Implementation and Simulation of Digital Control Compensators from Continuous Compensators Using MATLAB Software

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ABASTRACT

This paper describes the opportunity of implementing design procedures of Lead -lag compensators in continuous then use the designed implementation in discrete backward, forward and bilinear transform. The Compensators were simulated with different sampling times using MATLAB package, . The technique is used called Emulation Design, where the controller design is done in the continuous-time domain followed by controller discretization to produce a discrete-time controller for digital implementation and apply the sampling time . The paper shows that designing in continuous can be with the compensators first to calculate the parameters then replace them with discrete Coefficients and then simulate with appropriate sampling time with MATLAB package.

KEYWORDS

Digital Control , Discrete, Lead , Lag , Lead-Lag Compensators , Forward, Backward, Bilinear ,Z-transform , Zero Order Hold MATLAB Simulation.

1 INTRODUCTION

Lead and lag compensators are used quite extensively in control. A lead compensator can increase the stability or speed of response of a system; a lag compensator can reduce (but not eliminate) the steady state error. Depending on the effect desired, one or more lead and lag compensators may be used in various combinations. Lead, lag, and lead/lag compensators are usually designed for a system in transfer function form [1].

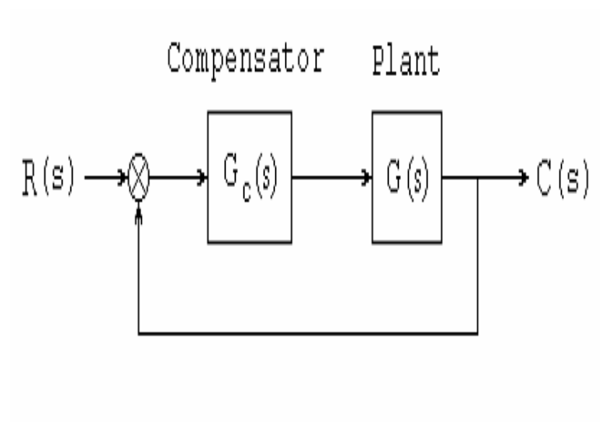

Figure 1. Closed loop system with compensator

The phase-lag, phase-lead, and phase-lead-lag compensator. These have characteristics of more ideal proportional integral (PI), proportional derivative (PD), and proportional integral derivative (PID) controllers, respectively. It is also possible to use, two phase-lead compensators in series in order to add more phase margin. [2].

The classical tuning methods for lead, lag and lead-lag compensators, employing Bode plot diagrams are based on trial-and-error and/or heuristic techniques, and are therefore approximate by nature. The main drawback of the trial-and-error method, which is usually based on considerations on the Bode plot, is that this method often leads to a controller that does not behave as expected [3] [4].

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2 DISCRETE IMPLEMENTATION OF LEAD COMPENSATO

For design lead compensator in equation (1) to discrete using difference approximation methods backward, forward and bilinear z-transformation

$$G_{Lead}(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \qquad (1)$$

Where

β is phase lead compensator parameter

T is phase lead compensator time constant

2.1 - Lead Compensator with Backward Approximation

Since $s = \frac{z-1}{zT'}$ in Backward approximation

$$G_{Lead}(z) = \frac{1}{\beta}\frac{\left(\frac{z-1}{zT'}\right)+\frac{1}{T}}{\left(\frac{z-1}{zT'}\right)+\frac{1}{\beta T}} \qquad (2)$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{z+\frac{zT'}{T}-1}{z+\frac{zT'}{\beta T}-1}$$

Where

T' is integration time.

$$G_{Lead}(z) = \frac{1}{\beta}\frac{z\left(1+\frac{T'}{T}\right)-1}{z\left(1+\frac{T'}{\beta T}\right)-1}$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{2(z-1)+\frac{T'(z+1)}{T}}{2(z-1)+\frac{T'(z+1)}{\beta T}}$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{2z+z\frac{T'}{T}+\frac{T'}{T}-2}{2z+z\frac{T'}{T}+\frac{T'}{\beta T}-2}$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{z\left(2+\frac{T'}{T}\right)+\left(\frac{T'}{T}-2\right)}{z\left(2+\frac{T'}{T}\right)+\left(\frac{T'}{\beta T}-2\right)}$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{za_3+a_2}{za_3+b_2} \qquad (3)$$

