

# EE116C/CS151B

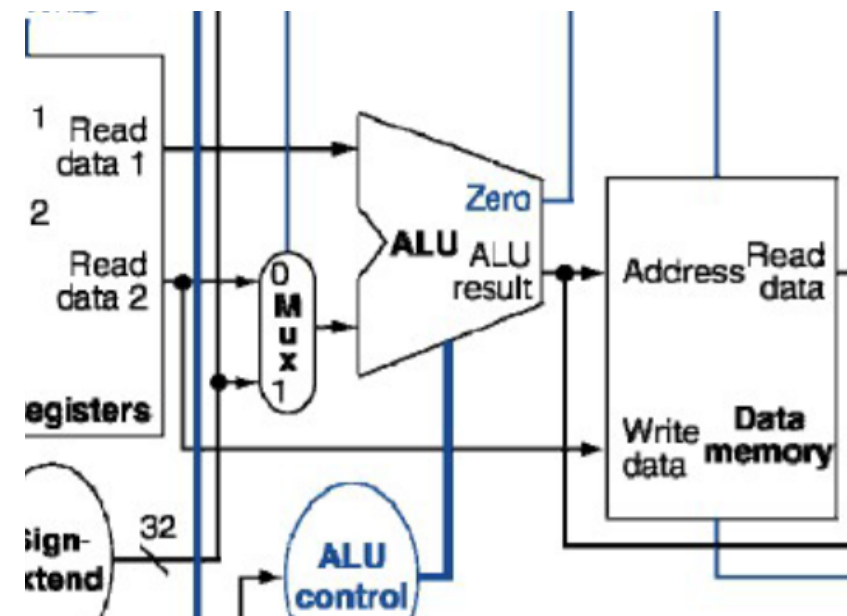
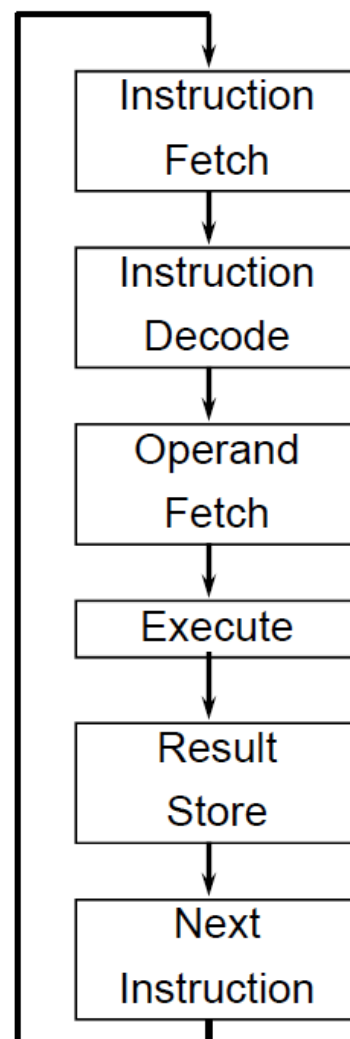
## Fall 2017

Instructor: Professor Lei He  
TA: Yuan Liang

# Review

- **Arithmetic Logic Unit (ALU) Design**

- Input: two operand (32-bits), ALU control signal
- Output: Zero, ALU results, overflow, carryout



# Review

- **Arithmetic Logic Unit (ALU) Design**

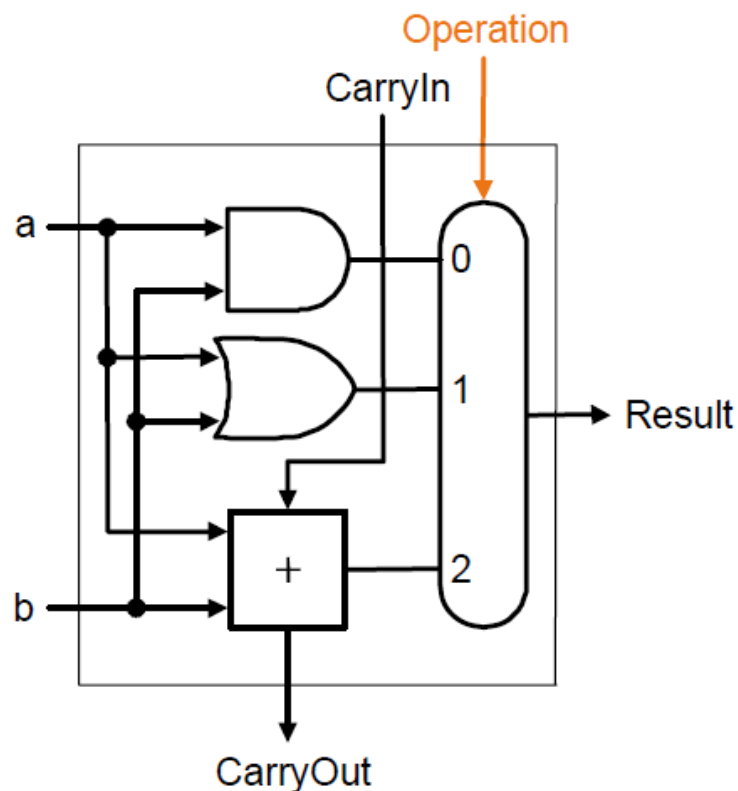
- Function:
  - Add
  - Subtract
  - Overflow dealing
  - And
  - Or
  - Branch
  - Set-On-Less-Than (SLT)

- **How?**

# Review

- **1-bit ALU**

- Input: two operand (1-bits), ALU control signal, **carryin**
- Output: Zero, ALU results, ~~**overflow**~~, carryout

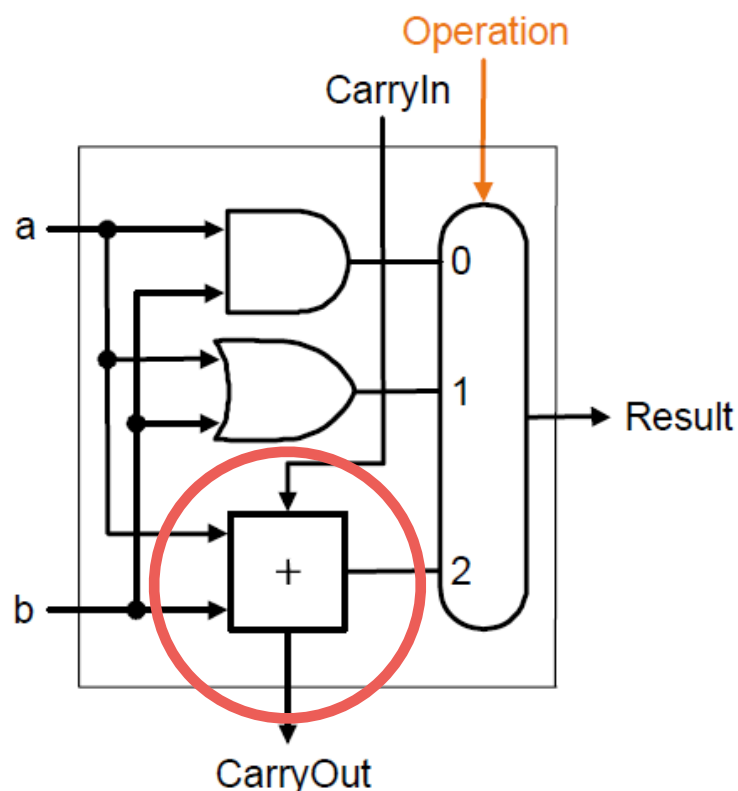


Operation?  
and,  
or,  
add with carryin,  
etc.

# Review

- **1-bit ALU**

- Input: two operand (1-bits), ALU control signal, **carryin**
- Output: Zero, ALU results, ~~**overflow**~~, carryout



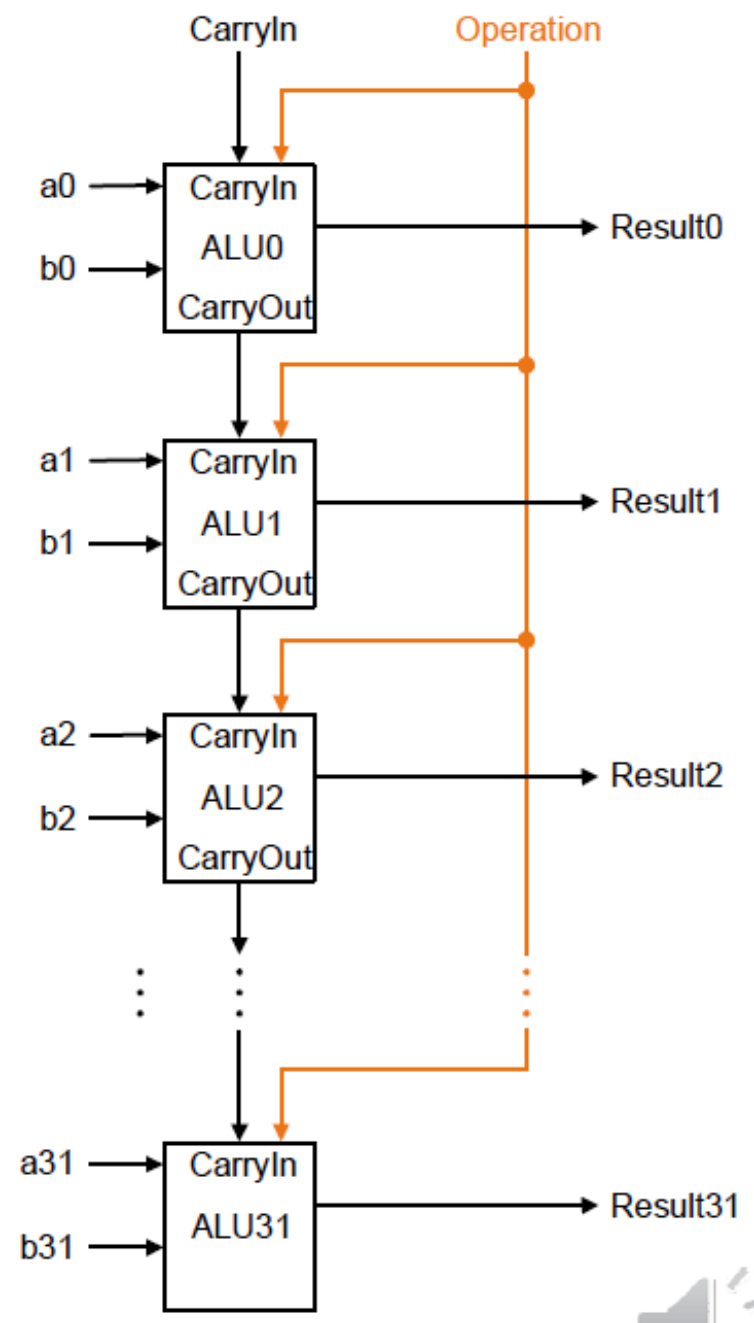
Composed of AND, OR, NOT gates.

$$\text{CarryOut} = (b \ \& \ \text{CarryIn}) \mid (a \ \& \ \text{CarryIn}) \mid (a \ \& \ b)$$

$$\text{Sum} = (!a \ \& \ !b \ \& \ \text{CarryIn}) \mid (!a \ \& \ b \ \& \ !\text{CarryIn}) \\ \mid (a \ \& \ !b \ \& \ !\text{CarryIn}) \mid (a \ \& \ b \ \& \ \text{CarryIn})$$

# Review

- **Style 1: Ripple Carry Adder (1-bit ALU -> 32-bit ALU)**



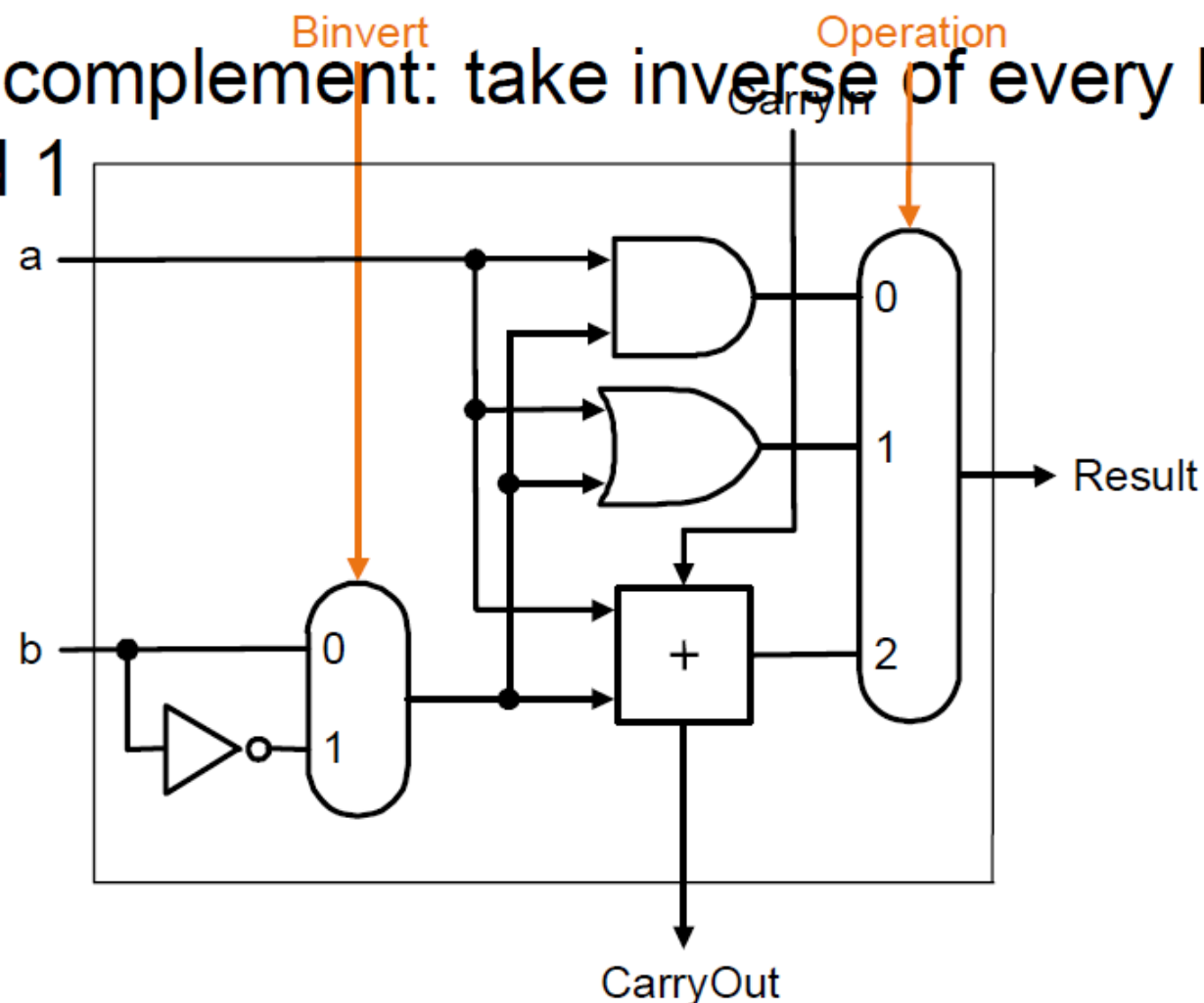
- **Function:**

- **Add**
- Subtract
- Overflow dealing
- **And**
- **Or**
- Branch
- Set-On-Less-Than (SLT)

