# MQE: Economic Inference from Data: Odds and Ends

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#### Odds and Ends

- Non-Standard Standard Errors
  - Robust standard errors
  - Clustered standard errors
  - Newey West Standard Errors
  - Conley Standard errors
- Confidence intervals for prediction
  - For a particular average
  - For a particular unit
- Standardizing

#### Non-standard standard errors

A standard error estimates the uncertainty around an estimated parameter.

Formally we have

$$se = \sqrt{\widehat{Var(\hat{\beta})}}.$$

Just like calculating point estimates, it is incredibly important to get your standard errors right.

You have to know what you don't know!

- Robust standard errors
- Clustered standard errors

### Using the diamonds data set from ggplot2:

knitr::kable(head(diamonds))

carat	cut	color	clarity	depth	table	price	X	у	z
0.23	Ideal	E	SI2	61.5	55	326	3.95	3.98	2.43
0.21	Premium	E	SI1	59.8	61	326	3.89	3.84	2.31
0.23	Good	E	VS1	56.9	65	327	4.05	4.07	2.31
0.29	Premium	- 1	VS2	62.4	58	334	4.20	4.23	2.63
0.31	Good	J	SI2	63.3	58	335	4.34	4.35	2.75
0.24	Very Good	J	VVS2	62.8	57	336	3.94	3.96	2.48

#### Regress price on carats and depth.

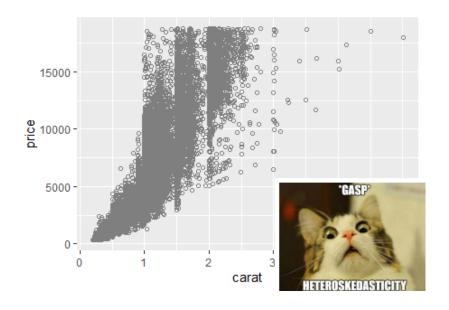
```
reg1<-felm(price-carat+depth, diamonds)
summary(reg1)</pre>
```

```
##
## Call:
     felm(formula = price ~ carat + depth, data = diamonds)
##
## Residuals:
       Min
                 10
                    Median
                                          Max
## -18238.9 -801.6 -19.6
                               546.3 12683.7
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4045.333 286.205 14.13 <2e-16 ***
## carat
              7765 141
                       14 009 554 28 <2e-16 ***
              -102.165 4.635 -22.04 <2e-16 ***
## depth
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1542 on 53937 degrees of freedom
## Multiple R-squared(full model): 0.8507 Adjusted R-squared: 0.8507
## Multiple R-squared(proj model): 0.8507 Adjusted R-squared: 0.8507
## F-statistic(full model):1.536e+05 on 2 and 53937 DF, p-value: < 2.2e-16
## F-statistic(proj model): 1.536e+05 on 2 and 53937 DF, p-value: < 2.2e-16
```

Cool.

## Plot the data to check OLS assumptions:

```
myPlot <- ggplot(data = diamonds, aes(y = price, x = carat)) +
geom_point(color = "gray50", shape = 21)</pre>
```



You should have the econometric heebie jeebies.

Homoskedastic assumption needed for OLS is not valid!

- ▶ The higher the carat, the greater the variance in price.
- ightharpoonup  $\Rightarrow$  OLS standard errors are likely to be wrong.

Thankfully all is not lost!

Lets relax the homoskedasticity assumption and allow for the variance to depend on the value of  $x_i$ .

We know that

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{\sum_{i=1}^n (x_i - \bar{x})^2 \sigma^2}{(\sum_{i=1}^n (x_i - \bar{x})^2)^2}$$

With heteroskedasticity  $\sigma^2$  is no longer constant and becomes a function of the particular value of  $x_i$  an observation has, so

$$Var(u_i|x_i) = \sigma_i^2$$

Where are we going to find all these  $\sigma_i^2$  for each individual observation?

## Eicker, Huber and White to the rescue!

Econometricians Eicker, Huber and White figured out a way to do this by basically using the square of the estimated residual of each observation,  $\hat{u}_i^2$ , as a stand-in for  $\sigma_i^2$ .

With this trick, a valid estimator for  $Var(b\hat{eta}_1)$ , with heteroskedasticity of **any** form (including homoskedasticity), is

$$Var(\hat{\beta}_1) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2 \hat{u}_i^2}{(\sum_{i=1}^{n} (x_i - \bar{x})^2)^2}$$

We commonly call the resulting standard errors "robust", or "heteroskedasticity-robust".

#### How can we find these in R?

```
reg1<-felm(price~carat+depth, diamonds)
summary(reg1, robust=TRUE)
##
## Call:
     felm(formula = price ~ carat + depth, data = diamonds)
##
## Residuals:
       Min
                1Q Median
                                          Max
## -18238 9 -801 6 -19 6
                               546.3 12683.7
##
## Coefficients:
              Estimate Robust s.e t value Pr(>|t|)
## (Intercept) 4045.333
                        369.176 10.96 <2e-16 ***
## carat
           7765.141
                       25.105 309.31 <2e-16 ***
            -102.165
                       5.946 -17.18 <2e-16 ***
## depth
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1542 on 53937 degrees of freedom
## Multiple R-squared(full model): 0.8507 Adjusted R-squared: 0.8507
## Multiple R-squared(proj model): 0.8507 Adjusted R-squared: 0.8507
## F-statistic(full model, *iid*):1.536e+05 on 2 and 53937 DF, p-value: < 2.2e-16
## F-statistic(proj model): 4.878e+04 on 2 and 53937 DF, p-value: < 2.2e-16
```

#### Or if you want to put them in a stargazer table:

```
stargazer(reg1, type = "latex" , se = list(reg1$rse), header=FALSE)
```

Table 2

	Dependent variable:		
	price		
carat	7,765.141***		
	(25.105)		
depth	-102.165***		
	(5.946)		
Constant	4,045.333***		
	(369.176)		
Observations	53,940		
$R^2$	0.851		
Adjusted R <sup>2</sup>	0.851		
Residual Std. Error	1,541.649 (df = 53937)		
Note:	*p<0.1; **p<0.05; ***p<0.01		

Note: robust standard errors are larger than regular standard errors, and thus more conservative (which is the right thing to be... you want to know what you don't know).

## Econometricians Haiku

T-stats looks too good

Try cluster standard errors significance gone.

from Angrist and Pischke 2008

Suppose that every observation belongs to (only) one of G groups.

The assumption we make when we cluster:

- there is no correlation across groups
- we allow for arbitrary within-group correlation.

Example: consider individuals within a village.

It may be reasonable to think that individuals' error terms are:

- correlated within a village
- aren't correlated across villages

I will spare you the matrix math needed to dive deeper into this.

Suffice to say that "cluster-robust" estimates allow for a more complicated set of correlations to exist within observations within a cluster.