Where

$$a_2 = \left(\frac{T'}{T}-2\right) \qquad (4)$$

$$b_2 = \left(\frac{T'}{\beta T}-2\right) \qquad (5)$$

2.2 - Lead Compensator with Forward Approximation

Since $s = \frac{z-1}{T'}$ in forward approximation

$$G_{Lead}(z) = \frac{1}{\beta}\frac{\left(\frac{z-1}{T'}\right)+\frac{1}{T}}{\left(\frac{z-1}{T'}\right)+\frac{1}{\beta T}} \qquad (6)$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{z+\frac{T'}{T}-1}{z+\frac{T'}{\beta T}-1}$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{z+(\frac{T'}{T}-1)}{z+(\frac{T'}{\beta T})-1)}$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{z+a_1}{z+b_1} \qquad (7)$$

Where

$$a_1 = \left(\frac{T'}{T}-1\right) \qquad (8)$$

$$b_1 = \left(\frac{T'}{\beta T}-1\right) \qquad (9)$$

2.3 - Lead Compensator with Bilinear Approximation

Since $s = \frac{2}{T'}\frac{z-1}{z+1}$ in bilinear approximation then

$$G_{Lead}(z) = \frac{1}{\beta}\frac{(\frac{2}{T'}\frac{z-1}{z+1})+\frac{1}{T}}{(\frac{2}{T'}\frac{z-1}{z+1})+\frac{1}{\beta T}} \qquad (10)$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{2(z-1)+\frac{T'(z+1)}{T}}{2(z-1)+\frac{T'(z+1)}{\beta T}}$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{2z+z\frac{T'}{T}+\frac{T'}{T}-2}{2z+z\frac{T'}{T}+\frac{T'}{\beta T}-2}$$

$$G_{Lead}(z) = \frac{1}{\beta}\frac{z\left(2+\frac{T'}{T}\right)+(\frac{T'}{T}-2)}{z\left(2+\frac{T'}{T}\right)+(\frac{T'}{\beta T}-2)}$$

$$G_{Lead}\,(z) = \frac{1}{\beta}\,\frac{za_3+a_2}{za_3+b_2} \qquad (11)$$

Where

$$a_2 = \left(\frac{T'}{T}-2\right) \qquad (12)$$

$$b_2 = \left(\frac{T'}{\beta T}-2\right) \qquad (13)$$

3-DISCRETE IMPLEMENTATION OF LAG COMPENSATOR

For design lag compensator in equation (14) to discrete using difference approximation methods backward, forward and bilinear z-transformation

$$G_{Lag}(s) = \frac{1}{\propto}\frac{s+\frac{1}{T}}{s+\frac{1}{\propto T}} \qquad (14)$$

Where

α is phase lag compensator parameter

T is phase lag compensator time constant

3.1 - Lag Compensator with Backward Approximation

Since $s = \frac{z-1}{zT'}$ in Backward approximation

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{\left(\frac{z-1}{zT'}\right)+\frac{1}{T}}{\left(\frac{z-1}{zT'}\right)+\frac{1}{\propto T}} \qquad (15)$$

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{z+\frac{zT'}{T}-1}{z+\frac{zT'}{\propto T}-1}$$

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{z(1+\frac{T'}{T})-1}{z(1+\frac{T'}{\propto T})-1}$$

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{zc-1}{zd-1} \qquad (16)$$

Where

$$c = \left(1+\frac{T'}{T}\right) \qquad (17)$$

$$d = \left(1+\frac{T'}{\propto T}\right) \qquad (18)$$

3.2 - Lag Compensator with Forward Approximation

Since $s = \frac{z-1}{T'}$ in forward approximation

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{(\frac{z-1}{T'})+\frac{1}{T}}{(\frac{z-1}{T'})+\frac{1}{\propto T}} \qquad (19)$$