# Review

- **Style 1: Ripple Carry Adder (1-bit ALU -> 32-bit ALU)**

2's complement: take inverse of every bit and add 1



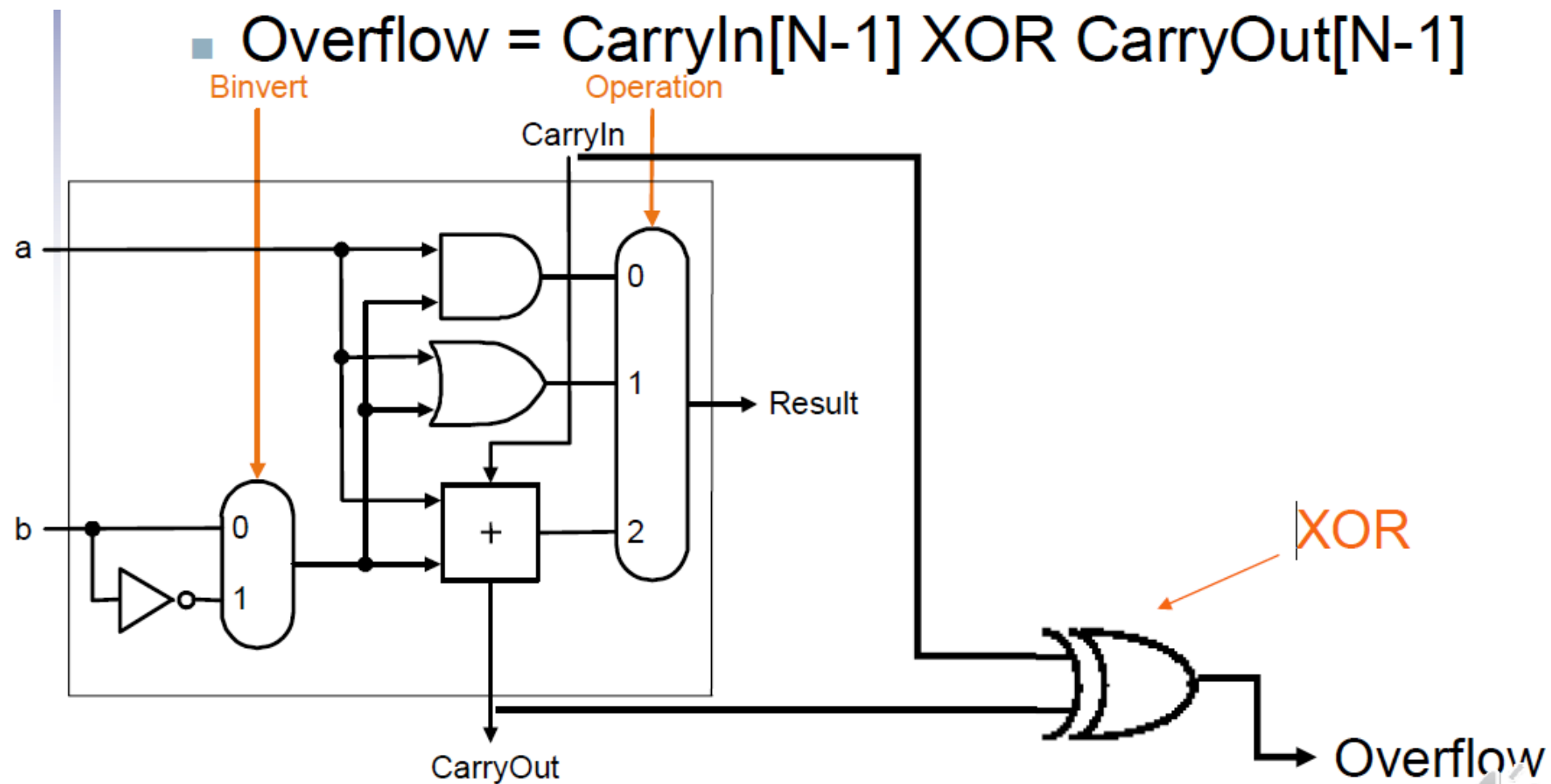
- **Function:**

- **Add**
- **Subtract**
- Overflow dealing
- **And**
- **Or**
- Branch
- Set-On-Less-Than (SLT)

# Review

- **Style 1: Ripple Carry Adder (1-bit ALU -> 32-bit ALU)**

- **Overflow = CarryIn[N-1] XOR CarryOut[N-1]**



- **Function:**

- **Add**
- **Subtract**
- **Overflow dealing**
- **And**
- **Or**
- **Branch**
- **Set-On-Less-Than (SLT)**

**only for the most significant bit ALU**



# Review

- **Style 1: Ripple Carry Adder (1-bit ALU -> 32-bit ALU)**

- bneq r1, r2, L
- beq r1, r2, L
- if (a == b)
- if (a != b)
- -> a - b == 0

One big NOR gate including all N results.

if any result(i) == 1 -> return 0  
if all result(i) == 0 -> return 1

- **Function:**

- **Add**
- **Subtract**
- **Overflow dealing**
- **And**
- **Or**
- **Branch**
- **Set-On-Less-Than (SLT)**

# Review

- **Style 1: Ripple Carry Adder (1-bit ALU -> 32-bit ALU)**

- `slt rd, rs, rt`

- if ( $rs < rt$ )  $rd = 1$

- else  $rd = 0$

If  $R[rs]$  is input A, and  $R[rt]$  is input B, we can subtract, and then take the most significant bit of the result.

.If  $MSB = 1$ , then the difference is negative and thus  $R[rs]$  is less than  $R[rt]$

• Function:

- **Add**

- **Subtract**

- **Overflow dealing**

- **And**

- **Or**

- **Branch**

- **Set-On-Less-Than (SLT)**

# Review

- **Style 1: Ripple Carry Adder (1-bit ALU -> 32-bit ALU)**

- **Function:**

- **Add**

- **Subtract**

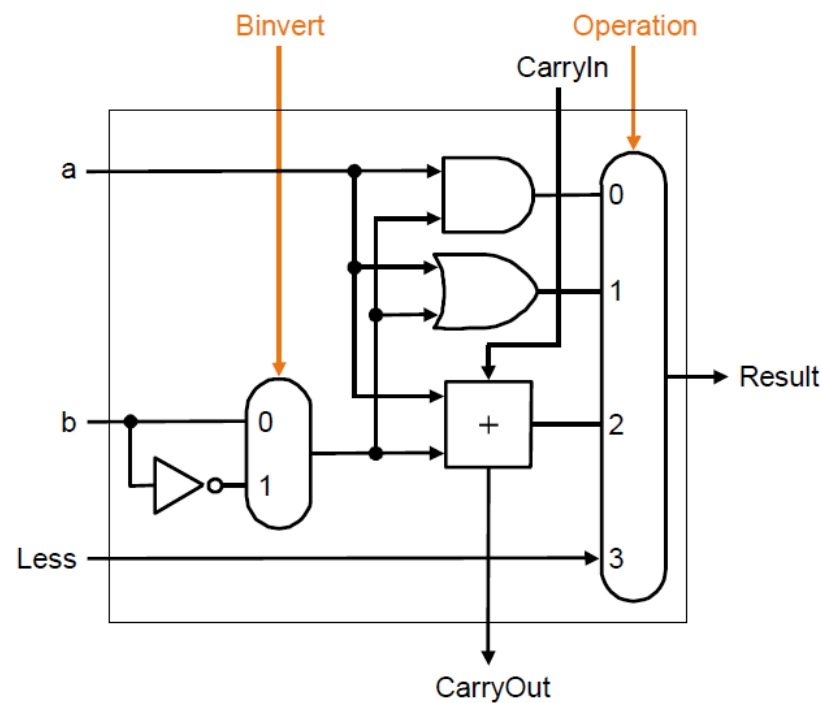
- **Overflow dealing**

- **And**

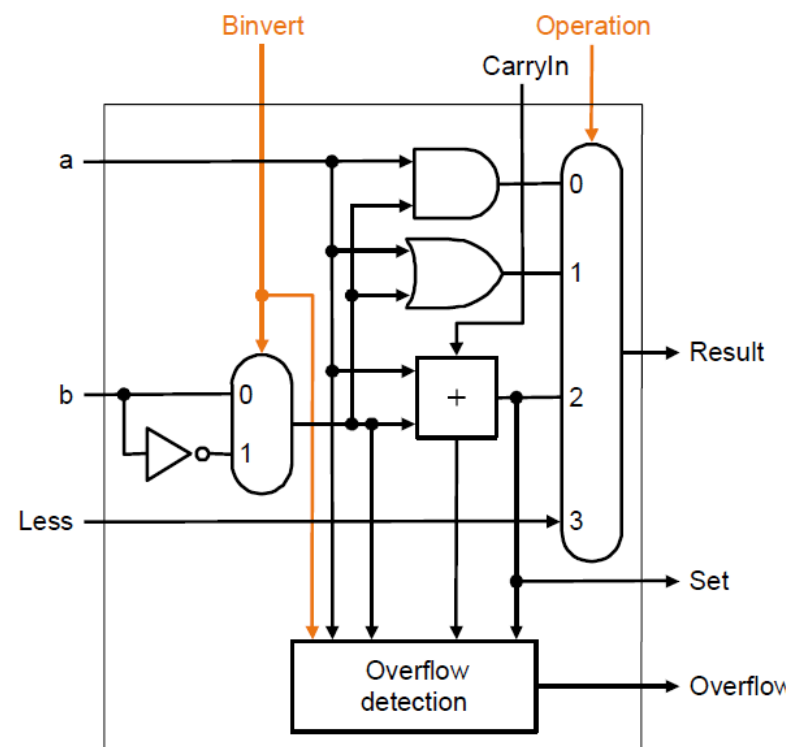
- **Or**

- **Branch**

- **Set-On-Less-Than (SLT)**



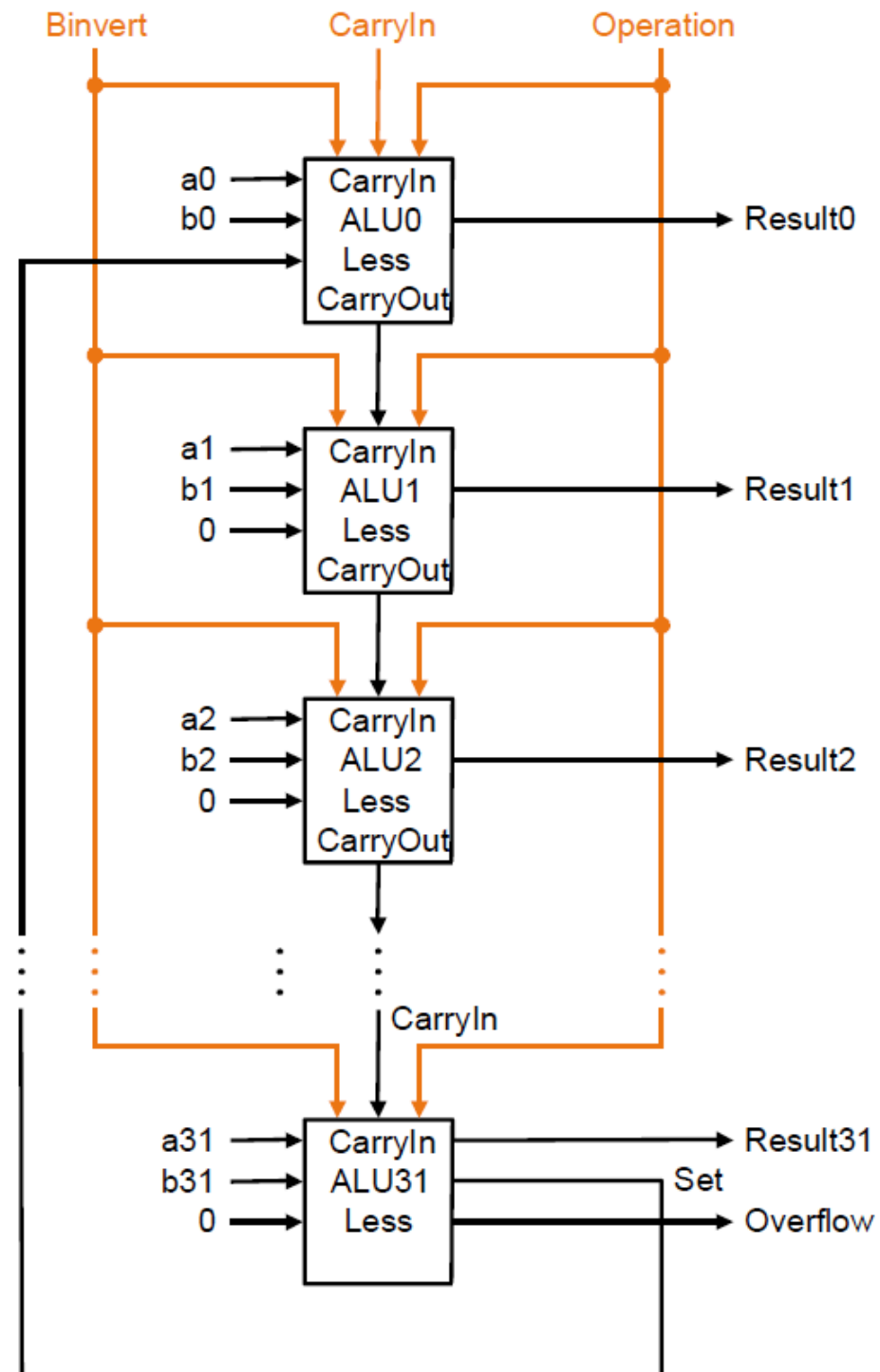
All but MSB



Most Significant Bit

# Review

- **Style 1: Ripple Carry Adder (1-bit ALU -> 32-bit ALU)**



- **Function:**

- **Add**

- **Subtract**

- **Overflow dealing**

- **And**

- **Or**

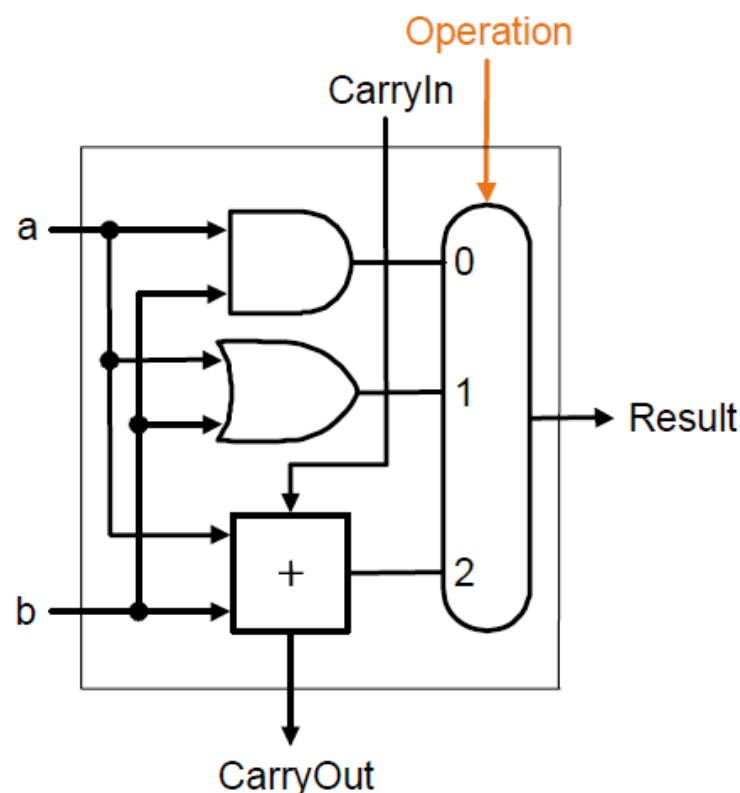
- **Branch**

- **Set-On-Less-Than (SLT)**

# Review

- **Discussion: ripple style 32-bit adder**

- Worst case delay for N-bit Ripple Carry Adder



$$\text{CarryOut} = (b \ \& \ \text{CarryIn}) \mid (a \ \& \ \text{CarryIn}) \mid (a \ \& \ b)$$

$$\text{Delay} = 2N$$

$$\begin{aligned} \text{Sum} = & (!a \ \& \ !b \ \& \ \text{CarryIn}) \mid (!a \ \& \ b \ \& \ !\text{CarryIn}) \\ & \mid (a \ \& \ !b \ \& \ !\text{CarryIn}) \mid (a \ \& \ b \ \& \ \text{CarryIn}) \end{aligned}$$

$$\text{Delay} = 2N$$

$$\text{Total delay} = 32 * 2N$$

# Review

- **Style 2: Carry Lookahead Adder (1-bit ALU -> 32-bit ALU)**
  - The principle of the Carry Lookahead Adder is to sacrifice space and complexity in the interest of speed.
  - For calculating Cout, we will use two additional signals at 1-bit adder:
    - Generate ( $G_n$ ) =  $A_n * B_n$
    - Propagate ( $P_n$ ) =  $A_n \wedge B_n$

# Review

- **Style 2: Carry Lookahead Adder (1-bit ALU -> 32-bit ALU)**

Ripple Carry Adder:

$$C1 = (A0 * B0) + (A0 * C0) + (B0 * C0)$$

$$2T + 3T$$

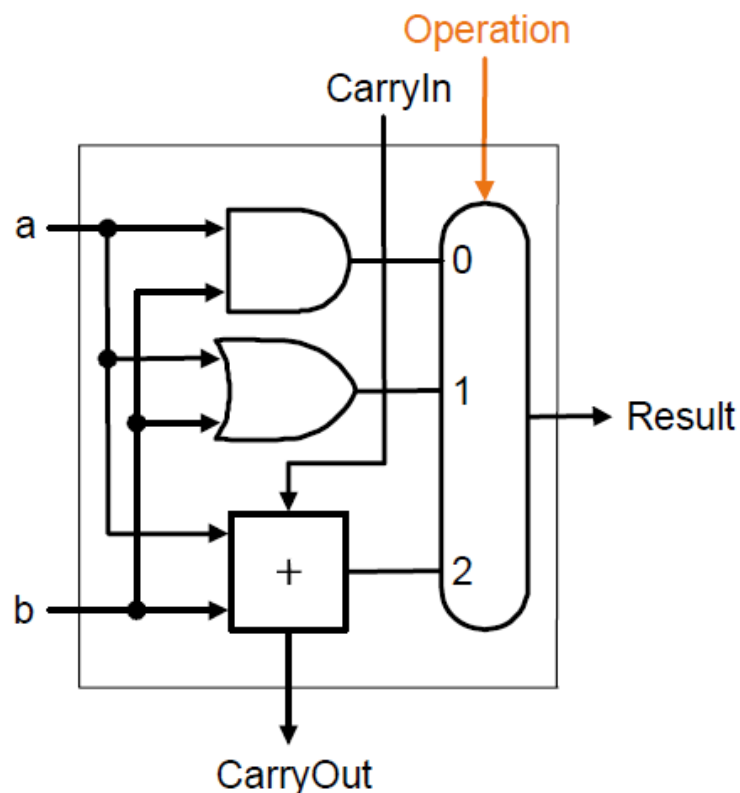
Carry Lookahead Adder:

$$G0 = A0 * B0$$

$$P0 = A0 \wedge B0$$

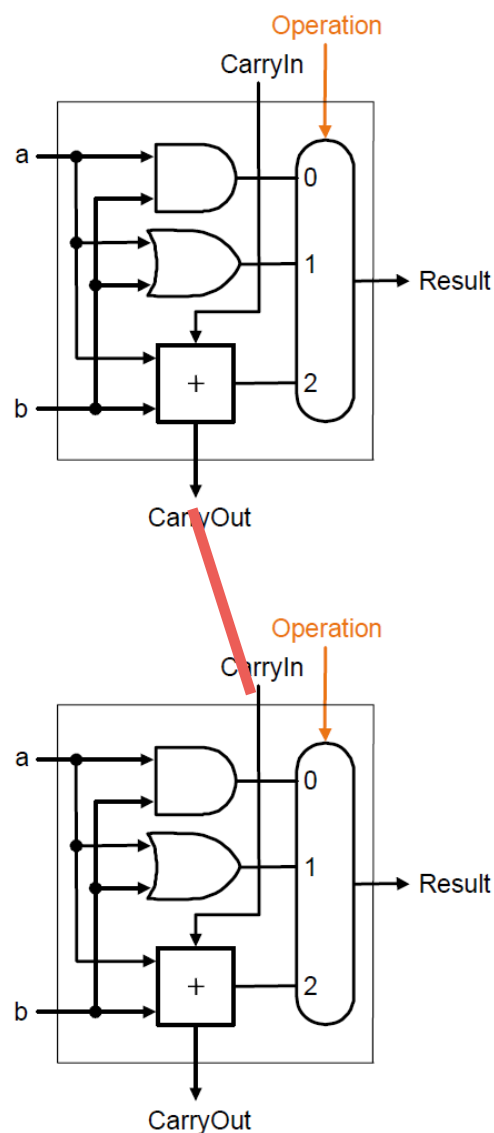
$$C1 = G0 + C0 * P0$$

$$2T + 2T + 2T$$



# Review

- Style 2: Carry Lookahead Adder (1-bit ALU -> 32-bit ALU)**



Ripple Carry Adder:

$$C1 = (A0 * B0) + (A0 * C0) + (B0 * C0)$$

$$C2 = (A1 * B1) + (A1 * C1) + (B1 * C1)$$

$$2T + 3T + 2T + 3T$$

Carry Lookahead Adder:

$$G0 = A0 * B0$$

$$P0 = A0 \wedge B0$$

$$C1 = G0 + C0 * P0$$

$$G1 = A1 * B1$$

$$P1 = A1 \wedge B1$$

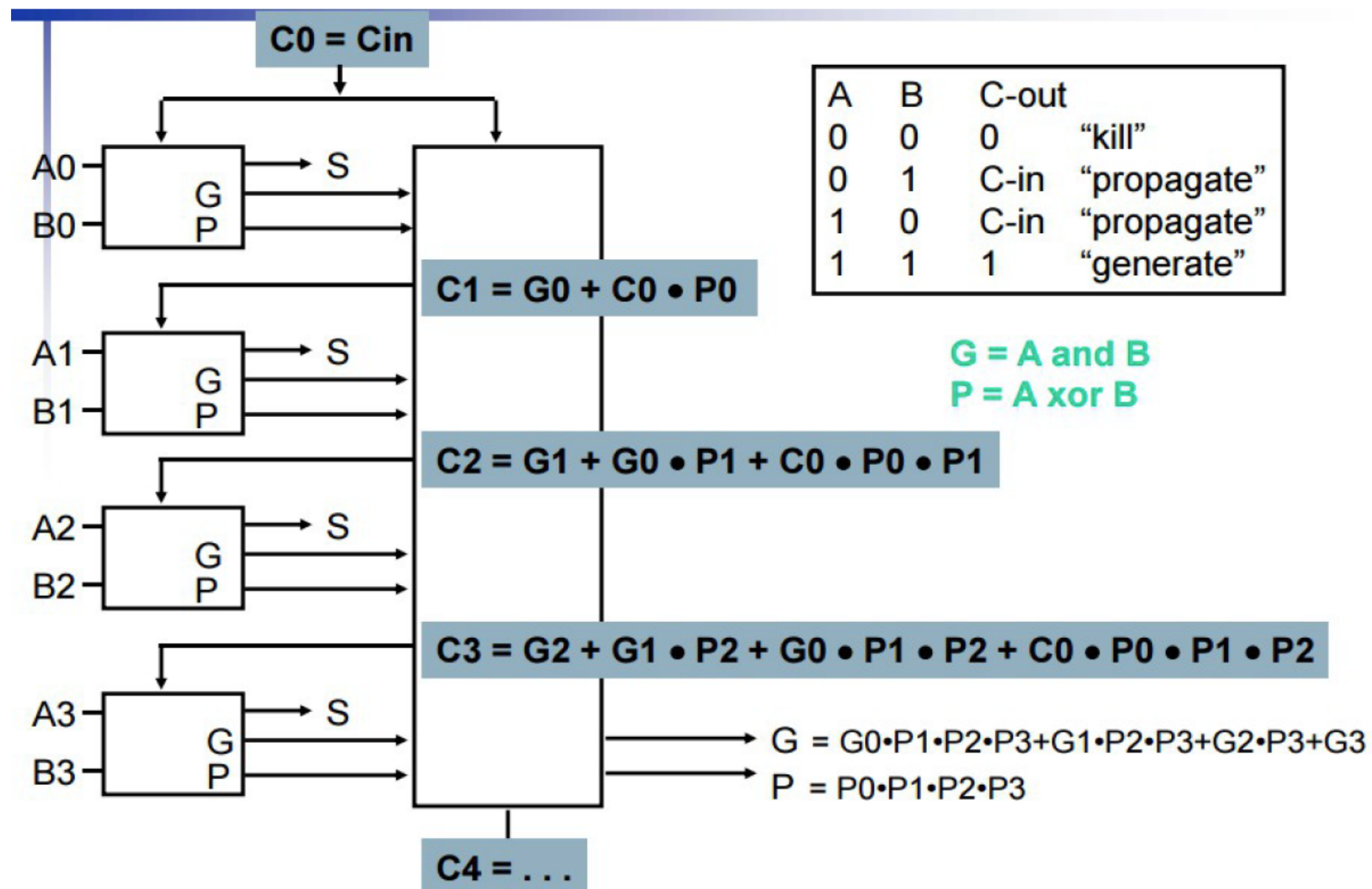
$$\begin{aligned} C1 &= G1 + C1 * P1 = G1 + (G0 + C0 * P0) * P1 \\ &= G1 + G0 * P1 + C0 * P0 * P1 \end{aligned}$$

