Brady Barlow

ENGR 4333/5333: Digital Signal Processing

HW 1: Ch 3

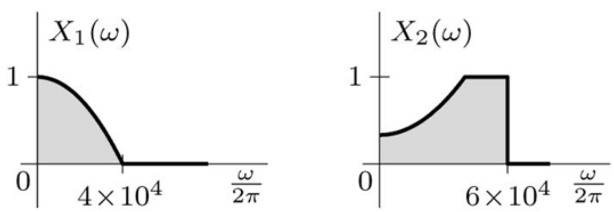
1) A sinusoid of frequency f_0 Hz is sampled at a rate $F_s = 20$ Hz. Find the apparent frequency of the sampled signal if f_0 is
a) 8 Hz **b)** 12 Hz **c)** 21 Hz **d)** 32 Hz

2) A signal $x(t) = 3\cos(6\pi t) + \cos(16\pi t) + 2\cos(20\pi t)$ is sampled at a rate 25% above the Nyquist rate. Sketch the spectrum of the sampled signal. How would you reconstruct $x(t)$ from these samples? If the sampling frequency is 25% below the Nyquist rate, what are the frequencies of the sinusoids present in the output of the filter with cutoff frequency equal to the folding frequency? Do not write the actual output; give just the frequencies of the sinusoids present in the output.

3) The signal $x(t) = \sin(0.7\pi t)\cos(0.5\pi t)$ is sampled using $F_s = 1$ Hz to yield a discrete time signal $x[n]$. Next, $x[n]$ is filtered using an ideal high-pass digital filter that eliminates all frequencies below $0.3F_s$, the output of which is called $y[n]$. Finally $y[n]$ is passed through a perfect reconstruction filter at the rate $F_s = 1/2$ Hz. Find a simplified expression for $y(t)$, the output of the reconstructor. Can this system operate in "real time"?

4) The Figure below shows Fourier spectra of real signals $x_1(t)$ and $x_2(t)$. Determine the Nyquist sampling rates for signals

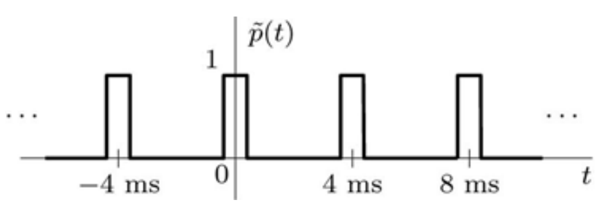
(a) $x_1(t)$ **(b)** $x_2(t/2)$ **(c)** $x_1^2(3t)$ **(d)** $x_1(t)x_2(t)$ **(e)** $1 - x_1(t)$



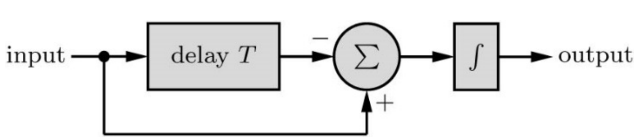
5) A pulse $p(t) = \Pi(1250t)$ is periodically replicated to form the pulse train shown below. Mathematically,

$$\tilde{p}(t) = \sum_{n=-\infty}^{\infty} p(t - 4\times10^{-3}n)$$

Using $x(t) = \text{sinc}(200t)$, find and sketch the spectrum of the sampled signal $x_{\tilde{p}}(t) = \tilde{p}(t)x(t)$. Explain whether it is possible to reconstruct $x(t)$ from these samples. If the sampled signal is passed through an ideal lowpass filter of bandwidth 100 Hz and unit gain, find the filter output. What is the filter output if its bandwidth B Hz is between 100 and 150 Hz? What happens if the filter bandwidth exceeds 150 Hz?



6) Show that the block diagram below represents a realization of a causal ZOH operation. You can do this by showing that the unit impulse response h(t) of this system is $\Pi\left(\frac{t-T/2}{T}\right)$.



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1) A sinusoid of frequency f_0 Hz is sampled at a rate $F_s = 20$ Hz. Find the apparent frequency of the sampled signal if f_0 is
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a) $f_a = \left\langle f_0 + \frac{f_s}{2} \right\rangle_{f_s} - \frac{f_s}{2}$

$f_a = \left(8 + \frac{20}{2} \right)_{20} - \frac{20}{2} = 18 - 10 = \boxed{8\,HZ = f_a}$

b) $f_a = \left(12 + \frac{20}{2} \right)_{20} - \frac{20}{2} = 2 - 10 = \boxed{-8\,HZ = f_a \text{ or } |f_a| = 8\,HZ}$

c) $f_a = \left\langle 21 + \frac{20}{2} \right\rangle_{20} - \frac{20}{2} = 11 - 10 = \boxed{1\,HZ = f_a}$

d) $f_a = \left\langle 32 + \frac{20}{2} \right\rangle_{20} - \frac{20}{2} = 2 - 10 = \boxed{-8\,HZ = f_a \text{ or } |f_a| = 8\,HZ}$

