

# Propositional Logic

CS161

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# What are Logic-Based Systems?

- Structure: Reasoning 'engine'



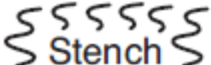



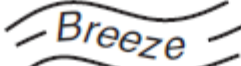
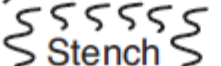
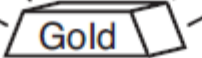


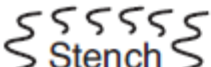





- Knowledge base: declarative sentences
- Dominant paradigm 1958-1988
  - AI proved a new theorem in lattice theory!
- Current research on unifying logic, prob., neural

# Wumpus World

- Grid world
- Pit causes breeze in adjacent cells
- Wumpus causes stench in adjacent cells
- Find the gold



# Wumpus World

 Stench		 Breeze	 PIT
	  Stench  Gold	 PIT	 Breeze
 Stench		 Breeze	
 START	 Breeze	 PIT	 Breeze

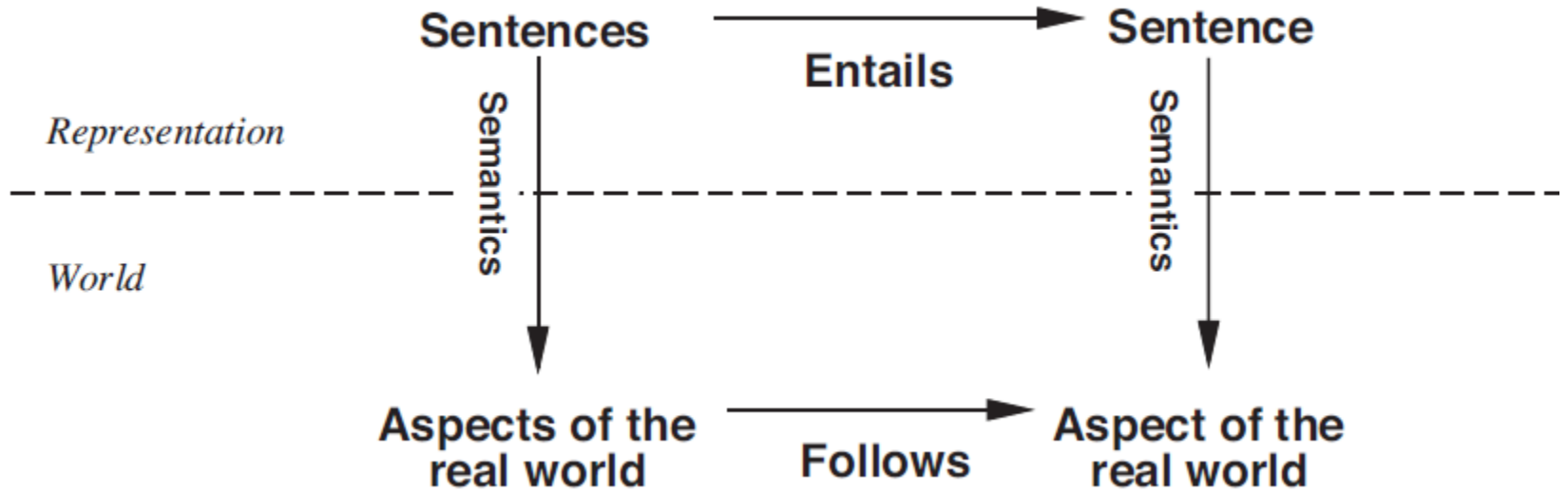
# Wumpus World: Discussion

- Overcome ignorance about the world by reasoning
- Conclusions guaranteed to be correct
- Inferred new knowledge
  - from observations
  - from lack of observations
  - at different point in time