$$2T + 3T + 3T$$



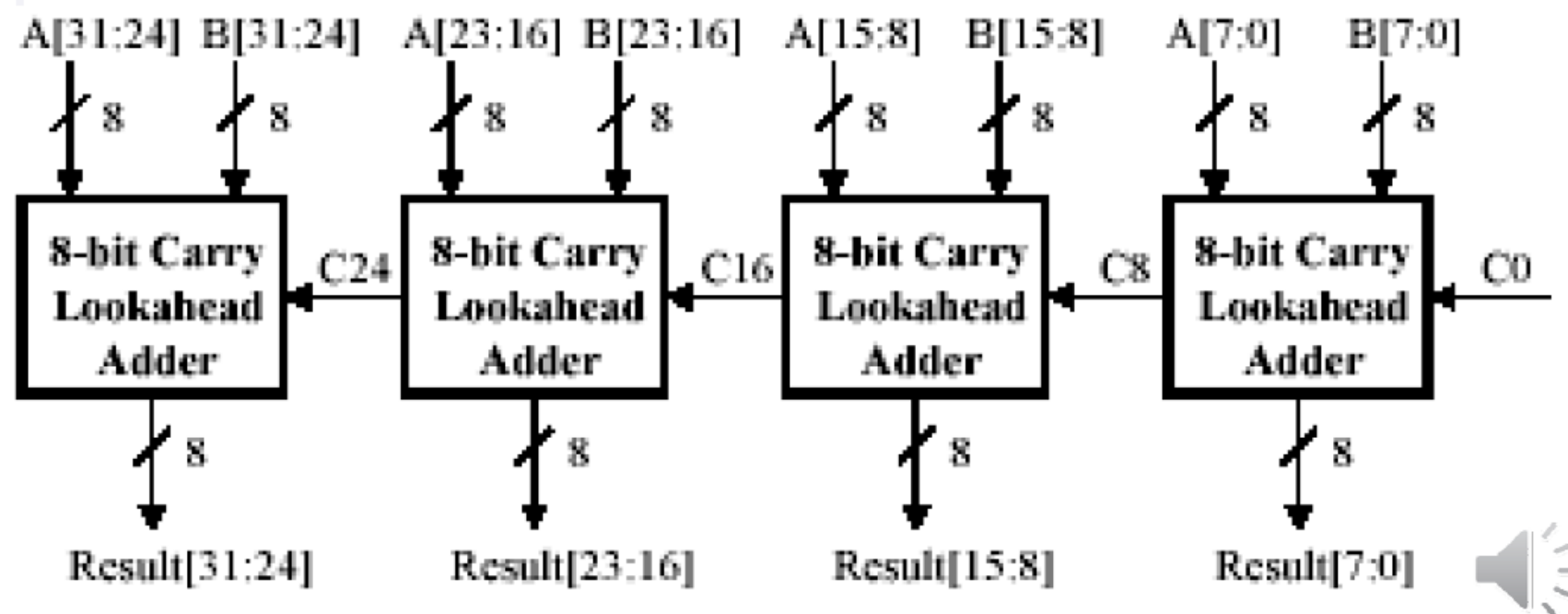
# Review

- **Style 2: Carry Lookahead Adder (1-bit ALU -> 32-bit ALU)**



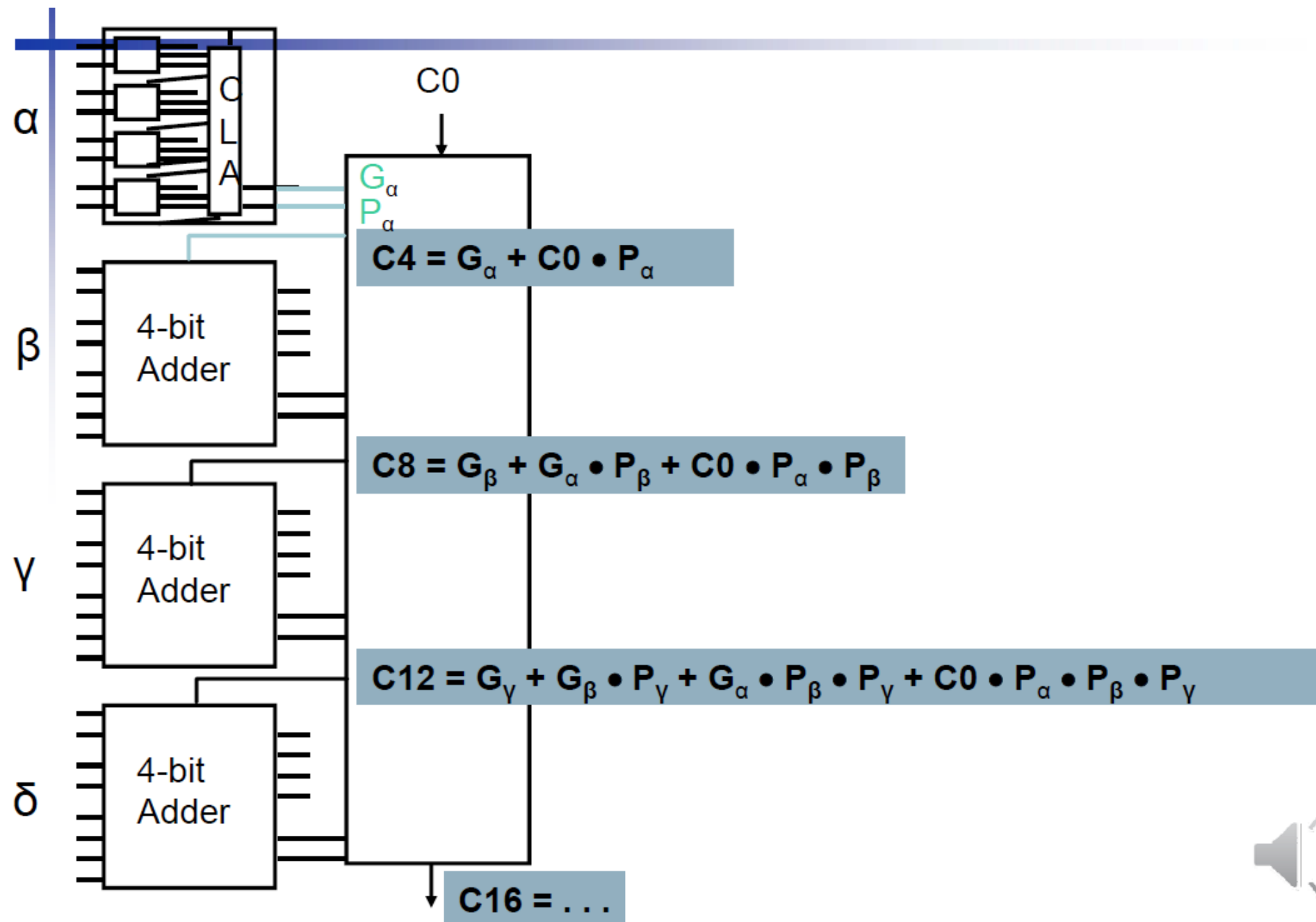
# Review

- **Style 2.1: [Partial] Carry Lookahead Adder**
  - Connect several N-bit Lookahead Adders together
  - Four 8-bit carry lookahead adders can form a 32-bit partial carry lookahead adder



# Review

- Style 2.2: [Hierarchical] Carry Lookahead Adder**



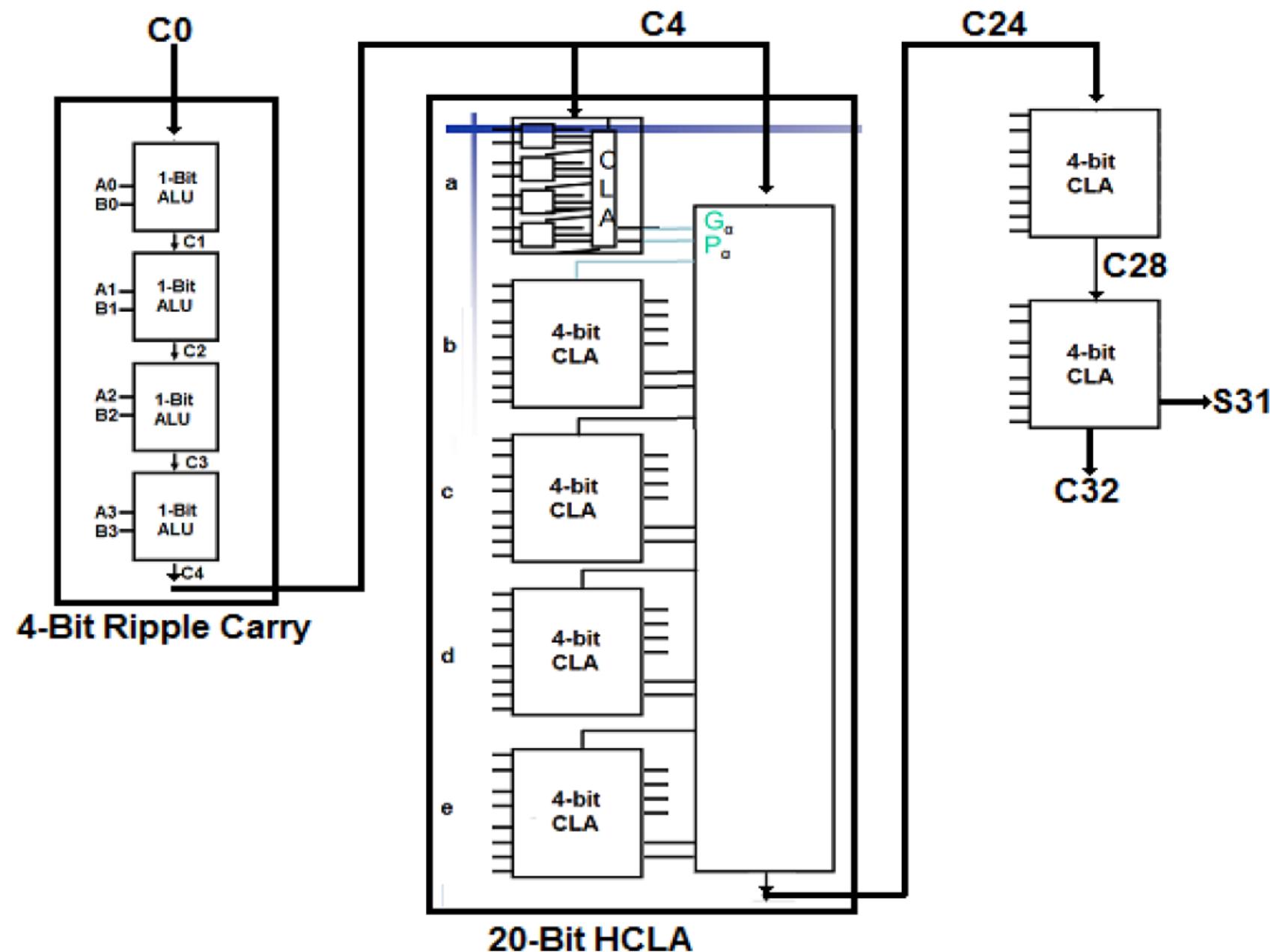
# Sample Question 1

Consider the 32-bit adder as shown.

Assume the following.

