Lecture Notes: Causal Inference Fall 2020

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Contents

1 The Hodgepodge

1.1 Non-standard standard errors

A standard error is, of course, an estimate of the uncertainty around an estimated parameter. Formally we have

$$se = \sqrt{\widehat{Var(\hat{\beta})}}$$

in other words, the standard error is the square root of the estimated variance of the estimated parameter.

Just like calculating point estimates, it is incredibly important to get your standard errors right. You have to know what you don't know!

1.1.1 Robust standard errors

We are going to use the diamonds data set from `ggplot2` for this exercise so we don't need to load an external data set.

```
knitr::kable(head(diamonds))
```

carat	cut	color	clarity	depth	table	price	x	y	z
0.23	Ideal	E	SI2	61.5	55	326	3.95	3.98	2.43
0.21	Premium	E	SI1	59.8	61	326	3.89	3.84	2.31
0.23	Good	E	VS1	56.9	65	327	4.05	4.07	2.31
0.29	Premium	I	VS2	62.4	58	334	4.20	4.23	2.63
0.31	Good	J	SI2	63.3	58	335	4.34	4.35	2.75
0.24	Very Good	J	VVS2	62.8	57	336	3.94	3.96	2.48

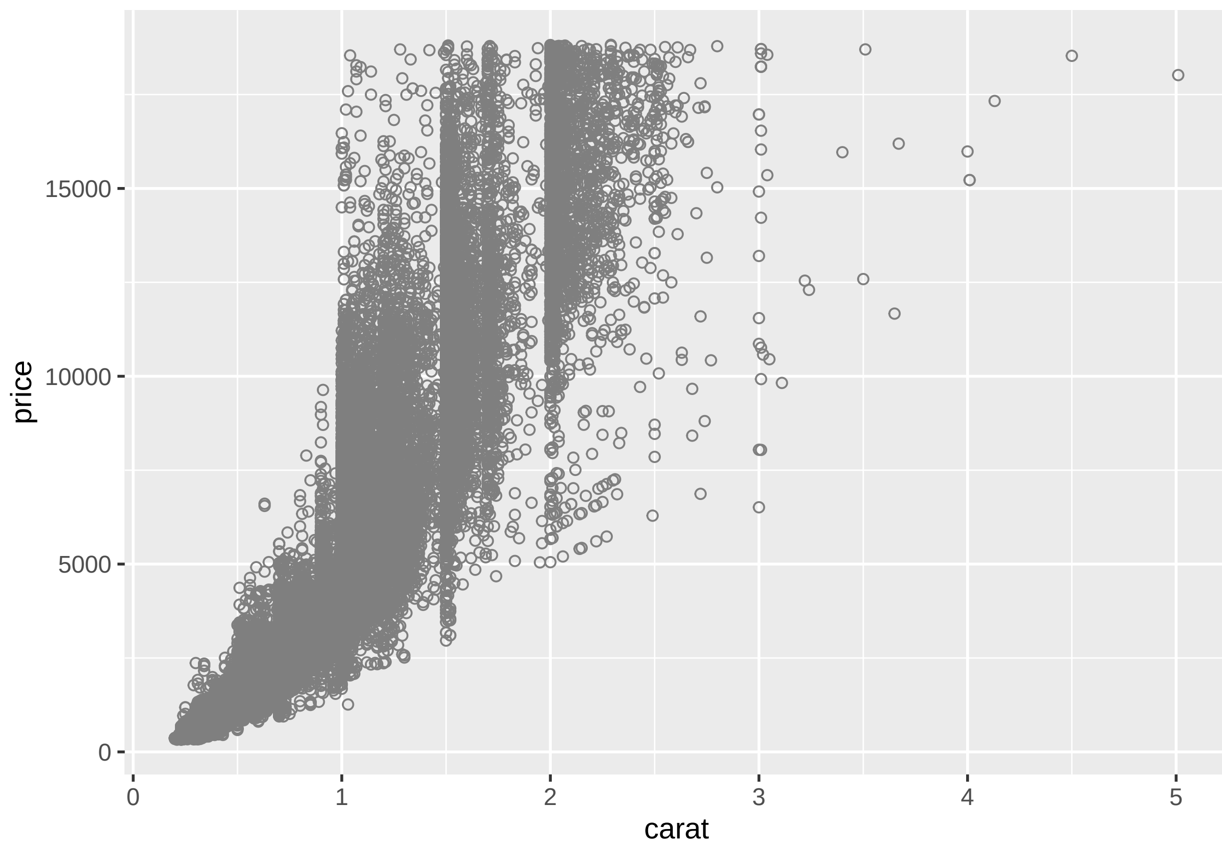
Lets regress price on carats and depth.

```r
reg1<-felm(price~carat+depth, diamonds)

summary(reg1)
```

```
##
## Call:
##    felm(formula = price ~ carat + depth, data = diamonds)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18238.9   -801.6    -19.6    546.3  12683.7
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4045.333    286.205   14.13   <2e-16 ***
## carat       7765.141     14.009  554.28   <2e-16 ***
## depth       -102.165      4.635  -22.04   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1542 on 53937 degrees of freedom
## Multiple R-squared(full model): 0.8507   Adjusted R-squared: 0.8507
## Multiple R-squared(proj model): 0.8507   Adjusted R-squared: 0.8507
## F-statistic(full model):1.536e+05 on 2 and 53937 DF, p-value: < 2.2e-16
## F-statistic(proj model): 1.536e+05 on 2 and 53937 DF, p-value: < 2.2e-16
```

Cool. But of course, we should make sure that our OLS assumptions make sense. One easy way to do this is to plot the data:

```r
myPlot <- ggplot(data = diamonds, aes(y = price, x = carat)) +
geom_point(color = "gray50", shape = 21)

myPlot
```



There are a bunch of things about this plot that should give you the econometric heebie jeebies. From an OLS perspective, you should be very afraid that these data are definitely not homoskedastic. The higher the carat, the greater the variance in price. This means that our OLS standard errors are likely going to get things wrong.

Heteroskedasticity is scary- but thankfully all is not lost. All we have to do is tweak our original assumptions a little bit to relax the homoskedasticity assumption and allow for the variance to depend on the value of x_i.

We know that

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sigma^2}{(\sum_{i=1}^{n}(x_i - \bar{x})^2)^2}$$

With heteroskedasticity σ^2 is no longer constant and becomes a function of the particular value of x_i an observation has, so

$$Var(u_i|x_i) = \sigma_i^2$$

Where are we going to find all these σ_i^2 for each individual observation?

