Sequences in GCSE Mathematics

GCSE Maths

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Introduction to Sequences

Definition: A sequence is an ordered list of numbers following a specific pattern.

Types of Sequences:

- Arithmetic Sequences
- Geometric Sequences
- Quadratic Sequences
- Special Sequences (Fibonacci, Triangular Numbers, etc.)

Arithmetic Sequences

Definition: A sequence where the difference between consecutive terms is constant.

General Formula:

$$a_n = a_1 + (n-1)d$$

- a_n = nth term
- a_1 = first term
- d = common difference
- n = term position

Example:

- Sequence: 3, 7, 11, 15, ...
- Common difference: $d = 4$
- Formula: $a_n = 3 + (n-1)4$

Geometric Sequences

Definition: A sequence where each term is found by multiplying the previous term by a constant ratio.

General Formula:

$$a_n = a_1 r^{(n-1)}$$

- a_n = nth term
- a_1 = first term
- r = common ratio
- n = term position

Example:

- Sequence: 2, 6, 18, 54, ...
- Common ratio: $r = 3$
- Formula: $a_n = 2 \times 3^{(n-1)}$

Quadratic Sequences

Definition: A sequence where the second difference between terms is constant.

General Formula:

$$a_n = an^2 + bn + c$$

- Identify the first and second differences.
- Use simultaneous equations to find a, b, c.

Example:

- Sequence: 2, 6, 12, 20, ...
- Second difference $= 2$, so formula is quadratic.

Special Sequences

Fibonacci Sequence:

- Each term is the sum of the two preceding terms.
- Sequence: 1, 1, 2, 3, 5, 8, 13, ...

Triangular Numbers:

- Sequence: 1, 3, 6, 10, 15, 21, ...
- Formula: $T_n = \frac{n(n+1)}{2}$

Finding the nth Term

Steps:

- Identify the type of sequence (arithmetic, geometric, quadratic).
- Find the common difference (if arithmetic) or ratio (if geometric).
- Use the appropriate formula to express the nth term.

Example:

- Sequence: 4, 9, 14, 19, ...
- Common difference: 5
- Formula: $a_n = 4 + (n - 1)5$

Practice Questions

- Find the 10th term of the sequence: 5, 8, 11, 14, ...
- Determine the nth term of the sequence: 3, 9, 27, 81, ...
- Identify the first five terms of the quadratic sequence: $n^2 + 2n + 1$.

Answer to Question 1

Find the 10th term of the sequence: 5, 8, 11, 14, ...

- Common difference: 3
- Formula: $a_n = 5 + (n-1)3$
- $a_{10} = 5 + (10-1)3 = 32$

Answer to Question 2

Determine the nth term of the sequence: 3, 9, 27, 81, ...

- This is a geometric sequence with a common ratio of 3.
- Formula: $a_n = 3 \times 3^{(n-1)}$

Answer to Question 3

Identify the first five terms of the quadratic sequence:
$n^2 + 2n + 1$

- Substituting $n = 1, 2, 3, 4, 5$:
- Terms: 4, 9, 16, 25, 36.