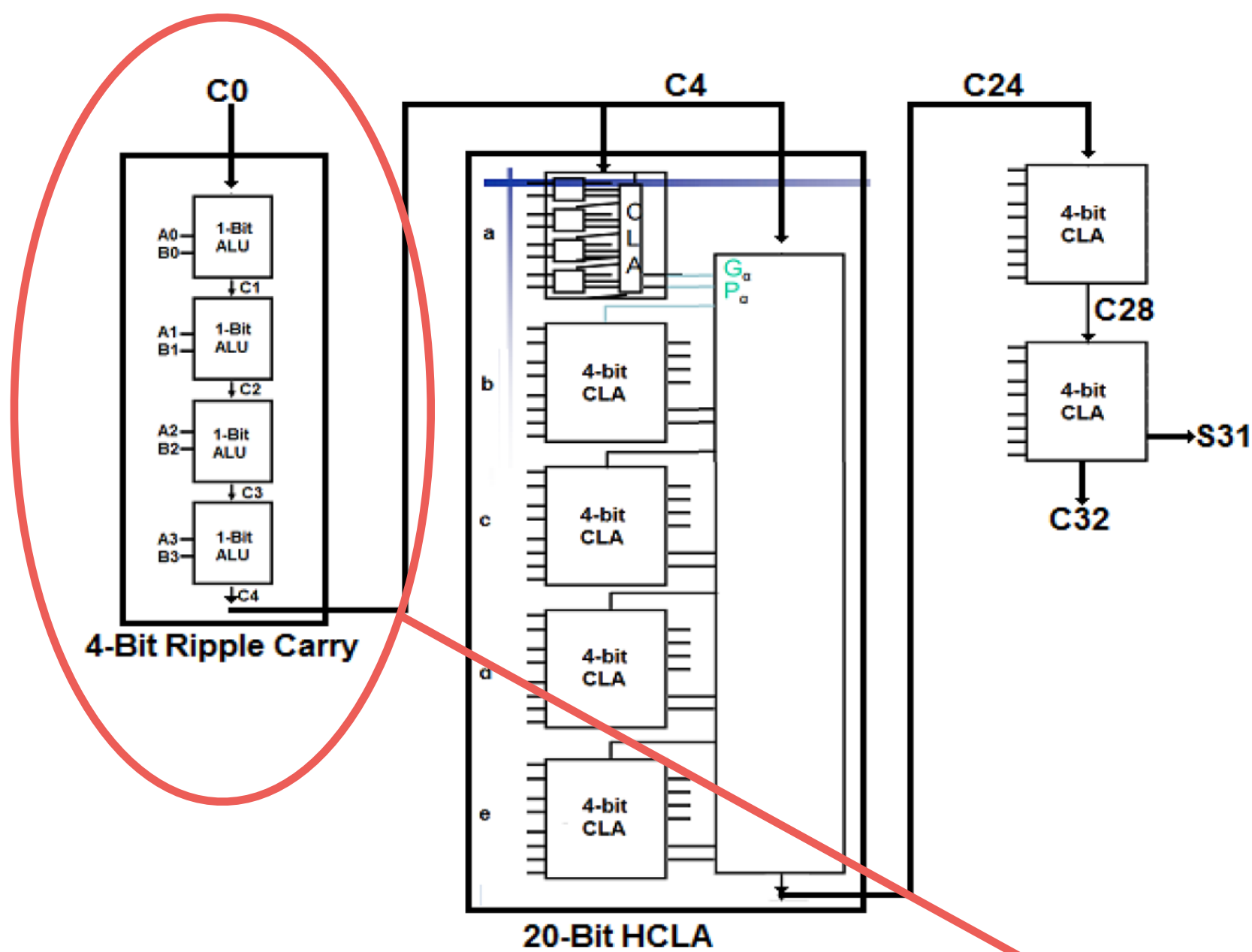
- Ripple Carry  $C_{n+1} = A_n * B_n + A_n * C_n + B_n * C_n$
- $S_n = C_n \wedge (A_n \wedge B_n)$
- delay time sheet:

Fan-In	Delay
1	1T
2	2T
3	3T
4	5T
5	8T
6	13T

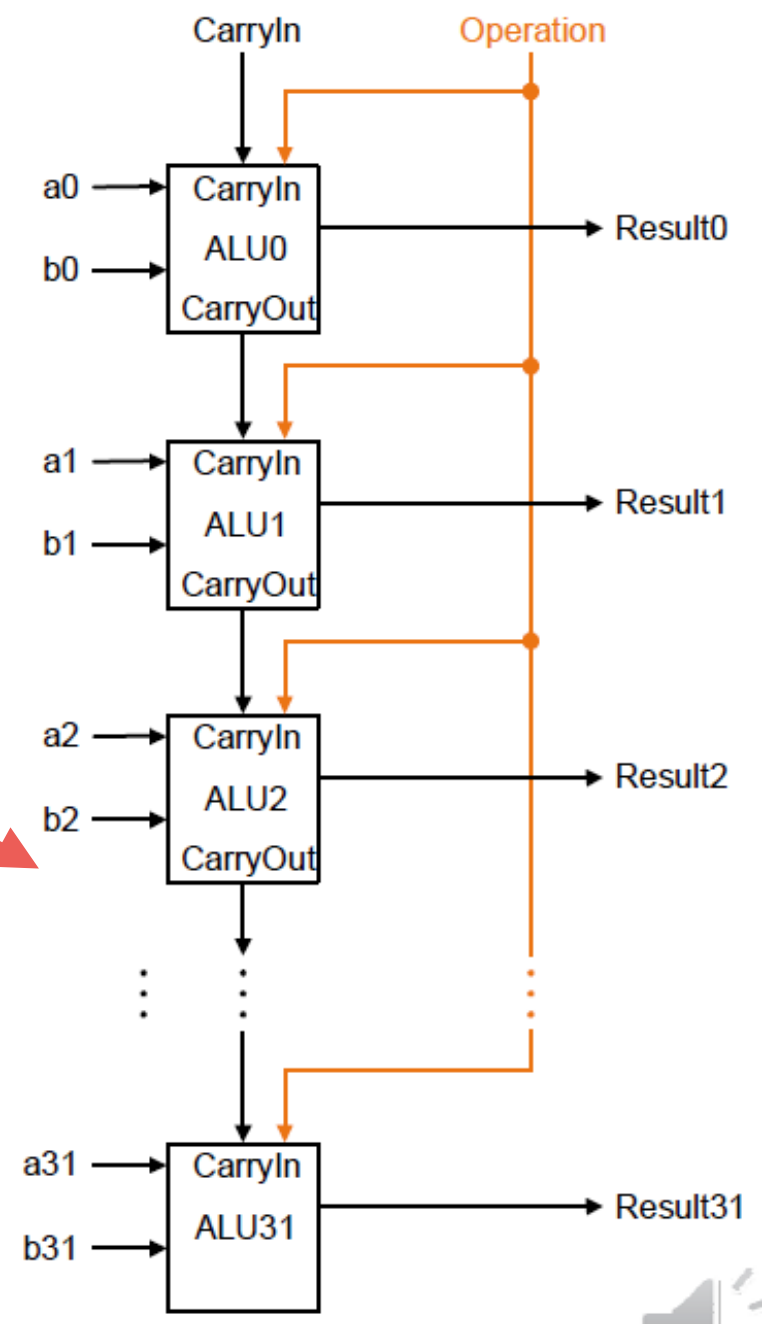


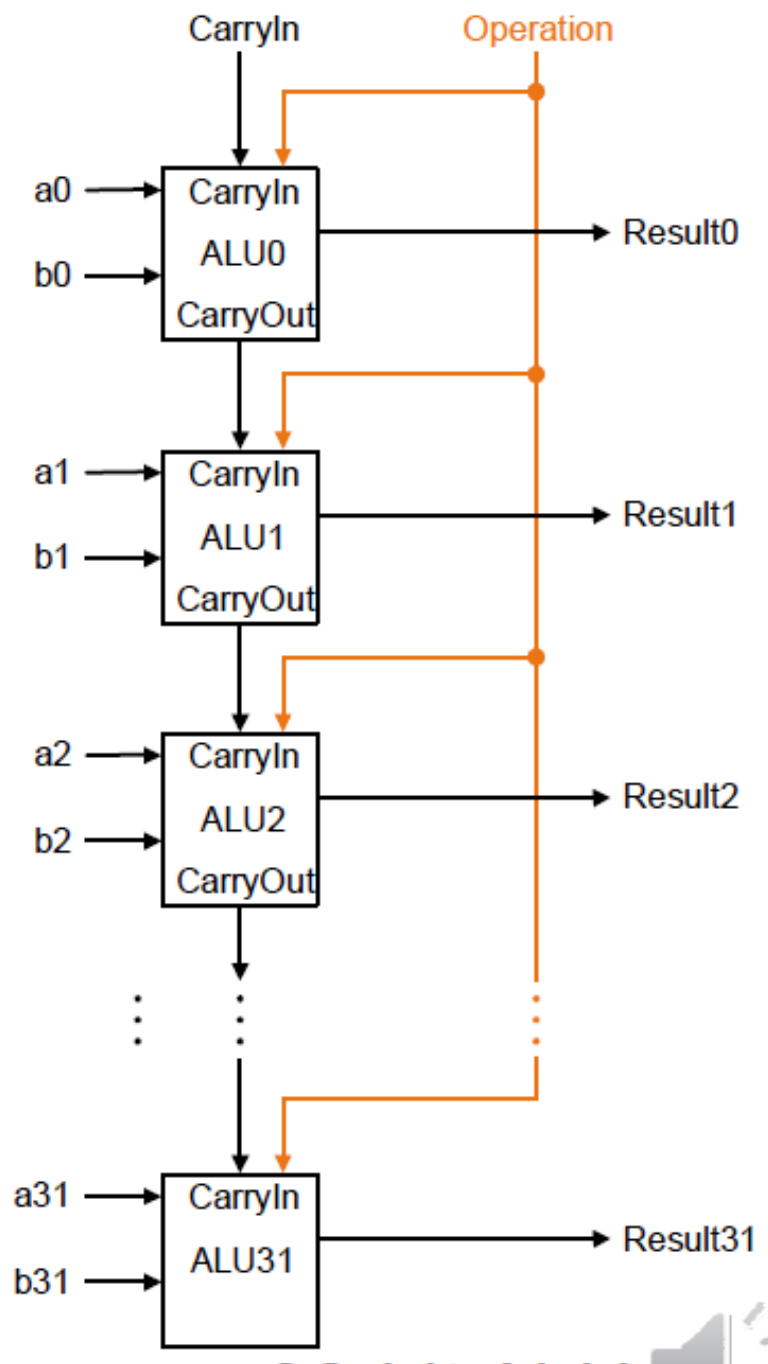
- **Steps:**

- 1. Find the critical path;
- 2. Understand the components of the ALU;
- 3. Calculate time delay for different parts in order.
  - When calculating time delay for different part, decompose it as a simple ALU as we have discussed before.