Econometricians Eicker, Huber and White figured out a way to do this by basically using the square of the estimated residual of each observation, $\hat{u}_i^2$, as a stand-in for σ_i^2. With this trick, a valid estimator for $Var(be\hat{t}a_1)$, with heteroskedasticity of **any** form (including homoskedasticity), is

$$Var(\hat{\beta}_1) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2 \hat{u}_i^2}{(\sum_{i=1}^{n}(x_i - \bar{x})^2)^2}$$

We commonly call the resulting standard errors "robust", or "heteroskedasticity-robust".

How can we find these in R?

```
reg1<-felm(price~carat+depth, diamonds)

summary(reg1, robust=TRUE)
```

```
##
## Call:
##    felm(formula = price ~ carat + depth, data = diamonds)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18238.9   -801.6    -19.6    546.3  12683.7
##
## Coefficients:
##              Estimate Robust s.e t value Pr(>|t|)
## (Intercept) 4045.333    369.176   10.96   <2e-16 ***
## carat       7765.141     25.105  309.31   <2e-16 ***
## depth       -102.165      5.946  -17.18   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1542 on 53937 degrees of freedom
## Multiple R-squared(full model): 0.8507   Adjusted R-squared: 0.8507
## Multiple R-squared(proj model): 0.8507   Adjusted R-squared: 0.8507
## F-statistic(full model, *iid*):1.536e+05 on 2 and 53937 DF, p-value: < 2.2e-16
## F-statistic(proj model): 4.878e+04 on 2 and 53937 DF, p-value: < 2.2e-16
```

Or if you want to put them in a stargazer table:

```
stargazer(reg1, type = "latex" , se =  list(reg1$rse), header=FALSE)
```

Table 2:

	Dependent variable:
	price
carat	7,765.141***
	(25.105)
depth	−102.165***
	(5.946)
Constant	4,045.333***
	(369.176)
Observations	53,940
R^2	0.851
Adjusted R^2	0.851
Residual Std. Error	1,541.649 (df = 53937)
Note:	*p<0.1; **p<0.05; ***p<0.01

It is worth noting that robust standard errors are larger than regular standard errors, and thus more conservative (which is the right thing to be… you want to know what you don't know).

1.1.2 Clustered standard errors

Econometricians Haiku

T-stats looks too good

Try cluster standard errors

significance gone.

from Angrist and Pischke 2008

Suppose that every observation belongs to (only) one of G groups. The assumption we make when we cluster is that there is no correlation across groups- but we will allow for arbitrary within-group correlation. A great example: consider individuals within a village. In many cases it's pretty reasonable to think that individuals' error terms are correlated within a village, but that individuals' errors aren't correlated across villages.

I will spare you the matrix math needed to dive deeper into this. Suffice to say that "cluster-robust" estimates allow for a more complicated set of correlations to exist within observations within a cluster. One thing to be aware of though is that you will need to have a fairly large number of clusters (40+) for the estimate to be credible.

Clustering in R:

I use the `NOxEmissions` dataset from the `robustbase` package. This is a dataset of hourly NO_x readings, including NO_x concentration, auto emissions and windspeed. We are going to use the observation date as our cluster variable. This allows for arbitrary dependence between observations in the same day, and zero correlation across days. Is this reasonable? … Maybe. But we'll go with it for now:

```r
nox <- as.data.frame(NOxEmissions) %>%
mutate(ones = 1)
noClusters <- felm(data = nox, LNOx ~ sqrtWS )

Clusters <- felm(data = nox, LNOx ~ sqrtWS |0|0| julday)

stargazer(noClusters,Clusters, type = "latex" , header=FALSE)
```

Table 3:

	Dependent variable:	
	LNOx	
	(1)	(2)
sqrtWS	-0.864^{***}	-0.864^{***}
	(0.020)	(0.048)
Constant	5.559^{***}	5.559^{***}
	(0.029)	(0.065)
Observations	8,088	8,088
R^2	0.185	0.185
Adjusted R^2	0.185	0.185
Residual Std. Error (df = 8086)	0.846	0.846
Note:	*p<0.1; **p<0.05; ***p<0.01	

In this case, the regular standard errors are smaller than the clustered standard errors. Be aware that this need not necessarily be the case - depending on the correlation between observations within a cluster, clustered standard errors can be smaller than regular standard errors.

1.1.3 Newey West Standard Errors

For time series data.

1.1.4 Conley Standard Errors

For spatial data.

1.2 Confidence intervals for predictions

You've already had to "predict" a value of the dependent variable, y, given certain values of the independent variables. But this prediction is just a guess, and we can construct a confidence interval to give a range of possible values for this prediction, to demonstrate this uncertainty.

There are two kinds of predictions we can make:

- A confidence interval for the **average** y given x_1, $x_2 \ldots$ x_k.
- A confidence interval for a **particular** y given x_1, $x_2 \ldots$ x_k.

We will use Wooldridge's birth weight data to construct both kinds of confidence intervals to demonstrate the (subtle) difference between them.

$$bweight = \beta_0 + \beta_1 lfaminc + \beta_2 meduc + \beta_3 parity + u$$

where bwght is birth weight in ounces, lfaminc is the log of family income in \$1000s, meduc is the education of the mother in years, and parity is the birth order of the child.

Estimating this equation in R, we get the following results:

```r
#using the bwght data from the wooldridge package
reg1<-lm(bwght~lfaminc+motheduc+parity, bwght)

summary(reg1)
```

```
##
## Call:
## lm(formula = bwght ~ lfaminc + motheduc + parity, data = bwght)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -94.533 -11.888   0.779  13.136 151.477
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 105.5652     3.3666  31.356  < 2e-16 ***
## lfaminc       2.1313     0.6506   3.276  0.00108 **
## motheduc      0.3172     0.2520   1.259  0.20829
## parity        1.5261     0.6119   2.494  0.01275 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.21 on 1383 degrees of freedom
##   (1 observation deleted due to missingness)
```

```
## Multiple R-squared:  0.01633,    Adjusted R-squared:  0.0142
## F-statistic: 7.654 on 3 and 1383 DF,  p-value: 4.482e-05
```

1.2.0.1 Confidence interval for the average Birthweight Recall that our model gives us the expected value:

$$E[bweight|lfaminc, meduc, parity] = \beta_0 + \beta_1 log(faminc) + \beta_2 meduc + \beta_3 parity$$

and our regression gives us an estimate of this:

$$\hat{E}[bweight|faminc, meduc, parity] = \hat{y} = \hat{\beta_0} + \hat{\beta_1} log(faminc) + \hat{\beta_2} meduc + \hat{\beta_3} parity$$

In words: when we plug in particular values of the independent variables, we obtain a predication for y, which is an estimate for the expected value of y given the particular values for the explanatory variables.

