MQE: Economic Inference from Data: Module 3: Instrumental Variables

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6/9/2020

You can't always get what you want

Even with fixed effects, certain types of unobservables can still bias our estimates.

For OVB to not be a problem, we want a treatment variable x_i where we know that there does not exist some omitted variable x_{ov} such that

- $cor(x_i, x_{ov}) \neq 0$ and $cor(y_i, x_{ov}) \neq 0$.

This is a tall order...

You can't always get what you want

But if you try sometimes,

You can't always get what you want

But if you try sometimes,

you just might find,

You can't always get what you want

But if you try sometimes,

you just might find,

you get what you need: a good instrumental variable.

An instrument for what?

I am interested in the relationship between y and x_1.

The true data generating process looks like this:

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

- x_1 and x_2 are uncorrelated with ϵ
- x_1 and x_2 are correlated with each other such that $Cov(x_1, x_2) \neq 0$

So whats the problem?

- you don't actually observe x_2.

Uh oh.

The problem:

The naive approach (but you of course know better then to do this. . .)

Regress y on just x_1:

$$y_i = \beta_0 + \beta_1 x_1 + \nu$$

where

$$\nu = \beta_2 x_2 + \epsilon.$$

Cov Math Rules:

$$Cov(X + Y, Z) = Cov(X, Z) + Cov(Y, Z)$$

$$Cov(X, X) = Var(X)$$

$$Cov(cX, Y) = cCov(X, Y)$$

The problem:

$$
\begin{aligned}
\hat{\beta}_{1,OLS} &= \frac{cov(x_1, y)}{var(x_1)} \\
&= \frac{cov(x_1, \beta_0 + \beta_1 x_1 + \nu)}{var(x_1)} \\
&= \frac{cov(x_1, \beta_0) + cov(x_1, \beta_1 x_1) + cov(x_1, \nu)}{var(x_1)} \\
&= \frac{\beta_1 var(x_1) + cov(x_1, \nu)}{var(x_1)} \\
&= \beta_1 + \frac{cov(x_1, \nu)}{var(x_1)}
\end{aligned}
$$

$cov(x_1, \nu) \neq 0$ since
$cov(x_1, \nu) = cov(x_1, \beta_2 x_2 + \epsilon) = \beta_2 cov(x_1, x_2) + cov(x_1, \epsilon)$ and
$cov(x_1, x_2) \neq 0$

$\Rightarrow \hat{\beta}_{1,OLS}$ is biased.

All is not lost!

An **instrumental variable** (IV) is a variable that

- is correlated with the "good" or "*exogenous*" variation in x_1
- is unrelated to the "bad" or "*endogenous*" or "*related-to-*x_2" variation in x_1.

Formally

An IV is a variable, z that satisfies two important properties:

- $Cov(z, x_1) \neq 0$ (the first stage).
- $Cov(z, \nu) = 0$ (since $Cov(z, x_2) = 0$) (the exclusion restriction)

.

The First Stage

$Cov(z, x_1) \neq 0$

- z and x_1 are correlated
- the IV is useless without a first stage.

We are trying to get a $\hat{\beta}_1$ such that $E[\hat{\beta}_1] = \beta_1$. If our instrument is totally unrelated to x_1, we won't have any hope of using it to get at β_1.

The exclusion restriction

$Cov(z, \nu) = 0$

- z has to affect y **only** through x_1.
- Since $Cov(z, \nu) = \beta_2 Cov(z, x_2) + Cov(z, \epsilon)$

  $\Rightarrow Cov(z, x_2) = 0$ and $Cov(z, \epsilon) = 0$

  $\Rightarrow$ all of the influence of z on y must be chanelled through x_1.

The IV estimator

$$
\begin{aligned}
\hat{\beta}_{1,IV} &= \frac{cov(z,y)}{cov(z,x)} \\
&= \frac{cov(z,\beta_0+\beta_1 x_1+\nu)}{cov(z,x_1)} \\
&= \beta_1 \frac{cov(z,x_1)}{cov(z,x_1)} + \frac{cov(z,\nu)}{cov(z,x_1)} \\
&= \beta_1 + \frac{cov(z,\nu)}{cov(z,x_1)}.
\end{aligned}
$$

With the exclusion restriction: $cov(z,\nu)=0 \Rightarrow E[\hat{\beta}_{1,IV}]=\beta_1$

Woot Woot! We have an unbiased estimator!

