$A \in \mathbb{R}^{m \times n}$ ← given.

$b \in \mathbb{R}^m$ ← given.

Find $x \in \mathbb{R}^n$ s.t $Ax = b$.

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

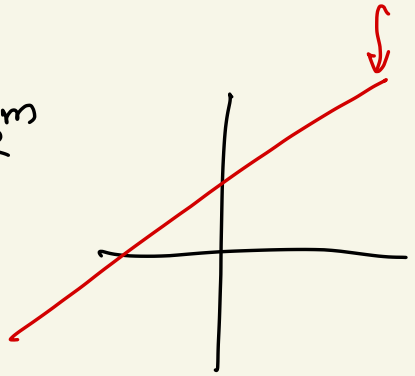
$$A = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix}$$

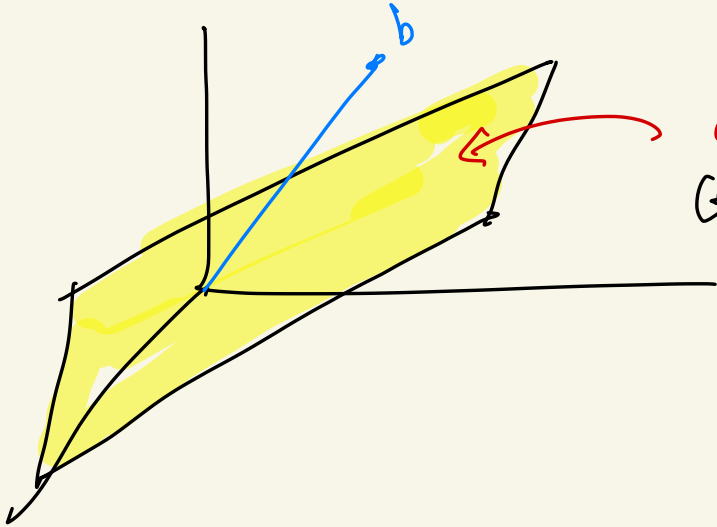
$$Ax = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1 a_1 + x_2 a_2 + \cdots + x_n a_n$$

Colspace (A) = span of columns of A

$$= \{ y_1 a_1 + \cdots + y_n a_n = Ay \mid y \in \mathbb{R}^n \} \subseteq \mathbb{R}^m$$

subspace of $\mathbb{R}^m$.



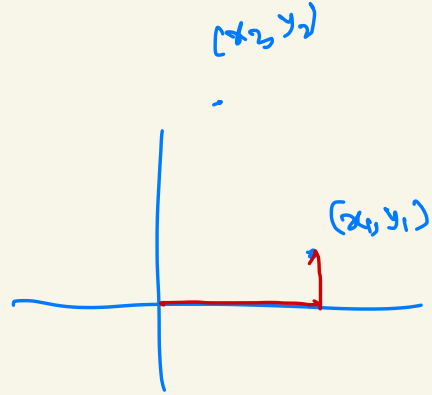


colspace (A)
(estimation space)

$x \in \mathbb{R}^n$

$\|x\|_2 = \sqrt{x^T x}$

$\|x - y\|_2 = \sqrt{(x-y)^T (x-y)}$



$b \notin \text{colspan}(A)$ $\quad A \in \mathbb{R}^{m \times n}$

every vector in colspan(A) can be written Ax for some $x \in \mathbb{R}^n$.

Define: $r \in \mathbb{R}^m$ $\quad r = b - Ax$

$\therefore b \notin \text{colspan}(A)$, $\quad r \neq 0$ for $x \in \mathbb{R}^n$

Aim: Choose an $x \in \mathbb{R}^n$ s.t. $\|r\|_2$ is as small as possible.

optimization problem:

$$\min_{x \in \mathbb{R}^n} \|r\|_2$$

$$\Downarrow$$

$$\min_{x \in \mathbb{R}^n} \|r\|_2^2$$

$r \in \mathbb{R}^m$

$$r = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{pmatrix}$$

$$\|r\|_2 = \sqrt{r^T r} = \sqrt{r_1^2 + \cdots + r_m^2}$$

$$\mathcal{D} = \{(x_i, y_i)\}_{i=1}^{m}$$

↓

n-dimensional input.