Constraint Satisfaction

CS161

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Constraint Satisfaction Problems

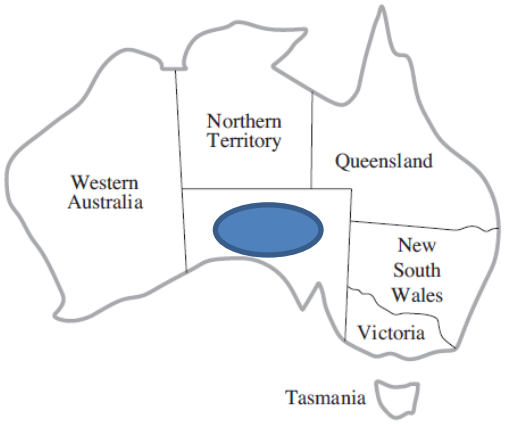
- What? n-queens, cryptoarithmetic, map coloring, SAT, etc.
- Factored problem formulation

  Look inside state structure (no longer black box)
- Representation (formal language)
  - Variables X
  - Domains D
  - Constraints (=goal test)
- More powerful general-purpose solvers

Example: Graph Coloring

What is the benefit of a factorized representation?

- Example: Coloring Australia
  - Suppose we know that SA=blue
  - Still 3^6 possible states left
  - No neighboring state can be blue
  - Really only 2^5*3 states left!



- Check inconsistency early
- Be smarter about which actions to try

Constraint Satisfaction Problems

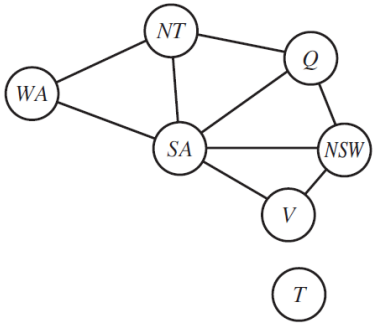
- Constraint types
  - Unary
  - Binary
  - Higher-order
  - Global constraints
  - Soft constraints
- Binary CSPs
- Constraint Graphs

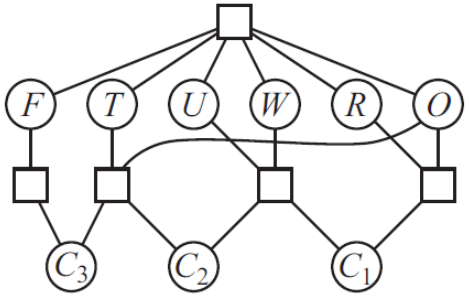
Examples

- Cryptarithmetic
- SAT solving

Constraint graphs

- Binary constraints:
  Binary constraint graphs
- Higher-order constraints:
  Constraint hypergraphs/bipartite graphs





Real-World Examples

- Assignment problems (who teaches this class)
- Timetabling problems (when and where)
- Hardware configuration
- Transportation scheduling
- Factory scheduling
- Floorplanning
- Etc.

CSP Search, the Naïve Way

- Initial state: empty assignment {}
- Actions: assign to a variable X a value v
  - Any choice of unassigned X
  - Any choice of v
  - Only allowed if no constraint becomes violated
- Successor function: add {X=v} to state
- Goal test:
  - all variables get assignment
  - all constraints are satisfied

CSP Search, the Naïve Way

- Search tree
- Which search algorithm to use?
  - Why not IDS? DLS? BFS?
  - DFS is perfect for CSP: optimal and complete
- Complexity?

