

AERMXN BERM min  $11 \text{Ax} - \text{b11}_2^2$ MEIR<sup>n</sup>

S.t. Cx = dCER J der  $C = \begin{bmatrix} C_i \\ C_2^T \end{bmatrix}$  where  $C_i^T \in \mathbb{R}$ Construct the Lagrangian function:  $L(x, 2) = ||Ax-b||_2^2 + 2|(c_1^7x-d_1) + 2|(c_2^7x-d_2) + -+ 2|(c_p^7x-d_p)|$ where  $Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \in \mathbb{R}^p$  is a vector of Lagrange multipliers. If is a solution of CLP, then there exist lagrange multipliers 2 s.t.

Constrained LS (CLS)

$$\frac{\partial L}{\partial x_{i}}(\hat{x},\hat{z}) = 0 \qquad \text{for } i=1,2,...,p$$

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KKT conditions

(Karush, Kuhn, Tucker)

fer i=1,2, --, n

97: (x, 5) = 0

 $\begin{bmatrix} 2A^TA & C^T \end{bmatrix} \in \mathbb{R}$   $C \qquad O \qquad = C$ xxt matrix c has linearly invertible if and only if independent rows and [A] has linearly independent columns.

Led ( ) be a vector such that char lin.

indep rows.  $\begin{bmatrix} 2\vec{A}^T A & c^T \\ c & o \end{bmatrix} \begin{bmatrix} \vec{z} \\ \vec{z} \end{bmatrix} = \begin{bmatrix} 0 \\ o \end{bmatrix}$   $\begin{bmatrix} R & A \\ a \end{bmatrix}$ hous linearly indep.

Lold.

KKT

To prove.

modrix is invertible. 2 ATA x + CT = 0 -(1) = 27 ATAZ+ 7 TCT == 0 7211Ax112+ 1927=

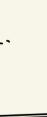
$$\Rightarrow 211A\overline{x}|_{2}^{2} = 0$$

$$\Rightarrow A\overline{x} = 0 \quad \text{and} \quad C\overline{x} = 0$$

$$\Rightarrow \left[ \begin{matrix} A \\ C \end{matrix} \right] \overline{x} = 0$$

$$\overrightarrow{\chi} = 0$$

From (1), 
$$c^{T}\overline{2}=0 \Rightarrow \overline{2}=0$$



columns of ct (rows 9 c)

Recursive LS problem: (12.4, Pg. 242, Nmls Book)
$$\begin{bmatrix}
-a_1^T - \\
b_2
\end{bmatrix}$$

$$A = \begin{bmatrix} -\alpha_1^T - \\ -\alpha_2^T - \\ \vdots \\ -\alpha_k^T - \end{bmatrix}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix}$$

$$\left( -\bar{\alpha}_{\nu}^{T} - \right)$$

$$= S^{-1} - S^{-1} u (1+v^{-1}u)^{-1} v^{-1}$$

$$= S^{-1} - \frac{(S^{-1}u)^{-1}}{(1+v^{-1}u)}$$

$$\Rightarrow A = \begin{bmatrix} a_1^T \\ \vdots \\ a_k^T \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_k \end{bmatrix}$$

$$a_k^{(k)} = \begin{bmatrix} (A^k)^T & (A^k)^T & b^k \end{bmatrix}$$

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ox = ([A(k+1)] (h+1)) (k+1)) (k+1)

$$a = (A)$$

$$A = A$$

$$A = A$$

$$\begin{bmatrix} \alpha_i^T \end{bmatrix} \begin{bmatrix} A^k \end{bmatrix}$$



$$\left(A^{(k+1)}\right)^{T} \left(A^{(k+1)}\right) = \left(A^{(k)}\right)^{T} \left(A^{(k)}\right)^{T}$$

$$= \left(A^{(k)}\right)^{T} \left(A^{(k)}\right)^{T}$$

$$= \left(A^{(k)}\right)^{T} \left(A^{(k)}\right) + \left(A^{(k)}\right)^{T}$$

$$= \left(A^{(k)}$$

$$= (A^{(k)})^T b^{(k)} + a_{k+1} b_{k+1}$$

C(k+1)  $R = \left[ \left( A^{(k+1)} \right)^{T} A^{(k+1)} \right]^{T} \left( A^{(k+1)} \right)^{T} b^{(k+1)}$