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{z+\frac{T'}{T}-1}{z+\frac{T'}{\propto T}-1}$$

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{z+\left(\frac{T'}{T}-1\right)}{z+\left(\frac{T'}{\propto T}-1\right)}$$

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{z+c_1}{z+d_1} \qquad (20)$$

Where

$$c_1 = \left(\frac{T'}{T}-1\right) \qquad (21)$$

$$d_1 = (\frac{T'}{\propto T}-1) \qquad (22)$$

3.3 - Lag Compensator with Bilinear Approximation

Since $s = \frac{2}{T'}\frac{z-1}{z+1}$ in bilinear approximation

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{(\frac{2}{T'}\frac{z-1}{z+1})+\frac{1}{T}}{(\frac{2}{T'}\frac{z-1}{z+1})+\frac{1}{\propto T}} \qquad (23)$$

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{2(z-1)+\frac{T'(z+1)}{T}}{2(z-1)+\frac{T'(z+1)}{\propto T}}$$

$$G_{Lag}(z) = \frac{1}{\propto}\,\frac{2z+z\frac{T'}{T}+\frac{T'}{T}-2}{2z+z\frac{T'}{T}+\frac{T'}{\propto T}-2}$$

$$G_{Lag}(z) = \frac{1}{\propto} \frac{z\left(2+\frac{T'}{T}\right)+(\frac{T'}{T}-2)}{z\left(2+\frac{T'}{T}\right)+(\frac{T'}{\propto T}-2)}$$

$$G_{Lag}(z) = \frac{1}{\propto} \frac{zc_3+c_2}{zc_3+d_2} \quad (24)$$

Where

$$c_2 = \left(\frac{T'}{T}-2\right) \quad (25)$$

$$d_2 = \left(\frac{T'}{\propto T}-2\right) \quad (26)$$

$$c_3 = \left(2+\frac{T'}{T}\right) \quad (27)$$

4-LEAD-LAG COMPENSATOR

A lead-lag compensator combines the effects of a lead compensator with those of a lag compensator. The result is a system with improved transient response, stability, and steady-state error. To implement a lead-lag compensator, first design the lead compensator to achieve the desired transient response and stability, and then design a lag compensator to improve the steady-state response of the lead-compensated system [5][6]

$$G_{Lead-lag}(s) = G_{Lead}(s) * G_{Lag}(s) \quad (28)$$

$$G_{Lead-lag}(z) = G_{Lead}(z) * G_{Lag}(z) \quad (29)$$

5-SIMULATION OF CONTINUOUS AND DIGITAL COMPENSATORS

For digital design, after the continuous system is designed with compensator in time domain and the response is satisfactory, emulation digital design take place for discretezation the system using digital compensators, Backward, Forward and Bilinear deference approximations, with Zero Order Hold transform for the plant at different sampling times in closed loop system using MATLAB CODE

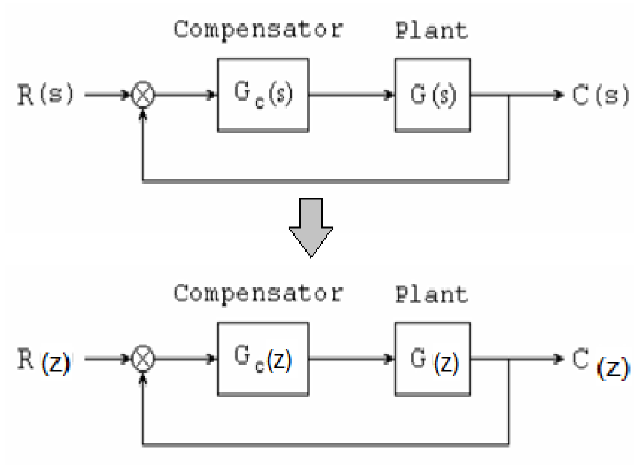

Figure 2. Continuous and digital closed loop systems with compensators

EXAMPLE 1

For the second order system given in equation (30).Design a compensator to meet the following specification: overshot of 5% and steady state error =0.02.