Do the following variables satisfy the properties of a good instrument?

1. You are interested how the length of a prison sentence affects recidivism 5 years later. You instrument with how strict the randomly assigned judge to the case is.
2. You are interested in how college attendance influences wages. You instrument with family home proximity to a college.
3. You are interested in how stress affects blood pressure. You instrument with COVID rates in the local community.

Some instruments:

An (old) survey of instrumental variables: Angrist and Krueger (2001)

Table 1

Examples of Studies That Use Instrumental Variables to Analyze Data From Natural and Randomized Experiments

Outcome Variable	*Endogenous Variable*	*Source of Instrumental Variable(s)*	*Reference*
		1. Natural Experiments	
Labor supply	Disability insurance replacement rates	Region and time variation in benefit rules	Gruber (2000)
Labor supply	Fertility	Sibling-Sex composition	Angrist and Evans (1998)
Education, Labor supply	Out-of-wedlock fertility	Occurrence of twin births	Bronars and Grogger (1994)
Wages	Unemployment insurance tax rate	State laws	Anderson and Meyer (2000)
Earnings	Years of schooling	Region and time variation in school construction	Duflo (2001)
Earnings	Years of schooling	Proximity to college	Card (1995)
Earnings	Years of schooling	Quarter of birth	Angrist and Krueger (1991)
Earnings	Veteran status	Cohort dummies	Imbens and van der Klaauw (1995)
Earnings	Veteran status	Draft lottery number	Angrist (1990)
Achievement test scores	Class size	Discontinuities in class size due to maximum class-size rule	Angrist and Lavy (1999)
College enrollment	Financial aid	Discontinuities in financial aid formula	van der Klaauw (1996)
Health	Heart attack surgery	Proximity to cardiac care centers	McClellan, McNeil and Newhouse (1994)
Crime	Police	Electoral cycles	Levitt (1997)
Employment and Earnings	Length of prison sentence	Randomly assigned federal judges	Kling (1999)
Birth weight	Maternal smoking	State cigarette taxes	Evans and Ringel (1999)

Chasing Unicorns

- z's that satisfy the first condition are easy to find, and we can test that $Cov(z, x_1) \neq 0$
- z's that satisfy the exclusion restriction are rare and we cannot test that $Cov(z, \nu) = 0$ since we don't observe ϵ.

Chasing Unicorns

A good IV is not unlike a unicorn. It is quite powerful/magical as it will allow you to recover a consistent estimate of β_1 in a situation that was otherwise hopeless.

Chasing Unicorns

It is also a rare, (some may argue imaginary) beast, that often turns out to be a horse with an optimistic rider (author).

- be skeptical of instrumental variables regressions
- be wary of trying them yourself
- be prepared to convince people the exclusion restriction is satisfied

A simulation

I generate some simulated data, with properties I fully understand:

The DGP: Y depends on two variables, X_1 and X_2 such that

$$Y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$$

- x_1 and x_2 covary with $Cov(x_1, x_2) = 0.75$
- z covaries with x_1 such that $Cov(x_1, z) = 0.25$
- z does not covary with x_2 (so $Cov(x_2, z) = 0$).

A simulation

```
library(MASS)
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.6.2
```

```
library(stargazer)
```

```
##
## Please cite as:

##  Hlavac, Marek (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables.

##  R package version 5.2.2. https://CRAN.R-project.org/package=stargazer
```

```
sigmaMat<-matrix(c(1,0.75,0.25,0.75,1,0,0.25,0,1), nrow=3)
sigmaMat
```

```
##      [,1] [,2] [,3]
## [1,] 1.00 0.75 0.25
## [2,] 0.75 1.00 0.00
## [3,] 0.25 0.00 1.00
```

```
set.seed(5000)
ivdat<- as.data.frame(mvrnorm(10000, mu = c(0,0,0),
                      Sigma = sigmaMat))

names(ivdat)<-c("x_1","x_2","z")
cov(ivdat)
```

```
##           x_1         x_2           z
## x_1 1.0036443 0.745103511 0.246358965
## x_2 0.7451035 0.981180725 0.002127948
## z   0.2463590 0.002127948 0.991118191
```