  - Number of variables N=|X|
  - Number of values V=|D|
  - Size of tree: (NV)*((N-1)*V)*((N-2)*V)*… = N!V^N
  - Note: only V^N distinct states! ☹

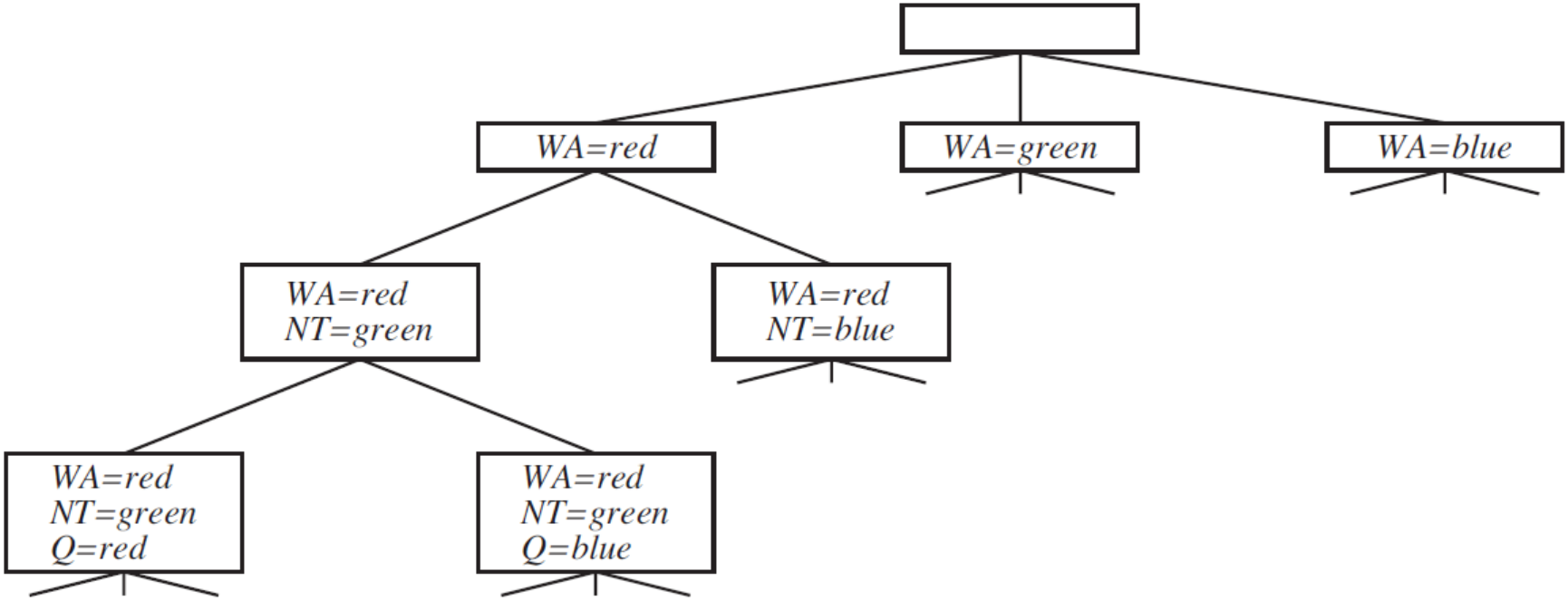
Can we do better?

- Variable assignments are commutative

- Per node, only need consider assignment to **one** variable
- Branching factor V
- Complexity: V^N

- This is called “backtracking search”.

Backtracking Search



Improvements?

