Stochastic Simulation (MIE1613H) - Homework 4

Due: April 13th

- Submit your homework to Quercus in PDF format by the above deadline. Late submissions are penalized 10% each day the assignment is late.
- At the top of your homework include your name, student number, department, and program.
- You may discuss the assignment with other students, but each student must solve the problems, write the code and the solutions individually.
- Code must be in Python unless stated otherwise in the problem statement. You must include both the source code (including comments to make it easy to follow) and the output.
- **Full mark is given to answers that are correct and clearly explained. Write a brief and clear explanation of your solution for each problem.**

Problem 1. Consider the $M/G/1$ example of section 4.3. Using a single replication and batching, estimate and provide a 95% confidence interval for the 0.8 quantile of the steady-state waiting time Y.

Problem 2. Problem 16 from Chapter 8 of the textbook.

Problem 3. For the Asian Option pricing example of section 4.5 (and assuming the same parameters), we are interested in estimating the sensitivity of the value of the option to the initial price value $X(0) = 50$. That is, if we denote the option value as a function of the initial price, i.e.,

$$\nu(X(0)) = E[Y(X(0))],$$

with

$$Y(X(0)) = e^{-rt}(\bar{X}(T) - K)^+,$$

we are interested in estimating the derivative $d\nu(X(0))/dX(0)$ at $X(0) = 50$.

Use the finite difference method to estimate the above derivative using simulation. Change the value of $\delta \in \{0.1, 0.5, 1, 5, 10\}$ in your finite-difference estimator, i.e.,

$$FD(X(0)) = \frac{Y(X(0) + \delta) - Y(X(0))}{\delta},$$

and discuss how it affects your estimate and its variability by reporting a confidence interval for each estimate. Use $n = 10,000$ replications and common random numbers when estimating the differnece between the value of the option under differnet initial prices.

Problem 4. Problem 27 from Chapter 8 of the textbook. Report a confidence interval for your estimate. (Use the same parameters as in the original Asian Option pricing example of section 4.5. The formula for the control-variate estimator confidence-interval is given on pages 232-233 of the textbook. See also Section 8.5.3)