Stochastic Simulation (MIE1613H) - Homework 2

Due: Feb 19th, 2019

- Submit your homework to Quercus in PDF format by the above deadline. Late submissions are penalized 20% each day.
- At the top of your homework include your name, student number, department, and program.
- Code must be in Python unless you have permission from instructor to use another language. You must include both the source code (including comments to make it easy to follow) and the output.
- You may discuss the assignment with other students, but each student must solve the problems, write the code and the solutions individually.
- **Full mark is given to answers that are correct and clearly explained. Write a brief and clear explanation of your solution for each problem.**

Problem 1. (Chapter 4, Exercise 8) In the simulation of the Asian option, the sample mean of 10,000 replications was 2.198270479 and the standard deviation was 4.770393202. *Approximately* how many replications would it take to decrease the reletive error to less than 1%?

Note: The relative error of a sample mean is the standard error divided by the mean.

Problem 2. (Down-and-in call option) Another variation of European options are barrier options. For a down-and-in call option, if we denote the stock price at time t by $X(t)$, the holder of the option receives payoff $(X(T) - K)^+$ at maturity time T only if the stock price has crossed below some barrier $b < X(0)$ before time T. Assume that the stock price evolves according to a Geometric Brownian Motion (GBM) with drift parameter $\mu = r = 0.05$ and volatility parameter $\sigma = 0.4$.

(a) Using simulation, estimate the expected payoff of a down-and-in call option for $T = 1$ year, assuming the initial price of $X(0) = 95$; strike price of $K = 100$ and barrier $b = 90$. Use 10,000 replications, and 64 steps when discretizing the GBM. Report a 95% confidence interval for the estimate.

(b) Compare the estimated expected payoff with that of a standard European call option (for the same parameters). Provide an intuitive explanation for your observation.

Problem 3. (Chapter 4, Exercise 4) Beginning with the PythonSim event-based $M/G/1$ simulation, implement the changes necessary to make it an $M/G/s$ simulation (a single queue with any number of servers). Keeping $\lambda = 1$ and $\tau/s = 0.8$, simulate the system for $s = 1, 2, 3$ and (a) report the estimated expected number of customers in the system (including customers in the queue and service), expected system time, and expected number of busy servers in each case. (b) Compare the results and state clearly what you observe. What you're doing is comparing queues with the same service capacity, but with 1 fast server as compared to 2 or more slow servers.

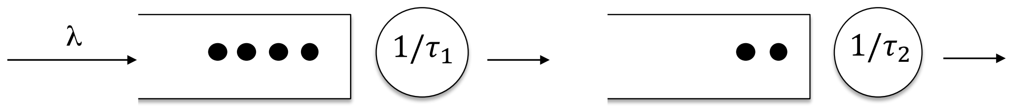

Figure 1: Tandem queue (problem 5).

HINT: The attribute "NumberOfUnits" of the Resource object returns the number of available units for any instance of the object.

Problem 4. (Chapter 4, Exercise 5) Modify the PythonSim event-based simulation of the $M/G/1$ queue to simulate a $M/G/1/c$ retrial queue. This means that customers who arrive to find c customers in the system (including the customer in service) leave immediately, but arrive again after an exponentially distributed amount of time with mean MeanTR. (You do not need to report any outputs for this problem.)

HINT: The existence of retrial customers should not affect the arrival process for first-time arrivals.

Problem 5. (Two queues in tandem) Customers arrive at a two-stage service system according to a stationary Poisson process with rate $\lambda = 1$. There is only 1 server working at each stage. Each customer has to go through both stages (first stage 1 and then stage 2) before departing the system. Service times are exponentially distributed with mean $\tau_1 = 0.8$ at stage 1 and $\tau_2 = 0.7$ at stage 2. Develop a simulation model of the system using PythonSim and estimate the quantities specified below in steady-state. Simulate the system for 50,000 time units with a warmup period of 5000 time units. Use 20 replications to obtain the estimates.

(a) The expected number of customers waiting at each stage (not including the customer in service).

(b) The expected total time customers spend at each stage (including both the waiting time and time spent in service).

(c) Expected utilization of each server.