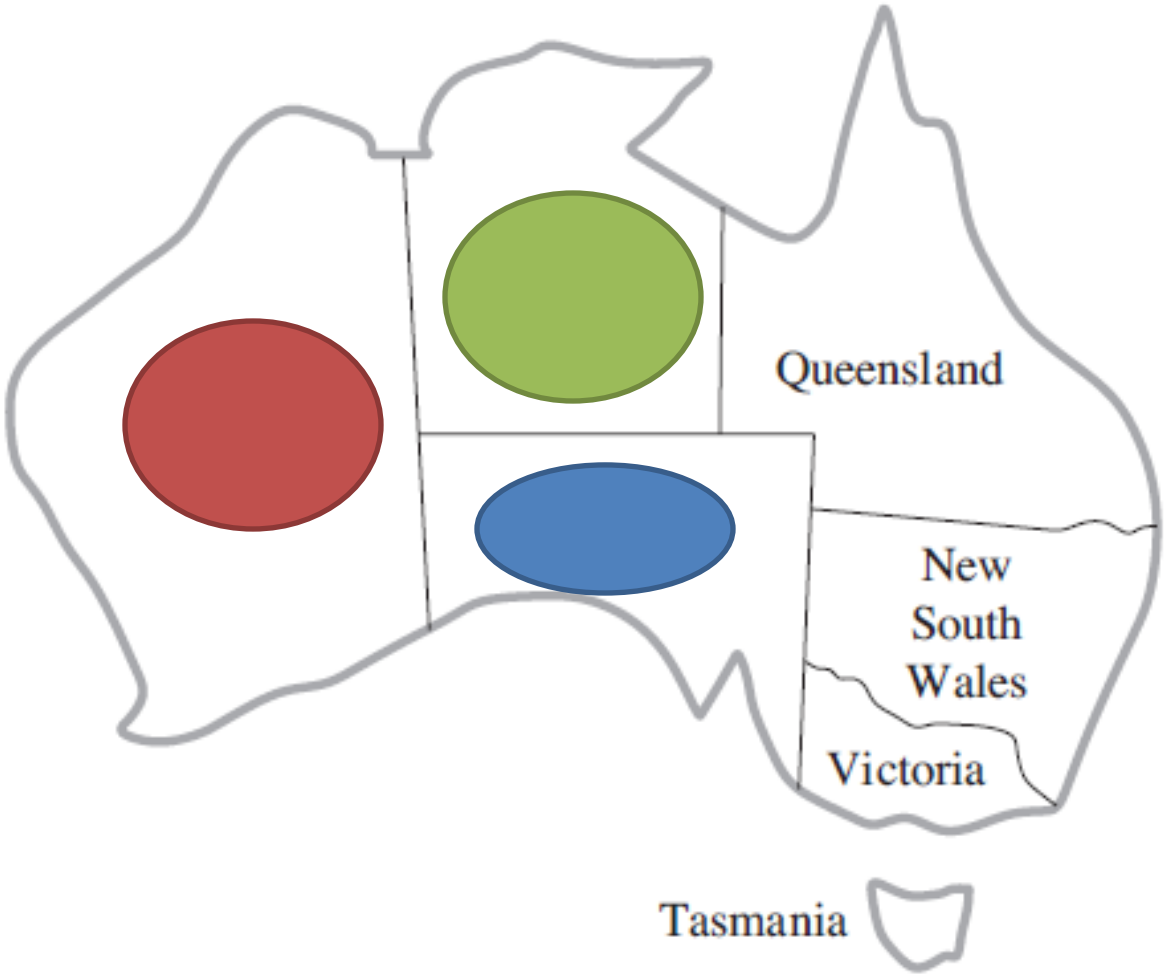
1. Which variable next?
2. Which value next?
3. Detect inevitable failure early
4. Exploit problem structure

Effect of Variable Order?

- Constraints: X=Y, Y=Z
- Constraint graph: X – Y – Z
- Which order to pick?
  - Order A: XZY
  - Order B: XYZ

- Variable order permutes the tree and changes its size!

