Step 4 -> ¬ Horned ∨ Magical

CNF => P1 ∧ P2 ∧ P3 ∧ P4

c) **50pt – See if the KB entails each of the following conclusions: "unicorn is mythical", "unicorn is magical", "unicorn is horned".** Use the indicated method, if available.

For each of them, **please indicate the inference method you are using and clearly indicate your conclusion at the beginning:**

Mythical	Immortal	Mammal	Horned	Magical	KB
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	TRUE	TRUE	FALSE	FALSE
TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
TRUE	TRUE	TRUE	FALSE	FALSE	FALSE
TRUE	TRUE	FALSE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	TRUE	FALSE	FALSE
TRUE	TRUE	FALSE	FALSE	TRUE	FALSE
TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
TRUE	FALSE	TRUE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	TRUE	FALSE	FALSE
TRUE	FALSE	TRUE	FALSE	TRUE	FALSE
TRUE	FALSE	TRUE	FALSE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	FALSE	FALSE	TRUE	FALSE
TRUE	FALSE	FALSE	FALSE	FALSE	FALSE
FALSE	TRUE	TRUE	TRUE	TRUE	FALSE
FALSE	TRUE	TRUE	TRUE	FALSE	FALSE
FALSE	TRUE	TRUE	FALSE	TRUE	FALSE
FALSE	TRUE	TRUE	FALSE	FALSE	FALSE
FALSE	TRUE	FALSE	TRUE	TRUE	FALSE
FALSE	TRUE	FALSE	TRUE	FALSE	FALSE
FALSE	TRUE	FALSE	FALSE	TRUE	FALSE
FALSE	TRUE	FALSE	FALSE	FALSE	FALSE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	TRUE	TRUE	FALSE	FALSE
FALSE	FALSE	TRUE	FALSE	TRUE	FALSE
FALSE	FALSE	TRUE	FALSE	FALSE	FALSE
FALSE	FALSE	FALSE	TRUE	TRUE	FALSE
FALSE	FALSE	FALSE	TRUE	FALSE	FALSE
FALSE	FALSE	FALSE	FALSE	TRUE	FALSE
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

I. **10pts - "unicorn is mythical":**

Conclusion: Using truth table, we can show that we can not conclude that the unicorn is mythical.

Show your work or state your argument:

As we can see by looking to truth table above, there is a modal that KB is true when it is not mythical.

II. **30pt - "unicorn is horned":**

Use either:

- simple resolution: that is apply resolution many times to see whether you can infer Horned, or

- **resolution refutation** (aka proof by contradiction): that is, add the negated form of whatever you want to check for entailment (e.g. ¬Horned), and see if you can reach a contradiction. This would show that the KB+negative is unsatisfiable, hence the conclusion can be inferred).

To show KB |= Horned, we will show that KB ∧ ¬Horned is not satisfiable.
So when we look at to truth table, KB is true for 3 times and whenever KB is true, horned is true too. So when we calculate KB ∧ ¬Horned, we get true and false which is false. So we proved that KB ∧ ¬Horned is not satisfiable.
In conclusion, we can say that KB |= Horned, which means unicorn is horned.

III. **10pt - "unicorn is magical":**

Use Modus Ponens method and state your conclusion.

As proved in part II of question 3, unicorn is Horned.
And we know this modal; P4 : Horned => Magical
With using Modus Ponens method, we can say that unicorn is Magical because Horned is given.