$$G(s) = \frac{2}{s^2+0.0833s+20.2} \quad (30)$$

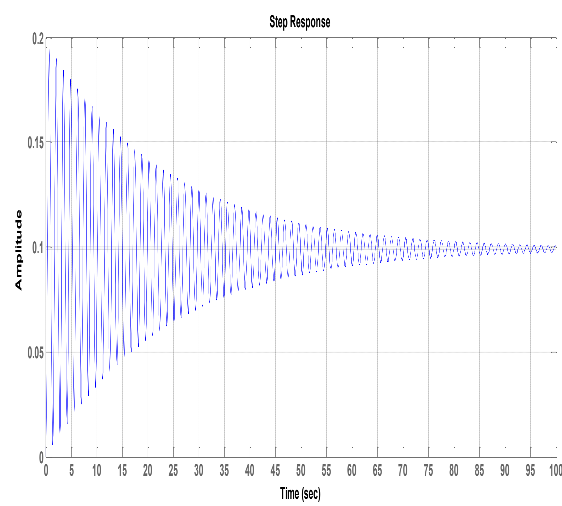

Figure 3. open loop unit step of equation (30)

The system as shown has large steady state error and oscillating at high frequencies so by following some steps compensator would by designed
First start to calculate the gain which satisfy the steady state error using unit step response of equation (31)

$$e_{ss}(\infty) = \frac{1}{1+Kp} \quad (31)$$

$$\text{kp} = \lim_{s\to 0} G(s)$$

$$\text{kp} = \lim_{s\to 0} \frac{2k}{s^2 + 0.0833s + 20.2}$$

$$\text{kp} = \frac{2k}{20.2} = 0.099k$$

$$since \quad \frac{1}{1 + 0.099k} = 0.02$$

From the calculation the gain k=494.95

The step response in the figure below of uncompensated system in equation (30) with gain k to satisfy the steady state error of 0.02.

$$\mathrm{G}(s) = \frac{989.89}{s^2+0.0833s+20.2} \tag{32}$$

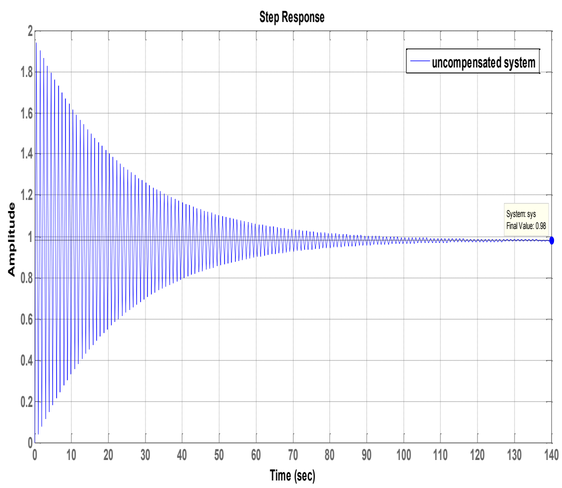

Figure 4. Closed loop response of uncompensated system of equation (30)

5.1-LEAD DESIGN

Since the purpose of phase lead compensator design in the frequency domain generally is to satisfy specifications on steady-state accuracy and phase margin. There may also be a specification on gain crossover frequency or closed-loop bandwidth. A phase margin specification can represent a requirement on relative stability due to pure time delay in the system, or it can represent desired transient response characteristics that have been translated from the time domain into the frequency domain

From bode plot below the phase margin of the uncompensated system is 0.153°.