**Part 1:**  
Ripple type of ALU  
composed of 1-bit adder





$$C_{n+1} = A_n * B_n + A_n * C_n + B_n * C_n$$

$$\text{Delay\_1} = 2T + 3T = 5T;$$

$$S_n = C_n \wedge (A_n \wedge B_n)$$

$$\text{Delay\_2} = 2T + 2T = 4T;$$

$$\text{Delay}(C_{in}) = 0;$$

$$\text{Delay}(C_1) = 5T$$

$$\text{Delay}(S_0) = 4T$$

$$\text{Delay}(C_2) = 5T + 5T = 10T$$

$$\text{Delay}(S_1) = 5T + 4T = 9T$$

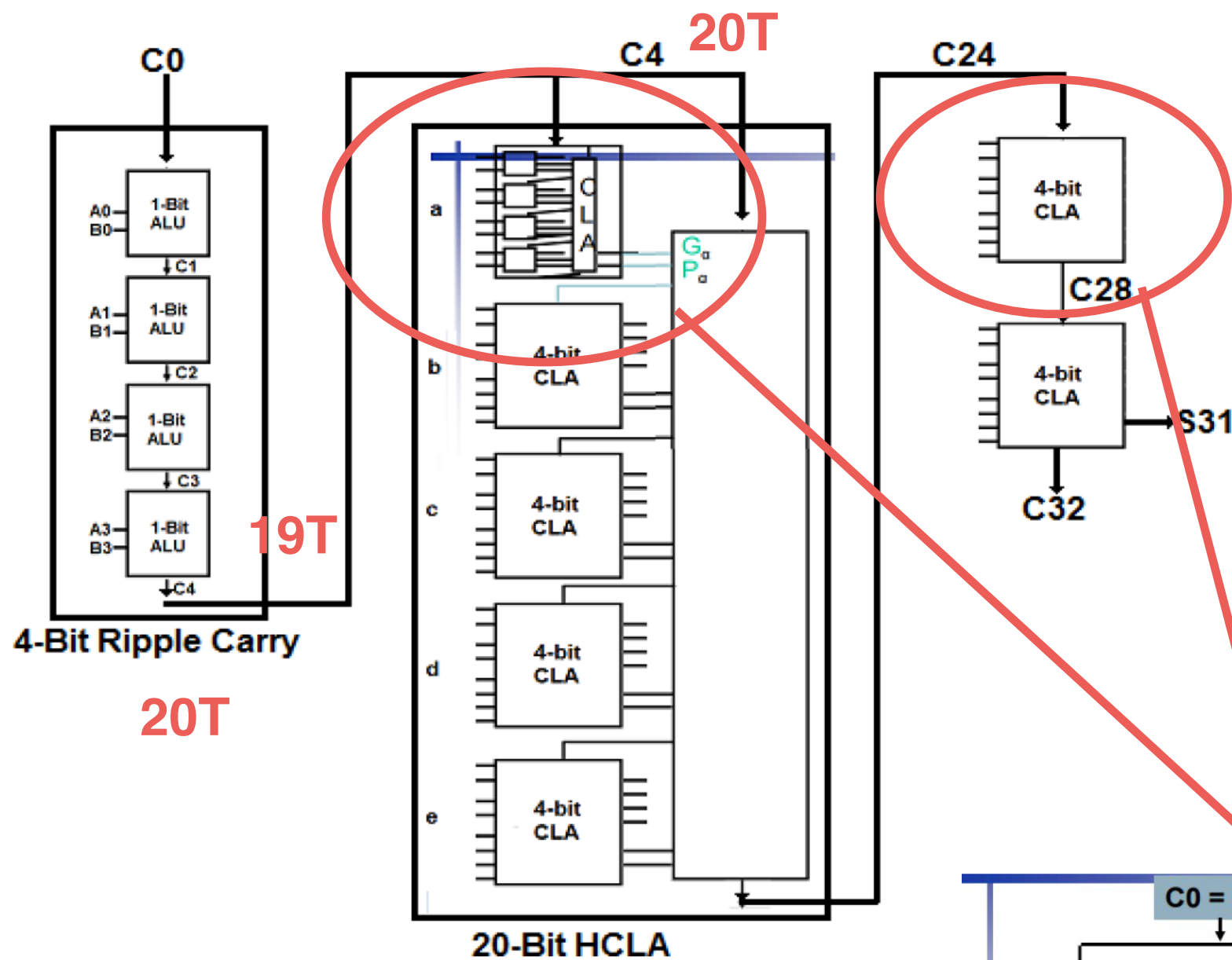
$$\text{Delay}(C_3) = 10T + 5T = 15T$$

$$\text{Delay}(S_2) = 10T + 4T = 14T$$

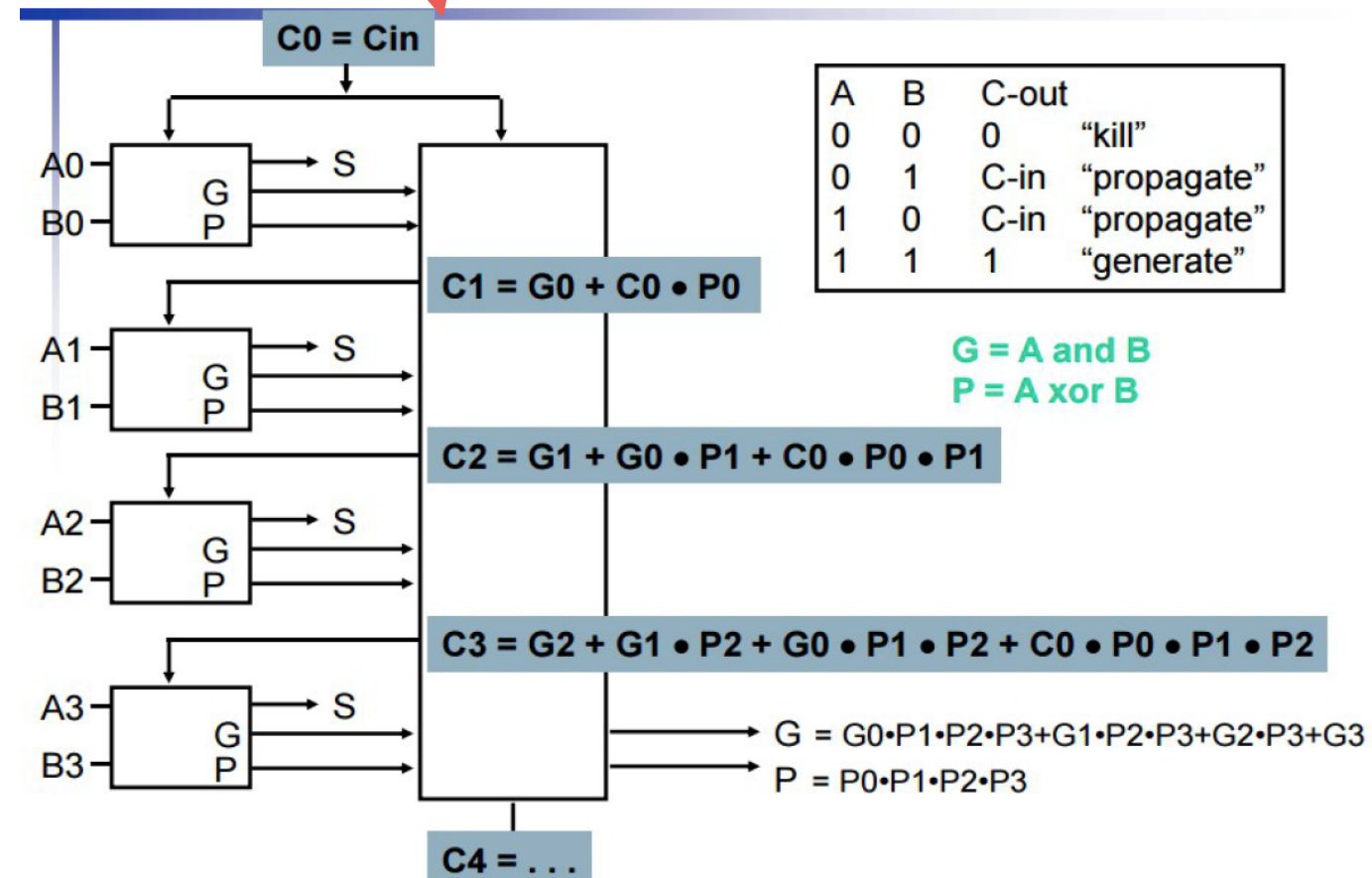
$$\text{Delay}(C_4) = 15T + 5T = 20T$$

$$\text{Delay}(S_3) = 15T + 4T = 19T$$

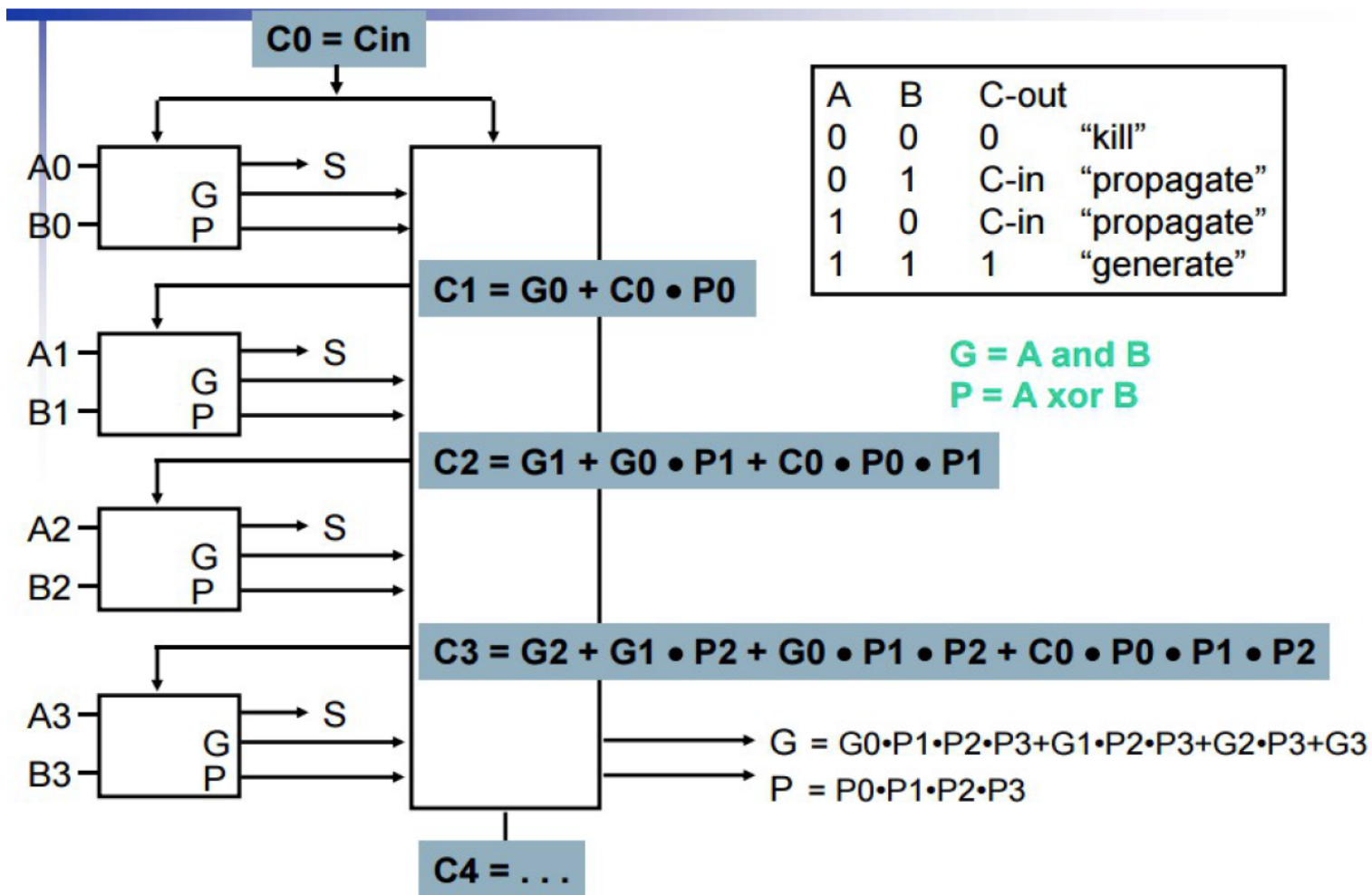
Fan-In	Delay
1	1T
2	2T
3	3T
4	5T
5	8T
6	13T