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2) A signal $x(t) = 3\cos(6\pi t) + \cos(16\pi t) + 2\cos(20\pi t)$ is sampled at a rate 25% above the Nyquist rate. Sketch the spectrum of the sampled signal. How would you reconstruct $x(t)$ from these samples? If the sampling frequency is 25% below the Nyquist rate, what are the frequencies of the sinusoids present in the output of the filter with cutoff frequency equal to the folding frequency? Do not write the actual output; give just the frequencies of the sinusoids present in the output.

Highest $w = 20\pi = 10\ Hz \times 2 = 20\ Hz$ Nyquist rate

$w_0 = 2\pi f$

$\times 1.25$

$\boxed{25\ Hz \text{ is } 1.25 \text{ above nyquist rate}}$

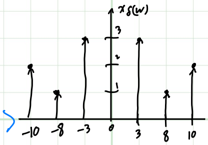
$x(t) \qquad x(w)$

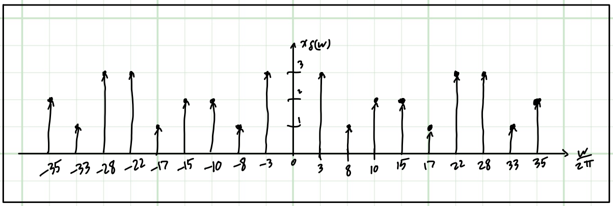
$\cos(w_0 t) \Rightarrow \pi[\delta(w-w_0) + \delta(w+w_0)]$

$$x(w) = \pi[3(w-\underset{Hz}{3}) + 3\delta(w+\underset{Hz}{3}) + \delta(w-\underset{Hz}{8}) + \delta(w+\underset{Hz}{8}) + 2\delta(w-\underset{Hz}{10}) + 2\delta(w+\underset{Hz}{10})]$$

$$x(w) = \pi[3\delta(w \pm \underset{Hz}{3}) + \delta(w \pm \underset{Hz}{8}) + 2\delta(w \pm \underset{Hz}{10})]$$

$$\boxed{x(w) = \pi[3\delta(w \pm 6\pi) + \delta(w \pm 16\pi) + 2\delta(w \pm 20\pi)]}$$





Reconstruct

$$x(t) = \left(\frac{3\pi}{2\pi} e^{-j6\pi t} + \frac{\pi}{2\pi} e^{-j16\pi t} + \frac{2\pi}{2\pi} e^{-j20\pi t}\right) 2$$

$$x(t) = 3\left(\frac{e^{-j6\pi t} + e^{j6\pi t}}{2}\right) + \left(\frac{e^{-j16\pi t} + e^{j16\pi t}}{2}\right) + 2\left(\frac{e^{-j20\pi t} + e^{j20\pi t}}{2}\right)$$

$$\boxed{x(t) = 3\cos(6\pi t) + \cos(16\pi t) + 2\cos(20\pi t)}$$

$20\ Hz * 0.75 = 15\ Hz$ 25% below

$Fs = 15\ Hz - 10\ Hz = 5\ Hz$

$- 8\ Hz = 7\ Hz$

$- 3\ Hz = 12\ Hz$

$\boxed{f_c = 10\ Hz \quad \text{frequency} = 3, 5, 7\ Hz}$

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3) The signal $x(t) = \sin(0.7\pi t)\cos(0.5\pi t)$ is sampled using $F_s = 1$ Hz to yield a discrete time signal $x[n]$. Next, $x[n]$ is filtered using an ideal high-pass digital filter that eliminates all frequencies below $0.3F_s$, the output of which is called $y[n]$. Finally $y[n]$ is passed through a perfect reconstruction filter at the rate $F_s = 1/2$ Hz. Find a simplified expression for $y(t)$, the output of the reconstructor. Can this system operate in "real time"?