Say we are interested in a confidence interval for the **average birthweight** for babies with a family income of \$14,500 (ln(14.5)=2.674), mothers with 12 years of education, and with 2 older siblings (parity=3). In other words, we are interested in:

$$\begin{aligned}
\hat{E}[bweight|faminc = 14.5, meduc = 12, parity = 3] &= 105.66 + 2.13 ln(faminc) + 0.317 meduc + 1.53 parity \\
\hat{y}_{faminc=14.5, meduc=12, parity=3} &= 105.66 + 2.13(2.674) + 0.317(12) + 1.53(3) \\
&= 119.75 ounces
\end{aligned}$$

How do we find a standard error for our estimate of the expected value of y for these particular values of the explanatory variables? Notice that this standard error is complicated because $\widehat{bweight}$ is a function of our $\hat{\beta}$'s which are all random variables. To avoid this computation, we want to transform our data. Before proceeding with the formal steps, recall that we have the following regression in mind

$$bweight = \beta_0 + \beta_1 lfaminc + \beta_2 meduc + \beta_3 parity + u$$

Then

$$\hat{\beta_0} = \hat{E}(bweight|lfaminc = 0, meduc = 0, parity = 0)$$

If we modify the regression by subtracting our particular values (specified above) for each of the independent variables, then we get the following regression

$$bweight = \beta_0 + \beta_1(lfaminc - 2.674) + \beta_2(meduc - 12) + \beta_3(parity - 3) + u$$

Then

$$\hat{\beta_0} = \hat{E}(bweight|lfaminc = 2.674, meduc = 12, parity = 3).$$

In other words, the new intercept is the predicted birthweight for babies with family incomes of \$14,500 (ln(14.5)=2.674), mothers with 12 years of education and with 2 older siblings. That's perfect! If we run this regression, `R` always outputs a standard error for the intercept coefficient. We can then grab the standard errors from there.

So step by step we need to:

1) Generate new variables: $\tilde{x}_j = x_j - \alpha_j$
2) Run the regression: $y = \tilde{\beta_0} + \tilde{\beta_1}\tilde{x_1} + ... + \tilde{\beta_k}\tilde{x_k} + \tilde{u}$
3) Then $\hat{E}[y|x_1 = \alpha_1, ..., x_k = \alpha_k] = \tilde{\beta_0}$

4) Plug these values into the formula for confidence intervals and interpret.

Below is the code for steps 1 and 2:

```
#Step 1: generate new variables
bwght$lfaminc_0<-bwght$lfaminc-2.674
bwght$motheduc_0<-bwght$motheduc-12
bwght$parity_0<-bwght$parity-3

#step 2: run the new regression

reg2<-lm(bwght~lfaminc_0+motheduc_0+parity_0,bwght)

summary(reg2)
```

```
##
## Call:
## lm(formula = bwght ~ lfaminc_0 + motheduc_0 + parity_0, data = bwght)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -94.533 -11.888   0.779  13.136 151.477
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 119.6491     1.0066 118.864  < 2e-16 ***
## lfaminc_0     2.1313     0.6506   3.276  0.00108 **
## motheduc_0    0.3172     0.2520   1.259  0.20829
## parity_0      1.5261     0.6119   2.494  0.01275 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.21 on 1383 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.01633,    Adjusted R-squared:  0.0142
## F-statistic: 7.654 on 3 and 1383 DF,  p-value: 4.482e-05
```

Using this output, the 95% confidence interval for the average birthweight for babies given a family income of \$14,500, a mother with 12 years of education and with 2 older siblings is:

$$[119.64 - 1.96(1.007), 119.64 + 1.96(1.007)] = [117.6653, 121.6158]$$

1.2.0.2 Confidence Interval for a particular individual's birthweight

A confidence interval for the average of observations with specific characteristics is not the same as a confidence interval for a particular individual observation. In forming a confidence interval for a particular unit, we must account for another very important source of variation: the variance in the unobserved error, which measures our ignorance of the unobserved factors that affect y_i.

We would like to construct a confidence interval for the birthweight of baby $i = 1$. Let $bweight_{i=1}$ denote that particular baby's birthweight with

$$bweight_{i=1} = \beta_0 + \beta_1 lfaminc_{i=1} + \beta_2 meduc_{i=1} + \beta_3 parity_{i=1} + u_{i=1}$$

Our best prediction of $bweight_{i=1}$ is $\widehat{bwieght}_{i=1}$ where

$$\widehat{bweight}_{i=1} = \hat{\beta}_0 + \hat{\beta}_1 lfaminc_{i=1} + \hat{\beta}_2 meduc_{i=1} + \hat{\beta}_3 parity_{i=1}$$

Now, there is some error, $\hat{u}_{i=1}$ associated with using $\widehat{bweight}_{i=1}$ to predict $bweight_{i=1}$ where

$$\begin{aligned}
\hat{u}_{i=1} &= bweight_{i=1} - \widehat{bweight}_{i=1} \\
&= (\beta_0 + \beta_1 lfaminc_{i=1} + \beta_2 meduc_{i=1} + \beta_3 parity_{i=1} + u_{i=1}) - (\hat{\beta}_0 + \hat{\beta}_1 lfaminc_{i=1} + \hat{\beta}_2 meduc_{i=1} + \hat{\beta}_3 parity_{i=1})
\end{aligned}$$

Finding the expected value, we get:

$$\begin{aligned}
E[\hat{u}_{i=1}] &= E[bweight_{i=1} - \widehat{bweight}_{i=1}] \\
&= (\beta_0 + \beta_1 lfaminc_{i=1} + \beta_2 meduc_{i=1} + \beta_3 parity_{i=1} + E[u_{i=1}]) - (E[\hat{\beta}_0] + E[\hat{\beta}_1] lfaminc_{i=1} + E[\hat{\beta}_2] meduc_{i=1} + E[\hat{\beta}_3] parity_{i=1}) \\
&= 0
\end{aligned}$$

Finding the variance we get

$$\begin{aligned}
Var(\hat{u}_{i=1}) &= Var(bweight_{i=1} - \widehat{bweight}_{i=1}) \\
&= Var(\beta_0 + \beta_1 lfaminc_{i=1} + \beta_2 meduc_{i=1} + \beta_3 parity_{i=1} + u_{i=1} - \widehat{bweight}_{i=1}) \\
&= Var(\widehat{bweight}_{i=1}) + Var(u_{i=1}) \\
&= Var(\widehat{bweight}_{i=1}) + \sigma^2 \\
\widehat{Var}(\hat{u}_{i=1}) &= Var(\widehat{bweight}_{i=1}) + \hat{\sigma}^2 \\
&= Var(\widehat{bweight}_{i=1}) + \frac{\sum \hat{u}_i^2}{n-k-1} \\
&= Var(\widehat{bweight}_{i=1}) + \frac{SSR}{n-k-1}
\end{aligned}$$

So you should see that there are two sources of variation in $\hat{u}_{i=1}$. First we have the sampling error in $\widehat{bweight}_{i=1}$ which arises because we have estimated the population parameters β. Second we have the variance of the error in the population ($u_{i=1}$).

