GCSE Maths: Vectors

Third Space Learning

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Introduction to Vectors

What are Vectors?

- Vectors describe movement from one point to another.
- Two key characteristics: **magnitude** (size) and **direction**.
- Used in geometry and physics to describe position and movement.

Vector Notation

Key Points:

- Vectors are represented by directed line segments.
- Boldface notation $\mathbf{a}$ or arrow notation $\overrightarrow{AB}$.
- Reversing the arrow gives the negative of the vector.

Example:

- $\overrightarrow{AB} = \mathbf{a}$, then $\overrightarrow{BA} = -\mathbf{a}$.

Column Vectors

Components:

- **Horizontal component** (x-axis movement) is the top number.
- **Vertical component** (y-axis movement) is the bottom number.

Example:

- $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ means 5 right, 2 up.
- $\mathbf{b} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ means 3 left, 4 down.

Magnitude of a Vector

Formula:

$$|\mathbf{a}| = \sqrt{x^2 + y^2}$$

Example:

- Given $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$,
- $|\mathbf{a}| = \sqrt{5^2 + 2^2} = \sqrt{25 + 4} = \sqrt{29}$.

Vector Arithmetic

Addition:

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$

Subtraction:

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}$$

Example:

- If $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$,
- $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$,
- $\mathbf{a} - \mathbf{b} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$.

Vector Multiplication

Multiplication by a Scalar:

$$k\mathbf{a} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

Example:

- Given $\mathbf{a} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, find $3\mathbf{a}$.
- $3\mathbf{a} = \begin{pmatrix} 12 \\ -6 \end{pmatrix}$.

Vector Geometry

Parallel Vectors:

- Two vectors are parallel if one is a scalar multiple of the other.
- Example: $\mathbf{b} = 2\mathbf{a}$ means $\mathbf{a}$ and $\mathbf{b}$ are parallel.

Vectors in Shapes:

- Equal vectors exist in parallelograms and triangles.
- Midpoints and bisectors use vector averages.

Using Vectors in Proofs

Key Techniques:

- Expressing points in terms of vectors.
- Proving collinearity (same straight line).
- Proving midpoints and ratios in triangles.

Example:

- Show that points are collinear using vector ratios.
- Find unknown points using vector expressions.