A simulation

```
ivdat$error<-rnorm(10000, mean=0, sd=1)

#The data generating process
B1<-10
B2<-(-20)

ivdat$Y<-ivdat$x_1*B1+ivdat$x_2*B2+ivdat$error

knitr::kable(head(ivdat))
```

x_1	x_2	z	error	Y
-0.1480778	-0.5218521	1.2549131	-0.0470256	8.909239
-0.8240697	-1.1709528	-0.7069437	0.3014639	15.479822
-2.2167131	-1.7736810	-0.7409791	0.5561088	13.862597
1.2223555	0.2629376	-0.0761498	0.2886668	7.253469
-0.1527584	-0.5661063	-0.6523414	0.8400475	10.634589
-0.4315813	-0.7802528	1.3235641	-0.3380069	10.951237

A simulation:

```
simiv1<-lm(Y~x_1+x_2, data=ivdat)
simiv2<-lm(Y~x_1, data=ivdat)
```

How will our estimate of $\hat{\beta}_1$ in model 2 compare to the true β?

$\Rightarrow$ Top Hat

A simulation:

```
cov(ivdat$Y,ivdat$x_2)
```

```
## [1] -12.1676
```

```
cov(ivdat$x_1,ivdat$x_2)
```

```
## [1] 0.7451035
```

How will our estimate of $\hat{\beta}_1$ in model 2 compare to the true β?

- $Cov(Y, x_2) < 0$
- $Cov(x_1, x_2) > 0$
- $\hat{\beta}_1$ is downward biased

A simulation:

```
stargazer(simiv1, simiv2, header=FALSE, type='latex', omit.stat = "all", single.row = TRUE)
```

Table 2

	Dependent variable:	
	Y	
	(1)	(2)
x_1	10.035^{***} (0.015)	-4.829^{***} (0.131)
x_2	-20.021^{***} (0.015)	
Constant	-0.023^{**} (0.010)	0.195 (0.131)
Note:		*p<0.1; **p<0.05; ***p<0.01

- With the correctly specified model $E[\hat{\beta}_1] = \beta_1$.
- If I do not observe x_2, the naive approach is biased.

A simulation:

Suppose there exists a variable z that satisfies the two conditions outlined above:

- $Cov(z, x_1) \neq 0$ (the first stage).
- $Cov(z, \nu) = 0$ (the exclusion restriction).

Our simulated data includes z, a variable with these properties

A simulation:

I instrument my endogenous variable, x_1, with my instrument z:

```
library(lfe)
```

```
## Loading required package: Matrix
```

```
simiv3<-felm(Y~1|0|(x_1~z),ivdat)
```

A simulation:

- I get an unbiased estimate of β_1!
- Careful: R^2 values get real funky (negative!?!)– don't use.

```
stargazer(simiv1, simiv2, simiv3, header=FALSE,
          type='latex', omit.stat = c("n", "f","ser"))
```

Table 3

	Dependent variable:		
	Y		
	OLS		*felm*
	(1)	(2)	(3)
x_1	10.035***	−4.829***	
	(0.015)	(0.131)	
x_2	−20.021***		
	(0.015)		
'x_1(fit)'			9.828***
			(0.796)
Constant	−0.023**	0.195	0.141
	(0.010)	(0.131)	(0.197)
R^2	0.995	0.119	−0.981
Adjusted R^2	0.995	0.119	−0.981
Note:		*p<0.1; **p<0.05; ***p<0.01	

2SLS:

How does β_{IV} use the instrumental variable to retrieve an unbiased estimate?

To build intuition, let's look at the two-stage least squares (2SLS) estimator β_{2SLS}.

When we are working with only one instrument and one endogenous regressor, $\beta_{IV} = \beta_{2SLS}$.