One thing to be aware of though is that you will need to have a fairly large number of clusters (40+) for the estimate to be credible.

#### Clustering in R:

I use the NOxEmissions dataset from the robustbase package.

- ▶ hourly  $NO_x$  readings, including  $NO_x$  concentration, auto emissions and windspeed.
- use the observation date as our cluster variable.

This allows for arbitrary dependence between observations in the same day, and zero correlation across days.

Is this reasonable? ... Maybe. But we'll go with it for now:

Table 3

	Dependent variable:				
	LNOx				
	(1)	(2)			
sqrtWS	-0.864***	-0.864***			
	(0.020)	(0.048)			
Constant	5.559***	5.559***			
	(0.029)	(0.065)			
Note:	Vote: *p<0.1; **p<0.05; ***p<0				

Here, the regular standard errors are smaller than the clustered standard errors.

This need not necessarily be the case and depends on the correlation between observations within a cluster.

# Newey West Standard Errors

For time series data.

# Conley Standard Errors

For spatial data.

## Confidence intervals for predictions

You know how to "predict" a value of the dependent variable, y, given certain values of the independent variables.

This prediction is just a guess, with uncertainty.

We can construct a confidence interval to give a range of possible values for this prediction.

There are two kinds of predictions we can make:

- ▶ A confidence interval for the **average** y given  $x_1, x_2, ..., x_k$ .
- ▶ A confidence interval for a **particular** y given  $x_1, x_2...x_k$ .

## Confidence intervals for predictions

Using Wooldridge's birth weight data:

$$bweight = \beta_0 + \beta_1 Ifaminc + \beta_2 meduc + \beta_3 parity + u$$

- bwght is birth weight in ounces,
- ▶ Ifaminc is the log of family income in \$1000s,
- meduc is the education of the mother in years,
- parity is the birth order of the child.

## Confidence intervals for predictions

#### Estimating this equation in R, we get the following results:

```
#using the bught data from the wooldridge package
reg1<-lm(bwght~lfaminc+motheduc+parity, bwght)
summary(reg1)
##
## Call:
## lm(formula = bwght ~ lfaminc + motheduc + parity, data = bwght)
##
## Residuals:
              1Q Median 3Q
      Min
                                    Max
## -94 533 -11 888 0 779 13 136 151 477
##
## Coefficients:
##
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 105.5652 3.3666 31.356 < 2e-16 ***
## lfaminc
             2.1313 0.6506 3.276 0.00108 **
## motheduc 0.3172 0.2520 1.259 0.20829
## parity
           1.5261
                         0.6119 2.494 0.01275 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.21 on 1383 degrees of freedom
    (1 observation deleted due to missingness)
## Multiple R-squared: 0.01633, Adjusted R-squared: 0.0142
## F-statistic: 7.654 on 3 and 1383 DF. p-value: 4.482e-05
```

Our model gives us the expected value:

 $E[bweight|faminc, meduc, parity] = \beta_0 + \beta_1 log(faminc) + \beta_2 meduc + \beta_3 parity$ and our regression gives us an estimate of this:

 $\hat{\mathcal{E}}[\mathit{bweight}|\mathit{faminc},\mathit{meduc},\mathit{parity}] = \hat{y} = \hat{eta}_0 + \hat{eta}_1 log(\mathit{faminc}) + \hat{eta}_2 meduc + \hat{eta}_3 parity$ 

 $\hat{y}$  is the expected value of y given the particular values for the explanatory variables.

Say we are interested in a confidence interval for the **average birthweight** for babies with:

- ightharpoonup a family income of \$14,500 (ln(14.5)=2.674),
- mothers with 12 years of education,
- 2 older siblings (parity=3).

```
\begin{split} \ddot{E}[\textit{bweight} | \textit{faminc} &= 14.5, \textit{meduc} = 12, \textit{parity} = 3] = 105.66 + 2.13 \textit{ln}(\textit{faminc}) + 0.317 \textit{meduc} + 1.53 \textit{parity} \\ & \hat{y}_{\textit{faminc} = 14.5, \textit{meduc} = 12, \textit{parity} = 3} = 105.66 + 2.13 (2.674) + 0.317 (12) + 1.53 (3) \\ &= 119.75 \textit{ounces} \end{split}
```

How do we find a standard error for  $\hat{y}$  at these particular values of the explanatory variables?

This standard error is complicated because  $\widehat{bweight}$  is a function of our  $\hat{\beta}$ 's which are all random variables.

To avoid this computation, we want to transform our data.

Recall that we have the following regression in mind

$$bweight = \beta_0 + \beta_1 Ifaminc + \beta_2 meduc + \beta_3 parity + u$$

Then

$$\hat{eta}_0 = \hat{E}(\textit{bweight}|\textit{Ifaminc} = 0, \textit{meduc} = 0, \textit{parity} = 0)$$

If we modify the regression by subtracting our particular values from the independent variables, we get

$$\textit{bweight} = \beta_0 + \beta_1(\textit{Ifaminc} - 2.674) + \beta_2(\textit{meduc} - 12) + \beta_3(\textit{parity} - 3) + u$$

Then

$$\hat{\beta}_0 = \hat{E}(bweight|lfaminc = 2.674, meduc = 12, parity = 3).$$

The new intercept is the predicted birthweight for babies with the particular values we are interested in.

If we run this regression in R, we can then grab the standard errors for the intercept.

So step by step we need to:

- 1) Generate new variables:  $\tilde{x}_j = x_j \alpha_j$
- 2) Run the regression:  $y = \tilde{\beta}_0 + \tilde{\beta}_1 \tilde{x}_1 + ... + \tilde{\beta}_k \tilde{x}_k + \tilde{u}$
- 3) Then  $\hat{E}[y|x_1 = \alpha_1, ..., x_k = \alpha_k] = \tilde{\beta_0}$
- 4) Plug these values into the formula for confidence intervals and interpret.

```
#Step 1: generate new variables
bwght$lfaminc_0<-bwght$lfaminc-2.674
bwght$motheduc 0<-bwght$motheduc-12
bwght$parity_0<-bwght$parity-3
#step 2: run the new regression
reg2<-lm(bwght~lfaminc 0+motheduc 0+parity 0.bwght)
summary(reg2)
##
## Call:
## lm(formula = bwght ~ lfaminc_0 + motheduc_0 + parity_0, data = bwght)
##
## Residuals:
              1Q Median 3Q
      Min
## -94.533 -11.888 0.779 13.136 151.477
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 119.6491 1.0066 118.864 < 2e-16 ***
## lfaminc_0 2.1313 0.6506 3.276 0.00108 **
## motheduc_0 0.3172 0.2520 1.259 0.20829
## parity_0 1.5261
                          0.6119 2.494 0.01275 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.21 on 1383 degrees of freedom
    (1 observation deleted due to missingness)
## Multiple R-squared: 0.01633, Adjusted R-squared: 0.0142
## F-statistic: 7.654 on 3 and 1383 DF. p-value: 4.482e-05
```

The 95% confidence interval for the average birthweight for babies given a family income of \$14,500, a mother with 12 years of education and with 2 older siblings is:

[119.64 - 1.96(1.007), 119.64 + 1.96(1.007)] = [117.6653, 121.6158]

A confidence interval for a particular average is not the same as a confidence interval for a particular individual.