Example

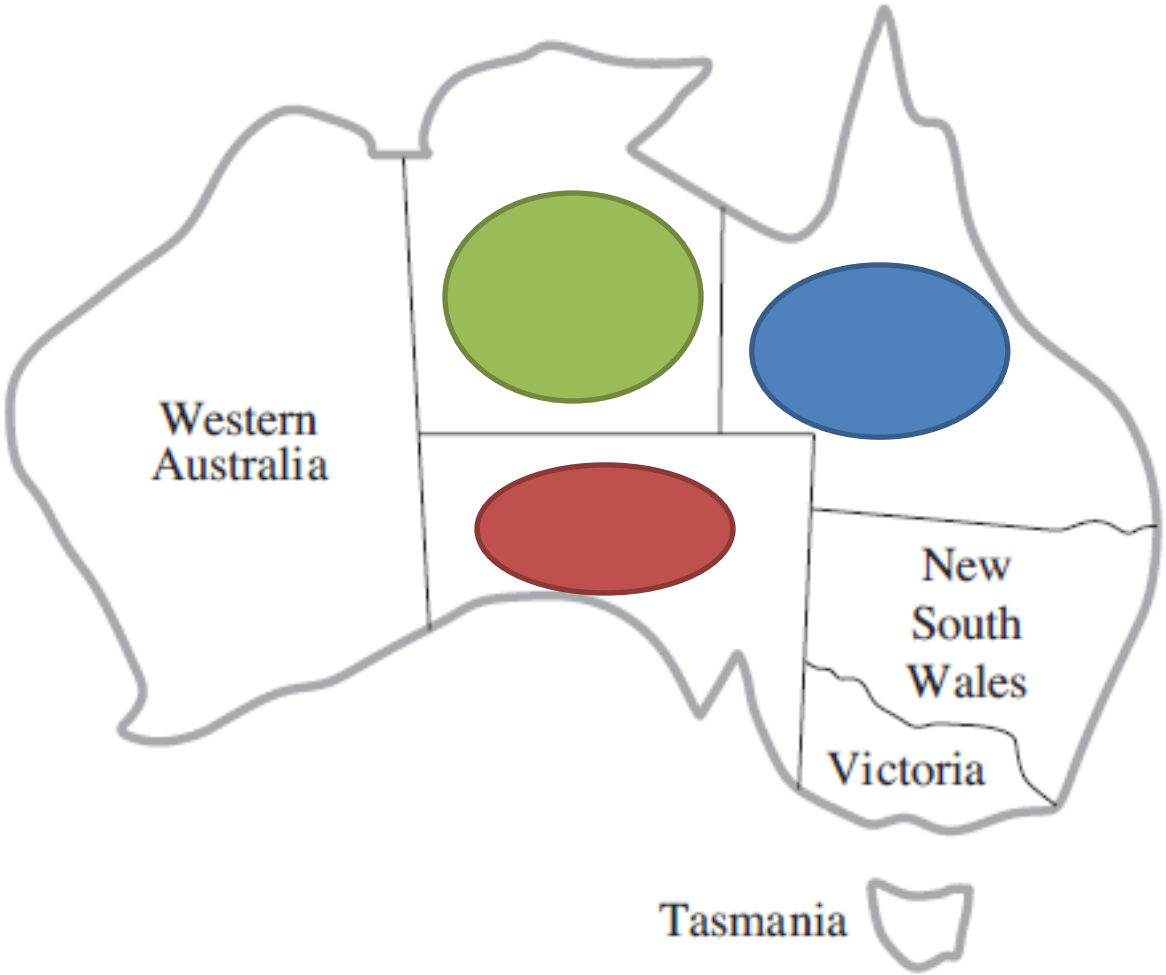


Variable Selection Heuristics

- Note: not a “heuristic” as in heuristic search
- General idea: first-fail principle

1. Most constrained variable heuristic:
   Pick variable with fewest legal values
   - aka Minimum remaining values heuristic
   - Reduces branching factor immediately