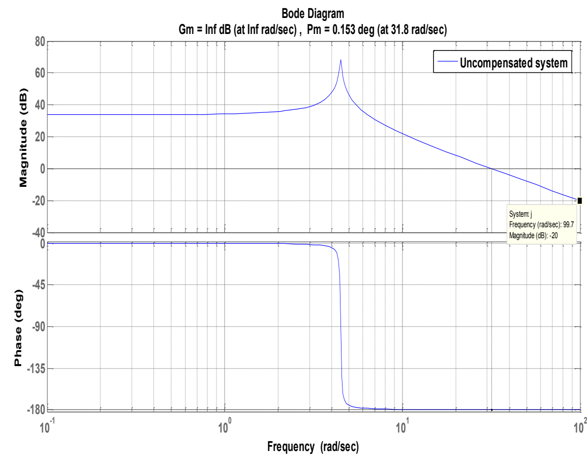

Figure 5. Bode plot of uncompensated system of equation (30)

The phase margin of the uncompensated system is 0.153° which cannot satisfy the value of ξ to have overshoot required. The required overshoot for design is 5% since

$$OS\% = e^{-(2\pi/\sqrt{1-\xi^2}\,)} \times 100 \tag{33}$$

From equation above to have OS=5% then ξ =0.69

$$\emptyset_{M=} tan^{-1} \frac{2\xi}{\sqrt{-2\xi + \sqrt{1 + 4\xi^2}}} \tag{34}$$

Where

ξ is damping ratio
OS% is the percentage overshoot
$\emptyset_M$ is the phase margin required

Using equation **(34)** to find the phase margin required, $\emptyset_M$ =64.61°

The maximum phase shift of the compensator is:
$$\phi_{max} = 64.619° - 0.153° + 15° = 79.466°$$

Where 10° to 15° is a correction factor.
Using equation (35) to find β

$$\beta = \frac{1 - \sin(\phi_{max})}{1 + \sin(\phi_{max})} \tag{35}$$

$$\beta = 8.498x10^{-3}$$

The compensator magnitude is:

$$Mag = \frac{1}{\sqrt{\beta}} = 10.8476$$

Compensator magnitude= **20 log** $\frac{1}{\sqrt{\beta}}$ (36)

= 20.7**dB**.

At the gain crossover frequency the magnitude of the compensator is **20.7dB**, however the magnitude of the compensated system should be **0 dB** at this point so the magnitude of the uncompensated system at this point should be (-**20.7dB**.). The value of (- **20.7dB)** in bode plot figure (5) above the gain crossover frequency is ($\boldsymbol{\omega}_{\max}$= 99.7 rad/sec)

To calculate the zero and pole of the compensator. The compensator should have unity gain in order to keep the steady state requirements as required.

$$\omega_{max} = \frac{1}{T\sqrt{\beta}} \quad (37)$$

$$T = \frac{1}{\omega_{max}\sqrt{\beta}} = 0.109$$

$$G_{Lead}(s) = \frac{1}{\beta}\frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$$

$$G_{Lead}(s) = 92.2\frac{s + 9.17}{s + 1079.58} \quad (38)$$

The figure below shows the step response of the compensator and the uncompensated system both in close loop with unity feedback.

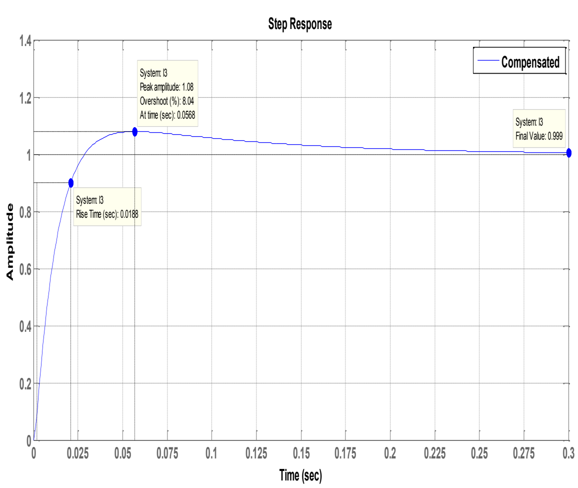

Figure 6. Step response of compensated system in continuous

The system improved in transient characteristics and steady state error very small .the lag compensator is not needed for this system.