2SLS:

2SLS proceeds in two (least squares regression) stages:

- the "first stage," a regression of our endogenous variable on our instrument

$$x_1 = \gamma_0 + \gamma_1 z + u.$$

- using the estimated $\hat{\gamma}$ coefficients we generate predicted values, $\hat{x}_1$:

$$\hat{x}_1 = \hat{\gamma}_0 + \hat{\gamma}_1 z$$

- the "second stage" where we regress our outcome on the predicted values of the endogenous variable

$$y = \beta_0 + \beta_1 \hat{x}_1 + \epsilon$$

The first stage:

```
sim2slsfs<-felm(x_1~z,ivdat)
summary(sim2slsfs)
```

```
##
## Call:
##    felm(formula = x_1 ~ z, data = ivdat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.1596 -0.6478  0.0059  0.6504  3.4744
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.002705   0.009708   0.279    0.781
## z           0.248567   0.009752  25.488   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9708 on 9998 degrees of freedom
## Multiple R-squared(full model): 0.06101   Adjusted R-squared: 0.06092
## Multiple R-squared(proj model): 0.06101   Adjusted R-squared: 0.06092
## F-statistic(full model):649.7 on 1 and 9998 DF, p-value: < 2.2e-16
## F-statistic(proj model): 649.7 on 1 and 9998 DF, p-value: < 2.2e-16
```

The second stage:

```
hatgamma0<-sim2slsfs$coefficients[1]
hatgamma1<-sim2slsfs$coefficients[2]
ivdat$hatx_1<-hatgamma0+hatgamma1*ivdat$z

sim2slsss<-felm(Y~hatx_1,ivdat)
```

The second stage:

- Math Magic! $\hat{\beta}_{1,2SLS}$ consistently estimates β_1 and $\hat{\beta}_{1,2SLS} = \hat{\beta}_{1,IV}$!

```
stargazer(simiv1, simiv3,sim2slsss, header=FALSE,  type='latex',
          omit.stat = "all", no.space=TRUE)
```

Table 4

	Dependent variable:		
	Y		
	OLS	*felm*	
	(1)	(2)	(3)
x_1	10.035***		
	(0.015)		
x_2	−20.021***		
	(0.015)		
‘x_1(fit)‘		9.828***	
		(0.796)	
hatx_1			9.828***
			(0.557)
Constant	−0.023**	0.141	0.141
	(0.010)	(0.197)	(0.138)
Note:		*p<0.1; **p<0.05; ***p<0.01	

- Note: The standard errors reported from the second stage of 2SLS will not be correct (because they are based on $\hat{x}_1$ rather than x_1).(There are ways to correct this but the math and coding are a bit complicated.)

The Reduced Form (and more cool IV intuition)

The **reduced form** regresses the outcome directly on the exogenous instrument (and any other exogenous variables if you have them):

$$y_i = \pi_0 + \pi_1 z_i + \eta$$

```
sim2slsrf<-felm(Y~z,ivdat)
stargazer(simiv3, sim2slsfs,  sim2slsrf,  type='latex', header=FALSE, omit.stat = "all")
```

Table 5

	Dependent variable:		
	Y	x_1	Y
	(1)	(2)	(3)
‘x_1(fit)‘	9.828***		
	(0.796)		
z		0.249***	2.443***
		(0.010)	(0.138)
Constant	0.141	0.003	0.167
	(0.197)	(0.010)	(0.138)
Note:			*p<0.1; **p<0.05; ***p<0.01

The Reduced Form (and more cool IV intuition)

We can recover $\hat{\beta}_1$ by taking the $\hat{\pi}_1$from the reduced form and dividing it by $\hat{\gamma}_1$ from the first stage:

$$\hat{\beta}_1 = \frac{\hat{\pi}_1}{\hat{\gamma}_1} = \frac{2.442}{0.249} = 9.807$$

- Math Magic!
- Why does this work?

The Reduced Form (and more cool IV intuition)

We can recover $\hat{\beta}_1$ by taking the $\hat{\pi}_1$from the reduced form and dividing it by $\hat{\gamma}_1$ from the first stage:

$$\hat{\beta}_1 = \frac{\hat{\pi}_1}{\hat{\gamma}_1} = \frac{2.442}{0.249} = 9.807$$

- Math Magic!
- Why does this work? We are taking the effect of z on y and scaling it by the effect of z on x_1 (since z affects y via x_1).

We saw the good. Now for the bad and ugly.

- The Forbidden Regression
- Weak Instruments

The Bad: The Forbidden Regression

Be weary of the **forbidden regression**!