Example

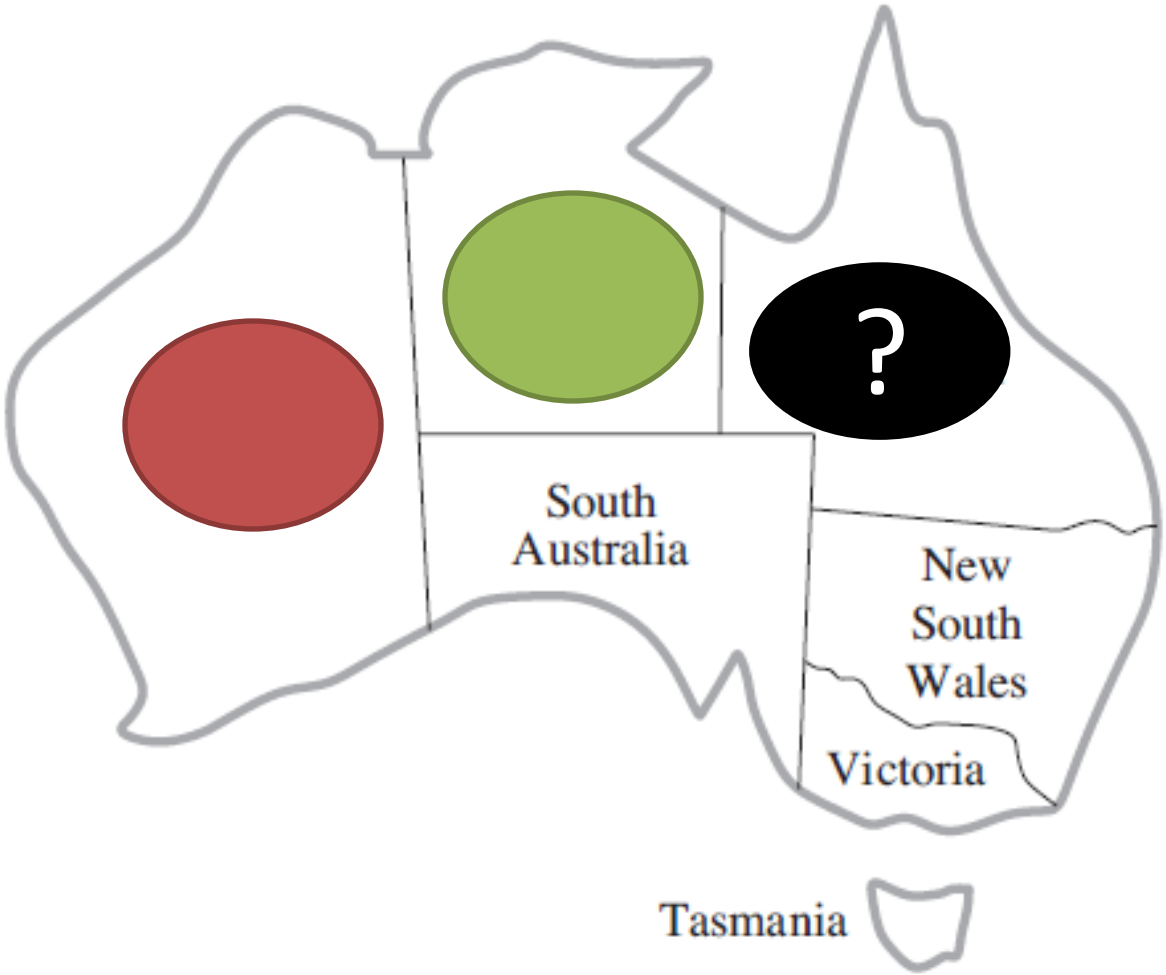


Variable Selection Heuristics

- Note: not a “heuristic” as in heuristic search
- General idea: first-fail principle

2. Degree heuristic: Pick variable with most constraints on remaining variables
   - Tie-breaker among most constrained variables
   - Easily computed on constraint graph (how?)
   - Reduces branching factor further down

Value Selection Heuristics



Value Selection Heuristics

- Note: not a “heuristic” as in heuristic search

1. Least constraining value heuristic:
   Pick value ruling out fewest values in remaining variables

Combined with value heuristics:
1000-queens feasible

Is there something wrong?