For individual observations, we must account for the variance in the unobserved error,  $u_i$ , which measures our ignorance of the unobserved factors that affect  $y_i$ .

We want a confidence interval for  $bweight_{i=1}$ , the birthweight of baby i=1, with

$$bweight_{i=1} = \beta_0 + \beta_1 Ifaminc_{i=1} + \beta_2 meduc_{i=1} + \beta_3 parity_{i=1} + u_{i=1}$$

Our best prediction of  $bweight_{i=1}$  is  $\widehat{bweight}_{i=1}$  where

$$\widehat{\text{bweight}}_{i=1} = \hat{\beta}_0 + \hat{\beta}_1 \text{Ifaminc}_{i=1} + \hat{\beta}_2 \text{meduc}_{i=1} + \hat{\beta}_3 \text{parity}_{i=1}$$

There is some error,  $\hat{u}_{i=1}$ , associated with using  $\hat{bweight}_{i=1}$  to predict  $bweight_{i=1}$  where

$$\begin{split} \hat{u}_{i=1} &= \textit{bweight}_{i=1} - \textit{bweight}_{i=1} \\ &= \left(\beta_0 + \beta_1 \textit{Ifaminc}_{i=1} + \beta_2 \textit{meduc}_{i=1} + \beta_3 \textit{parity}_{i=1} + u_{i=1}\right) \\ &- \left(\hat{\beta}_0 + \hat{\beta}_1 \textit{Ifaminc}_{i=1} + \hat{\beta}_2 \textit{meduc}_{i=1} + \hat{\beta}_3 \textit{parity}_{i=1}\right) \end{split}$$

Finding the expected value, we get:

$$\begin{split} E[\hat{u}_{i=1}] &= E[\textit{bweight}_{i=1} - \textit{bweight}_{i=1}] \\ &= \left(\beta_0 + \beta_1 \textit{Ifaminc}_{i=1} + \beta_2 \textit{meduc}_{i=1} + \beta_3 \textit{parity}_{i=1} + E[u_{i=1}]\right) \\ &- \left(E[\hat{\beta}_0] + E[\hat{\beta}_1] \textit{Ifaminc}_{i=1} + E[\hat{\beta}_2] \textit{meduc}_{i=1} + E[\hat{\beta}_3] \textit{parity}_{i=1}\right) \\ &= 0 \end{split}$$

Finding the variance we get

$$\begin{aligned} Var(\hat{u}_{i=1}) &= Var(bweight_{i=1} - bweight_{i=1}) \\ &= Var(\beta_0 + \beta_1 Ifaminc_{i=1} + \beta_2 meduc_{i=1} + \beta_3 parity_{i=1} + u_{i=1} - bweight_{i=1}) \\ &= Var(bweight_{i=1}) + Var(u_{i=1}) \\ &= Var(bweight_{i=1}) + \sigma^2 \\ \widehat{Var(\hat{u}_{i=1})} &= Var(bweight_{i-1}) + \hat{\sigma}^2 \end{aligned}$$

There are two sources of variation in  $\hat{u}_{i=1}$ .

- ▶ the sampling error in  $\widehat{bweight}_{i=1}$  which arises because we have estimated the population parameters  $\beta$ .
- ▶ the variance of the error in the population  $(u_{i=1})$ .

#### We can compute:

- ►  $Var(\widehat{bweight}_{i=1})$  exactly the way we did before.
- ightharpoonup  $\hat{\sigma}^2$  from our regression output.

The 95% confidence interval for  $bweight_{i=1}$  is then

$$\hat{y} \pm 1.96 * se(\hat{u}_{i=1})$$

## Confidence Interval for prediction: a specific unit

Steps in computing a confidence interval for a particular y when  $x_j = \alpha_j$ :

- 1) Generate new variables:  $\tilde{x}_j = x_j \alpha_j$
- 2) Run the regression:  $y = \tilde{\beta_0} + \tilde{\beta_1}\tilde{x_1} + ... + \tilde{\beta_k}\tilde{x_k} + \tilde{u}$
- 3) Then  $\hat{E}[y|x_1 = \alpha_1, ..., x_k = \alpha_k] = \tilde{\beta}_0$  and the standard error of the estimate is  $se(\tilde{\beta}_0)$
- 4) Get an estimate for the variance of  $\hat{u} = \hat{\sigma}^2$  from the R output
- 5) compute the standard error:  $\sqrt{se(\tilde{\beta}_0)^2 + \hat{\sigma}^2}$
- Plug these values into the formula for confidence intervals and interpret.

## Confidence Interval for prediction: a specific unit

```
#Step 1: generate new variables
bwght$lfaminc_0<-bwght$lfaminc_0<-bwght$motheduc_0<-bwght$motheduc_0<-bwght$parity-3

#step 2: run the new regression
reg2<-lm(bwght-lfaminc_0+motheduc_0+parity_0,bwght)
summary(reg2)</pre>
```

```
##
## Call:
## lm(formula = bwght ~ lfaminc 0 + motheduc 0 + parity 0, data = bwght)
##
## Residuals:
##
     Min
             10 Median
                           30
                                 Max
## -94.533 -11.888 0.779 13.136 151.477
##
## Coefficients:
##
            Estimate Std. Error t value Pr(>|t|)
## lfaminc_0 2.1313 0.6506 3.276 0.00108 **
## motheduc 0 0.3172 0.2520 1.259 0.20829
## parity_0 1.5261 0.6119 2.494 0.01275 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.21 on 1383 degrees of freedom
## (1 observation deleted due to missingness)
## Multiple R-squared: 0.01633. Adjusted R-squared: 0.0142
## F-statistic: 7.654 on 3 and 1383 DF, p-value: 4.482e-05
```

## Confidence Interval for prediction: a specific unit

```
#step 4: get the estimate of the variance
summary(lm(bwght~lfaminc_0+motheduc_0+parity_0,bwght))$sigma^2
```

## [1] 408.5987

The 95% confidence interval for a particular baby's birthweight with:

- ► family income of \$14,500 (ln(14.5=2.674)),
- a mother with 12 years of education
- 2 older siblings is:

$$SE = \sqrt{se(\tilde{\beta}_0)^2 + \hat{\sigma}^2} = \sqrt{(1.007^2) + 408.59} = 20.239$$

$$CI = [119.64 - 1.96 * (20.239); 119.64 + 1.96 * (20.239)]$$

$$= [79.972; 159.308]$$

#### Standardizing variables eliminates the units:

- makes it possible to compare the magnitude of estimates across independent variables.
- makes interpretation easier if you have variables with weird arbitrary units that are unfamiliar to people.