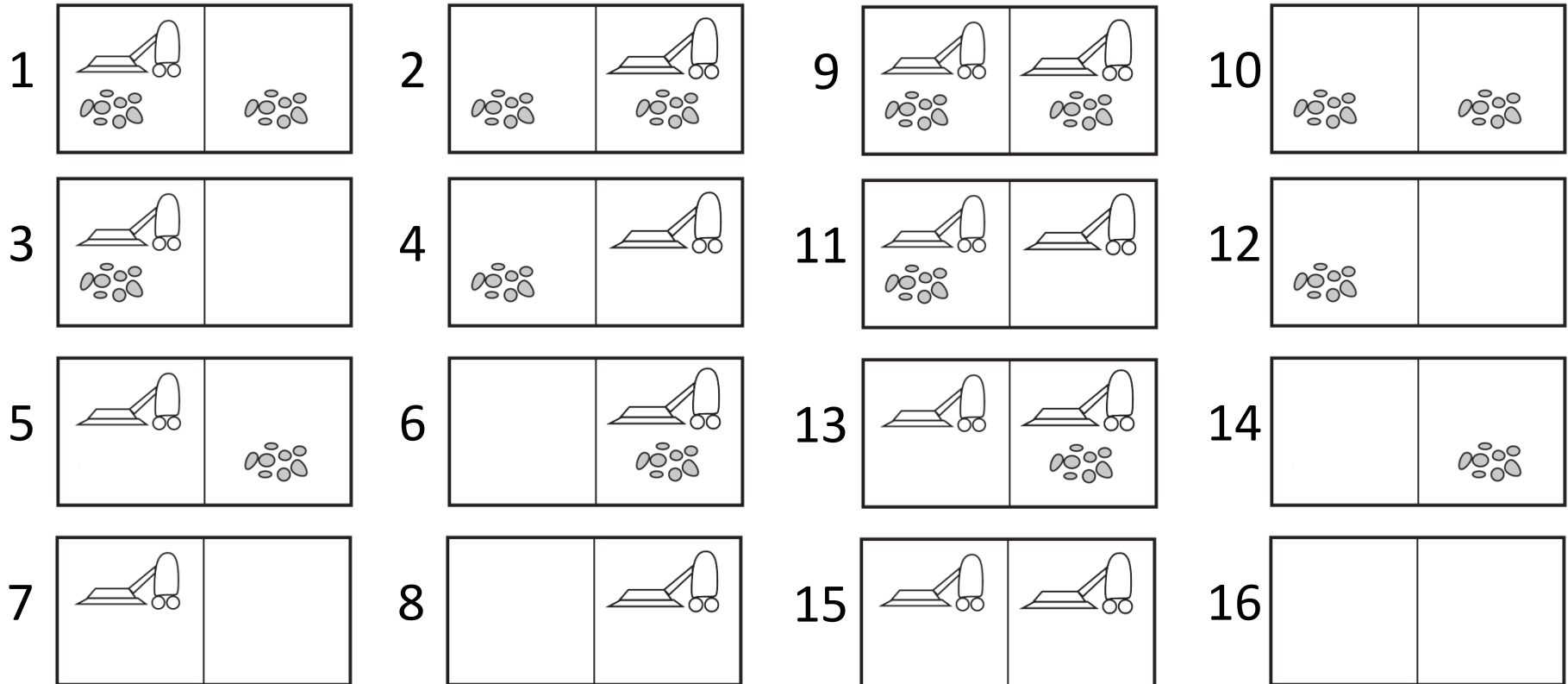
# Syntax: What am I allowed to write?

- $x+5=y$  or  $x5y+=?$
- Variables/propositions:  $X, Y, Z$ , Rain, Sun
- Grammar of sentences:
  - Variables are sentences (called atoms)
  - If  $\alpha$  is a sentence, so is  $\neg\alpha$
  - If  $\alpha$  and  $\beta$  are sentences, so are  
 $(\alpha \vee \beta), \alpha \wedge \beta, \alpha \Rightarrow \beta, \alpha \Leftrightarrow \beta, \dots$  (not all required)
- Negated atoms are literals

