Eigenvalues and eigenvectors of a square matrix.

$A \in \mathbb{R}^{n \times n}$

Then a nonzero $x \in \mathbb{R}^n$ is called as an eigenvector of A if for some $\lambda \in \mathbb{R}$,

$$Ax = \lambda x$$

$$\Rightarrow \quad (A - \lambda I)x = 0$$

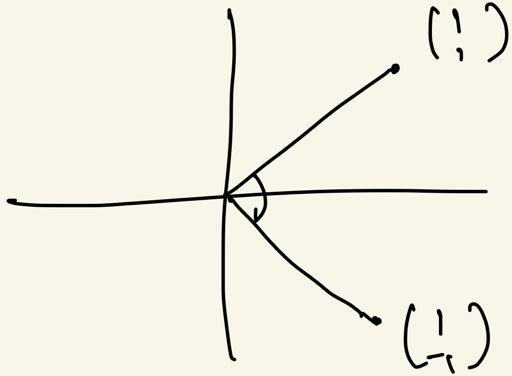
Ex: $A \in \mathbb{R}^{2 \times 2}$

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$$

$\lambda_1 = -2, \quad \lambda_2 = 4$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

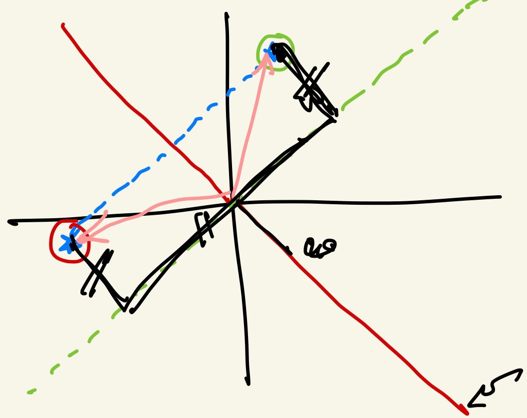
$$A\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad ; \quad A\begin{pmatrix} 1 \\ -1 \end{pmatrix} = -2\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$



$V^\perp = \{y \in \mathbb{R}^2 \mid y^T x = 0 \quad \forall x \in V\}$

$\mathbb{R}^2$

Ex:



$V \subseteq \mathbb{R}^2$

T = matrix representation of orthogonal reflection thru' V.

$T \in \mathbb{R}^{2\times 2}$

$\{v, w\}$ is an orthonormal basis of $\mathbb{R}^2$.

any, $x \in \mathbb{R}^2$, $\quad x = \alpha v + \beta w$ for $\alpha, \beta \in \mathbb{R}$.

$$T(x) = T(\alpha u + \beta w) = \alpha T(u) + \beta T(w)$$

$$= \alpha u - \beta w$$

$u \in \mathbb{R}^{2\times 1}$

Look at $\underset{\overset{\shortparallel}{T_2}}{uu^T} \in \mathbb{R}^{2\times 2}$ & $\underset{\overset{\shortparallel}{T_1}}{ww^T} \in \mathbb{R}^{2\times 2}$

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} (u_1\ u_2)^T = \begin{pmatrix} u_1^2 & u_1 u_2 \\ u_1 u_2 & u_2^2 \end{pmatrix}$$

$$T_1 u = (ww^T)\, u = w(w^T u) = 0$$

$$T_1 w = (ww^T)\, w = w(w^T w) = w$$

$$T = I - 2T_1$$

$$Tu = (I - 2T_1)\, u = Iu - 2T_1 u = u$$

$$Tw = (I - 2T_1)\, w = Iw - 2T_1 w = w - 2w = -w$$

$Q = T$.

$$Q^T Q = (I - 2ww^T)^T (I - 2ww^T) = (I - 2ww^T)(I - 2ww^T)$$

$$= I - 2ww^T - 2ww^T + 4w(w^T w)w^T = I$$