Suppose we have a regression with two variables,  $x_1$  and  $x_2$ :

$$y = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{u}$$

A linear regression must go through the point of averages:

• if we plugged in  $\bar{x}_1$  and  $\bar{x}_2$ , we would predict  $\bar{y}$ :

$$\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}_1 + \hat{\beta}_2 \bar{x}_2$$

We can subtract the second equation from the first to get:

$$\hat{y} - \bar{y} = (\hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{u}) - (\hat{\beta}_0 + \hat{\beta}_1 \bar{x}_1 + \hat{\beta}_2 \bar{x}_2)$$
  
=  $\hat{\beta}_1 (x_1 - \bar{x}_1) + \hat{\beta}_2 (x_2 - \bar{x}_2) + \hat{u}$ 

Dividing both sides of this equation by the standard deviation of y,  $\sigma_y$  and multiplying each independent variable by  $1=\frac{\sigma_x}{\sigma_x}$ , we can get the regression into standard units:

$$\left(\frac{y-\bar{y}}{\hat{\sigma}_y}\right) = \frac{\hat{\sigma}_{x_1}}{\hat{\sigma}_y} \hat{\beta}_1\left(\frac{x_1-\bar{x}_1}{\hat{\sigma}_{x_1}}\right) + \frac{\hat{\sigma}_{x_2}}{\hat{\sigma}_y} \hat{\beta}_2\left(\frac{x_2-\bar{x}_2}{\hat{\sigma}_{x_2}}\right) + \frac{\hat{u}}{\hat{\sigma}_y}$$

Controlling for  $x_2$  a **one standard deviation** increase in  $x_1$  leads to a  $\frac{\hat{\sigma}_{x_1}}{\hat{\sigma}_y}\hat{\beta}_1$  **standard deviation** increase in the predicted y.

This is the standardized coefficient or "beta" coefficient.

It is also possible to only standardize some variables ( you will need an intercept in this case).

In R, we can get these coefficients by using the scale() command in R.

We use the bwght2 dataset to look at how parent ages correlate with birth weights. (Note: birth weights here will be measured in grams).

I estimate four different regressions of the type

$$birthweight_i = \beta_0 + \beta_1 motherage_i + \beta_2 fatherage_i + \epsilon_i$$

scaling either the dependent and/or independent variables.

```
bwght<-bwght2
reg1<-lm(bwght-mage+fage, bwght)
reg2<-lm(scale(bwght)-scale(mage)+scale(fage), bwght)
reg3<-lm(scale(bwght)-mage+fage, bwght)
reg4<-lm(bwght-scale(mage)+scale(fage), bwght)
meandep1<-round(mean(bwght$bwght),2)
meandep2<-round(mean(scale(bwght$bwght)),2)
sddep1<-round(sd(bwght$bwght),2)
sddep2<-round(sd(bwght$bwght),2)</pre>
```

Table 4

	Dependent variable:			
	bwght	scale(bwght)		bwght
	(1)	(2)	(3)	(4)
mage	-3.992 (3.943)		-0.007 (0.007)	
fage	9.313*** (3.291)		0.016*** (0.006)	
scale(mage)		-0.033 (0.033)		-19.044 (18.812)
scale(fage)		0.092*** (0.033)		53.205*** (18.803)
Constant	3,221.030*** (87.703)	-0.001 (0.023)	-0.312** (0.152)	3,400.304*** (13.448)
Mean SD	3401.12 576.54	0 1	0 1	3401.12 576.54
Note:	*p<0.1; ***p<0.05; ****p<0.01			

#### Interpreting column 1:

- A mother that is a year older predicts a birthweight that is 3.992 grams less (not significant).
- A father that is a year older predicts a birthweight that is 9.313 grams more (significant).

#### Interpreting column 2:

- A mother whose age is one standard deviation higher predicts a birthweight that is 0.033 standard deviations lower (not significant).
- A father whose age is one standard deviation higher predicts a birthweight that is 0.092 standard deviations higher (significant).

#### Interpreting column 3:

- A mother that is a year older predicts a birthweight that is 0.007 standard deviations lower (not significant).
- A father that is a year older predict a birthweight that is 0.016 standard deviations higher (significant).

#### Interpreting column 4:

- A mother whose age is one standard deviation higher predicts a birthweight that is 19.044 grams lower (not significant).
- A father whose age is one standard deviation higher predicts a birthweight that is 53.205 grams higher (significant).

# Binary Dependent variables

What happens if the outcome variable is binary?

- ► Linear Probability Models
- Logits

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + u$$

- It no longer makes sense to interpret  $\beta_j$  as the unit change in y given a one-unit increase in  $x_i$  holding all other factors fixed:
  - y either changes from  $0 \rightarrow 1$ , from  $1 \rightarrow 0$  or doesn't change.
- $\Rightarrow \beta_j$  measures the change in the probability of success when  $x_j$  changes by one unit holding all other factors constant.

$$Pr(y = 1|x) = \beta_0 + \beta_1 x_1 + ... + \beta_k x_k$$

$$inlf = \beta_0 + \beta_1 nwifeinc + \beta_2 educ + \beta_3 exper + \beta_4 exper^2 + \beta_5 age + \beta_6 kidslt6 + \beta_6 educ + \beta_6 educ$$

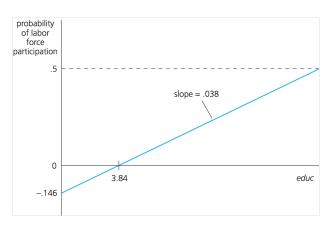
- ▶ inlf ("in the labor force") is a binary variable indicating labor force participation by a married woman in 1975.
- husbands earnings (nwifeinc, measured in thousands of dollars)
- years of education (educ)
- past years of labor market experience (exper)
- ► age (age)
- number of children less than six years old (kidslt6)
- number of kids between 6 and 18 years of age (kidsge6)