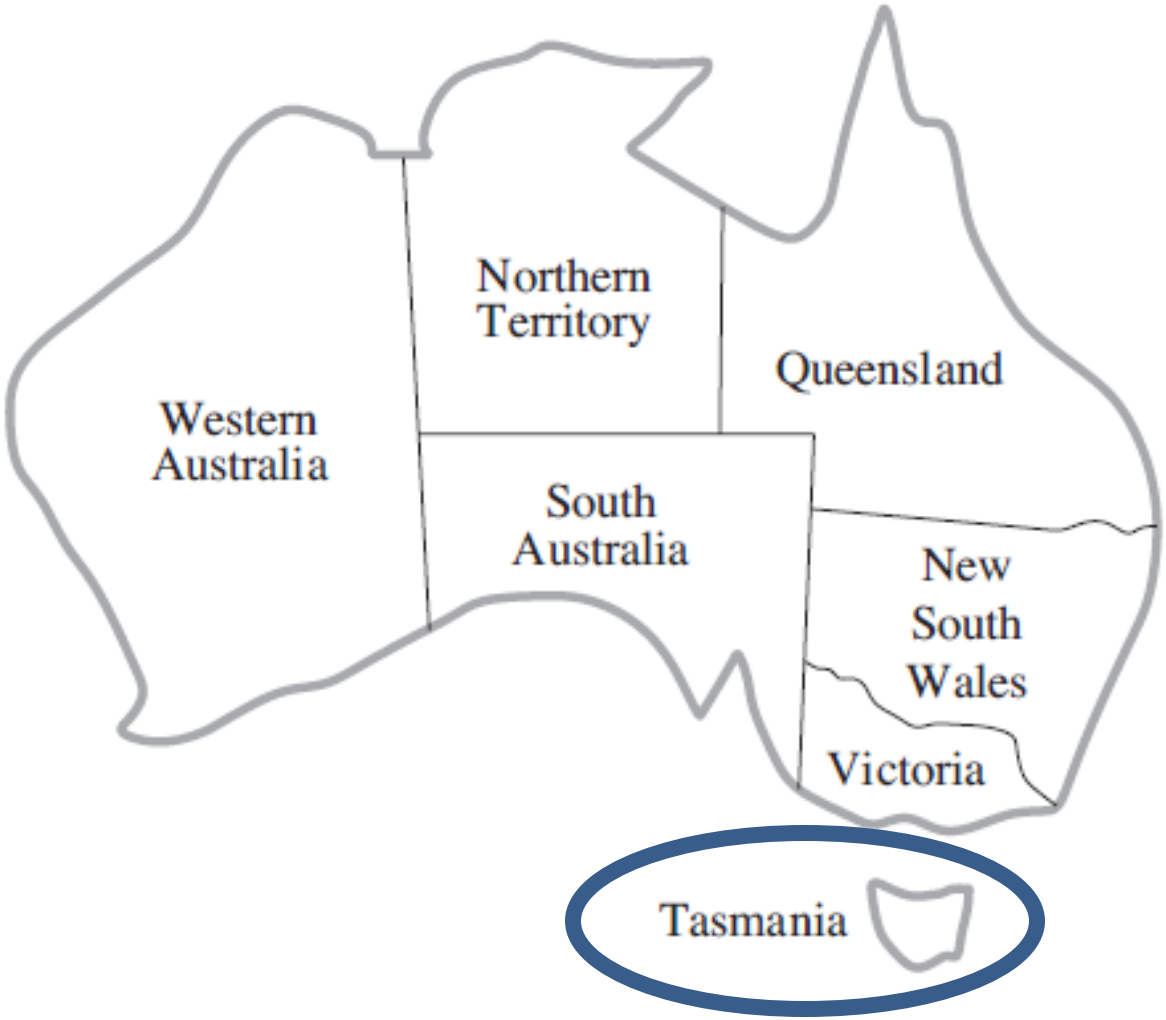
- Most constrained variable
- Least constraining value

Are these contradicting?