People sometimes try to run a logit, probit, or some other non-linear regression as the first stage of a 2SLS procedure. This is a bad idea. Don't do it.

The Ugly: Weak Instruments

Recall that

$$\hat{\beta}_{IV} = \beta + \frac{cov(z, \nu)}{cov(z, x_1)}.$$

A weak instrument is an instrument with a weak first stage: $Cov(z, x_1)$, is small.

Why is this a problem?

The Ugly: Weak Instruments

A weak instrument will amplify any endogeneity that exists in your model.

For $E[\hat{\beta}_{IV}] = \beta_1$, we need $cov(z, \nu) = 0$ (the exclusion restriction) to hold.

Suppose this assumption is violated in a small way, meaning that $cov(z, \nu) \neq 0$ but that it was a very small value.

If $cov(z, x_1)$ is also small, the violation of the exclusion restriction will get amplified leading to potentially severe bias in our estimator.

The Ugly: Simulation

I generate a simulated dataset with:

- a week first stage $cov(z, x_1) = 0.03$
- a small violation of the exclusion restriction, $cov(z, x_2) = 0.01$

```
sigmaMat<-matrix(c(1,0.75,0.03,0.75,1,0.01,0.03,0.01,1), nrow=3)
sigmaMat
```

```
##      [,1] [,2] [,3]
## [1,] 1.00 0.75 0.03
## [2,] 0.75 1.00 0.01
## [3,] 0.03 0.01 1.00
```

```
set.seed(5000)
ivdatwk<- as.data.frame(mvrnorm(10000, mu = c(0,0,0),
                    Sigma = sigmaMat))

names(ivdatwk)<-c("x_1","x_2","z")
cov(ivdatwk)
```

```
##            x_1        x_2          z
## x_1 0.98285285 0.74457303 0.02472936
## x_2 0.74457303 1.00432318 0.01277121
## z   0.02472936 0.01277121 0.98892020
```

```
ivdatwk$error<-rnorm(10000, mean=0, sd=1)
ivdatwk$nu=(-20)*ivdatwk$x_2+ivdatwk$error
```

The Ugly: Simulation

```
#The data generating process
B1<-10
B2<-(-20)
ivdatwk$Y<-ivdatwk$x_1*B1+ivdatwk$x_2*B2+ivdatwk$error

simivweakfs<-lm(x_1~z,ivdatwk)
simivweak<-felm(Y~1|0|(x_1~z),ivdatwk)
stargazer(simivweakfs,simivweak,  type='latex', omit.stat = c("n", "adj.rsq", "rsq", "ser"),
          header=FALSE)
```

Table 6

	Dependent variable:	
	x_1	Y
	OLS	*felm*
	(1)	(2)
z	0.025**	
	(0.010)	
‘x_1(fit)‘		−0.415
		(5.685)
Constant	0.007	0.137
	(0.010)	(0.146)
F Statistic	6.295** (df = 1; 9998)	
Note:	*p<0.1; **p<0.05; ***p<0.01	