#### Using the mroz data that is part of the wooldridge package:

```
mroz$exper2<-mroz$exper^2
reg1<-lm(inlf~nwifeinc+educ+exper+exper2+age+kidslt6+kidsge6, mroz)
summary(reg1)
##
## Call:
## lm(formula = inlf ~ nwifeinc + educ + exper + exper2 + age +
      kidslt6 + kidsge6, data = mroz)
##
## Residuals:
       Min
                10 Median
                                 30
                                          Max
## -0.93432 -0.37526 0.08833 0.34404 0.99417
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.5855192 0.1541780 3.798 0.000158 ***
## nwifeinc -0.0034052 0.0014485 -2.351 0.018991 *
## educ 0.0379953 0.0073760 5.151 3.32e-07 ***
## exper 0.0394924 0.0056727 6.962 7.38e-12 ***
## exper2 -0.0005963 0.0001848 -3.227 0.001306 **
## age
          -0.0160908 0.0024847 -6.476 1.71e-10 ***
## kidslt6 -0.2618105 0.0335058 -7.814 1.89e-14 ***
            0.0130122 0.0131960 0.986 0.324415
## kidsge6
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4271 on 745 degrees of freedom
## Multiple R-squared: 0.2642, Adjusted R-squared: 0.2573
## F-statistic: 38.22 on 7 and 745 DF, p-value: < 2.2e-16
```

For a woman with: nwifeinc = 50, exper = 5, age = 30, kidslt6 = 1 and kidsge6 = 0, the relationship between years of education and the probability of being in the labor force is given by:

$$Pr(inlf = 1|educ) = (0.585 - 0.003 * 50 + 0.039 * 5 - 0.001 * 5^2 - 0.016 * 30 - 0.262 * 1) + 0.038 * educ$$
  
=  $-0.146 + 0.038educ$ 



## Linear Probability Model Drawbacks:

- 1) The predicted probabilities from our regression aren't bound between 0 and 1.
- 2) The outcome is implied to be linearly related to the independent variables. This is not possible with probabilities. (Ex: going from 0 to 4 young children reduces the probability by 0.262\*4=1.048, 104.8 percentage points, which is impossible).
- 3) When y is a binary variable Var(y|x) = p(x)[1 p(x)]. This means that there must be heteroskedasticity in the linear probability model. Thus you should always use **heteroskedasticity robust standard errors** with linear probability models.

How do we address the drawbacks of linear probability models?

With **logistic regressions**, which are estimated via maximum likelihood methods rather than OLS.

Logits differ from the linear probability model in the type of function used for the estimated probability.

In linear probability models, we have

$$Pr(y = 1|x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2.$$

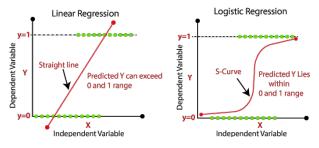
With a logit, we have

$$Pr(y = 1|x_1, x_2) = \frac{e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}{1 + e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}} = \Lambda(\beta_0 + \beta_1 x_1 + \beta_2 x_2)$$

where  $\Lambda$  is commonly used notation to represent the complex logit function.

Why would we ever pick a functional form for the probability that looks as complicated as  $\Lambda$ ?

- ▶ Because it is bounded between 0 and 1 (so our predictions make sense),
- it has nice statistical properties (which we will not get into).



In linear probability models, the marginal effect,  $\frac{\partial Pr(y=1|x)}{\partial x_j}$ , of  $x_j$  on Pr(y=1|x) is constant.

Logit models are **non-linear**: the exact marginal effect of  $x_j$  on Pr(y = 1|x) changes depending on the other values of x.

We can select where we calculate the marginal effects at:

- In R, the default is to compute the average marginal effect which is the average of the marginal effect for each observation in the sample.
- we could also specify any values of our independent variables to evaluate our marginal effects precisely at those values (Ex: at the mean values of the  $x_i$ 's).

#### Working with Logits:

- ▶ R output for a logit will not give you a marginal effect. Instead it reports the **log-odds**.
- ► To get marginal effects, we need to request the marginal effects and specify which marginal effects we are interested.

Looking at labor force participation using the mroz data.

- 1) estimate a logistic regression with the glm() function.
- 2) Running summary() will then get us the log-odds.

```
\label{lem:condition} reglogit1 <-glm(inlf-nwifeinc+educ+exper+exper2+age+kidslt6+kidsge6, mroz, family="binomial") \\ summary(reglogit1)
```

```
##
## Call:
## glm(formula = inlf ~ nwifeinc + educ + exper + exper2 + age +
##
      kidslt6 + kidsge6, family = "binomial", data = mroz)
##
## Deviance Residuals:
##
      Min
               10 Median
                                30
                                       Max
## -2.1770 -0.9063 0.4473
                            0.8561
                                     2.4032
##
## Coefficients:
##
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 0.425452 0.860365 0.495 0.62095
## nwifeinc -0.021345 0.008421 -2.535 0.01126 *
         0.221170 0.043439 5.091 3.55e-07 ***
## educ
## exper 0.205870 0.032057 6.422 1.34e-10 ***
## exper2 -0.003154 0.001016 -3.104 0.00191 **
## age
         -0.088024 0.014573 -6.040 1.54e-09 ***
## kidslt6 -1.443354 0.203583 -7.090 1.34e-12 ***
## kidsge6
            0.060112
                       0.074789 0.804 0.42154
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
```

#### To get the marginal effects:

- 3) run the estimated glm() object through the margins() function (from the margins package).
- the default returns the average marginal effect

```
library(margins)
## Warning: package 'margins' was built under R version 3.6.3
marg_reglogit1<-margins(reglogit1)
summary(marg_reglogit1)
      factor
                                            lower
                                                    upper
##
         age -0.0157 0.0024 -6.6027 0.0000 -0.0204 -0.0111
        educ 0.0395 0.0073 5.4145 0.0000 0.0252 0.0538
##
       exper 0.0368 0.0052 7.1386 0.0000 0.0267 0.0469
##
##
     exper2 -0.0006 0.0002 -3.1759 0.0015 -0.0009 -0.0002
    kidsge6 0.0107 0.0133 0.8051 0.4207 -0.0154 0.0369
##
    kidslt6 -0.2578 0.0319 -8.0696 0.0000 -0.3204 -0.1951
    nwifeinc -0.0038 0.0015 -2.5714 0.0101 -0.0067 -0.0009
```

you can instead compute the marginal effect for a particular type of observation:

```
DF <- data.frame(age=30,
                 nwifeinc=50.
                 exper=5,
                 exper2=25.
                 kidsge6=0.
                 kidslt6=1.
                 educ=12,
                 stringsAsFactors=FALSE)
marg_specific <- margins(reglogit1, data = DF)
summary(marg_specific)
##
      factor
                 AME
                         SE
                                           lower
                                                     upper
                                  z
         age -0.0163 0.0049 -3.3268 0.0009 -0.0259 -0.0067
##
##
        educ 0.0410 0.0088 4.6729 0.0000 0.0238 0.0582
##
       exper 0.0382 0.0089 4.2893 0.0000 0.0207 0.0556
##
     exper2 -0.0006 0.0002 -2.8205 0.0048 -0.0010 -0.0002
    kidsge6 0.0111 0.0130 0.8554 0.3924 -0.0144 0.0367
##
##
    kidslt6 -0.2675 0.0567 -4.7195 0.0000 -0.3787 -0.1564
   nwifeinc -0.0040 0.0011 -3.5885 0.0003 -0.0061 -0.0018
```

# Contrasting a logit to a linear probability model (LPM)

In sports betting, the Las Vegas point spread is the predicted scoring differential between two opponents as quoted in Las Vegas. We are interested in the probability that the favored team actually wins.