**Part 2:**  
**Carry Lookahead Adder**  
 composed of 1-bit adder







Fan-In	Delay
1	1T
2	2T
3	3T
4	5T
5	8T
6	13T

$$C0 = 0$$

$$G0 = A0 * B0$$

$$P0 = A0 \wedge B0$$

$$C1 = G0 + C0 * P0$$

$$S0 = C0 \wedge (A0 \wedge B0)$$

$$\text{Delay}(C1) = 2T + 2T = 4T$$

$$\text{Delay}(S0) = 2T + 2T = 4T$$

$$G1 = A1 * B1$$

$$P1 = A1 \wedge B1$$

$$C2 = G1 + C1 * P1$$

$$= G1 + G0 * P1$$

$$+ C0 * P0 * P1$$

$$S1 = C1 \wedge (A1 \wedge B1)$$

$$= G0 \wedge (A1 \wedge B1) +$$

$$C0 * P0 \wedge (A1 \wedge B1)$$

$$\text{Delay}(C2) = 8T$$

$$\text{Delay}(S1) = 6T$$

$$G2 = A2 * B2$$

$$P2 = A2 \wedge B2$$

$$C3 = G2 + C2 * P2$$

$$S2 = C2 \wedge (A2 \wedge B2)$$

$$\text{Delay}(C3) = 12T$$

$$\text{Delay}(S2) = 10T$$

$$G = 12T$$

$$P = 7T$$

$$G3 = A3 * B3$$

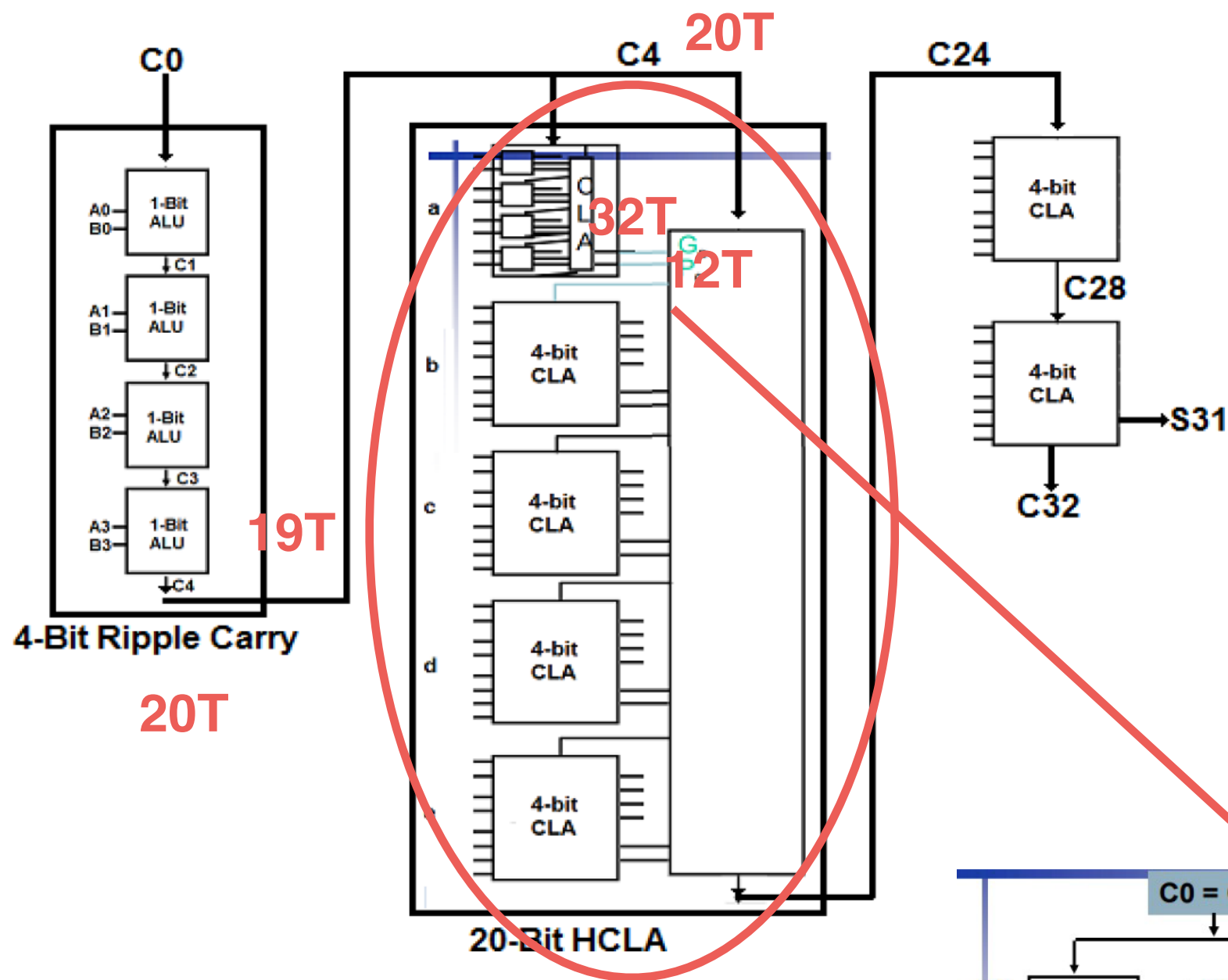
$$P3 = A3 \wedge B3$$

$$C4 = G3 + C3 * P3$$

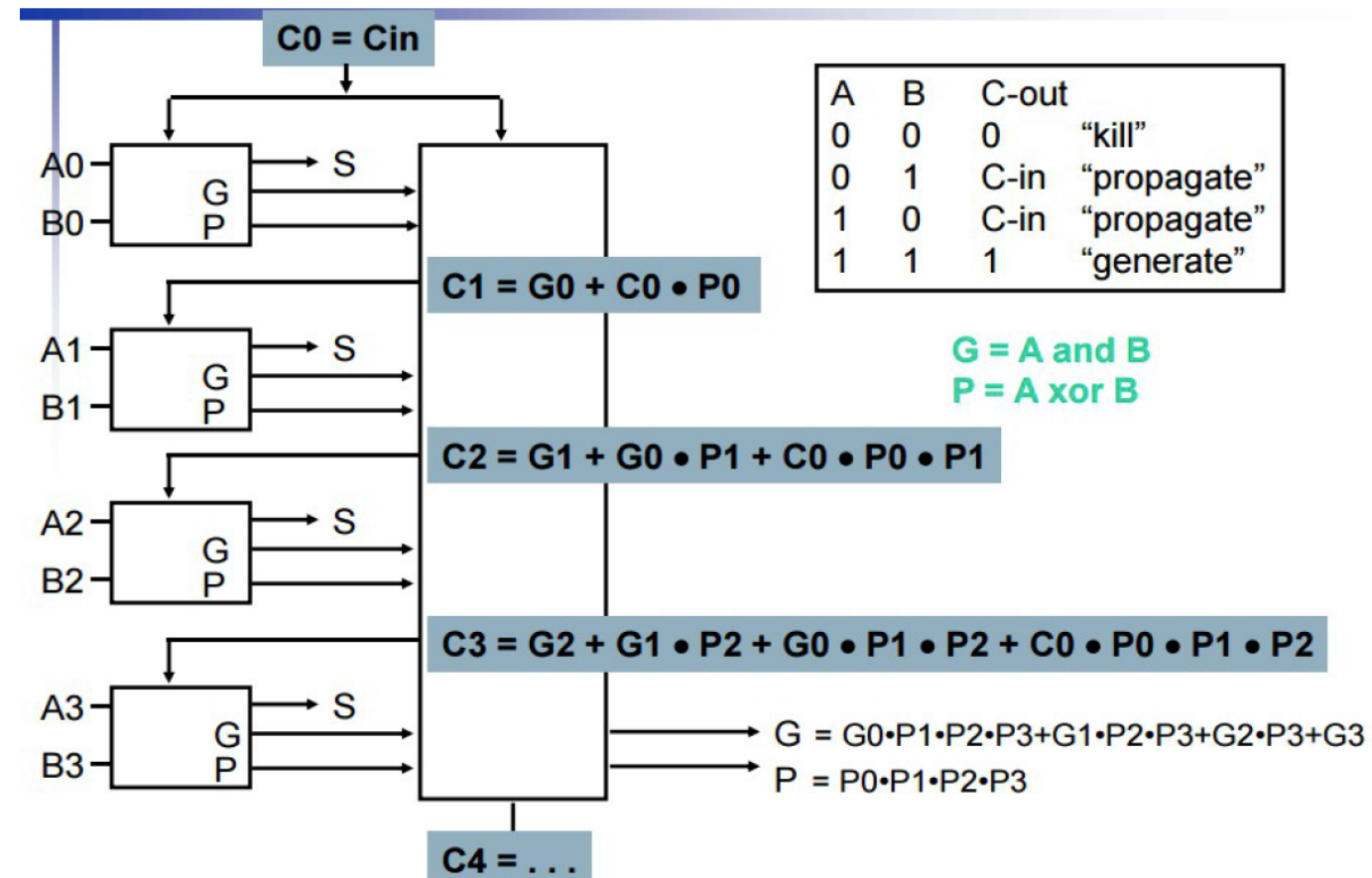
$$S3 = C3 \wedge (A3 \wedge B3)$$

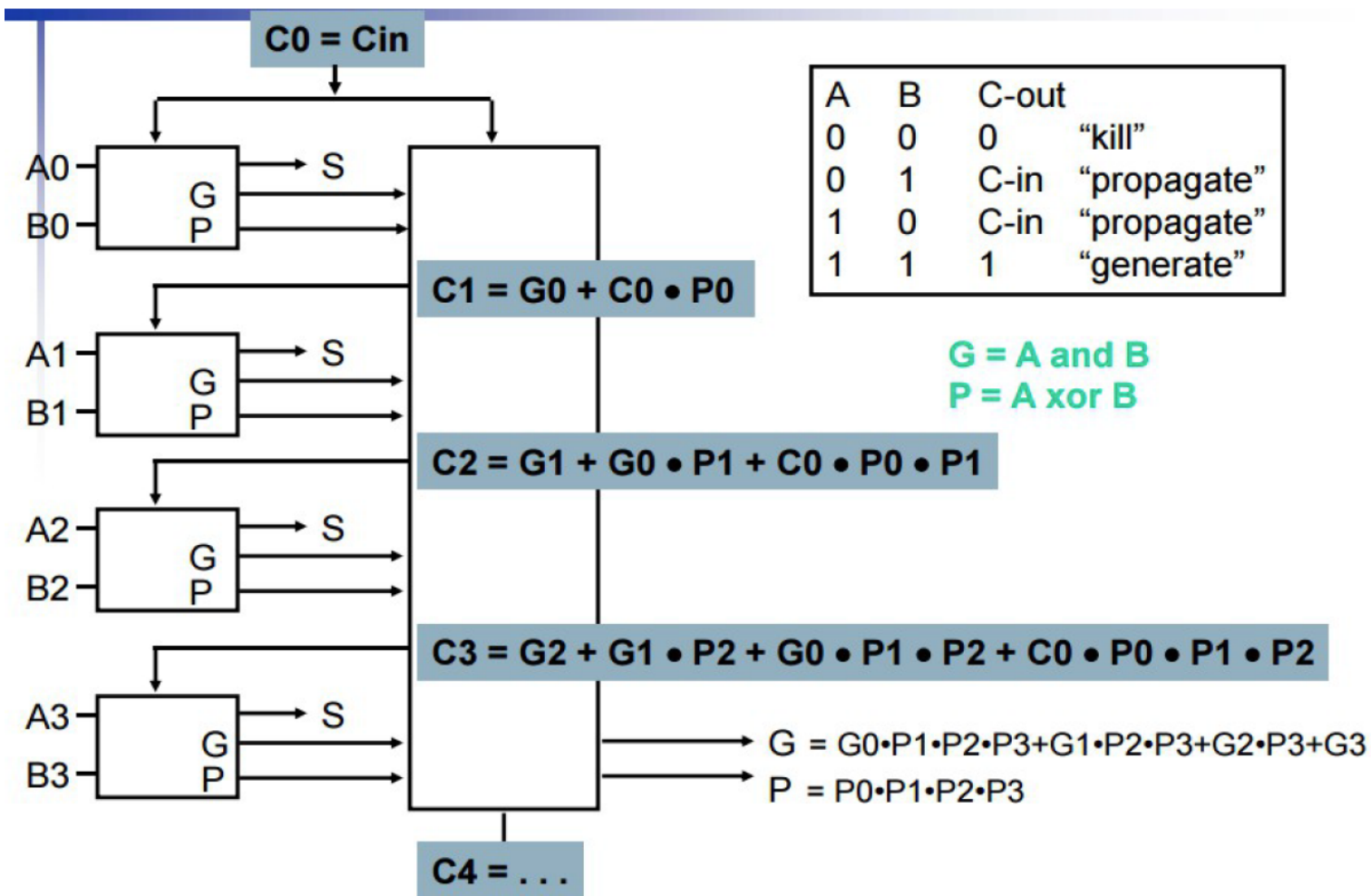
$$\text{Delay}(C4) = 16T$$

$$\text{Delay}(S3) = 12T$$



## Part 2: Carry Lookahead Adder composed of 1-bit adder

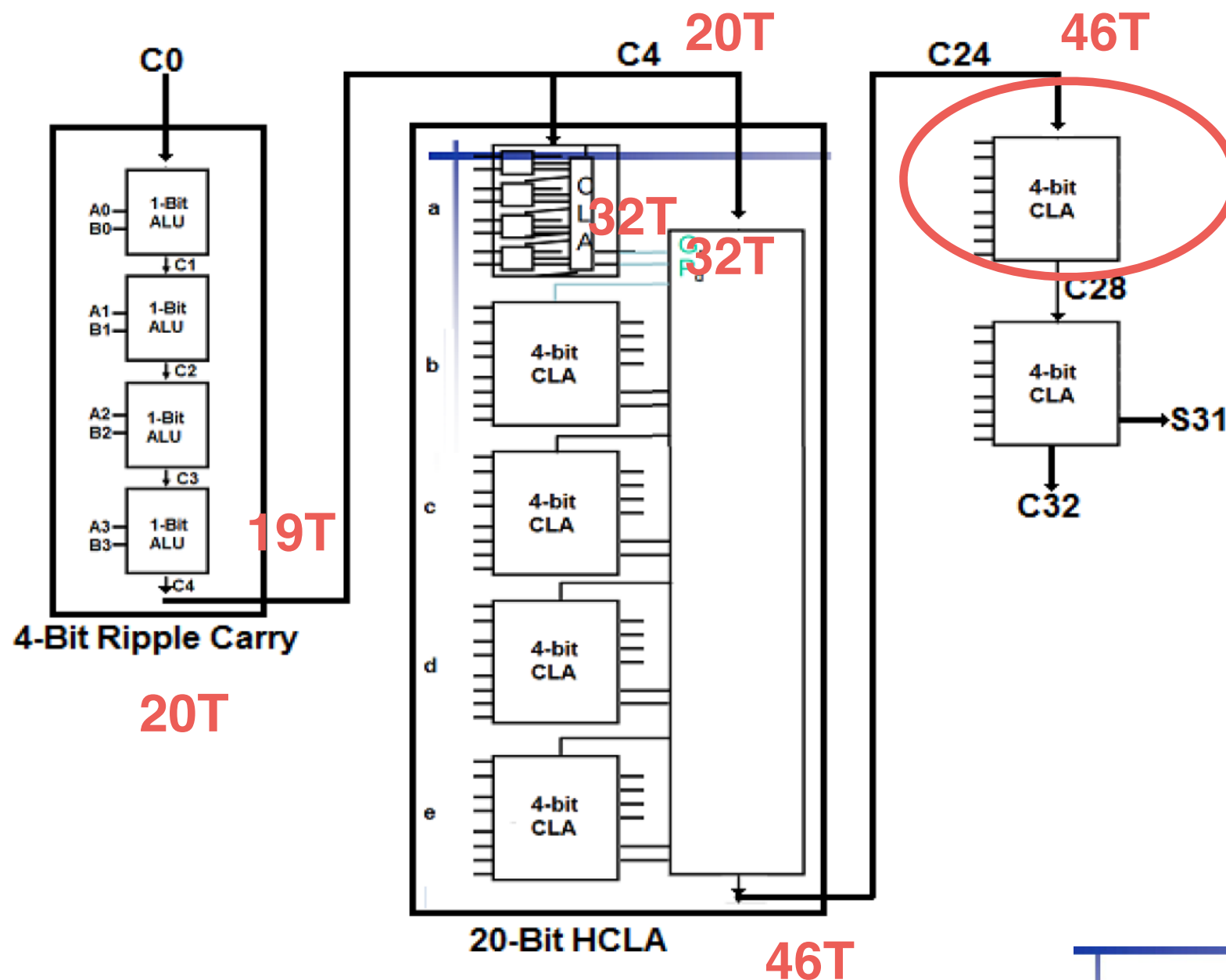




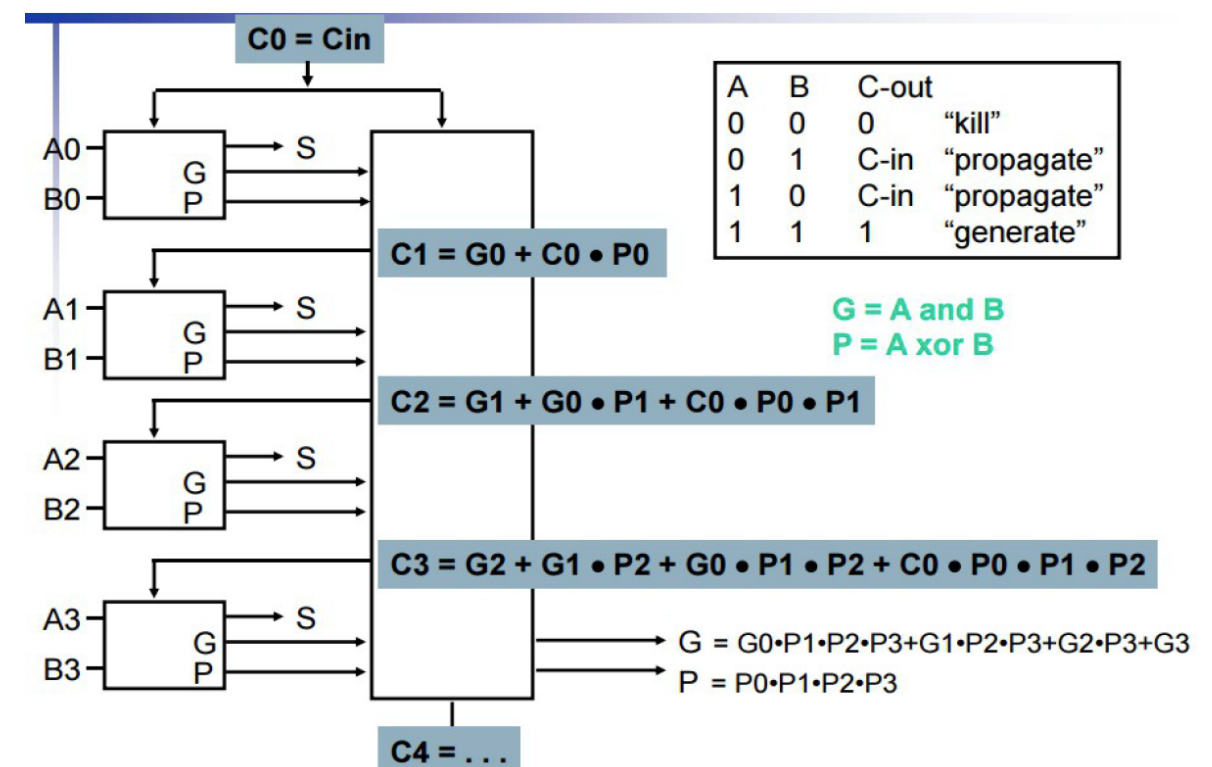
Fan-In	Delay
1	1T
2	2T
3	3T
4	5T
5	8T
6	13T