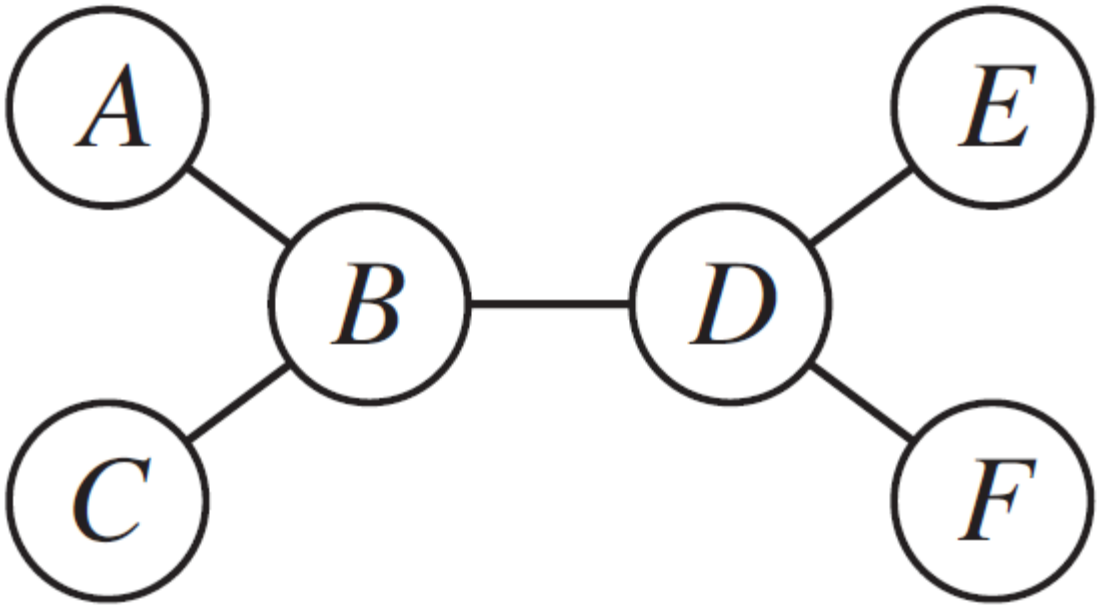
What if all assignments are solutions?

What if there is no solution?