We can run the following regression:

$$\widehat{\mathit{favwin}}_i = \hat{eta}_0 + \hat{eta}_1 \mathit{spread}_i + \hat{eta}_2 \mathit{favhome}_i + \hat{eta}_3 \mathit{fav} 25_i + \hat{eta}_4 \mathit{und} 25_i + \hat{u}_i$$

- favwin is a dummy variable indicating whether the favored team won,
- spread is the Las Vegas point spread,
- favhome is a dummy variable indicating whether the favored team is playing at home,
- ► fav25 and und25 indicate whether the favored team and the underdog team are in the top 25 teams respectively.

#### Using the pntsprd data from the wooldridge package:

```
lpm<-felm(favwin~spread+favhome+fav25+und25, data=pntsprd)
summarv(lpm, robust=TRUE)
##
## Call:
##
     felm(formula = favwin ~ spread + favhome + fav25 + und25, data = pntsprd)
##
## Residuals:
##
      Min
               10 Median
                                     Max
## -0.9972 -0.1105 0.1470 0.2991 0.4869
##
## Coefficients:
##
               Estimate Robust s.e t value Pr(>|t|)
## (Intercept) 0.558815 0.040422 13.825
                                            <2e-16 ***
## spread
               0.017763 0.002056 8.637 <2e-16 ***
## favhome
               0.054353 0.040923 1.328 0.185
## fav25
               0.010982 0.039100 0.281 0.779
## und25
            -0.101104 0.089530 -1.129
                                           0.259
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4016 on 548 degrees of freedom
```

## Multiple R-squared(full model): 0.116 Adjusted R-squared: 0.1095
## Multiple R-squared(proj model): 0.116 Adjusted R-squared: 0.1095
## F-statistic(full model, \*iid\*):17.97 on 4 and 548 DF, p-value: 7.007e-14
## F-statistic(proj model): 26.2 on 4 and 548 DF, p-value: < 2.2e-16

```
## factor AME SE z p lower upper

## fav25 0.0076 0.0434 0.1748 0.8612 -0.0774 0.0926

## favhome 0.0425 0.0362 1.1740 0.2404 -0.0284 0.1134

## spread 0.0243 0.0033 7.3360 0.0000 0.0178 0.0308

## und25 -0.0579 0.0650 -0.8912 0.3728 -0.1852 0.0694
```

All else constant, a 1 point increase in the Vegas point spread is estimated to increase the predicted probability of winning:

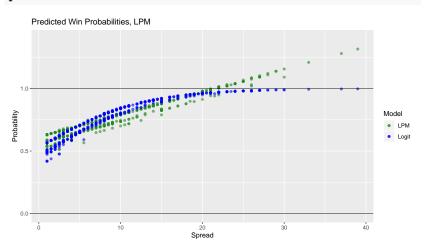
- by 1.78 percentage points using the linear probability model
- by 2.43 percentage points on average using the logit model.

It is generally the case that results using these two different methods will often be quite similar.

To see the difference, we plot the predicted probabilities

```
#generating the fitted values for both models
df<- mutate(pntsprd, lpm prob=lpm$fitted.values,</pre>
            logit_prob=logit$fitted.values)
plot<-df%>%
  ggplot(aes(x=spread))+
  geom point(aes(y=lpm prob, colour="LPM"), alpha=0.6)+
  geom point(aes(y=logit prob, colour="Logit"), alpha=0.6)+
  geom hline(yintercept = 1, alpha=0.7)+
  geom hline(yintercept = 0,alpha=0.7)+
  scale_colour_manual("Model",
                      breaks=c("LPM", "Logit"),
                      values=c("blue", "forestgreen"))+
  lims(y=c(0,1.4))+
  labs(title="Predicted Win Probabilities, LPM",
       x="Spread",
       v="Probability")
```

#### plot





Lots of new spatial data becoming available.

With the right tools these can be used to explore all kinds of questions.

Spatial data is just data like anything else. A

The main difference is that spatial data usually come in 2 (or even 3) dimensions (usually latitude and longitude).

To deal with spatial data, we'll need a bunch of new packages:

- ▶ GISTools, rgdal, rgeos, maptools, raster (all spatial utilities),
- broom (this will let us turn spatial data into a format that ggplot2 can handle).
- lubridate to more easily deal with dates (because why not)
- ► RColorBrewer for new color palettes

#### Notes:

- The order in which you load packages can matter.
- other programs (often proprietary) are better at dealing with spatial data analysis. R does not have a unified spatial toolkit, meaning that different data types and formats and functions don't necessarily get along very well.

```
library(raster)
#library(vctrs)
#library(tibble)
library(GISTools)
#library(readr)
library(dplyr)
library(lubridate)
#library(xtable)
library(ggplot2)
library(rgeos)
library(rgdal)
library(maptools)
library(broom)
```

#### What the frack?

We're going to use georeferenced data to look at unconventional oil and gas drilling in Pennsylvania.

We'll start with a spatial data file from the US Cansus Bureau with the shape of each county in the state.

This data comes in ArcGIS's the shapefile format.

```
counties <- shapefile("data/PA counties.shp")
counties
               : SpatialPolygonsDataFrame
## class
## features
               . 67
               : -80.51989, -74.6895, 39.7198, 42.51607 (xmin, xmax, ymin, ymax)
## extent
## crs
               : +proj=longlat +datum=NAD83 +no_defs
## variables
               : 13
               : STATEFPEC, COUNTYFPEC, CNTYIDFPEC, NAMEEC, NAMELSADEC, LSADEC, CLASSFPEC, MTFCCEC, FUN
## names
                                  001,
## min values :
                       42,
                                            42001, Adams, Adams County,
                                                                             06,
                                                                                        H1,
                                                                                              G4020.
                       42.
                                  133.
                                          42133. York, York County.
## may values :
                                                                             06.
                                                                                        H6.
                                                                                              G4020.
```

## Shapefiles

#### What's in this object?

- This file contains polygons outlines of shapes.
- ▶ We can check it was read correctly into R by checking its class:

#### class(counties)

```
## [1] "SpatialPolygonsDataFrame"
## attr(,"package")
## [1] "sp"
```

Perfect. This is R's version of a polygon shapefile.

## **Shapefiles**

What's actually in this dataset?

- ► We can figure this out by looking at its slots usingslotNames()
- ▶ This is like calling names() on a simpler object

```
slotNames(counties)
## [1] "data" "polygons" "plotOrder" "bbox" "proj4string"
```

What is all this?

