First-Order Logic

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Overview

1. First-Order Logic
   - Practice Problem 1
   - Practice Problem 2
   - Practice Problem 3

Practice Problem 1 - Which of the following are correct? I

This exercise uses the function *MapColor* and predicates $In(x, y)$, $Borders(x, y)$, and $Country(x)$, whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

Practice Problem 1 - Which of the following are correct? II

1. Paris and Marseilles are both in France.
    1. $In(Paris \wedge Marseilles, France)$
    2. $In(Paris, France) \wedge In(Marseilles, France)$
    3. $In(Paris, France) \vee In(Marseilles, France)$

Practice Problem 1 - Which of the following are correct? III

2. There is a country that borders both Iraq and Pakistan.
    1. $\exists c, Country(c) \wedge Border(c, Iraq) \wedge Border(c, Pakistan)$
    2. $\exists c, Country(c) \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$
    3. $[\exists c, Country(c)] \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$
    4. $\exists c, Border(Country(c), Iraq \wedge Pakistan)$

Practice Problem 1 - Which of the following are correct? IV

3. All countries that border Ecuador are in South America
    1. $\forall c, Country(c) \wedge Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)$
    2. $\forall c, Country(c) \Rightarrow [Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)]$
    3. $\forall c, [Country(c) \Rightarrow Border(c, Ecuador)] \Rightarrow In(c, SouthAmerica)$
    4. $\forall c, Country(c) \wedge Border(c, Ecuador) \wedge In(c, SouthAmerica)$

Practice Problem 1 - Which of the following are correct? V

4. No region in South America borders any region in Europe
    1. $\neg[\exists c, d\ In(c, SouthAmerica) \wedge In(d, Europe) \wedge Borders(c, d)]$
    2. $\forall c, d\ [In(c, SouthAmerica) \wedge In(d, Europe)] \Rightarrow \neg Borders(c, d)]$
    3. $\neg\forall c\ In(c, SouthAmerica) \Rightarrow \exists d\ In(d, Europe) \wedge \neg Borders(c, d)$
    4. $\forall c\ In(c, SouthAmerica) \Rightarrow \forall d\ In(d, Europe) \Rightarrow \neg Borders(c, d)$

Practice Problem 1 - Which of the following are correct? VI

5. No two adjacent countries have the same map color
   1. $\forall x, y\ \neg Country(x) \vee \neg Country(y) \vee \neg Borders(x, y) \vee \neg(MapColor(x) = MapColor(y))$
   2. $\forall x, y\ (Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge \neg(x = y)) \Rightarrow \neg(MapColor(x) = MapColor(y))$
   3. $\forall x, y\ Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge \neg(MapColor(x) = MapColor(y))$
   4. $\forall x, y\ (Country(x) \wedge Country(y) \wedge Borders(x, y)) \Rightarrow MapColor(x \neq y)$

Practice Problem 2 - First Logic for family tree I

Given an example of family tree as shown in Figure 1:

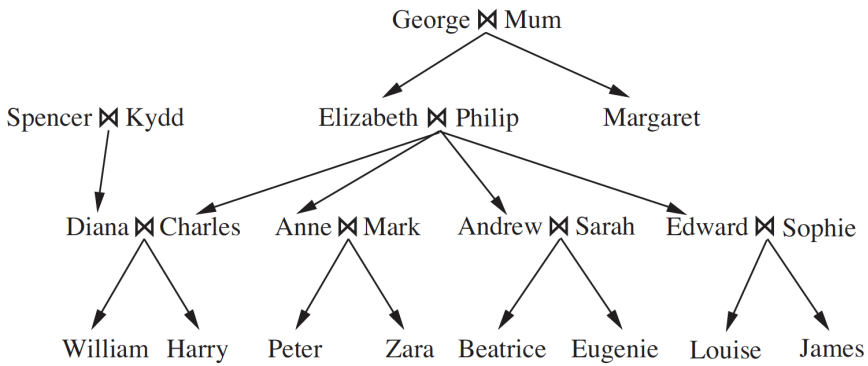

Figure: A typical family tree, the symbol ⋈ connects spouses and arrows point to children

Practice Problem 2 - First Logic for family tree II

Write axioms describing the predicates *Grandchild*, *Greatgrandparent*, *Ancestor*, *Brother*, *Sister* , *Daughter*, *Son*, *FirstCousin*, *BrotherInLaw*, *SisterInLaw*, *Aunt*, and *Uncle*. Find out the proper definition of m^th cousin n times removed, and write the definition in first-order logic. Now write down the basic facts depicted in the family tree in Figure 1. Using a suitable logical reasoning system, *TELL* it all the sentences you have written down, and *ASK* it who are Elizabeths grandchildren, Dianas brothers-in-law, Zaras great-grandparents, and Eugenies ancestors.

Practice Problem 3 - Use *First Order Logic* to inference I

Suppose you are given the following axioms:

1. $0 \leq 3$
2. $7 \leq 9$
3. $\forall x\ x \leq x$
4. $\forall x\ x \leq x + 0$
5. $\forall x\ x + 0 \leq x$
6. $\forall x, y\ x + y \leq y + x$
7. $\forall w, x, y, z\ w \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$
8. $\forall x, y, z\ x \leq y \wedge y \leq z \Rightarrow x \leq z$

Practice Problem 3 - Use *First Order Logic* to inference II

(1) Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps.

Practice Problem 3 - Use *First Order Logic* to inference III

(2) Give a forward-chaining proof of the sentence $7 \leq 3 + 9$. Again, show only the steps that lead to success.

The End