

# Constraint Satisfaction Problem

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## 1 Constraint Satisfaction Problems

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- Practice Problem 1
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# Practice Problem 1 - Course Scheduling I

You are in charge of scheduling for computer science classes that meet Mondays, Wednesdays and Fridays. There are 5 classes that meet on these days and 3 professors who will be teaching these classes. You are constrained by the fact that each professor can only teach one class at a time. The classes are:

- Class 1 - Intro to Programming: meets from 8:00-9:00am
- Class 2 - Intro to Artificial Intelligence: meets from 8:30-9:30am
- Class 3 - Natural Language Processing: meets from 9:00-10:00am
- Class 4 - Computer Vision: meets from 9:00-10:00am
- Class 5 - Machine Learning: meets from 10:30-11:30am

The professors are:

- Professor A, who is qualified to teach Classes 1, 2, and 5.
- Professor B, who is qualified to teach Classes 3, 4, and 5.
- Professor C, who is qualified to teach Classes 1, 3, and 4.

# Practice Problem 1 - Course Scheduling

## Question 1

Formulate this problem as a CSP problem in which there is one variable per class, stating the domains, and constraints. Constraints should be specified formally and precisely, but may be implicit rather than explicit.

# Practice Problem 1 - Course Scheduling

## Question 2

Draw the constraint graph associated with your CSP.

# Practice Problem 1 - Course Scheduling

## Question 2

Your CSP should look nearly tree-structured. Briefly explain (one sentence or less) why we might prefer to solve tree-structured CSPs.

## Practice Problem 2 - General questions

The graph below is a constraint graph for a CSP that has only binary constraints. Initially, no variables have been assigned. For each of the following scenarios, mark all variables for which the specified filtering might result in their domain being changed.

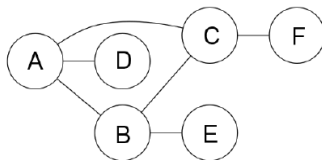


Figure: CSP

# Practice Problem 2 - General questions I

Q1

A value is assigned to A. Which domains might be changed as a result of running forward checking for A?

Q2

A value is assigned to A, and then forward checking is run for A. Then a value is assigned to B. Which domains might be changed as a result of running forward checking for B?



## Practice Problem 2 - General questions II

Q3

A value is assigned to A. Which domains might be changed as a result of enforcing arc consistency after this assignment?

Q4

A value is assigned to A, and then arc consistency is enforced. Then a value is assigned to B. Which domains might be changed as a result of enforcing arc consistency after the assignment to B?

# Practice Problem 3 - Trapped Pacman I

Pacman is trapped! He is surrounded by mysterious corridors, each of which leads to either a pit ( $P$ ), a ghost ( $G$ ), or an exit ( $E$ ). In order to escape, he needs to figure out which corridors, if any, lead to an exit and freedom, rather than the certain doom of a pit or a ghost.

The one sign of what lies behind the corridors is the wind:

- a pit produces a strong breeze ( $S$ ),
- and an exit produces a weak breeze ( $W$ ),
- while a ghost doesn't produce any breeze at all.

## Practice Problem 3 - Trapped Pacman II

Unfortunately, Pacman cannot measure the strength of the breeze at a specific corridor. Instead, he can stand between two adjacent corridors and feel the max of the two breezes. For example,

- if he stands between a pit and an exit he will sense a strong ( $S$ ) breeze,
- while if he stands between an exit and a ghost, he will sense a weak ( $W$ ) breeze.

The measurements for all intersections are shown in the figure below 2.

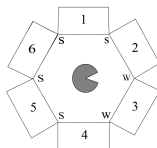


Figure: trapped Pacman

## Practice Problem 3 - Trapped Pacman III

Also, while the total number of exits might be zero, one, or more, Pacman knows that two neighboring squares will not both be exits.

Pacman models this problem using variables  $X_i$  for each corridor  $i$  and domains  $P$ ,  $G$ , and  $E$ .

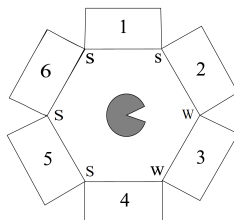


Figure: Trapped Pacman (repeated)

# Practice Problem 3 - Trapped Pacman

## Question 1

State the binary and/or unary constraints for this CSP (either implicitly or explicitly).

## Question 2

Cross out the values from the domains of the variables that will be deleted in enforcing arc consistency.

$X_1$	$P$	$G$	$E$
$X_2$	$P$	$G$	$E$
$X_3$	$P$	$G$	$E$
$X_4$	$P$	$G$	$E$
$X_5$	$P$	$G$	$E$
$X_6$	$P$	$G$	$E$

# Practice Problem 3 - Trapped Pacman

## Question 3

According to MRV, which variable or variables could the solver assign first?

## Question 4

Assume that Pacman knows that  $X_6 = G$ . List all the solutions of this CSP or write none if no solutions exist.

# Practice Problem 2 - Trapped Pacman

## Question 5

The CSP described below has a circular structure with 6 variables. Now consider a CSP forming a circular structure that has  $n$  variables ( $n > 2$ ), as shown below. Also assume that the domain of each variable has cardinality  $d$ . Explain precisely how to solve this general class of circle-structured CSPs efficiently (i.e. in time linear in the number of variables), using methods covered in class. Your answer should be at most two sentences.

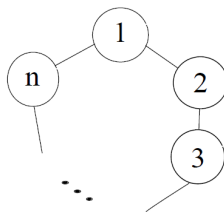


Figure: Circular CSP

# Practice Problem 2 - Trapped Pacman

## Question 6

If standard backtracking search were run on a circle-structured graph, enforcing arc consistency at every step, what, if anything, can be said about the worst-case backtracking behavior (e.g. number of times the search could backtrack)?



# Games

# Games - Problem 1

Consider the game tree below, which contains maximizer nodes, minimizer nodes, and chance nodes. For the chance nodes the probability of each outcome is equally likely.

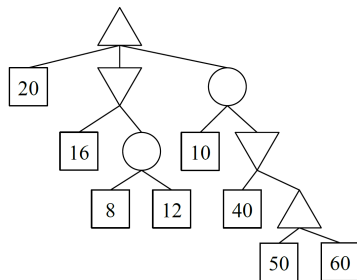


Figure: Min-Max

# Questions

Q1

Fill in the values of each of the nodes.

Q2

Is pruning possible?

## Practice Problem 2 - $\alpha - \beta$ Pruning

Consider the game tree shown below 6. For what range of  $U$  will the indicated pruning take place?

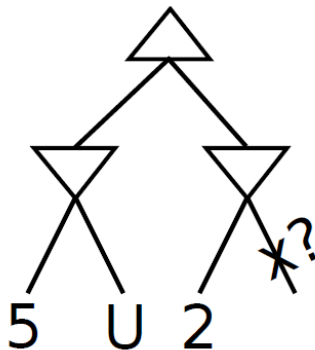


Figure:  $\alpha - \beta$  Pruning

# The End