- polygons: the spatial component of our spatial dataset. Each polygon is a long two-variable dataframe with longitudes (x) and latitudes (y) (like a giant connect-the-dots).
- data: a dataframe, where each row is actually associated with one of the polygons in our shapefile.
- -bbox and plotOrder: tell the base plotting commands how to display the data.
  - > proj4string tells R how to display and project the data

## Shapefiles

#### Let's look at the data in this shapefile:

```
names(counties@data) <- tolower(names(counties@data))
head(counties@data)</pre>
```

```
statefpec countyfpec cntvidfpec
                                                      namelsadec lsadec
                                         nameec
## 0
            42
                       001
                                 42001
                                          Adams
                                                    Adams County
                                                                      06
## 1
            42
                       027
                                 42027
                                         Centre
                                                   Centre County
                                                                      06
## 2
            42
                       023
                                42023 Cameron Cameron County
                                                                      06
            42
## 3
                       025
                                42025
                                         Carbon
                                                   Carbon County
                                                                      06
## 4
            42
                       029
                                42029 Chester Chester County
                                                                      06
## 5
            42
                       045
                                 42045 Delaware Delaware County
                                                                      06
     classfpec mtfccec funcstatec
                                       alandec awaterec
                                                         intptlatec
                                  A 1342811635
## 0
                  G4020
                                                 8560860 +39.8694707
                  G4020
## 1
            H<sub>1</sub>
                                  A 2877281248
                                                 7879260 +40 9091673
## 2
            H1
                  G4020
                                  A 1026698470
                                                 5664145 +41.4382882
                  G4020
## 3
                                     988299800 15353755 +40.9178324
## 4
            H1
                  G4020
                                  A 1943960881 22601992 +39 9737772
## 5
            H1
                  G4020
                                     476398058 17503777 +39.9160594
       intptlonec
     -077 2177295
   1 -077 8478195
   2 -078.1983134
  3 -075.7094276
## 4 -075.7503816
## 5 -075.4008573
```

- Mostly identifying information about each county
- Not that interesting: if we want good stuff we will have to merge it in.

## Shapefiles: Projections

The world is a sphere (sorry Kyrie Irving) that we are trying to represent on our (flat) computer screens.

- ► The projection defines what dimensions to stretch to make this representation happen.
- ► Spatial objects (should) have a projection attached to them counties@proj4string

```
## CRS arguments: +proj=longlat +datum=NAD83 +no_defs
```

This tells us that we are using:

- latitude and longitude to identify our data,
- ▶ the North American Datum 83 (a common choice for the US).

If you're going to combine multiple geographic datasets, it's important that they all use the same projection.

- 1) we extract the shapefile data so we can use it later
- 2) we convert our polygon into a data frame that ggplot2 can handle, using broom's tidy() function.
- 3) we merge, or join(), the data that came with the shapefile, into this new dataframe.

We are going to do this process several times, so we'll write a function to take care of it for us:

```
mapToDF <- function(shapefile) {
    # first assign an identifier to the main dataset
    shapefile@data$id <- rownames(shapefile@data)
    # now "tidy" our data to convert it into a dataframe that
    #is usable by ggplot2
    mapDF <- tidy(shapefile) %>%
    # and this data onto the information attached to the shapefile
    left_join(., shapefile@data, by = "id") %>%
    as.data.frame()
return(mapDF)
}
```

```
paCounties <- mapToDF(counties)
## Regions defined for each Polygons
head(paCounties)
         long
                   lat order hole piece group id statefpec countyfpec
## 1 -76.99950 39.79106
                           1 FALSE
                                           0.1 0
                                                         42
                                                                   001
## 2 -76.99943 39.79044
                           2 FALSE
                                           0.1 0
                                                         42
                                                                   001
## 3 -76.99939 39.79013
                           3 FALSE
                                           0.1 0
                                                         42
                                                                   001
                        4 FALSE
## 4 -76.99939 39.79003
                                       1 0.1 0
                                                         42
                                                                   001
                        5 FALSE
                                           0.1 0
## 5 -76.99931 39.78907
                                                         42
                                                                   001
## 6 -76.99929 39.78852
                           6 FALSE
                                           0.1 0
                                                         42
                                                                   001
    cntyidfpec nameec
                        namelsadec lsadec classfpec mtfccec funcstatec
         42001 Adams Adams County
## 1
                                       06
                                                 H1
                                                      G4020
## 2
         42001 Adams Adams County
                                       06
                                                      G4020
## 3
      42001 Adams Adams County
                                       06
                                                      G4020
## 4
      42001 Adams Adams County
                                       06
                                                      G4020
## 5
       42001 Adams Adams County
                                       06
                                                 H1
                                                      G4020
## 6
         42001 Adams Adams County
                                       06
                                                 Н1
                                                      G4020
##
       alandec awaterec intptlatec
                                      intptlonec
## 1 1342811635 8560860 +39.8694707 -077.2177295
## 2 1342811635 8560860 +39.8694707 -077.2177295
## 3 1342811635 8560860 +39.8694707 -077.2177295
## 4 1342811635 8560860 +39.8694707 -077.2177295
## 5 1342811635 8560860 +39.8694707 -077.2177295
## 6 1342811635 8560860 +39.8694707 -077.2177295
```

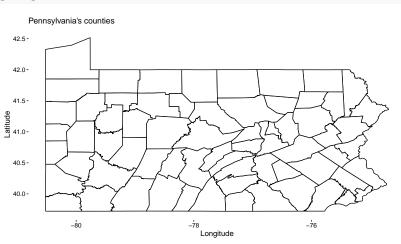
Once we have this dataset, it's easy to plot using ggplot2.

I start by defining a ggplot theme for my maps and then plot the data:

```
myMapThemeStuff <- theme(panel.background = element_rect(fill = NA),
    panel.border = element_blank(),
    panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(),
    axis.ticks = element_line(color = "gray5"),
    axis.text = element_text(color = "black", size = 10),
    axis.title = element_text(color = "black", size = 12),
    legend.key = element_blank()
)

paMap <- ggplot(data = paCounties, aes(x = long, y = lat, group = id)) +
    geom_polygon(color = "black", fill = "white") +
    myMapThemeStuff +
    ggtitle("Pennsylvania's counties") +
    xlab("Longitude") +
    ylab("Latitude")</pre>
```