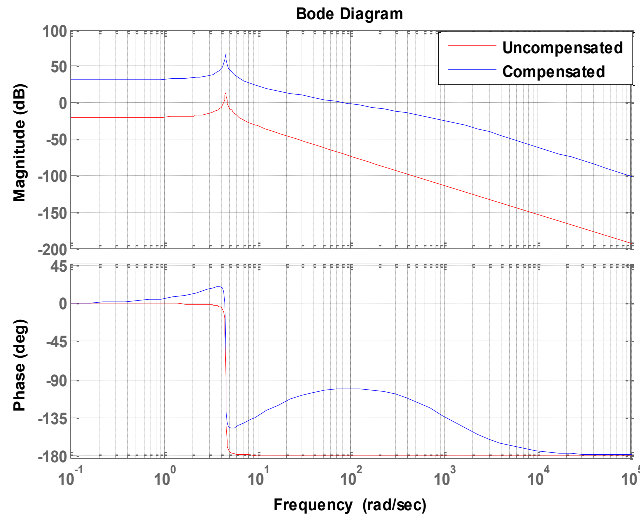

Figure 7. Bode plot of compensated and uncompensated system equation (30)

The bode plot of figure (7) shows where the compensator effected in the phase plot and phase margin is 79^{o} instead of 0.153^{o}.

For the digital design , after the system is designed with lead- compensator and the response was satisfactory, emulation digital design take place for discretezation the system using three approximation methods of compensators , Backward, Forward, Bilinear at different sampling times in closed loop system

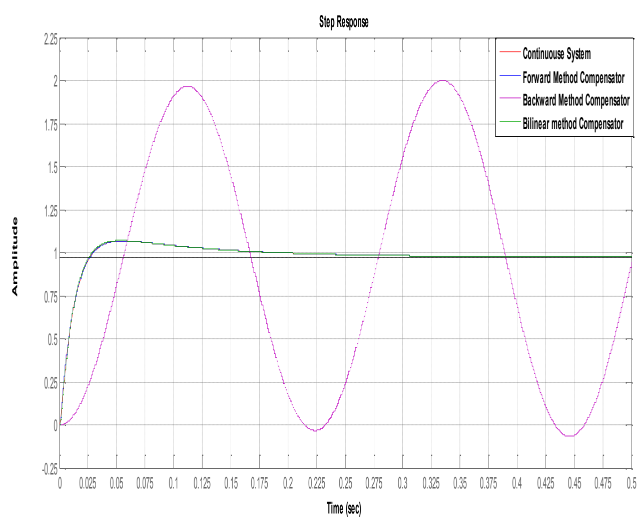

Figure 8. Step response of compensator in continuous, digital forward, backward, bilinear with Z.O.H system at Ts =0.001 sec"

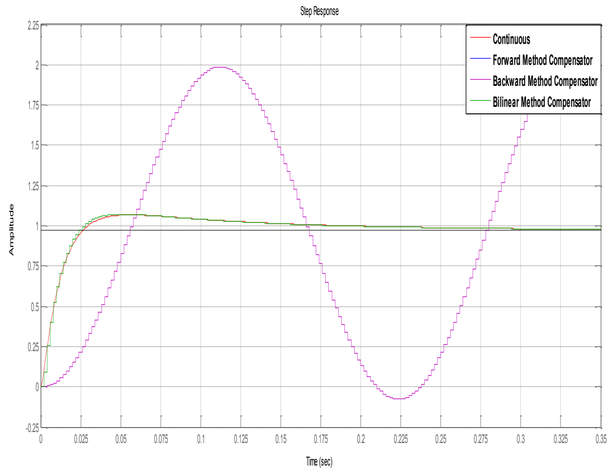

Figure 9. Step response of compensator in continuous, digital forward, backward , bilinear with Z.O.H system at Ts =0.002 sec”