Now we can compute the $Var(\widehat{bweight}_{i=1})$ exactly the way we did before (by subtracting the specific values we are interested in and re-running the regression and looking at the intercept term's standard errors). Second we can compute $\frac{SSR}{n-k-1}$ from our regression output. The 95% confidence interval for $bweight_{i=1}$ is then

$$\hat{y} \pm 1.96 * se(\hat{u}_{i=1})$$

Steps in computing a confidence interval for a particular y when $x_j = \alpha_j$:

1) Generate new variables: $\tilde{x}_j = x_j - \alpha_j$
2) Run the regression: $y = \tilde{\beta}_0 + \tilde{\beta}_1 \tilde{x_1} + ... + \tilde{\beta}_k \tilde{x_k} + \tilde{u}$
3) Then $\hat{E}[y|x_1 = \alpha_1, ..., x_k = \alpha_k] = \tilde{\beta}_0$ and the standard error of the estimate is $se(\tilde{\beta}_0)$
4) Get an estimate for the variance of $\hat{u} = \hat{\sigma}^2$ from the R output
5) compute the standard error: $\sqrt{se(\tilde{\beta}_0)^2 + \hat{\sigma}^2}$
6) Plug these values into the formula for confidence intervals and interpret.

Below is the code:

```r
#Step 1: generate new variables
bwght$lfaminc_0<-bwght$lfaminc-2.674
bwght$motheduc_0<-bwght$motheduc-12
bwght$parity_0<-bwght$parity-3

#step 2: run the new regression

reg2<-lm(bwght~lfaminc_0+motheduc_0+parity_0,bwght)

summary(reg2)
```

```
##
## Call:
## lm(formula = bwght ~ lfaminc_0 + motheduc_0 + parity_0, data = bwght)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -94.533 -11.888   0.779  13.136 151.477
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 119.6491     1.0066 118.864  < 2e-16 ***
## lfaminc_0     2.1313     0.6506   3.276  0.00108 **
## motheduc_0    0.3172     0.2520   1.259  0.20829
## parity_0      1.5261     0.6119   2.494  0.01275 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.21 on 1383 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.01633,    Adjusted R-squared:  0.0142
## F-statistic: 7.654 on 3 and 1383 DF,  p-value: 4.482e-05
```

```r
#step 4: get the estimate of the variance

summary(lm(bwght~lfaminc_0+motheduc_0+parity_0,bwght))$sigma^2
```

```
## [1] 408.5987
```

The confidence interval for a particular baby's birthweight with a family income of $14,500 (ln(14.5=2.674)), a mother with 12 years of education and with 2 older siblings we have:

$$\begin{aligned} SE &= \sqrt{se(\tilde{\beta}_0)^2 + \hat{\sigma}^2} = \sqrt{(1.007^2) + 408.59} = 20.239 \\ CI &= [119.64 - 1.96 * (20.239); 119.64 + 1.96 * (20.239)] \\ &= [79.972; 159.308] \end{aligned}$$

1.3 Variable Transformations

1.3.1 Scaling

1.3.2 Standardizing

Standardizing variables eliminates the units in order to be able to compare the magnitude of estimates across independent variables. This can make interpretation easier if you have variables with weird arbitrary units that are unfamiliar to people. We can solve this issue by standardizing the variables.

Suppose we have a regression with two variables, x_1 and x_2:

$$y = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{u}$$

We know that our regression must go through the point of averages, or think if we average the previous equation, and use the fact that the $\hat{u}_i$'s have a zero sample average, or if we plugged in $\bar{x}_1$ and $\bar{x}_2$, we would predict $\bar{y}$:

$$\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}_1 + \hat{\beta}_2 \bar{x}_2$$

We can subtract the second equation from the first to get:

$$\begin{aligned}\hat{y} - \bar{y} &= (\hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{u}) - (\hat{\beta}_0 + \hat{\beta}_1 \bar{x}_1 + \hat{\beta}_2 \bar{x}_2) \\ &= \hat{\beta}_1 (x_1 - \bar{x}_1) + \hat{\beta}_2 (x_2 - \bar{x}_2) + \hat{u}\end{aligned}$$

With a little bit of additional algebra, dividing both sides of this equation by the standard deviation of y, σ_y and multiplying each independent variable by $1 = \frac{\sigma_x}{\sigma_x}$. we can get the entire regression into standard units:

$$(\frac{y - \bar{y}}{\hat{\sigma}_y}) = \frac{\hat{\sigma}_{x_1}}{\hat{\sigma}_y} \hat{\beta}_1 (\frac{x_1 - \bar{x}_1}{\hat{\sigma}_{x_1}}) + \frac{\hat{\sigma}_{x_2}}{\hat{\sigma}_y} \hat{\beta}_2 (\frac{x_2 - \bar{x}_2}{\hat{\sigma}_{x_2}}) + \frac{\hat{u}}{\hat{\sigma}_y}$$

Now we can say that controlling for x_2 a one standard deviation increase in x_1 leads to a $\frac{\hat{\sigma}_{x_1}}{\hat{\sigma}_y} \hat{\beta}_1$ standard deviation increase in the predicted y. We call this new term the standardized coefficient or "beta" coefficient. In R, we can get these coefficients by using the `lm.beta` command following a regression using `lm`.

We use the bwght2 dataset to look at how parent ages correlate with birth weights. (Note: birth weights here will be measured in grams). I estimate four different regressions of the type

$$birthweight_i = \beta_0 + \beta_1 motherage_i + \beta_2 fatherage_i + \epsilon_i$$

scaling either the dependent and/or independent variables.

```r
bwght<-bwght2

reg1<-lm(bwght~mage+fage, bwght)
reg2<-lm(scale(bwght)~scale(mage)+scale(fage), bwght)
reg3<-lm(scale(bwght)~mage+fage, bwght)
reg4<-lm(bwght~scale(mage)+scale(fage), bwght)

meandep1<-round(mean(bwght$bwght),2)
meandep2<-round(mean(scale(bwght$bwght)),2)
sddep1<-round(sd(bwght$bwght),2)
sddep2<-round(sd(scale(bwght$bwght)),2)

stargazer(reg1,reg2, reg3, reg4, type = "latex" , header=FALSE,
          add.lines=list(c("Mean",meandep1,meandep2, meandep2, meandep1 ),
                         c("SD",sddep1,sddep2, sddep2, sddep1 )))
```

Interpreting column 1:

- Having a mother that is **a year** older at birth predicts a birthweight that is 3.992 **grams** less (not statistically significant).
- Having a father that is **a year** older at birth predicts a birthweight that is 9.313 **grams** more (highly statistically significant).