The Ugly: Simulation

```
cov(ivdatwk$z,ivdatwk$x_1)
```

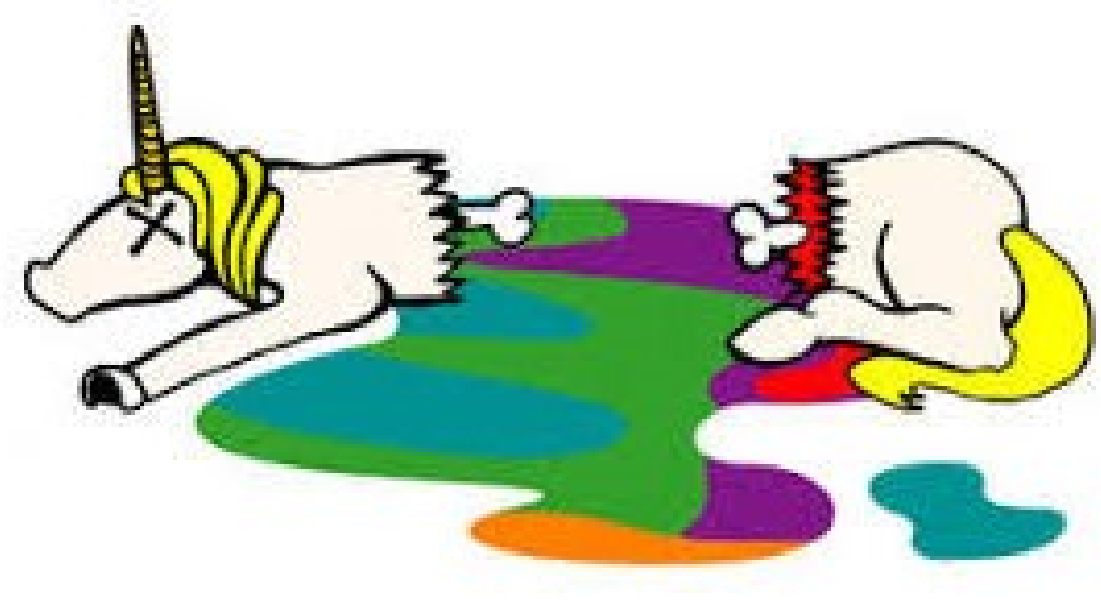
[1] 0.02472936

```
cov(ivdatwk$z,ivdatwk$nu)
```

[1] -0.2575614

We can see that $cov(z, x_1) = 0.02473$ and $cov(z, \nu) = -0.25756$ so

$$\hat{\beta}_{1,IV} = 10 + \frac{-0.25756}{0.02473} = -0.415 \neq \beta_1 = 10.$$



The Ugly: Weak Instruments

This is a major problem because:

- It is rare that an instrument would be perfectly independent of all confounding factors, and
- it is impossible to test the exclusion restriction.

$\Rightarrow$ be very cautious about results when there is a weak first stage.

What constitutes a "weak" instrument?

The standard benchmark is a first stage F-test that is less than 10 (ie you want the F-stat to be large).

Dealing with Multiples

See the lecture notes for example of how to deal with more complicated specifications:

- Control variables
- Multiple Instruments
- Multiple endogenous variables and multiple instruments

Unicorns and Work-horses

The real Siberian unicorn, *Elasmotherium sibiricum*, 29,000 BC.

Unicorns and Work-horses

IV estimations show up in two different types of situations:

- IV projects:
  - the validity of the instrumental variable is central to the identification strategy
  - can be very interesting because they are often looking at an important but highly endogenous variable
  - the validity of the causal claims, depends **heavily** on the validity of the instrument.
- Cameo appearances in other projects:
  - in randomized control trials (RCT)
  - in regression discontinuity (RD) projects
  - the random assignment of treatment is used as an instrument to estimate treatment effects

"Work Horse" IV intuition and medical trials (RCT)

Medical trials are a fantastic example of an application of instrumental variables:

- socially important (perhaps the most important application of IV to date)
- very clean experimental design

And this is a good segway to the RCT module.

Medical trials (RCT)

The model for a medical trial:

$$Y_i = \alpha + \tau D_i + \epsilon_i.$$

- Y_i represents a medical outcome (continuous or discreet).
- D_i is generally a dummy variable (1 if treated and 0 if not).
- The error term, ϵ_i represents all other factors that affect the health outcome

Medical trials (RCT)

This regression model corresponds to the potential outcome model with constant treatment effects:

$$\begin{aligned} Y_i(D_i) &= D_i Y(1) + (1 - D_i) Y(0) \\ Y_i(0) &= \alpha + \epsilon_i \\ Y_i(1) &= Y_i(0) + \tau. \end{aligned}$$

Our goal:

- estimate the effect that the treatment (say a pill) has on our outcome (say blood pressure)
- our hope is that τ is negative and large in magnitude.

Medical trials (RCT)

Options: Non-Experimental estimates:

- Selling the drug to the general population.
- Collect some data
- Regress blood pressure on whether or not you take the pill
- **What is the problem with this approach?**

Medical trials (RCT)

Options: Non-Experimental estimates:

- Selling the drug to the general population.
- Collect some data
- Regress blood pressure on whether or not you take the pill
- **What is the problem with this approach?**
  - people who take the pill are the ones who have high blood pressure to begin with!
  - We will likely get a positive estimate of τ (even with controls).