$$\sin a \cos b = \frac{\sin(a+b) + \sin(a-b)}{2}$$

$$\sin(\underset{0.35}{0.7\pi t})\cos(\underset{0.25}{0.5\pi t}) = \frac{\sin(0.35+0.25)2\pi t \quad \sin(0.35-0.25)2\pi t}{2}$$

$$x(t) = \tfrac{1}{2}\left[\sin(2\pi\, 0.1 t) + \sin(2\pi\, 0.6\, t)\right]$$

$$f_{a_1} = \langle 0.1 + \tfrac{1}{2}\rangle_1 - \tfrac{1}{2} = 0.6 - 0.5 = 0.1\ \text{Hz}$$

$$f_{a_2} = \langle 0.6 + \tfrac{1}{2}\rangle_1 - \tfrac{1}{2} = 0.1 - 0.5 = -0.4\ \text{Hz}$$

$y(t)$ with HPF of $0.3FS = 0.3Hz$ will make $x(t)$ into the following:

$y(t) = \frac{1}{2}\sin(2\pi 0.6 t)$ but the 0.6 Hz is aliased by $-0.4Hz$

$= y(t) = -\frac{1}{2}\sin(2\pi\, 0.4\, t)$ now if $fs = \frac{1}{2}Hz$

then $\boxed{y(t) = -\frac{1}{2}\sin(2\pi\, 0.2\, t)}$

It can't run real time due to the input being twice as fast as the output.

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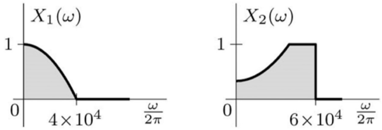

a) $FS = \text{Nyquist rate} = 2 \times \max Hz$

$FS = 40000 HZ = 40 kHZ \times 2 = \boxed{80 kHZ}$

b) $FS = 60000 HZ = \frac{60 kHZ}{2} \times 2 = \boxed{60 kHZ}$

c) $x_1^2(3t) = x_1(3t) * x_1(3t) = x_1(3 \cdot 2t) = 40 \cdot 6 = 240 \, kHZ$

$= 240 \times 2 = \boxed{480 kHZ}$

for nyquist

$x_1(t)\, x_2(t) =$ add frequency components

d) $(40 + 60) kHZ = 100 kHZ \times 2 = \boxed{200 kHZ}$

e) $1 - x_1(t)$ Just applys an offset but the nyquist rate is the same as A

$= \boxed{80 kHZ}$

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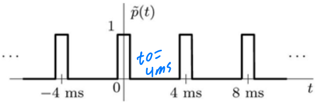

$$x(t) = \text{sinc}(200t) \Longleftrightarrow X(\omega) = \frac{\pi}{200}\left(\frac{\omega}{400\pi}\right)$$

$$X_{\tilde{p}}(\omega) = \sum_{k=-\infty}^{k=\infty} \tilde{P}_k X(\omega - k\omega_s)$$

$$\frac{1}{1250} / 2 = \pm 0.0004$$

$$\tilde{P}_k = X_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{jk\omega_0 t}\, dt$$

$$\tilde{P}_k = \frac{1}{4ms}\int_{-0.0004}^{0.0004} (1)\, e^{j\overbrace{\frac{2\pi}{4ms}}^{500\pi} kt}\, dt$$

$$\tilde{P}_k = 2\cancel{5}0\, \frac{e^{j500\pi k t}}{j\cancel{500}\pi k}\Bigg|_{-0.0004}^{0.0004} = (1)\left(\frac{e^{\overbrace{j500\pi k(0.0004)}^{j0.2\pi k}} - e^{\overbrace{j500\pi k(-0.0004)}^{-j0.2\pi k}}}{j2\pi k}\right)$$

$$= 0.2\, \text{sinc}(0.2k)$$

$$\boxed{X_{\tilde{p}}\omega = \sum_{k=-\infty}^{k=\infty} 0.2\, \text{sinc}(0.2k)\, \frac{\pi}{200}\left(\frac{\omega - k\omega_s}{400\pi}\right)}$$

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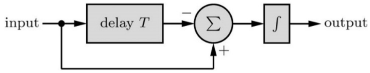
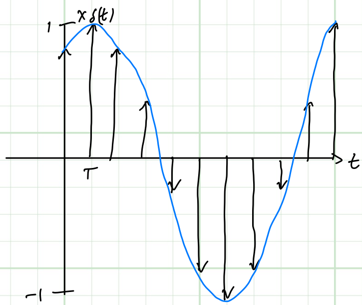


$$x(t) = \delta(t) \longrightarrow \int \delta(t) - \delta(t-T)\, dt$$

$$h(t) = u(t) - u(t-T)$$

$$h(t) = \Pi\left(\frac{t - T/2}{T}\right)$$



$$\hat{x}(t) = \sum_{n=-\infty}^{\infty} x(nT)\, h(t-nT) = \sum_{n=-\infty}^{\infty} x(nT)\, \Pi\left(\frac{t - nT - T/2}{T}\right)$$