Table 4:

	Dependent variable:			
	bwght	scale(bwght)		bwght
	(1)	(2)	(3)	(4)
mage	−3.992		−0.007	
	(3.943)		(0.007)	
fage	9.313***		0.016***	
	(3.291)		(0.006)	
scale(mage)		−0.033		−19.044
		(0.033)		(18.812)
scale(fage)		0.092***		53.205***
		(0.033)		(18.803)
Constant	3,221.030***	−0.001	−0.312**	3,400.304***
	(87.703)	(0.023)	(0.152)	(13.448)
Mean	3401.12	0	0	3401.12
SD	576.54	1	1	576.54
Observations	1,826	1,826	1,826	1,826
R^2	0.005	0.005	0.005	0.005
Adjusted R^2	0.004	0.004	0.004	0.004
Residual Std. Error (df = 1823)	574.677	0.997	0.997	574.677
F Statistic (df = 2; 1823)	4.912***	4.912***	4.912***	4.912***
Note:		*p<0.1; **p<0.05; ***p<0.01		

Interpreting column 2:

- Having a mother whose age is **one standard deviation higher** at birth predicts a birthweight that is 0.033 **standard deviations** lower (not statistically significant).
- Having a father whose age is **one standard deviation higher** at birth predicts a birthweight that is 0.092 **standard deviations** higher (highly statistically significant).

Interpreting column 3:

- Having a mother that is **a year** older at birth predicts a birthweight that is 0.007 **standard deviations** lower (not statistically significant).
- Having a father that is **a year** older at birth predict a birthweight that is 0.016 **standard deviations** higher (highly statistically significant).

Interpreting column 4:

- Having a mother whose age is **one standard deviation higher** at birth predicts a birthweight that is 19.044 **grams** lower (not statistically significant).
- Having a father whose age is **one standard deviation higher** at birth predicts a birthweight that is 53.205 **grams** higher (highly statistically significant).

Notes:

- You can also standardize y, x_1 and x_2 directly in the data set and then run your regression on the standardized variables though this involves more coding.
- You do not need to standardize all the variables. You could just standardize x_1 and adjust your interpretation accordingly (you will need an intercept in this case).

1.3.3 Logs

1.3.4 Quadratics

1.3.5 Interactions with continuous variables

1.4 Binary Dependent variables

1.4.1 Linear Probability Models

Most of the models we have seen have had continuous dependent variables: the y variable was some quantitative amount that took on a range of possible values (a test score, a weight,...). Sometimes though y can be a dummy variable, taking on either the value 1 or 0. What changes when we have a binary variable on the left hand side?

$$y = \beta_0 + \beta_1 x_1 + ... + \beta_k x_k + u$$

- It no longer makes sense to interpret β_j as the unit change in y given a one-unit increase in x_j holding all other factors fixed. Indeed, y either changes from $0 \to 1$, from $1 \to 0$ or doesn't change.
- The β_j's still have a useful interpretation though: each β_j measures the change in the probability of success when x_j changes by one unit holding all other factors constant.

$$Pr(y = 1|x) = \beta_0 + \beta_1 x_1 + ... + \beta_k x_k$$

1.4.1.1 Example: Let $inlf$ ("in the labor force") be a binary variable indicating labor force participation by a married woman in 1975. $inlf = 1$ if the woman reports working for a wage outside the home at some point during the year, and zero otherwise. Our independent variables might include:

- husbands earnings ($nwifeinc$, measured in thousands of dollars)

- years of education ($educ$)
- past years of labor market experience ($exper$)
- age (age)
- number of children less than six years old ($kidslt6$)
- number of kids between 6 and 18 years of age ($kidsge6$)

We estimate the following model using the `mroz` data that is part of the `wooldridge` package:

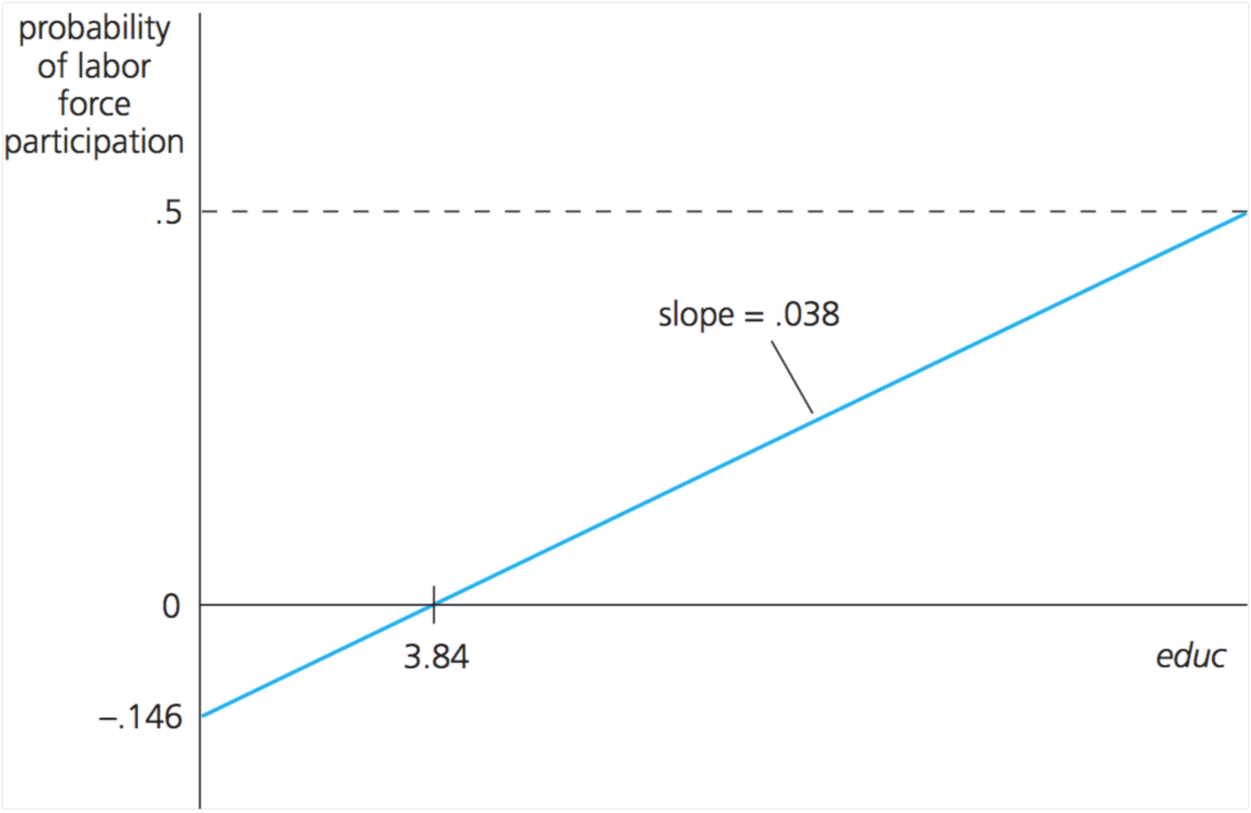
$$inlf = \beta_0 + \beta_1 nwifeinc + \beta_2 educ + \beta_3 exper + \beta_4 exper^2 + \beta_5 age + \beta_6 kidslt6 + \beta_7 kidsge6 + u$$

```
mroz$exper2<-mroz$exper^2
reg1<-lm(inlf~nwifeinc+educ+exper+exper2+age+kidslt6+kidsge6, mroz)
summary(reg1)
```

```
##
## Call:
## lm(formula = inlf ~ nwifeinc + educ + exper + exper2 + age +
##     kidslt6 + kidsge6, data = mroz)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.93432 -0.37526  0.08833  0.34404  0.99417
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.5855192  0.1541780   3.798 0.000158 ***
## nwifeinc    -0.0034052  0.0014485  -2.351 0.018991 *
## educ         0.0379953  0.0073760   5.151 3.32e-07 ***
## exper        0.0394924  0.0056727   6.962 7.38e-12 ***
## exper2      -0.0005963  0.0001848  -3.227 0.001306 **
## age         -0.0160908  0.0024847  -6.476 1.71e-10 ***
## kidslt6     -0.2618105  0.0335058  -7.814 1.89e-14 ***
## kidsge6      0.0130122  0.0131960   0.986 0.324415
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4271 on 745 degrees of freedom
## Multiple R-squared:  0.2642, Adjusted R-squared:  0.2573
## F-statistic: 38.22 on 7 and 745 DF,  p-value: < 2.2e-16
```