Medical trials (RCT)

Options: A Medical Trial:

- randomly assign some patients to the treatment group and others to the control group.
- Treatment group is given the pill and told to take it
- Control group are given a placebo (or nothing at all).

Medical trials (RCT)

Back in the old days: the **intention to treat** (or ITT) estimate:

- Calculate $\bar{Y}_{i,treat} - \bar{Y}_{i,control}$
- This is the equivalent to estimating $Y_i = \alpha + \tau D_i + \epsilon_i$ where $D_i = 1$ if in the treatment group and 0 if in the control group
- **intention to treat** because you compare the group that you intended to treat and the group that you do not intend to treat.

What is the problem with this?

Medical trials (RCT)

Non-compliance:

- some (selected) people in the treatment group would fail to take the pill
- some (selected) people in the control group would obtain the pill from another source (even though they were not supposed to)

Non-compliance can bias the estimate of τ.

How can this bias be corrected?

The "Work Horse" IV:

This is actually a simple IV problem:

- The instrument, Z_i is the intention to treat:
  - $Z_i = 1$ if you are assigned to the treatment group (we intend to treat you)
  - $Z_i = 0$ if you are assigned to the control group (we do not intend to treat you)

Does Z_i satisfies the two properties of a good instrument?

The "Work Horse" IV:

Does Z_i satisfies the two properties of a good instrument?

- Z_i is randomly assigned so by construction will be uncorrelated with ϵ_i so $cov(Z_i, \epsilon_i) = 0$ (the exclusion restriction).
- Z_i is correlated with D_i, because you are going to be more likely to take the pill if you are in the treatment group so $cov(Z_i, D_i) \neq 0$ (the first stage).

$\Rightarrow Z_i$ is a valid instrument for D_i and the IV estimator gives us a consistent estimate of τ, the effect of taking the pill on blood pressure.

The "Work Horse" IV: Intuition

How does this fix the non-compliance problem?

Example:

- Assume the non-compliance problem only exists for the people in the treatment group.
- Only half the people in the treatment group take the pill (ie half of the treatment group fails to comply and does not take the pill while the other half takes the pill as they were told to)

What will the IV look like?

The "Work Horse" IV: Intuition

The first stage will regress whether you took the pill on whether you were in the treatment group D_i on Z_i:

$$D_i = \gamma_0 + \gamma_1 Z_i + u_i$$

Zero people in the control group took the pill while half in the treatment group took the pill:

$\Rightarrow E[\hat{\gamma}_0] = 0$ and $E[\hat{\gamma}_1] = 0.5$.

The "Work Horse" IV: Intuition

The IV estimate is the reduced form scaled by the first stage:

The reduced form is a regression of Y_i (your blood pressure) on Z_i(whether you were assigned to the treatment or control group):

$$Y_i = \pi_0 + \pi_1 Z_i + v_i$$

Therefore our IV estimate,

$$\hat{\tau}_{IV} = \frac{\hat{\pi}_1}{\hat{\gamma}_1} = \frac{\hat{\pi}_1}{0.5}.$$

How is this fixing the non-complier problem?

The "Work Horse" IV: Intuition

How is this fixing the non-complier problem?

The reduced form is estimating the effect that being assigned to the treatment group has on blood pressure.

But this is not what we are interested in.

We want to know the effect of taking the pill.

The "Work Horse" IV: Intuition

How is this fixing the non-complier problem?

If there were perfect compliance:

- the reduced form estimate would be the effect of taking the pill
  - the first stage would give us $\hat{\gamma}_1 = 1$
  - the IV estimate would be $\hat{\tau}_{IV} = \frac{\hat{\pi}_1}{1} = \hat{\pi}_1$.