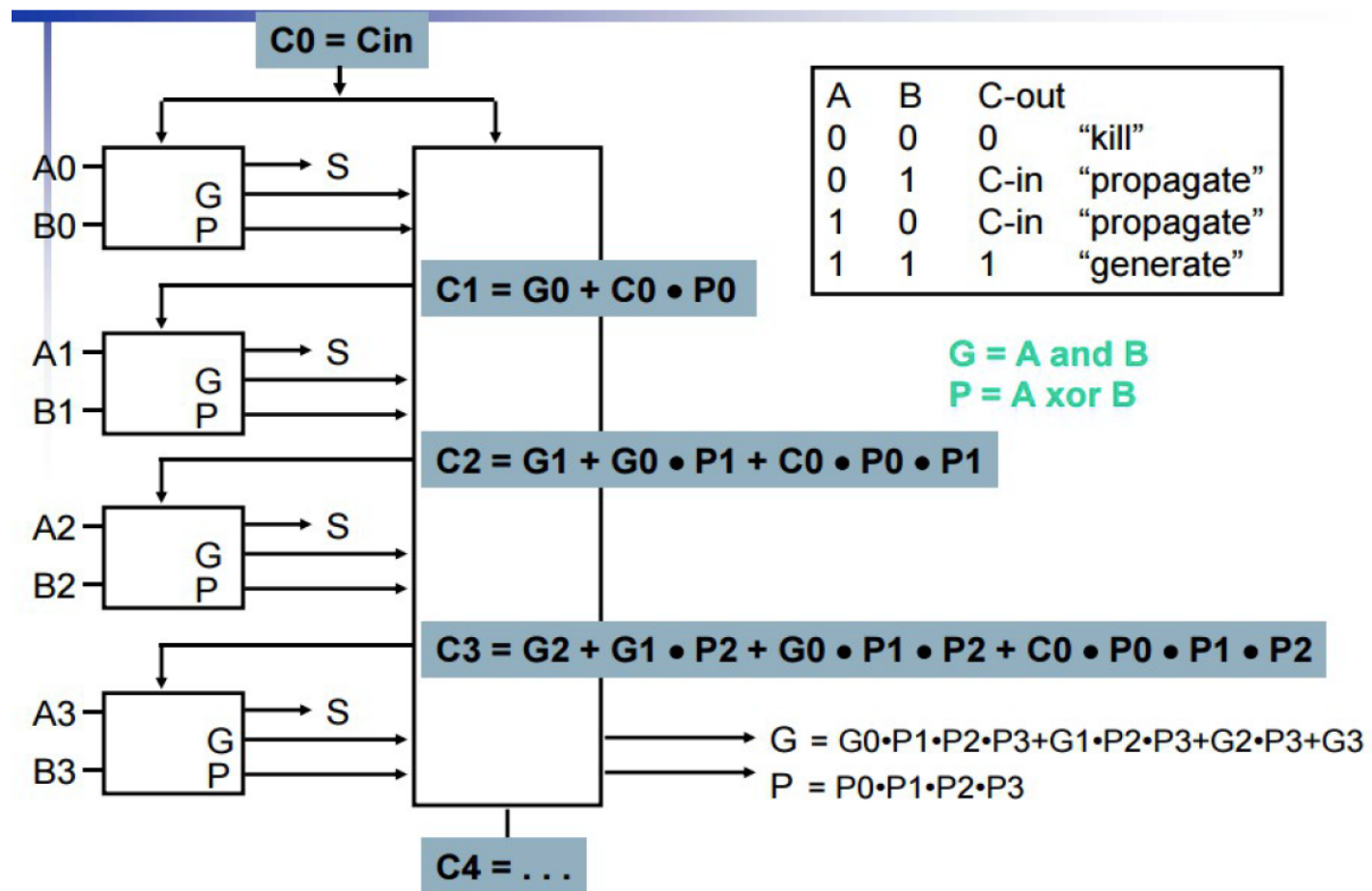
$$\begin{aligned}
 C_e = & G_e + G_d \cdot P_e + \\
 & G_c \cdot P_d \cdot P_e + \\
 & G_b \cdot P_c \cdot P_d \cdot P_e + \\
 & G_a \cdot P_b \cdot P_c \cdot P_d \cdot P_e + \\
 & C_4 \cdot P_a \cdot P_b \cdot P_c \cdot P_d \cdot P_e
 \end{aligned}$$

$$\text{Delay}(C_e) = 26T$$



## Part 2: Carry Lookahead Adder composed of 1-bit adder



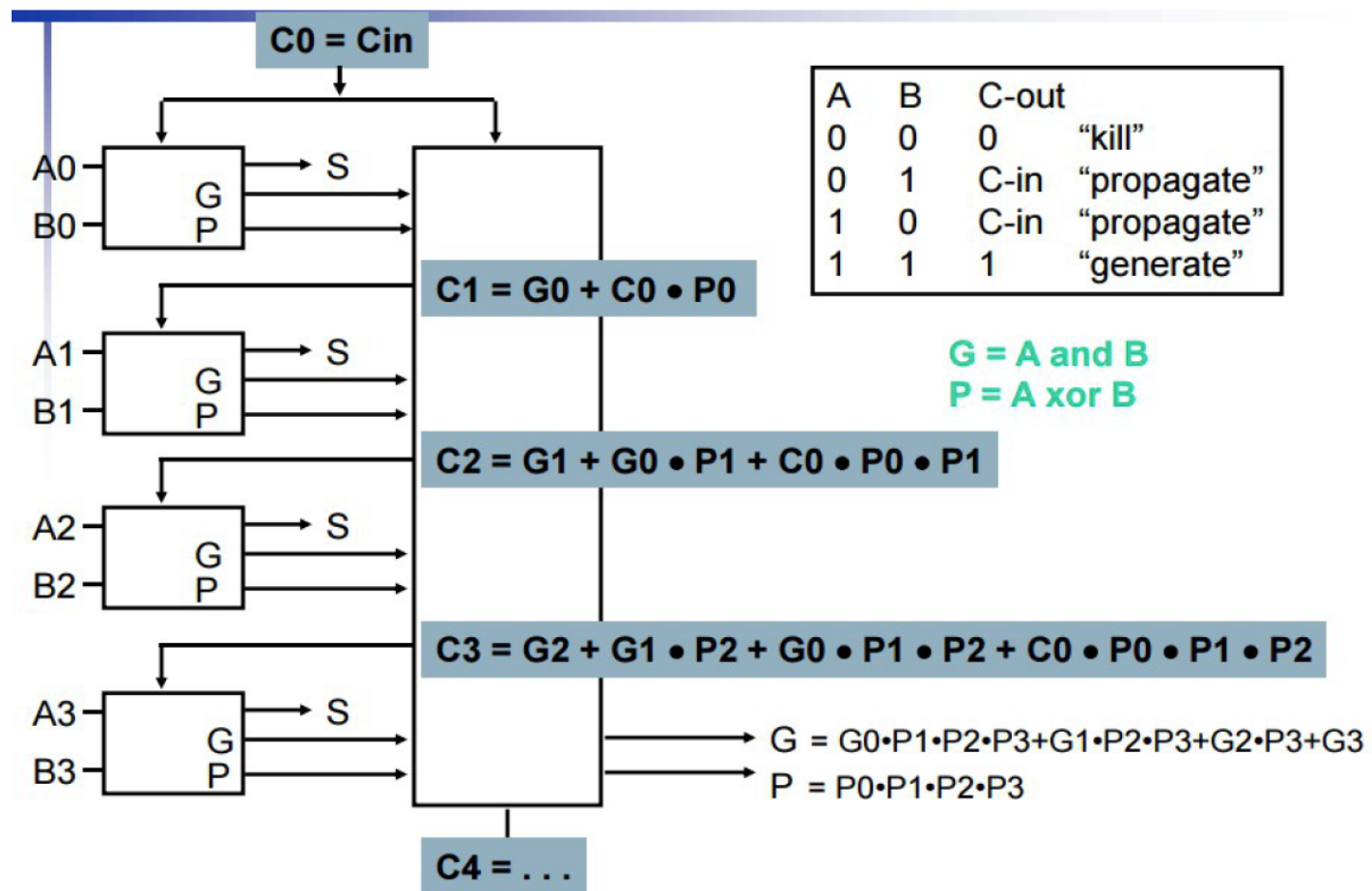


Fan-In	Delay
1	1T
2	2T
3	3T
4	5T
5	8T
6	13T

$$\text{Delay}(C4) = 16T$$



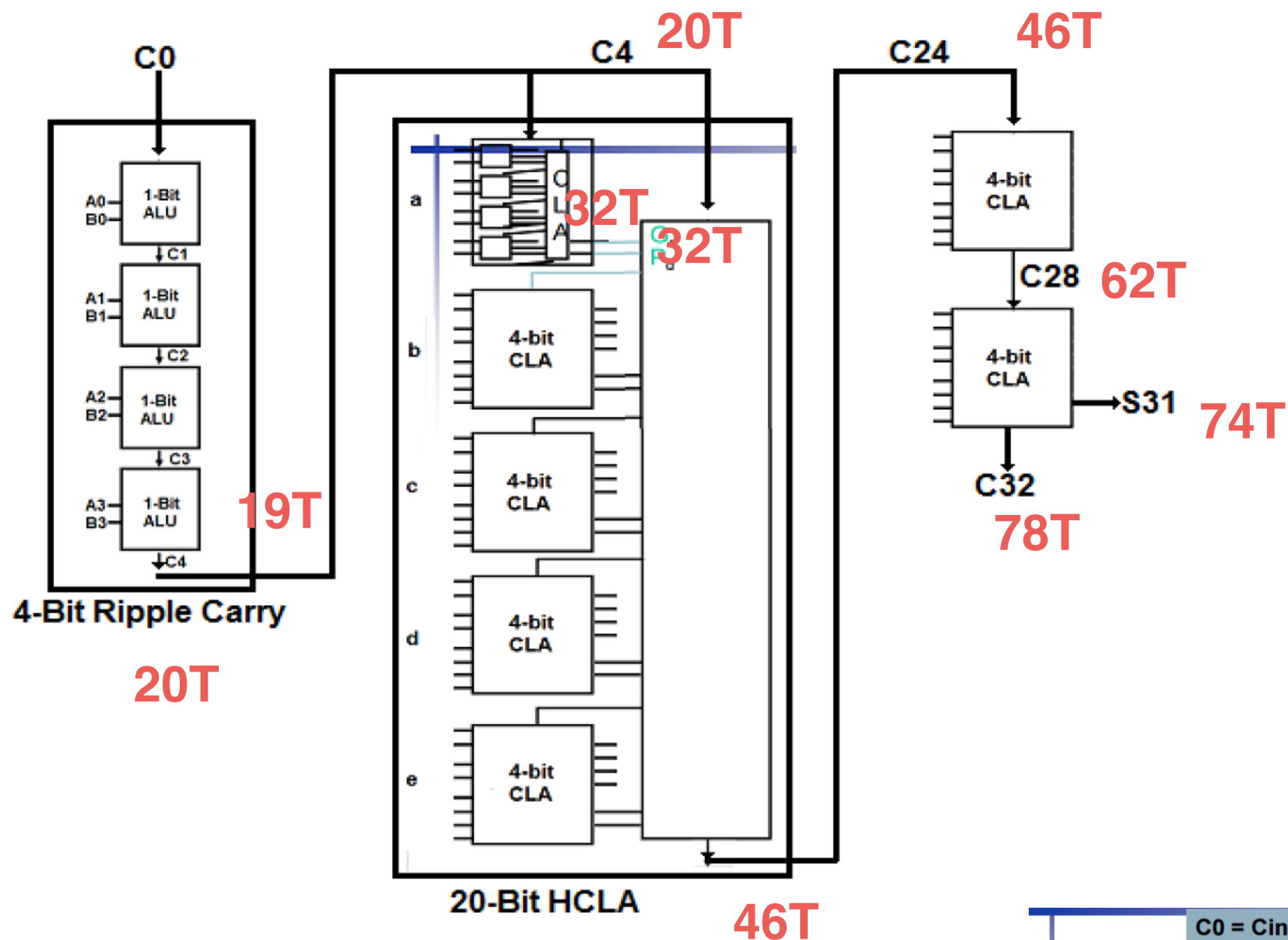




Fan-In	Delay
1	1T
2	2T
3	3T
4	5T
5	8T
6	13T

Delay(C4) = 16T

Delay(S3) = 13T



## Part 2: Carry Lookahead Adder composed of 1-bit adder