As seen above, the forward and Bilinear methods of the compensated system are sensitive to the sampling time, at 0.001sec and 0.002 s and identical to continuous with improvement in the transient characteristics and steady state error e_{ss}. For the backward method it is unstable and compensator did not satisfy the appropriate design

6-CONCLUSION

Discretizing the systems is very useful and important since we can chose one of the many digital transformations which can achieve the needed requirements for implementations , they are might not change the characteristics of the system in the small sampling times in different orders of systems , In closed loop another parameter is added for designing which is the sampling time Ts. The simulation of the MATLAB package shows the forward and Bilinear transformations systems have the same behavior of the continuous but in backward system is unstable at different sampling time.

7- MATLAB CODE

```
% Lead and Lag implementation from Continues design to digital implementation

% This Program is used for Lead, Lag, and Lead-lag compensators in continuous and digital.

% After designing all parameters needed for each compensator, the program asks to enter these parameters and uses step response the  system

%NOTE:
% 1- MAKE SURE TO ENTER THE ZERO AND POLE   VALUES OF THE COMPENSATORS AS BELOW
% (s+1) the s= -1 BECAUSE  OF USING ( zpk ) MATLAB command
% 2-Emulation digital design is used .

%For discrete Compensators, the program uses the same parameters entered before , and uses forward ,Backward or Bilinear for the compensators .and Zero Order Hold for the system
% 3- The sampling Time is the same for system and the compensators

clc
s=tf('s');
fprintf('This Program is used for Lead, Lag, and Lead-lag compensators in continuous and digital.\n  ');
%Transfer Function needed to be transferred
H1=input('Enter Your TF Function of S : \n')
h=figure(1);
hold;
%step(H1) %Step response of open-loop system
Kc=input('Enter the Gain Of the TF to Obtain the desired ess : \n')%Gain to obtain reqiured steady state error
O=input('Choose a compensator\n 1- Continuous Lead compensator ,\n 2- Continuous Lag comensator ,\n3- Continuous Lead-lag compensator\n')
if O==1
    g_l=input('Enter the Gain value of Lead Compensator : \n');
    zl=input('Enter the zero value of Lead Compensator : \n');
    pl=input('Enter the pole value of Lead Compensator : \n');
    G_lead =zpk(zl,pl,g_l)
    G_Fs=Kc*G_lead*H1;
    GTs= feedback (G_Fs,1) %Closed Loop System
    step (GTs)
elseif O==2
    g_g=input('Enter the Gain value of Lag Compensator : \n');
    zg=input('Enter the zero value of Lag Compensator : \n');
```

```
    pg=input('Enter the pole value of
Lag Compensator : \n');
    G_lag =zpk(zg,pg,g_g)
    G_Fs=Kc*G_lag*H1;
    GTs= feedback (G_Fs,1) %Closed Loop
System
    step (GTs)
else
    g_l=input('Enter the Gain value of
Lead Compensator : \n');
    zl=input('Enter the zero value of
Lead Compensator : \n');
    pl=input('Enter the pole value of
Lead Compensator : \n');
    G_lead =zpk(zl,pl,g_l)