### paMap



## 2 Vortical Woll

Vac

Let's bring in another dataset, with the locations of unconventional wells drilled in Pennsylvania between 2002 and 2013 (courtesy of FracTracker Alliance).

```
wells <- read.csv("data/PA wells.csv") %>%as.data.frame()
names(wells) <- tolower(names(wells))
head(wells)
    spud date
                    api ogo num
                                                        operator
## 1 5/24/2002 125-22033 DGD-49020
                                             BELDEN & BLAKE CORP
## 2 5/31/2003 125-22074 OGO-60915 RANGE RESOURCES APPALACHIA LLC
## 3 6/14/2003 063-33347 DGD-38958
                                                  XTO ENERGY INC
## 4 8/27/2003 129-25004 OGO-60915 RANGE RESOURCES APPALACHIA LLC
     9/6/2003 129-25012 DGD-60915 RANGE RESOURCES APPALACHTA LLC
## 6 2/27/2004 129-25209 DGD-33810
                                         GREAT OAK ENERGY INC
##
                   region
                          county
                                               municipality
                                                                  farm name
## 1 EP DOGO SWDO Dstr Off Washington
                                              Hanover Twp ANDERSON UNIT 1
## 2 EP DOGO SWDO Dstr Off Washington Mount Pleasant Twp
                                                                     RENZ 1
## 3 EP DOGO SWDO Dstr Off
                               Indiana
                                                  Grant Twp LEONARD MUMAU 2
## 4 EP DOGO SWDO Dstr Off Westmoreland South Huntingdon Twp HEPLER CALVIN 1
## 5 EP DOGO SWDO Dstr Off Westmoreland South Huntingdon Twp
## 6 EP DOGO SWDO Dstr Off Westmoreland
                                                  Salem Two EXPORT FUEL 1
    well code desc
                                well_status latitude longitude
## 1
               GAS
                                       Active 40.46467 -80.47789
## 2
               GAS
                                       Active 40.28316 -80.28402
               GAS
## 3
                                       Active 40.80562 -78.93993
## 4 COMB. OIL&GAS Regulatory Inactive Status 40.15899 -79.70181
## 5
     COMB. OIL&GAS
                                       Active 40.15602 -79.72340
## 6
               GAS
                                       Active 40.43544 -79.58450
    configuration unconventional
## 1 Vertical Well
```

## [1] NA

Before we add it to our plot we need to make sure that it uses the same projection.

It's currently a dataframe - let's turn it into a SpatialPointsDataFrame (like the polygon, except for points) and attach a projection.

```
#the coordinates() function sets spatial coordinates to define a spatial object
coordinates(wells) <--longitude + latitude
class(wells)

## [1] "SpatialPointsDataFrame"
## attr(,"package")
## [1] "sp"
## [1] "sp"
#the proj4string() function retreives the projection attributes of the wells object
proj4string(wells)</pre>
```

Since the wells data does not have a projection, we have to assign it ourselves.

The website did not specify a projection so we select WGS84 (a common global projection) and then re-project it to match our county data's.

```
# assign a projection (WGS&4)...check out https://rspatial.org/raster/spatial/6-crs.html
# for more info on coordinate reference systems
proj4string(wells) < CRS("hroj=longlat +datum=WGS84")
# re-project this to match the county data
wells <- spTransform(wells, CRS(proj4string(counties)))

## Warning in proj4string(counties): CRS object has comment, which is lost in
## output
#check
proj4string(wells)

## Warning in proj4string(wells): CRS object has comment, which is lost in
## output

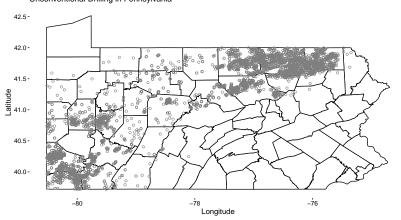
## [1] "+proj=longlat +datum=NAD83 +no_defs"
#convert back to a dataframe so that ggplot2 can handle it
wellsDF <- as.data.frame(wells)
```

### Now we'll plot both the counties and the wells on our map:

# A less boring map!

#### paMap





### Got the blue's?

Let's plot different colors by year of well drilling.

- 1) convert the spud date (drill date) variable to date format,
- 2) extract the year (let's actually make pairs of years)
- 3) convert this into a factor.

#### Got the blue's?

This is also a great excuse to bring in a great R packages: RColorBrewer.

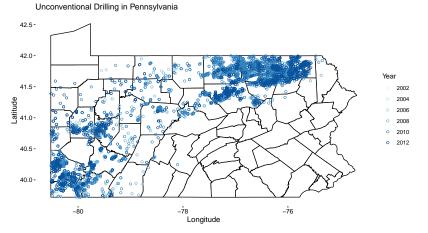
You can see all of the available color palettes by typeing display.brewer.all() into your consol.

```
paMap <- ggplot() +
    geom_polygon(data = paCounties, aes(x = long, y = lat,
    geom_point(data = wellsDF, aes(x = longitude, y = latin
    scale_color_brewer(palette="Blues") +
    myMapThemeStuff +
    ggtitle("Unconventional Drilling in Pennsylvania") +
    xlab("Longitude") +
    ylab("Latitude") +
    labs(color = "Year")</pre>
```

### Got the blue's?

### paMap





## Playing around

Let's bring in some new data from the EIA: a shapefile of the Marcellus shale play - the rock formation from which you can extract hydrocarbons.

playBdry <- shapefile("data/ShalePlay Marcellus Boundary EIA Aug2015 v2.shp")

```
playBdry
              : SpatialPolygonsDataFrame
## class
## features
## extent
              : -82.52247, -75.20568, 37.18328, 42.76125 (xmin, xmax, ymin, ymax)
              : +proj=longlat +datum=WGS84 +no defs
## crs
## variables
                     Basin, Lithology, Shale play, Source, Area sq mi, Area sq km,
## names
                                                                                     Age_shale
## value
              : Appalachian.
                               Shale, Marcellus,
                                                    EIA.
                                                              58326.
                                                                        151065, Middle Devonian
playBdry@proj4string
```

```
## CRS arguments: +proj=longlat +datum=WGS84 +no_defs
```

```
This file is WGS84. We'll have to convert it:
```

```
playBdry <- spTransform(playBdry, CRS(proj4string(counties)))</pre>
```

Use our mapToDF() function from earlier to convert this into a dataframe:

```
bdryDF <- mapToDF(playBdry)

## Regions defined for each Polygons
```

## Playing around

```
bigPlot <- ggplot(data = bdryDF, aes(x = long, y = lat)) +
        geom_polygon(data = paCounties, aes(x = long, y= lage)
        geom_path(data = bdryDF, aes(x = long, y = lat), co
        geom_point(data = wellsDF, aes(x = longitude, y = 1
        scale_color_brewer(palette="Blues") +
      # put in a bounding box to restrict ourselves to the
      # part of the play in PA
        xlim(counties@bbox[1,1], counties@bbox[1, 2]) +
        ylim(counties@bbox[2,1], counties@bbox[2, 2]) +
        ggtitle("Unconventional Drilling in Pennsylvania")
        xlab("Longitude") +
        ylab("Latitude") +
        myMapThemeStuff+
        labs(color = "Year")
```

# Playing around

#### bigPlot