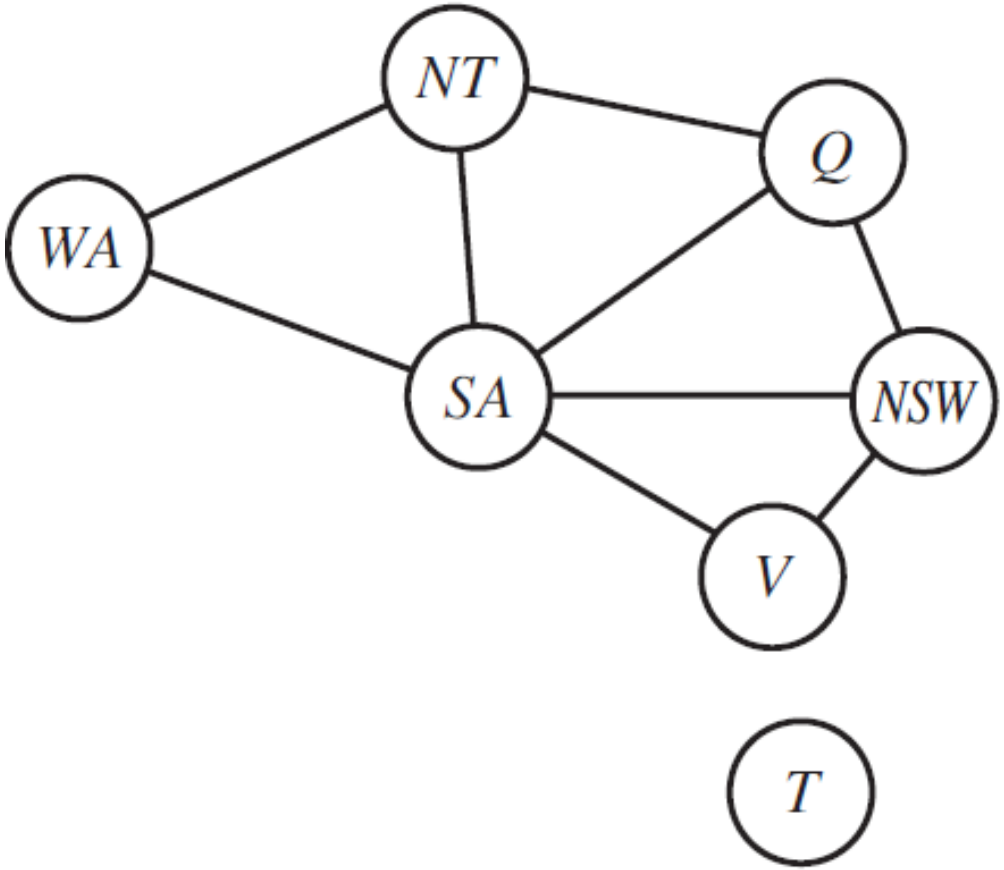
Structure


Northern Territory
Queensland
Western Australia
South Australia
New South Wales
Victoria
Tasmania

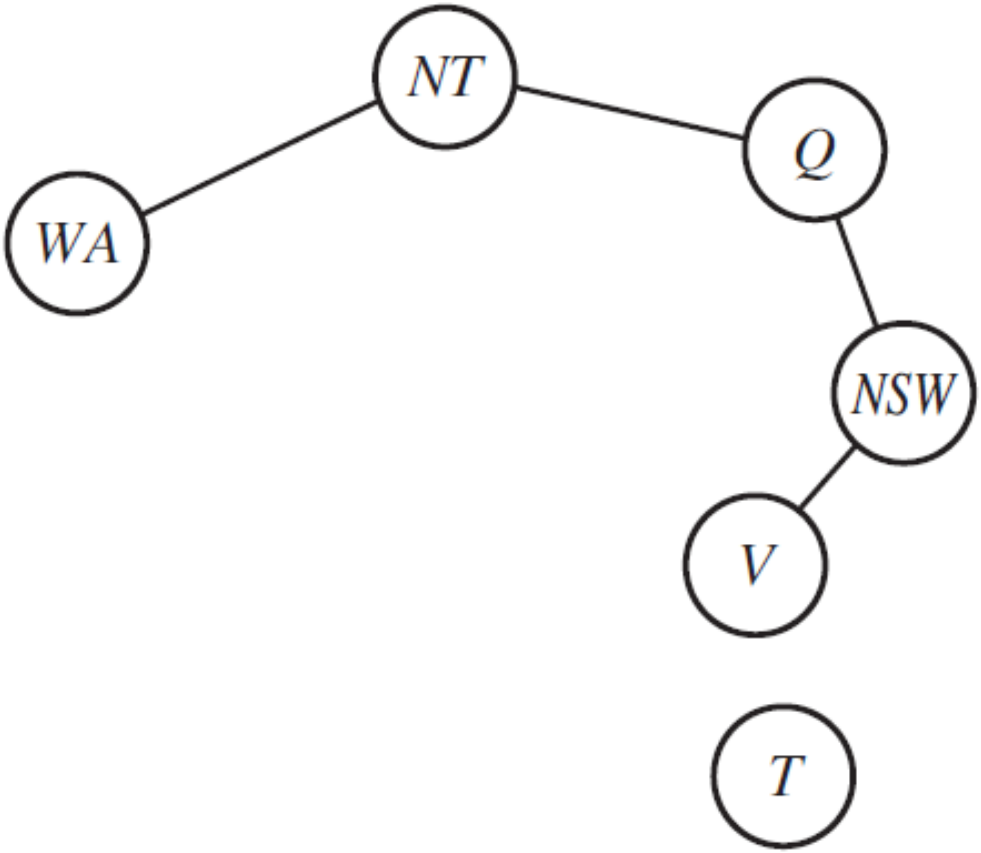
Structure: Tree



Structure?



Structure: Cutset Conditioning



Structure: Tree Decomposition