If compliance isn't perfect:

- the reduced form estimates the effect of increasing the probability you take the pill
  - This means that the reduced form is not estimating the full effect of taking the pill

The "Work Horse" IV: Intuition: Example

- 10 people in the treatment group. 5 take the pill and 5 do not.
  - The (expected) mean blood pressure for the treatment group:

$$E[Y_i|TreatGroup_i] = \frac{5\alpha + 5(\alpha + \tau)}{10} = \alpha + \frac{\tau}{2}$$

- 10 people in the control group, no one takes the pill.
  - The (expected) mean blood pressure for the control group:

$$E[Y_i|ContGroup_i] = \alpha$$

The "Work Horse" IV: Intuition: Example

- 10 people in the treatment group. 5 take the pill and 5 do not.
  - The (expected) mean blood pressure for the treatment group:

$$E[Y_i|TreatGroup_i] = \frac{5\alpha + 5(\alpha + \tau)}{10} = \alpha + \frac{\tau}{2}$$

- 10 people in the control group, no one takes the pill.
  - The (expected) mean blood pressure for the control group:

$$E[Y_i|ContGroup_i] = \alpha$$

So the reduced form coefficient(=the difference of means):

$$\hat{\pi}_1 = \alpha + \frac{\tau}{2} - \alpha = \frac{\tau}{2}$$

is half the effect of taking the pill (since only half the treated group takes it).

The "Work Horse" IV: Intuition: Example

Thus,

$$E[\tau_{IV}] = \frac{\pi_1}{\gamma_1} = \frac{0.5\tau}{0.5} = \tau.$$

The IV estimate gives us a consistent estimate because it is scaling the reduced form by the first stage.

We re-scale the reduced form because:

- being in the treatment group only increases your probability of taking the pill by 50 percentage points not by a full 100 percentage points.
- the reduced form only represents half of the effect of taking the pill

The "Work Horse" IV: Intuition:

Note:

- the IV is different from simply taking the mean of Y_i for those in the treatment group who took the pill and subtracting the mean of Y_i for those in the control group who did not take the pill.
- This would be affected by the same selection issues as a simple OLS regression of Y_i on d_i.

The "Work Horse" IV: In practice

Recall Arseneaux, Gerber and Green (2006):

Evaluate a "Get out the Vote" mobilization:

- Who gets called ($Call_i$) is random
- Who answers the call ($Contact_i$) is not

In the paper they generate:

- estimates that controlled for observable characteristics
- "experimental" estimates in order to gauge bias from unobservables

The "experimental" estimates use an instrumental variable.

The "Work Horse" IV: In practice

They are interested in estimating how getting contacted by the "Get out the Vote" mobilization affect the likelihood of actually voting:

$$Votes_i = \beta_0 + \beta_1 Contacted_i + \beta_j X_j + \epsilon_i.$$

But who gets contacted is not random (not everyone will pick up the phone!)

They instrument $Contacted_i$, with being randomly assigned to receive a call from the campaign. Thus the first stage is

$$Contacted_i = \gamma_0 + \gamma_1 Called_i + \gamma_j X_j + u_i$$

The "Work Horse" IV: In practice

Replicating results of columns 1 of p.49 and p.50:

```
library(haven)
```

```
## Warning: package 'haven' was built under R version 3.6.3
```

```
agg_data<-read_dta("../../data/data_M3_IV/IA_MI_merge040504.dta")

#scalling the vote02 variable to remove excess 0's from tables
agg_data$vote02<-100*as.numeric(agg_data$vote02)

regols1<-felm(vote02~contact+state+comp_mi+comp_ia,agg_data)
regiv1<-felm(vote02~state+comp_mi+comp_ia|0|(contact~treat_real+state+comp_mi+comp_ia),agg_data)
```

```
## Warning in chol.default(mat, pivot = TRUE, tol = tol): the matrix is either
## rank-deficient or indefinite
```

The "Work Horse" IV: In practice

Replicating results of columns 1 of p.49 and p.50:

```
stargazer(regols1,regiv1,type='latex', se = list( regols1$rse, regiv1$rse), header=FALSE,
          omit.stat = "all")
```

Table 7

	Dependent variable:	
	vote02	
	(1)	(2)
contact	6.207***	
	(0.306)	
state	6.671***	7.388***
	(0.347)	(0.350)
comp_mi	4.836***	4.911***
	(0.098)	(0.098)
comp_ia	6.353***	6.083***
	(0.177)	(0.178)
'contact(fit)'		0.360
		(0.498)
Constant	46.128***	46.081***
	(0.126)	(0.126)
Note:	*p<0.1; **p<0.05; ***p<0.01	