As an example, given that this is a linear probability model, we would interpret the coefficient on $educ$ as: holding all else fixed, an additional year of education increases predicted probability of labor force participation by 0.038, or 3.8 percentage points.

The graph below depicts the probability of labor force participation and $educ$. The other independent variables, for the purposes of this example, are fixed at the values $nwifeinc = 50$, $exper = 5$, $age = 30$, $kidslt6 = 1$ and $kidsge6 = 0$.



For a woman with these characteristics, the relationship between years of education and the probability of being in the labor force is given by:

$$\begin{aligned} Pr(inlf = 1|educ) &= (0.585 + (-0.003) * 50 + (0.039) * 5 + (-0.001) * 5^2 + (-0.016) * 30 + (-0.262) * 1) + 0.038 * educ \\ &= -0.146 + 0.038educ \end{aligned}$$

1.4.1.2 Linear Probability Model Drawbacks:

1) Because this is a linear regression, we are trying to fit a line to the data. This means that we can get predicted probabilities that are negative or greater than one. We can see here that the predicted probability of labor force participation for women with education that is below 3.84 years is negative. Similarly, if a woman has 31 years of education it would be greater than one. This isn't too much of a concern here because in the sample few women will fall in those ranges. Nevertheless, the predicted probabilities from our regression aren't bound between 0 and 1.

2) A related problem: a linear probability model implies that the outcome is linearly related to the independent variables. T His is not possible with probabilities. In the previous example, an additional child reduces the probability of working by 0.262. Thus our model implies that going from 0 to 4 young children reduces the probability by 0.262*4=1.048, 104.8 percentage points, which is impossible.

3) This model uses the idea that the probability that the dummy is equal to one is actually a function of our x's, which means the variance of the dummy is a function of our x's. Indeed, when y is a binary variable

$$Var(y|x) = p(x)[1 - p(x)]$$

where $p(x)$ is a shorthand for the probability of success $p(x) = \beta_0 + \beta_1 x_1 + ... + \beta_k x_k$. This means that there must be heteroskedasticity in the linear probability model. Thus you should always use **heteroskedasticity robust standard errors** with linear probability models.

1.4.2 Logits

How do we address the drawbacks of linear probability models? With **logistic regressions**, which are estimated via maximum likelihood methods rather than OLS.

Logits differ from the linear probability model in what kind of function is used for the estimated probability.

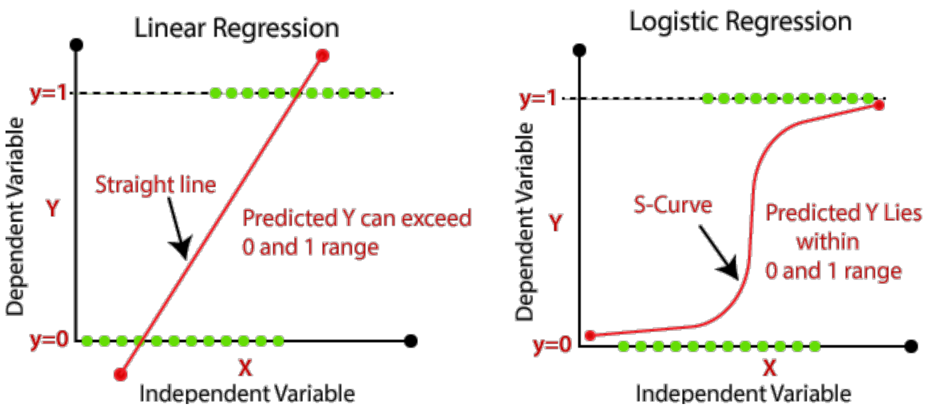
In linear probability models, we have

$$Pr(y=1|x_1,x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2.$$

With a logit, we have

$$Pr(y=1|x_1,x_2) = \frac{e^{(\beta_0+\beta_1 x_1+\beta_2 x_2)}}{1+e^{(\beta_0+\beta_1 x_1+\beta_2 x_2)}} = \Lambda(\beta_0 + \beta_1 x_1 + \beta_2 x_2)$$

where Λ is commonly used notation to represent the complex logit function.



Why would we ever pick a functional form for the probability that looks as complicated as Λ? Because it is bounded between 0 and 1 (so our predictions make sense), and it has nice statistical properties (which we will not get into).

One important thing to note about the estimation of logit models is that `R` output will not give you the marginal effect of one more unit of x_j on the probability of y taking the value 1 ($\frac{\partial Pr(y=1|x)}{\partial x_j}$). Instead it reports the **log-odds** by default, which are not the same as marginal effects!

