Trigonometry and Forming Equations

GCSE Mathematics

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Introduction to Trigonometry

Key Concept: Trigonometry deals with the relationships between angles and sides in right-angled triangles.

Three Main Ratios:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

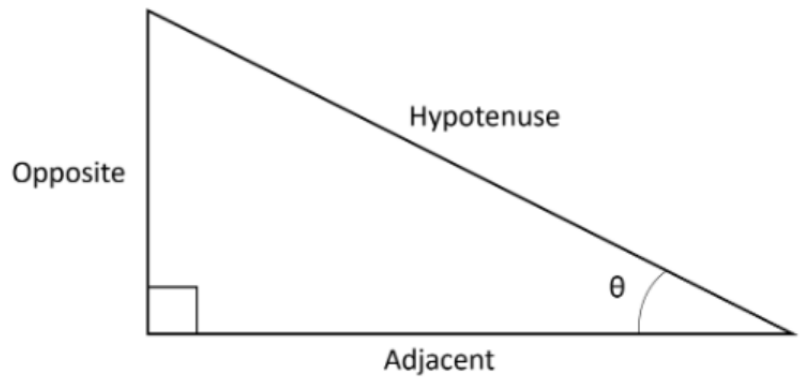
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Identifying Triangle Sides

In a right-angled triangle:

- The **hypotenuse** is always the longest side.
- The **opposite** side is across from the given angle.
- The **adjacent** side is next to the given angle.



Using Trigonometry to Find a Side

Example: Find the missing side in a triangle where $\theta = 30^\circ$ and hypotenuse is 10 cm.

Solution:

$$\sin 30^\circ = \frac{x}{10}$$
$$0.5 = \frac{x}{10}$$
$$x = 5 \text{ cm}$$

Using Trigonometry to Find an Angle

Example: Find θ in a triangle where $\frac{opposite}{hypotenuse} = 0.6$.

Solution:

$$\sin\theta = 0.6$$
$$\theta = \sin^{-1}(0.6)$$
$$\theta = 36.87^\circ$$

Forming and Solving Equations

Key Steps:

- Identify the given information (side lengths and angles).
- Choose the correct trigonometric ratio.
- Set up an equation and solve for the unknown.

Real-Life Applications

Trigonometry is used in:

- Engineering and construction.
- Navigation and GPS technology.
- Physics and wave analysis.

Practice Problems

1. Find x in a triangle where $\theta = 45^\circ$ and hypotenuse $= 10$ cm.
2. Find θ if $\frac{adjacent}{hypotenuse} = 0.8$.
3. A ladder leans against a wall. The base is 4 m away, and the ladder is 6 m long. Find the angle it makes with the ground.

Answers to Practice Problems

1. $x = 7.07$ cm using $x = 10 \times \cos 45^\circ$.
2. $\theta = 36.87^\circ$ using $\cos^{-1}(0.8)$.
3. $\theta = \sin^{-1}(\frac{4}{6}) = 41.41^\circ$.