    g_g=input('Enter the Gain value of
Lag Compensator : \n');
    zg=input('Enter the zero value of
Lag Compensator : \n');
    pg=input('Enter the pole value of
Lag Compensator : \n');
    G_lag =zpk(zg,pg,g_g)
    G_Fs=Kc*G_lead*G_lag*H1;
    GTs= feedback (G_Fs,1) %Closed Loop
System
    step (GTs)
end
T=input('\nEnter the sampling Time ');%
Sampling Time
Gz=c2d(H1,T,'zoh')
disp('The Zero Order Hold Method TF
above\n')
%step(Gz)
z=tf('z',T)
for r=1:3
O1=input('Enter the Method of
Approximation \n1-Forward Method
compensators \n2-Backward Method
compensators \n3-Bilinear Method
compensators\n4-Exite\n ');
if O1==1
disp('Now using Digital Forward Method
Approximation')
O=input('\n 1-Discrete Lead compensator
,\n2-Discrete Lag comensator ,\n3-
Discrete Lead-lag compensator')
if O==1
    Gz_lead=g_l*(((z-1)/T)-zl)/(((z-
1)/T)-pl)
    G_Fz=Kc*Gz_lead*Gz;
    GTz= feedback (G_Fz,1) %Closed Loop
System
    step (GTz)
elseif O==2
    Gz_lag=g_g*(((z-1)/T)-zg)/(((z-
1)/T)-pg)
    G_Fz=Kc*Gz_lag*Gz;
    GTz= feedback (G_Fz,1) %Closed Loop
System
```

```
    step (GTz)
else
    Gz_lead=g_l*(((z-1)/T)-zl)/(((z-
1)/T)-pl)
    Gz_lag=g_g*(((z-1)/T)-zg)/(((z-
1)/T)-pg)
    G_Fz=Kc*Gz_lead*Gz_lead*Gz;
    GTz= feedback (G_Fz,1) %Closed Loop
System
    step (GTz)
end
elseif O1==2
disp('Now using Digital Backwaed Method
Approximation')
O=input('/n 1-Discrete Lead compensator
,2-Discrete Lag comensator ,3-Discrete
Lead-lag compensator')
if O==1
    Gz_lead=g_l*(((z-1)/z*T)-zl)/(((z-
1)/z*T)-pl)
    G_Fz=Kc*Gz_lead*Gz;
    GTz= feedback (G_Fz,1) %Closed Loop
System
    step (GTz)
elseif O==2
    Gz_lag=g_g*(((z-1)/z*T)-zg)/(((z-
1)/z*T)-pg)
    G_Fz=Kc*Gz_lag*Gz;
    GTz= feedback (G_Fz,1) %Closed Loop
System
    step (GTz)
else
    Gz_lead=g_l*(((z-1)/z*T)-zl)/(((z-
1)/z*T)-pl)
    Gz_lag=g_g*(((z-1)/z*T)-zg)/(((z-
1)/z*T)-pg)
    G_Fz=Kc*Gz_lead*Gz_lead*Gz;
    GTz= feedback (G_Fz,1) %Closed Loop
System
    step (GTz)
end
elseif O1==3
disp('Now using Digital Bilinear Method
Approximation')
O=input('/n 1-Discrete Lead compensator
,2-Discrete Lag comensator ,3-Discrete
Lead-lag compensator')
if O==1
    Gz_lead=g_l*((2*(z-1))/(T*(z+1))-
zl)/((2*(z-1))/(T*(z+1))-pl)
    G_Fz=Kc*Gz_lead*Gz;
    GTz= feedback (G_Fz,1) %Closed Loop
System
    step (GTz)
elseif O==2
    Gz_lag=g_g*((2*(z-1))/(T*(z+1))-
zg)/((2*(z-1))/(T*(z+1))-pg)
    G_Fz=Kc*Gz_lag*Gz;
```

```
    GTz= feedback (G_Fz,1) %Closed Loop
System
    step (GTz)
else
    Gz_lead=g_l*((2*(z-1))/(T*(z+1))-
zl)/((2*(z-1))/(T*(z+1))-pl)
    Gz_lag=g_g*((2*(z-1))/(T*(z+1))-
zg)/((2*(z-1))/(T*(z+1))-pg)
    G_Fz=Kc*Gz_lead*Gz_lead*Gz;
    GTz= feedback (G_Fz,1) %Closed Loop
    step (GTz)
end
else
    button = questdlg('Ready to quit?',
...
                            'Exit
Dialog','Yes','No','No');
          switch button
            case 'Yes',
              disp('Exiting MATLAB');
              %Save variables to
matlab.mat
              save
            case 'No',
              quit cancel;
          end
end
end
```

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