To get marginal effects, we need to do a couple more steps. This is because these models are **non-linear**, such that the exact marginal effect of x_j on y **changes depending on the other values of x**. Therefore we have a bunch of options for where we calculate the marginal effects at. In `R`, the default is to compute the **average marginal effect** which is the average of the marginal effect for each observation in the sample. In `R` we can also specify any values of our independent variables to evaluate our marginal effects precisely at those values. Note that this is in contrast to the constant marginal effect of x_j on $Pr(y=1|x)$ found in the linear probability model, and is another potential benefit of running a logit model.

1.4.2.1 Example:

I return to the previous example, looking at labor force participation using the `mroz` data.

In `R` we can estimate a logistic regression with the `glm()` function. Running `summary()` will then get us the log-odds.

```
reglogit1<-glm(inlf~nwifeinc+educ+exper+exper2+age+kidslt6+kidsge6, mroz, family="binomial")
summary(reglogit1)
```

```
##
## Call:
```

```
## glm(formula = inlf ~ nwifeinc + educ + exper + exper2 + age +
##     kidslt6 + kidsge6, family = "binomial", data = mroz)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1770  -0.9063   0.4473   0.8561   2.4032
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.425452   0.860365   0.495  0.62095
## nwifeinc    -0.021345   0.008421  -2.535  0.01126 *
## educ         0.221170   0.043439   5.091 3.55e-07 ***
## exper        0.205870   0.032057   6.422 1.34e-10 ***
## exper2      -0.003154   0.001016  -3.104  0.00191 **
## age         -0.088024   0.014573  -6.040 1.54e-09 ***
## kidslt6     -1.443354   0.203583  -7.090 1.34e-12 ***
## kidsge6      0.060112   0.074789   0.804  0.42154
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1029.75  on 752  degrees of freedom
## Residual deviance:  803.53  on 745  degrees of freedom
## AIC: 819.53
##
## Number of Fisher Scoring iterations: 4
```

To get the marginal effects, we need to run the estimated `glm()` object through the `margins()` function from the **margins** package.

```r
library(margins)
```

```
## Warning: package 'margins' was built under R version 3.6.3
```

```r
marg_reglogit1<-margins(reglogit1)

summary(marg_reglogit1)
```

```
##    factor     AME     SE       z      p   lower   upper
##       age -0.0157 0.0024 -6.6027 0.0000 -0.0204 -0.0111
##      educ  0.0395 0.0073  5.4145 0.0000  0.0252  0.0538
##     exper  0.0368 0.0052  7.1386 0.0000  0.0267  0.0469
##    exper2 -0.0006 0.0002 -3.1759 0.0015 -0.0009 -0.0002
##   kidsge6  0.0107 0.0133  0.8051 0.4207 -0.0154  0.0369
##   kidslt6 -0.2578 0.0319 -8.0696 0.0000 -0.3204 -0.1951
##  nwifeinc -0.0038 0.0015 -2.5714 0.0101 -0.0067 -0.0009
```

Recall that by default this computes the average marginal effect for the sample. To instead compute the marginal effect for a particular type of observation

```r
DF <- data.frame(age=30,
                 nwifeinc=50,
                 exper=5,
                 exper2=25,
                 kidsge6=0,
                 kidslt6=1,
```

```
                educ=12,
                stringsAsFactors=FALSE)

marg_specific <- margins(reglogit1, data = DF)

summary(marg_specific)
```

```
##    factor     AME     SE       z      p   lower   upper
##       age -0.0163 0.0049 -3.3268 0.0009 -0.0259 -0.0067
##      educ  0.0410 0.0088  4.6729 0.0000  0.0238  0.0582
##     exper  0.0382 0.0089  4.2893 0.0000  0.0207  0.0556
##    exper2 -0.0006 0.0002 -2.8205 0.0048 -0.0010 -0.0002
##   kidsge6  0.0111 0.0130  0.8554 0.3924 -0.0144  0.0367
##   kidslt6 -0.2675 0.0567 -4.7195 0.0000 -0.3787 -0.1564
##  nwifeinc -0.0040 0.0011 -3.5885 0.0003 -0.0061 -0.0018
```

1.4.3 Example contrasting a logit to a linear probability model (LPM)

In sports betting, the Las Vegas point spread is the predicted scoring differential between two opponents as quoted by a sports book in Las Vegas. We are interested in the probability that the favored team actually wins.

We can run the following regression:

$$\widehat{favwin}_i = \hat{\beta}_0 + \hat{\beta}_1 spread_i + \hat{\beta}_2 favhome_i + \hat{\beta}_3 fav25_i + \hat{\beta}_4 und25_i + \hat{u}_i$$

Where $favwin$ is a dummy variable indicating whether the favored team won, $spread$ is the Las Vegas point spread, $favhome$ is a dummy variable indicating whether the favored team is playing at home, $fav25$ and $und25$ indicate whether the favored team and the underdog team are in the top 25 teams respectively.

I use the `pntsprd` data from the wooldridge package to first estimate the linear probability model:

```
lpm<-felm(favwin~spread+favhome+fav25+und25, data=pntsprd)

summary(lpm, robust=TRUE)
```

```
##
## Call:
##    felm(formula = favwin ~ spread + favhome + fav25 + und25, data = pntsprd)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.9972 -0.1105  0.1470  0.2991  0.4869
##
## Coefficients:
##              Estimate Robust s.e t value Pr(>|t|)
## (Intercept)  0.558815   0.040422  13.825   <2e-16 ***
## spread       0.017763   0.002056   8.637   <2e-16 ***
## favhome      0.054353   0.040923   1.328    0.185
## fav25        0.010982   0.039100   0.281    0.779
## und25       -0.101104   0.089530  -1.129    0.259
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4016 on 548 degrees of freedom
## Multiple R-squared(full model): 0.116   Adjusted R-squared: 0.1095
```

```
## Multiple R-squared(proj model): 0.116   Adjusted R-squared: 0.1095
## F-statistic(full model, *iid*):17.97 on 4 and 548 DF, p-value: 7.007e-14
## F-statistic(proj model):  26.2 on 4 and 548 DF, p-value: < 2.2e-16
```

Let's compare the LPM model results to those for the equivalent logit:

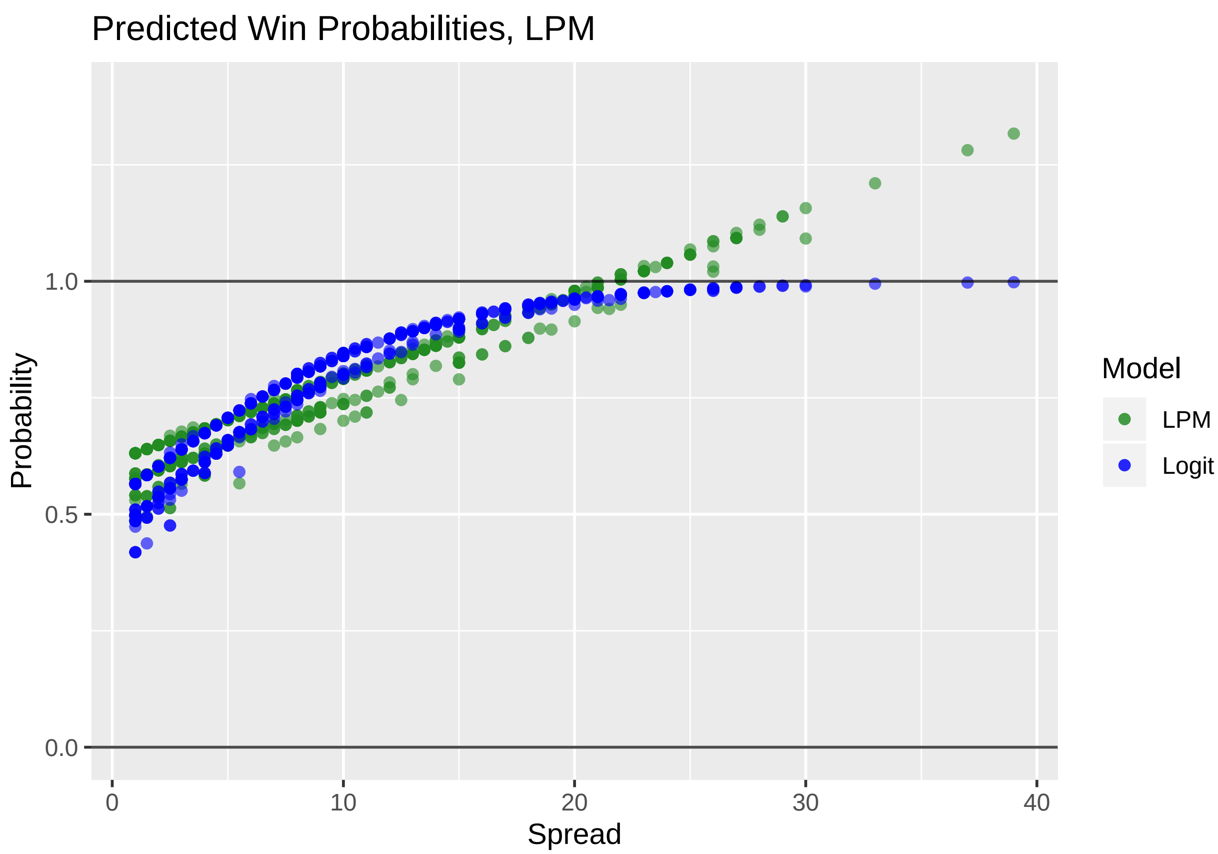
```
logit<-glm(favwin~spread+favhome+fav25+und25, data=pntsprd, family="binomial")
marg_logit<-margins(logit)
summary(marg_logit)
```

```
##   factor     AME     SE       z      p   lower  upper
##    fav25  0.0076 0.0434  0.1748 0.8612 -0.0774 0.0926
##  favhome  0.0425 0.0362  1.1740 0.2404 -0.0284 0.1134
##   spread  0.0243 0.0033  7.3360 0.0000  0.0178 0.0308
##    und25 -0.0579 0.0650 -0.8912 0.3728 -0.1852 0.0694
```

You can see that all else constant, a 1 point increase in the Vegas point spread is estimated to increase the predicted probability of winning by 1.78 percentage points using the linear probability model and 2.43 percentage points on average using the logit model. It is generally the case that results using these two different methods will often be quite similar.

To really see the difference between them, we can plot the predicted probabilities (obtained with `$fitted.values` on the `lm()` or `glm()` objects).

```
df<- mutate(pntsprd, lpm_prob=lpm$fitted.values,
            logit_prob=logit$fitted.values)

plot<-df%>%
  ggplot(aes(x=spread))+
  geom_point(aes(y=lpm_prob, colour="LPM"), alpha=0.6)+
  geom_point(aes(y=logit_prob, colour="Logit"), alpha=0.6)+
  geom_hline(yintercept = 1, alpha=0.7)+
  geom_hline(yintercept = 0,alpha=0.7)+
  scale_colour_manual("Model",
                      breaks=c("LPM", "Logit"),
                      values=c("blue", "forestgreen"))+
  lims(y=c(0,1.4))+
  labs(title="Predicted Win Probabilities, LPM",
       x="Spread",
       y="Probability")
plot
```



We can see straightaway that we estimated probabilities greater than one with the linear probability model (in green) for high point spreads, and that the logit probabilities (blue) never exceed one. Additionally, you can see that the effect of one additional point in the spread has a constant effect for linear probability models, while the slope of the logit probabilities changes depending on the level of the point spread.

1.5 Mapping in R

Increasingly, applied econometricians are embedding spatial data into our analyses. A whole host of research questions and designs become possible if you are able to take advantage of, and work with, the huge volume of spatial data that is rapidly becoming available. That said, working with spatial data takes a few new tricks. We'll first go through a variety of spatial data types, then show off the long-promised (and dramatically over-used) nighttime lights dataset. Coooooool.

First off, we often make a big deal out of spatial data - but it's just data like anything else. Anybody who tries to tell you that spatial data is *completely different*(!) from anything you've seen before is trying to sell you a textbook. The main difference between spatial datasets and any other type of dataset is that spatial data usually come in 2 (or even 3) dimensions. For practical purposes, this usually means that a spatial dataset has latitude and longitude variables in some form or another.

To deal with spatial data, we'll need a bunch of new packages: `GISTools`, `rgdal`, `rgeos`, `maptools`, `raster` (all spatial utilities), and `broom` (this will let us turn spatial data into a format that ggplot2 can handle). We'll also get a new package that will let us more easily deal with dates (because why not): `lubridate`, and a final one which gives us access to a bunch of new color palettes: `RColorBrewer`. The number of packages we're installing here should make you uncomfortable. While you certainly can do spatial data analysis in `R`, it's definitely a challenge compared with some other (proprietary) options out there (Matlab and ArcGIS, for example). The benefits are that you'll have a completely integrated work flow, all in one program; and that once you've wrangled your spatial data to turn it into something useful, `R` is great for everything else. The big problem with doing spatial data in `R` is that there's no unified spatial toolkit, meaning that different data types and formats and functions don't necessarily get along very well. I've tried to make this as headache-free as possible, but just be aware that (in my opinion) spatial data work is not necessarily R's comparative advantage.