# Semantics



# Worlds or Truth Assignments





# Sentences and Worlds

- Syntax:  
LR, RR, LD, RD
- Exactly one robot: sentences  $\alpha$
- Relationship between sentence  $\alpha$  and world  $\omega$ 
  - $\alpha$  is true at  $\omega$



$$\omega \models \alpha$$

$\alpha$  holds in  $\omega$

$\omega$  satisfies  $\alpha$

–  $\alpha$  is false at  $\omega$

$$\omega \not\models \alpha$$

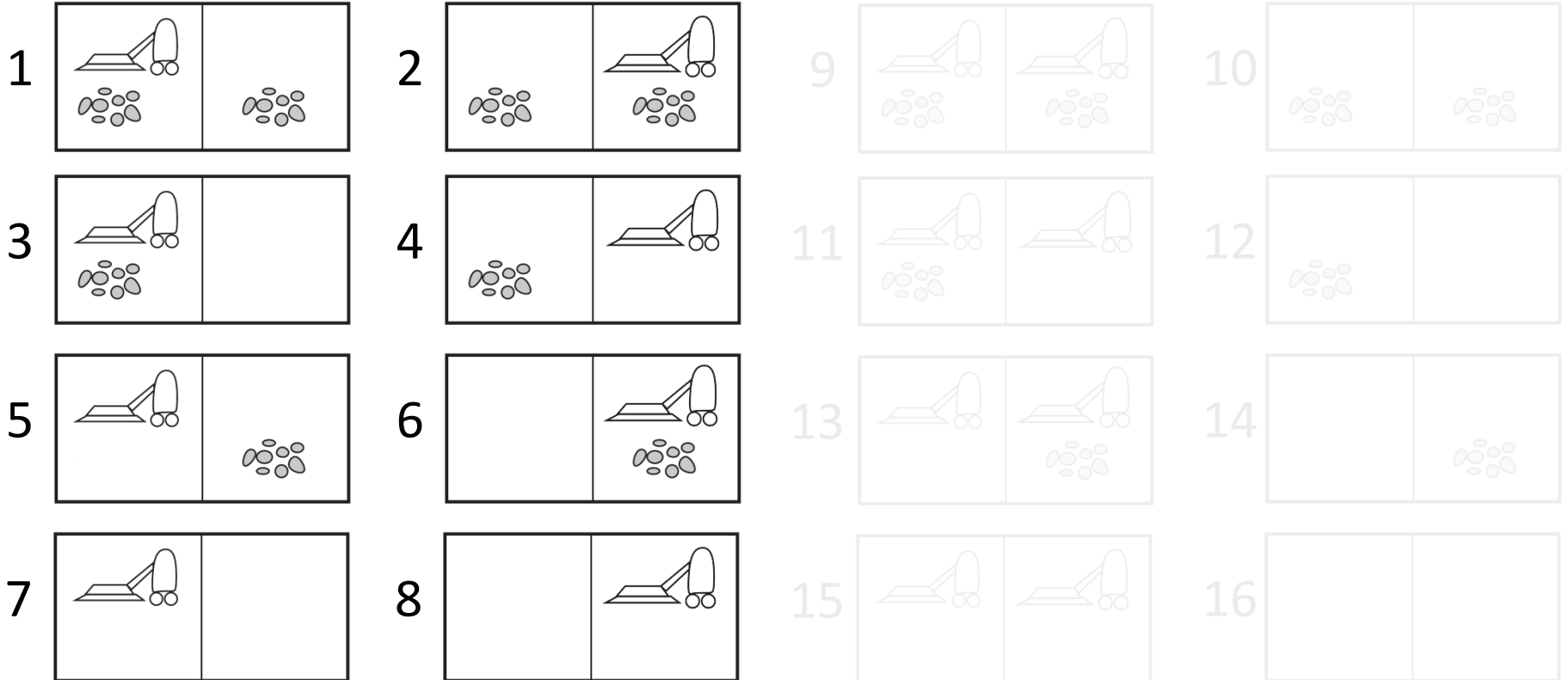
# Semantics

- Knowledge/meaning is a set of worlds  $\omega$
- Meaning of  $\alpha$  is that the “set of worlds  $\omega$  where  $\alpha$  is true” are the only ones possible:

$$M(\alpha) = \{\omega: \omega \models \alpha\}$$

- Example:
  - Initially all 16 worlds

# Meaning/Semantics



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  - Initially all 16 worlds
  - Sentence  $\alpha$  removes 10,12,14,16 and 9,11,13,15
  - Sentence: Dirt and robot cannot co-exist



