Stochastic Simulation (MIE1613H) - Homework 1

Due: Jan 29, 2019

- Submit your homework on Quercus as a single PDF file by the deadline. Late submissions are penalized 20% each day the homework is late.
- At the top of your homework include your name, student number, department, and program.
- You may discuss the assignment with other students in general terms, but each student must solve the problems, write the code and the solutions individually.
- The simulation models must be programmed in Python. You must include both the source code (including comments to make it easy to follow) and the output of the simulation in your submission.
- To make it easier for us to check your solutions, set the random seed to 1 in all simulations using `np.random.seed(1)`.

Problem 1. Assume that X is exponentially distributed with rate $\lambda = 0.2$. We are intreated in computing $E[(X-3)^+]$. (Note: $a^+ = \text{Max}(a, 0)$, i.e, if $a < 0$ then $a^+ = 0$ and if $a \geq 0$ then $a^+ = a$.)

(a) Compute the expected value exactly.

(b) Estimate the expected value using Monte Carlo simulation and provide a 95% confidence interval for your estimate. **Note**: You can generate a random sample of an exponentially distributed random variable with rate λ in Numpy using `np.random.exponential(1/λ)`.

(c) Create a plot that demonstrates the convergence of the Monte Carlo estimate to the exact value as the number of samples increases.

Problem 2. In the TTF example from the first class we simulated the system until the time of first failure. Modify the simulation model to simulate the system for a given number of days denoted by T. Assume that all other inputs and assumptions are the same as the original example.

(a) What is the average number of functional components until time $T = 1000$ based on one replication of the simulation?

(b) What is the average number of functional components until time $T = 2000$ based on one replication of the simulation? Compare the results from part (a) and (b) and summerize your observation in one sentence.

HINT: To get the simulation to stop at time T use the functional version of the code and create a new event called EndSimulation, with a variable NextEndSimulation as part of the event calendar. Note that you also need to modify the logic of the model in the failure and repair event functions.

Problem 3. Modify the TTF simulation assuming that there are three components, one active and two spares, but still only one can be repaired at a time. Repair time is 3.5 days. Run your simulation for 1000 replications and report a 95% confidence interval for the expected time to failure of the system.

Problem 4. The standard error of an estimator is defined as the standard deviaition of that estimator. In class we introduced the sample mean $\bar{X}_n = (1/n)\sum_{i=1}^{n} X_i$ as an estimator of $E[X]$ where X_i's are iid samples of the random variable X. What is the standard error of the estimator $\bar{X}_n$? Assume that the statndard deviation of X is σ.

Problem 5. If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is 1, what is the (approximate) probability that you will win a prize (a) at least once, (b) exactly once, (c) at least twice?