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- Example:
  - Initially all 16 worlds
  - Sentence  $\alpha$  removes 10,12,14,16 and 9,11,13,15
  - Sentence: Dirt and robot cannot co-exist
  - Sentence: Robot in left cell



# Meaning/Semantics



# Formal Semantics + Algorithm

$$\omega \models \alpha?$$





# Syntax Conversion Rules



# Wumpus World Example

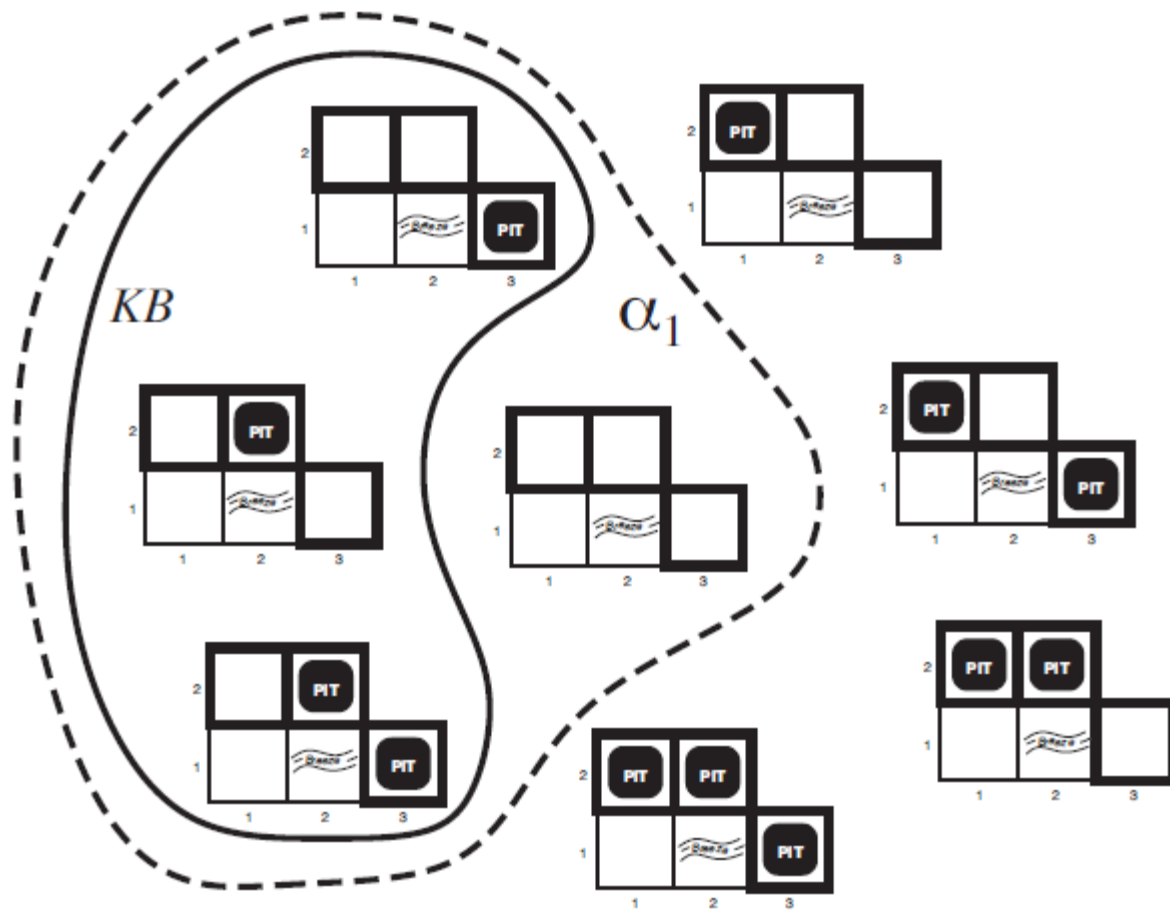


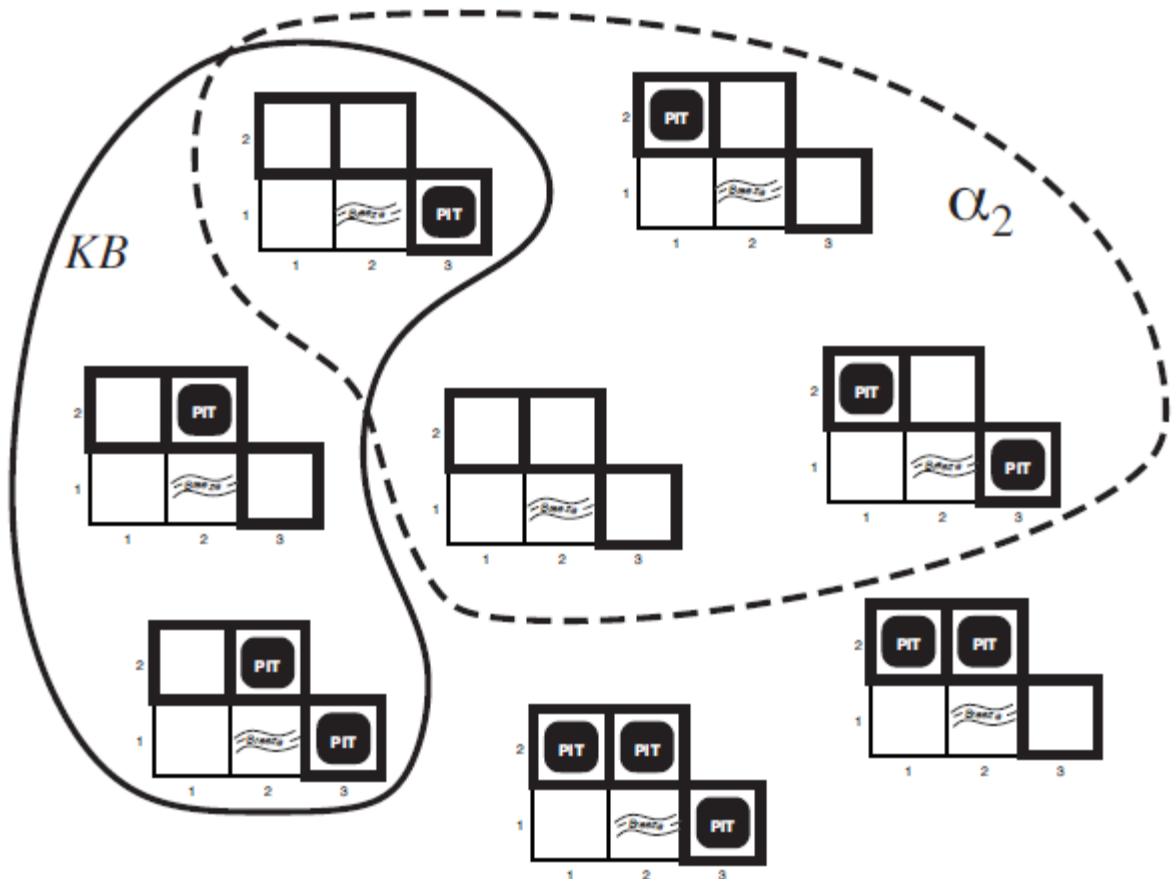
# Relationships Between Sentences



# Entailment

1		2		9		10	
3		4		11		12	
5		6		13		14	
7		8		15		16	





# Propositional Reasoning

